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Article

Influence of a curvature on wave run-up in a coastal dike line

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Abstract: Due to climatic change and the increased usage of coastal areas, there is an increasing risk of dike failures along the coasts worldwide. Wave run-up plays a key role in planning and design of a coastal structure. Coastal engineers use empirical equations for the determination of wave run-up. These formulae generally include the influence of various hydraulic, geometrical and structural parameters, but neglect the effect of the curvature of coastal dikes on wave run-up and overtopping. The scope of this research is to find the effects of the dike curvature on wave run-up for regular wave attack by employing numerical model studies for various dike-opening angles and comparing it with physical model test results. Numerical simulation is carried out using DualSPHysics, a mesh-less model and OpenFOAM, a mesh-based model. A new influence factor is introduced to determine the influence of curvature along a dike line. For convexly curved dikes (α_d = 210° to 270°) under perpendicular wave attack, a higher wave run-up was observed for larger opening angles at the center of curvature whereas for concavely curved dikes (α_d = 90° to 150°) under perpendicular wave attack, wave run-up increases at the center of curvature as the opening angle decreases. This research aims to contribute a more precise analysis and understanding the influence of the curvature in a dike line and thus ensuring a higher level of protection in the future development of coastal structures.

Keywords: Curved Dike, DualSPHysics, OpenFOAM, Physical model tests, Wave run-up.

1. Introduction

Wave run-up and wave overtopping are decisive parameters not only for designing freeboards of coastal structures but also for the safety and rehabilitation of coastal structures, which helps to reduce the risk of failure. Due to the stochastic nature of wave processes, an exact description of wave run-up or overtopping is not possible. Hence, some empirical equations based on physical or numerical model tests help to determine wave run-up or wave overtopping. In those empirical formulas for wave run-up and wave overtopping suggested in literature (EurOtop, 2018) [1], several factors based on the influences of berm, roughness, oblique wave attack and slope are already considered. However, geometrical characteristics like the curvature of the dike are not included.

Wave run-up is defined as the maximum distance a wave may travel up the face of the coastal structure (see Figure 1). The hypothesis is set if the wave run-up is influenced by the curvature in a dike line due to additional overlapping physical processes, i.e. refraction and diffraction. Yet there is limited information available on the influence of wave run-up on a curved dike and no detailed investigations have been done to include the factor based on curvature in the prediction formulae for wave run-up. The aim of this research is to provide an insight of wave run-up on a curved dike using numerical models validated with measurements from physical model tests. The numerical investigation is accomplished using DualSPHysics, a mesh-less model and OpenFOAM, a mesh-based model. Both of these numerical models are capable to simulate wave transformation, wave breaking and interaction with sloping structures, which made them a feasible alternative to experimental investigations to predict wave run-up numerically.

This study aims not only to discuss the influence of wave run-up on a curved dike but also to discuss the wave transformation processes on a convex and concave curvature dikes for regular waves in contrast to linear dike profiles.

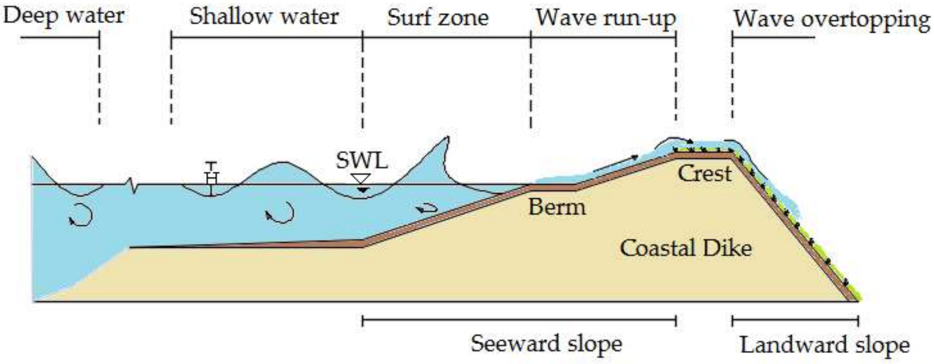

Figure 1. Definition sketch of wave run-up on coastal dikes

2. Influence of a Curvature in a Dike Line

The coastal dike lines are bent concave (bent to the landside) and convex (bent to the seaside) due to local geographical conditions of the coastline or geological characteristics. Figure 2 shows the opening angle, α_d and opening radius, r_d of a dike and they may influence the hydrodynamics of approaching waves. The opening angle, α_d is defined as the seaward angle between the tangents of the dike flanks. The opening radius, r_d is defined as the distance between the meeting point of the perpendiculars of both dike flanks and the limit of the dike curvature.

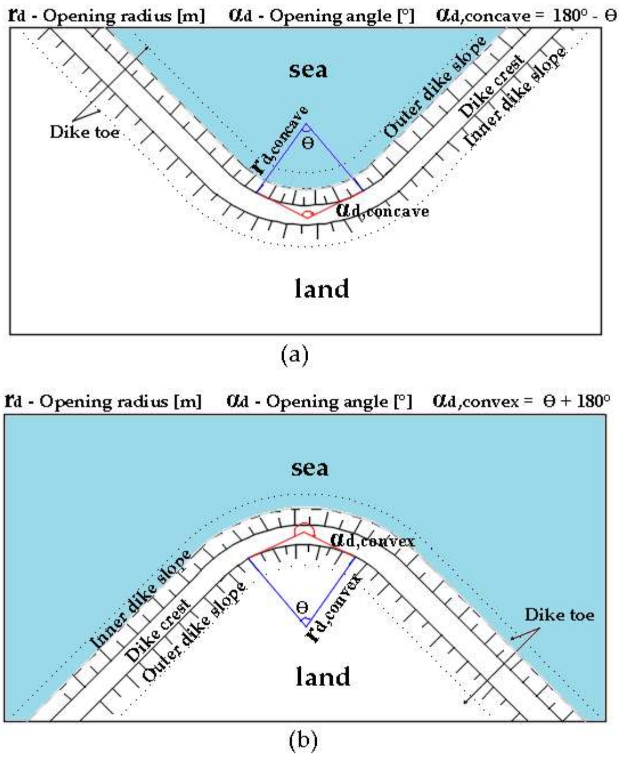

Figure 2. Definition of opening angle, α_d and opening radius, r_d of curvatures in a dike line

3. Literature Review

The contributions of Mayer at al (1994) [2], Goda (2000) [3], Napp et al (2004) [4], EurOtop (2018) [1] and Bornschein et al (2014) [5] are among the earlier investigations on concave or convex profiles. Mayer et al (1994) [2] attempted to predict run-up in concave profiles. They provided an analytical solution to estimate wave run-up in complex concave beach topographies by integrating Hunt's formula (1959) [6] with Saville's formula (1957) [7] iterative solution for composite slopes. However, this iterative method is complex and requires a prior determination of the wave breaking point. Goda (2000) [3] presented a numerical solution for the reflection effect on a concave seawall corner. According to his formulations presented, the wave height increases with a decreasing opening angle of the dike curvature and argued that this is due to the wave energy concentration inside a bay. Napp et al (2004) [4] stated that there is lower overtopping rate at 90° and 120° concave corner in a vertical wall. He referred the observed decrease is due to the influence of combination of different wave breaking processes in combination with the effects of reflection and refraction. EurOtop (2018) [1] assumes that a concave curvature (with respect to the seaward face) could lead to an accumulation of wave energy, thus an increase in wave run-up and wave overtopping. On the other hand, for convex curvature (with respect to the landward face), EurOtop (2018) [1] assumes that the wave run-up and overtopping will decrease due to the distribution of wave energy. Bornschein et al (2014) [5] observed visually a local increase in wave run-up and wave overtopping during a physical experiment model on a 270° convex dike. Except few speculations, neither mathematical expression that describes the effect of curvature nor an explanation on the hydrodynamic processes at curved dikes is available yet. Therefore, either in-depth experimental or numerical investigation is required to provide better understanding on the influence of curvature in a dike line on wave run-up of approaching waves.

4. Numerical Model

There are numerous computational models available to simulate hydrodynamic processes for coastal areas. However, very few software is suitable for this research due to the interaction of waves and structures, which involves many nonlinear phenomena like wave propagation, wave transformation, interaction among incident and reflected waves, wave breaking, wave run-up / run-down and wave overtopping. Numerical models based on the Navier-Stokes equation can be used to solve these complex phenomena. Thus, the software used for modelling wave run-up on a curved dike is chosen to be DualSPHysics, a mesh-less model and OpenFOAM, a mesh-based model. The other alternatives include REEF3D and SWASH. But, both the software has some limitations to execute this investigation and hence not considered for further research. An overview of the chosen software is described below.

4.1 DualSPHysics

DualSPHyics is based on the Smoothed Particle Hydrodynamics (SPH) method (Crespo et al, 2015) [8]. Smoothed Particle Hydrodynamics (SPH) is a mesh free Lagrangian particle method and has special advantages in modeling complex fluid flows, especially those with fluid-structure interactions and large fluid deformations. SPH was first invented by Monaghan et al, 1977 [9] to solve astrophysical problems in three-dimensional domain. DualSPHysics is a mesh-less model where the fluid is discretized into set of particles, which possess material properties and interact with each other within the range controlled by a smoothing function. For each particle, the physical quantities such as velocity, density, pressure etc., are computed as an interpolation of the values of the neighboring particles. Wave generation is included in DualSPHysics for both regular and random waves. In this way, the numerical model can be used to simulate a physical wave flume or a wave basin. Both active and passive wave absorption can be implemented in DualSPHysics. However, active wave absorption is possible only for a piston-type wave maker. A damping zone is implemented in DualSPHysics as passive wave absorption system. The model description, numerical simulation and results are described in section 6 and section 7.

4.2 OpenFOAM

OpenFOAM (Open Source Field Observation and Manipulation) is used for various science and engineering applications and it is most suitable for complex fluid flow. OpenFOAM is based on VOF (Volume of Fluid) method. To simulate free surface wave generation and absorption, a wave generation toolbox is available: Waves2Foam (Jacobsen et al, 2012) [10] and OlaFoam, an evolution of IHFOAm (Higuera et al, 2013) [11]. OpenFOAM in conjunction with olaFoam was assessed to be a suitable numerical tool for modling wave run-up on dikes for 3D cases. OlaFoam provides the possibility to generate regular, irregular and solitary waves as well as the wave maker type. Active wave absorption is implemented to avoid reflections of waves from boundaries. The numerical simulations and results from OpenFOAM with olaFoam are described in detail in section 6 and section 7.

For more details on the applied numerical software, the readers can refer the respective user guide DualSPHysics user guide [12] [13], OlaFoam manual [14] OpenFOAM user guide [15] [16].

5. Physical Model

The wave-induced response of a curvature in the dike line was studied in physical model tests with a 1:6 sloping beach and in three general model set-ups – a) a concave geometry, b) a convex geometry which have been contrasted in configuration and c) a straight geometry for reference. The tests were conducted in a multidirectional wave basin.

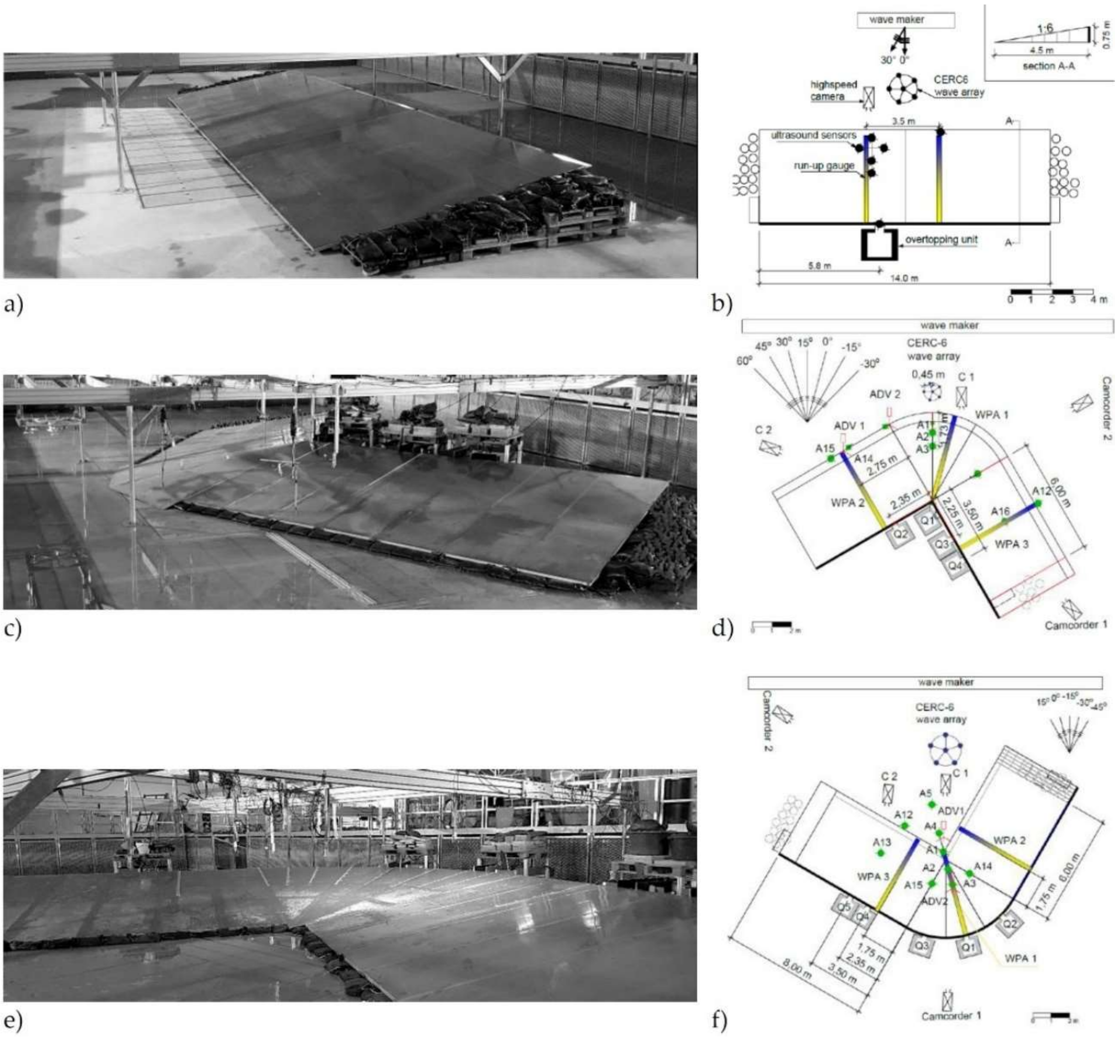

Figure 3. Physical model set-up of the impermeable 1:6 sloped dike model in the wave basin with straight front (a, b), convex (c, d) and concave curvature (e, f)

The basin has a test area of 30 m x 15 m over a horizontal floor. Waves are generated by a 72-element multidirectional wave maker including active reflection compensation routines on the long side of the basin. The three remaining boundaries of the basin are equipped with a passive eight-layer screen absorber.

The incident wave conditions are measured with a six-gauge array in front of the dike. Further gauges on the dike slope record the incident waves propagating onshore directed over the straight and curved slopes. Flow velocities are measured at several positions on the slope. Wave run-up gauges provide data for each wave run-up event. They are located on the corners and on both straight wings of the model. As some waves cause overtopping, the corresponding volumes are collected in up to five overtopping reservoirs equipped with load cells. The reservoirs are distributed along the crest of the dike and enable a quantification of the mean overtopping discharge in the corner sections and on the straight wings of the model.

The tested configurations are given in Table 1.

Table 1. Test configurations in the physical model

Curvature	Straight	Convex	Concave
Opening angle α_d [°]	180	240; 270	90; 120
Wave direction β [°]	0; 30; 45	– 30; – 15; 0; 15; 30; 45; 60	– 30; – 15; 0; 15; 30;

Tests have been conducted with regular and irregular waves (long- and short-crested). The freeboard height ($1.0 < R_c/(H_{m0}\ \xi_{m-1,0}) < 1.3$), the Iribarren number ($0.7 < \xi_{m-1,0} < 1.4$) and the angle of the incident waves (Table 1) was varied from test to test. For the present paper results from regular waves only given in Table 2 are considered to contrast numerical approaches.

6. Numerical Investigation on a Curved Dike Line

To analyze the wave run-up and wave transformation processes on curved dikes, a simulation program with different opening angles and angles of wave attack for various wave parameters was investigated. Table 2 shows the wave parameters for regular waves used for the numerical simulation in both DualSPHysics and OpenFOAM. The different opening angles, α_d chosen for the simulation are 90°, 120°, 150°, 180°, 210°, 240° and 270°. The first three opening angles are tested for concavely curved dikes and the last three opening angles are tested for convexly curved dikes. The different angles of wave attack, β included in the simulation are 0°, 30° and 45° respectively.

Table 2. Wave parameters for the numerical simulation in DualSPHysics and OpenFOAM

Wave height *H* [m]	Wave period *T* [s]	Water depth *d* [m]
0.07	1.22	0.55
0.10	1.46	0.55
0.10	1.79	0.55

6.1 Calibration study

The calibration is done for a 3D numerical model for a straight dike on both OpenFOAM and DualSPHysics. Calibration simulations were performed and post-processed according to the boundary conditions and numerical settings for various wave parameters. The results are compared in accordance with Hunt's formula (1959) [6] (see Figure 3). In addition to calibration, convergence study has been done to ensure the sensitivity of both numerical modelling attempts. The olaFoam convergence study includes appropriate cell size, staggered grids and the influence of Courant number whereas interparticle distance (dp) is investigated in DualSPHysics. A cell size of 0.025 m is chosen for further analysis in OpenFOAM. The Courant number is chosen to be 1.0 for analysis so that the fluid particles move maximally one cell within one-time step. In case of simulation in DualSPHysics, the size of the interparticle distance is chosen to be 0.03 m to reduce the computational time and high data storage volume. The chosen mesh size and interparticle size show reasonably good agreement for wave run-up results in accordance with Hunt's formula (1959) [6] and hence the error does not exceed ±15%.

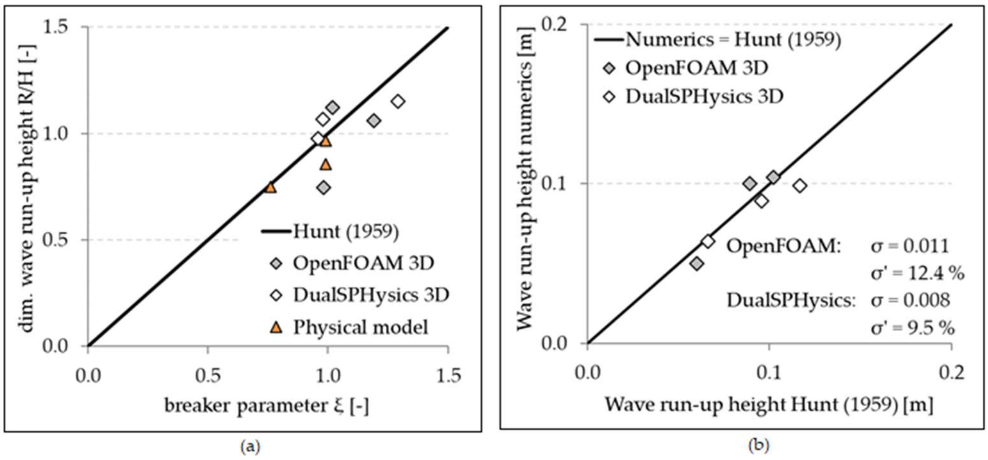

Figure 3. Results of the wave run-up calibration on a 3D straight dike compared to Hunt (1959)

6.2 Numerical model set-up

A 3D numerical model of 1:6 slope for both concavely and convexly curved dike is modelled on a numerical wave basin. Figure 4 illustrates the 3D models of a convexly and concavely curved dike with its boundaries using DualSPHysics. The left boundary corresponds to wave generation (piston type) and other boundaries act as wave absorbers. Both active wave absorber and a damping zone as a passive wave absorber are used in DualSPHysics model. These absorption systems allow generating long time series of waves in relatively short domains with negligible wave reflection (Altomare et al., 2017) [17].

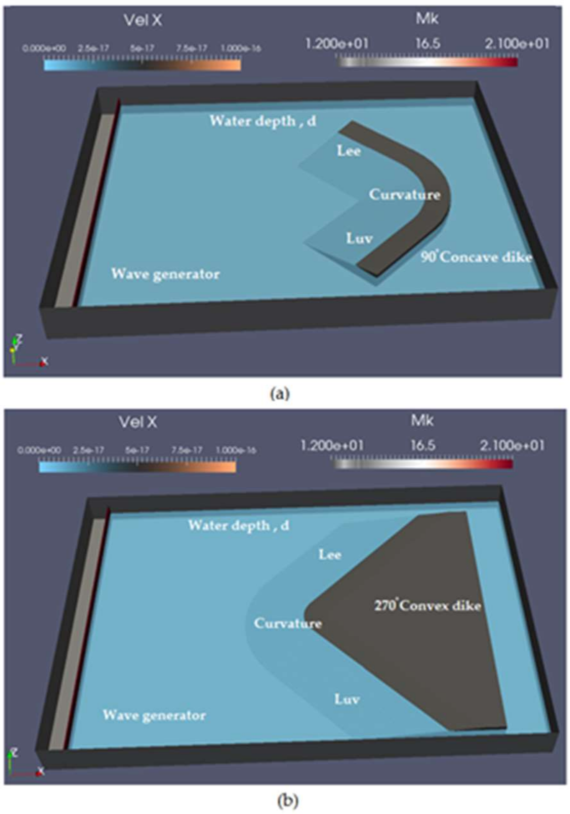

Figure 4. 3D numerical model of curved dikes in DualSPHysics: (a) 90° concave dike; (b) 270° convex dike

For simulations with oblique wave attack either the wave generation can be adapted (generate oblique waves) or the dike structure can be rotated. To avoid diffraction areas when generating oblique waves, the dike structures are rotated in both numerical models.

6.3 Transformation processes on a curved dike

Unlike the transformation processes on straight dikes that mainly include reflection, refraction, shoaling and breaking, additional effects appear at curved dikes. At convex corners, waves are firstly refracted (see Figure 5a) and concentrated at the curvature until they break (see Figure 5b) and then turn towards the dike flanks (see Figure 5c) where they finally superimpose with the incoming waves resulting in wave rollers (see Figure 5d). At concave corners, waves first encounter the dike flank (see Figure 6a) and then are redirected towards the curvature where they interact with further incoming waves influencing the wave breaking process (see Figure 6b) and finally inducing a rip current (see Figure 6d). Along with these complex transformation processes, an irregular wave run-up evolution along the dike line occurs. This pattern for convex and concave shaped dikes is applicable for the respective opening angle shown in Figure 5 and Figure 6. This may vary and the transformation processes are strongly dependent on the wave parameters, angle of wave attack and opening angle.

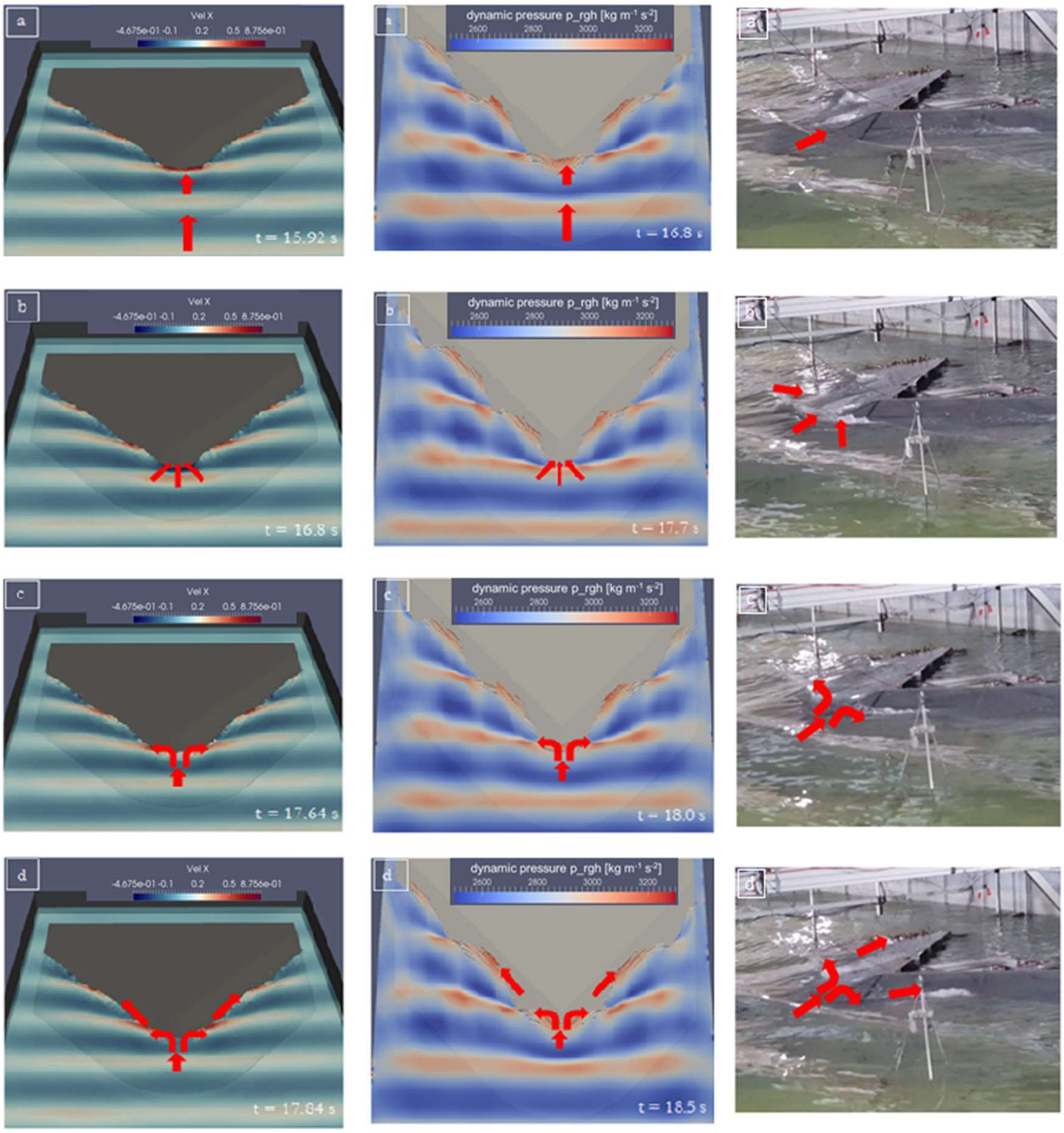

Figure 5. Wave transformation processes on a 270° convexly curved dike (Left: DualSPHysics, Middle: OpenFOAM, Right: Physical model); $H = 0.10$ m, $T = 1.46$ s, $\beta = 0°$

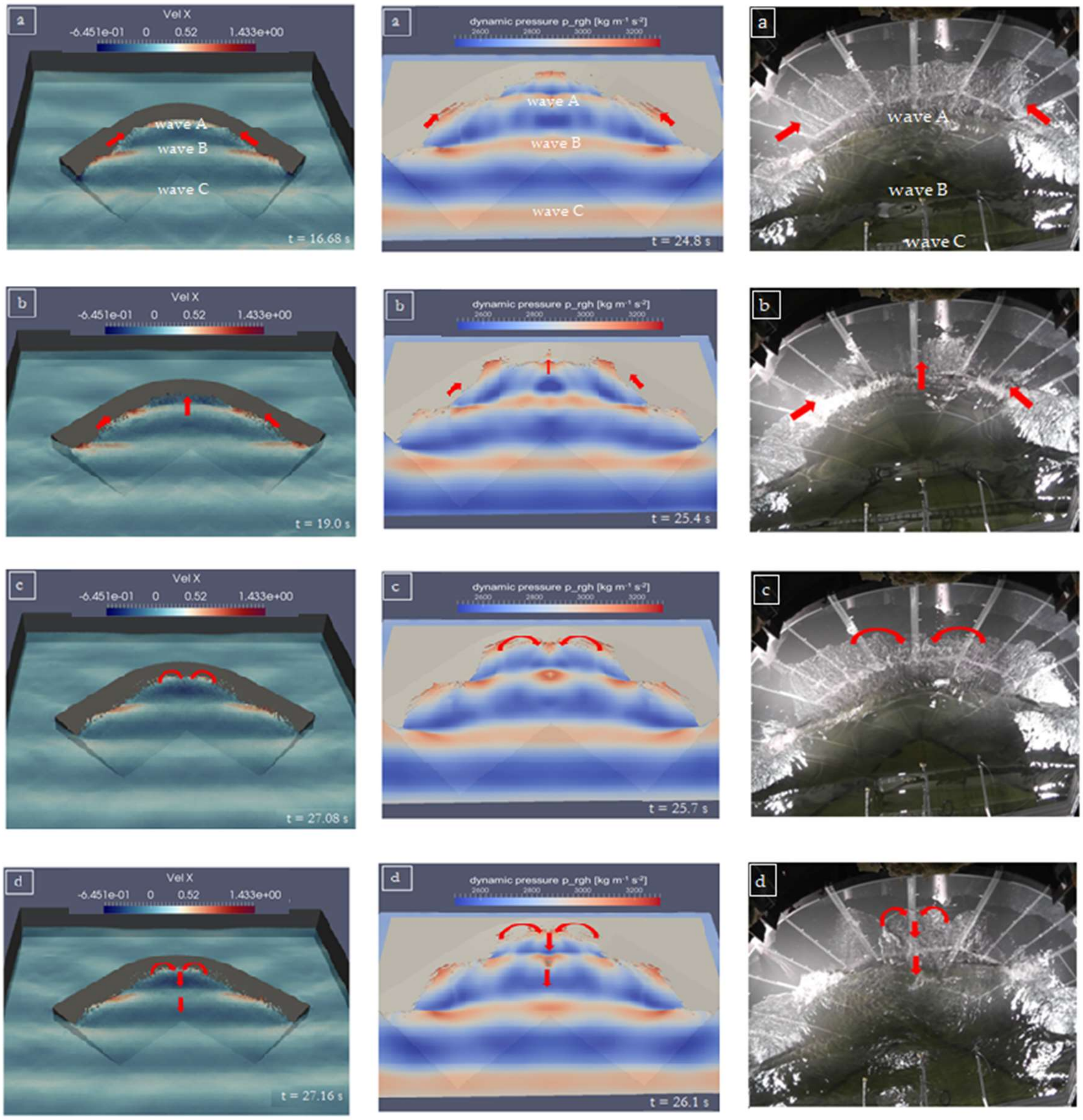

Figure 6. Wave transformation processes on a 90° concavely curved dike (Left: DualSPHysics, Middle: OpenFOAM, Right: Physical model); H = 0.10 m, T = 1.46 s, β = 0°

The wave transformation processes on a concavely curved dike profile are almost the same in the numerical and physical model. Figure 6 (a) shows the slope-parallel wave breaking of wave *A* until this is breaking in the corner of the concavely curved dike profile (b) causing a wave run-up. The wave run-down (c) of wave *A* causes a prematurely wave breaking of wave *B*. The wave run-down is forming of a recycling jet of water in the dike corner (d) which is interacting with wave *C*.

7. Analysis and Discussion

The effect of wave run-up due to oblique wave attack can be expressed by the influence factor γ_β, (EurOtop, 2018) [1].

$$\gamma_\beta = \frac{R_{influence}}{R_{no\ influence}} = \frac{R_{\beta i}}{R_{\beta=0^\circ}} \quad (1)$$

The straight dike analysis is validated for influence factor γ_β in both the numerical and physical models. Figure 7 gives the influence factor γ_β from numerical simulations, the physical model tests and literature plotted against the angle of wave attack β. Generally, the influence factor γ_β decreases with increasing angle of wave attack. Overall, the influence factors from the physical model tests, DualSPHysics and OpenFOAM reproduce formula from literature (Wassing (1957) [18], Van der Meer (1995) [19], Ohle et al. (2003) [20]) very well.

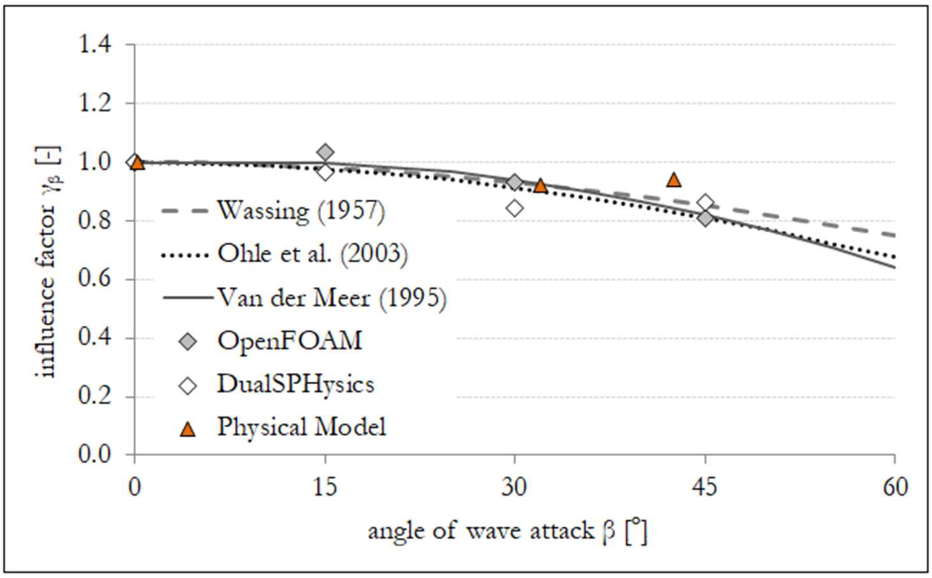

Figure 7. Influence factor γ_β for oblique, regular wave attack from numerical simulations and literature

To determine the influence of curvature along a coastal dike line, a new influence factor is introduced. This influence factor γ_c based on the influence of curvature is derived similar to the influence factor due to obliquity, γ_β introduced in the EurOtop (2018) [1] (see equation 1). Similar to equation 1, the run-up measurements from the curved dike that have an influence due to the curvature are compared to measurements from the straight dike that have no influence due to the curvature by using an influence factor γ_c. The correction factor describing the influence of curvature γ_c, which is defined as follows, is implemented for the analysis.

$$\gamma_c = \frac{R_{influence}}{R_{no\ influence}} = \frac{\left[R/\left(H * \xi_0\right)\right]_{\beta,\alpha_d}}{\left[R/\left(H * \xi_0\right)\right]_{\beta,\alpha_d=180^\circ}} \qquad (2)$$

Based on this new influence factor γ_c, the further analyses for convex and concave dikes are carried out.

7.1 Wave run-up on a curved dike line

The wave run-up on convexly and concavely curved dikes for regular waves under both perpendicular and oblique wave attack is analyzed at the center of curvature and the results are summarized in the following. The wave run-up heights from the convexly and concavely curved dikes are compared to the straight dike by using equation 2.

Figure 8 shows the influence of the curvature in the dike line on wave run-up for perpendicular wave attack. The bars in Figure 8 represent the standard deviation of different set of wave parameters. For convexly curved dikes (α_d = 210° to 270°), a higher run-up was observed for larger opening angles at the center of curvature. The wave energy focuses on the corner caused by wave refraction over the slope. For large opening angles (α_d = 270°) the increase of the wave run-up derived from physical model tests is larger than calculated with the numerical models. In case of concavely curved dikes (α_d = 90° to 150°), wave run-up increases at the center of curvature as the opening angle decreases.

The results from OpenFOAM and the physical model tests for a concavely curved dikes are almost in line whereas the wave run-up calculated with DualSPHysics is lower for α_d = 90° and 120°.

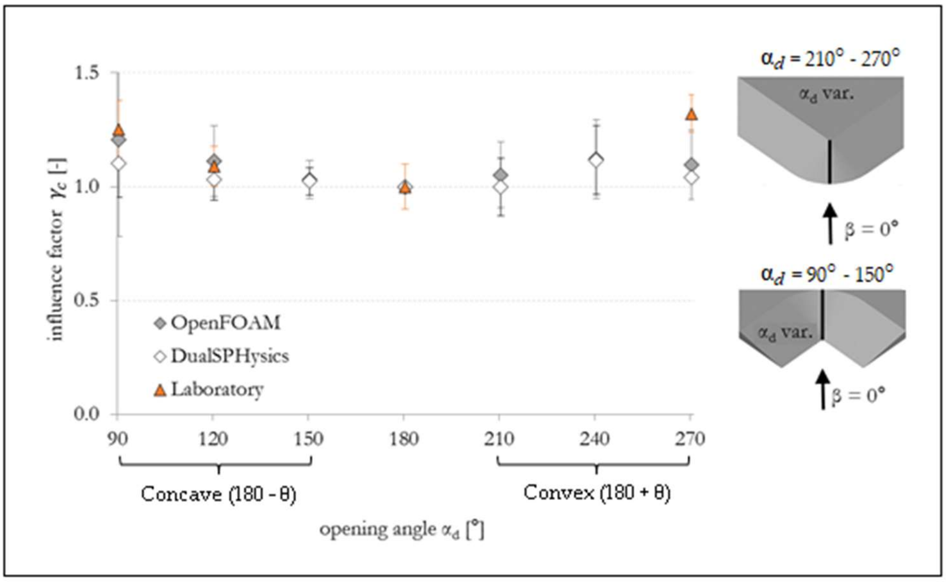

Figure 8. Influence of a curvature in the dike line on wave run-up for perpendicular wave attack – studied position: center of curvature

Figure 9 shows the influence of curvature in the dike line on wave run-up for a 30° oblique wave attack. The standard deviation of various set of wave parameters is also included as bars in Figure 9. For convexly curved dikes (α_d = 210° to 270°), a mild increase in wave run-up at the center of the curvature is observed for larger opening angles except for α_d = 210° in DualSPHysics simulations. Nevertheless, the increase is very little and data scatter around γ_c = 1.0. Similarly, at concave corners, a slight increase in wave run-up was noticed at α_d = 90°. For α_d = 120° the scatter in the wave run-up recorded from the physical model tests is high.

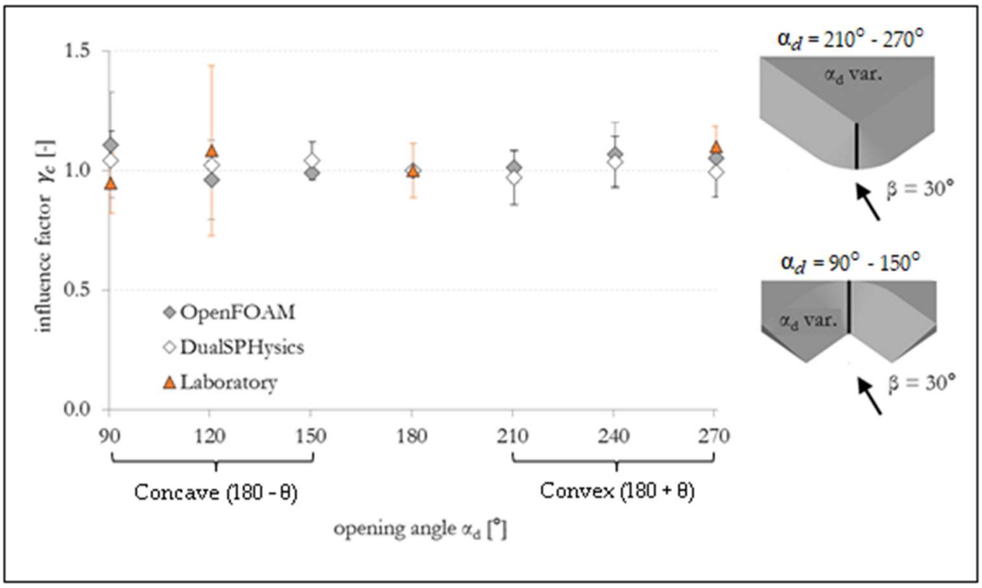

Figure 9. Influence of a curvature in the dike line on wave run-up for a 30° oblique wave attack – studied position: center of curvature

Figure 10 shows the influence of curvature in the dike line on wave run-up for a 45° oblique wave attack. The bars represent the standard deviation of different wave parameters in Figure 10. For convexly curved dikes (α_d = 210° to 270°), a higher run-up is observed at the center of curvature for larger opening angles. Results from physical model tests cause a significant higher wave run-up compared to the two numerical models for α_d = 270°. The extremely high results are due to swash running over the convex curve. For α_d = 90°, a very high run-up was observed at the center of the curvature in OpenFOAM simulations under 45° oblique wave attack. In contrast, in the physical model tests a significantly reduced wave run-up height is observed. This special case (α_d = 90°) might be distorted by model effects, as the incident waves propagate over the model boundary of the slope (side of the luv slope). As the model boundaries differ in numerical and physical model test runs a deviation is likely. In general, corresponding data points have to be evaluated with care.

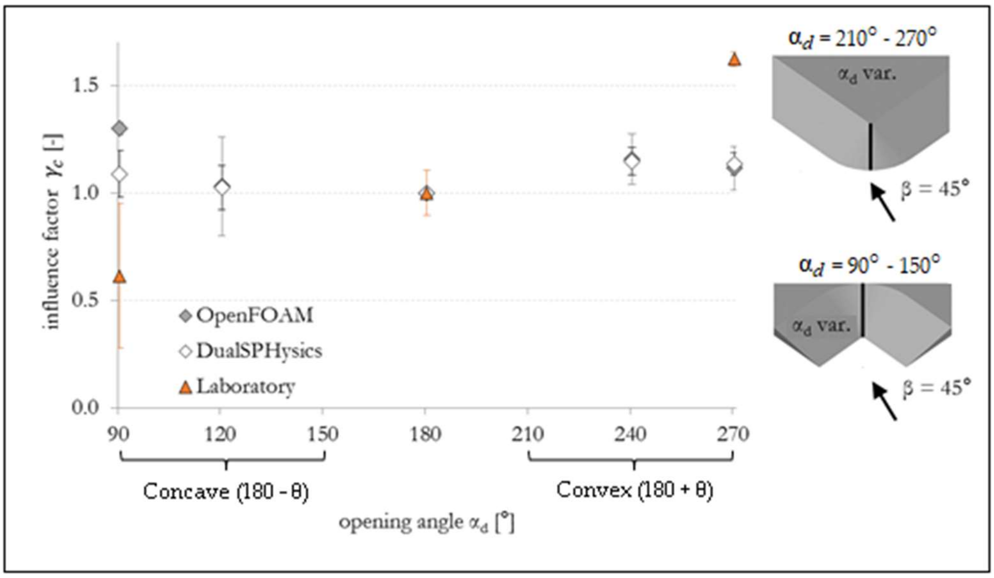

Figure 10. Influence of a curvature in the dike line on wave run-up for a 45° oblique wave attack – studied position: center of curvature

The influence factors for the center of curvature, which are derived from the run-up values from both numerical models and physical model, based on equation 2, are summarised in Table 3. These values confirm that there is an influence in curvature on wave run-up. The future work will include to derive a mathematical expression for the influence of curvature after performing some additional test cases.

Table 3. Influence factors for curvature γ_c for different opening angles with different angles of wave attack

Opening angle α_d	**Influence factors γ_c (Position: Center of Curvature)**								
	β = 0°			**β = 30°**			**β = 45°**		
	Open FOAM	*Dual SPH*	*Phys. model*	*Open FOAM*	*Dual SPH*	*Phys. model*	*Open FOAM*	*Dual SPH*	*Phys. model*
90°	1.20	1.10	1.25	1.11	1.04	0.95	1.30	1.09	0.61
120°	1.11	1.03	1.09	0.96	0.97	1.08	1.03	1.02	–
150°	1.03	1.02	–	0.99	1.04	–	–	–	–
180°	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
210°	1.05	1.00	–	1.01	0.97	–	–	–	–
240°	1.12	1.17	–	1.07	1.04	–	1.15	1.15	–
270°	1.09	1.04	1.32	1.05	1.00	1.10	1.12	1.17	1.62

8. Conclusions and Future Outlook

The influence of convex and concave curves in the dike line on wave run-up for a regular wave was investigated using numerical models. Findings are compared with a subset of results stemming from physical model tests. The numerical investigation was accomplished using DualSPHysics, a meshless model and OpenFOAM, a mesh-based model. A 3D numerical wave basin was set up and calibrated in both numerical models. The reference dike was chosen as a straight dike and further analysis of curved dikes was compared to the straight dike. The numerical analysis includes the estimation of wave run-up and the wave transformation processes at the curvature. For selected cases, reference tests with a physical model were conducted in a 3D wave basin. The analysis was done for both convex and concave curvatures with different opening angles, angles of wave attack and wave parameters for regular waves. From the analysis, it is observed that the underlying hydrodynamic flow processes at curved dike lines show complex wave processes like wave rollers in case of convex shaped dikes and multi-directional transformation processes in case of concave shaped dikes. A new influence factor γ_c is introduced to determine the influence of curvature along the coastal dike line. A mild increase in wave run-up at the center of curvature on a curved dike is observed in most of the test cases under perpendicular or oblique wave attack. For perpendicular wave attack clear trends are visible for different opening angles. For inclined wave attack the influence of the curvature decreases and the run-up for curvatures scatters in the range of straight slopes. Differences between the different numerical and physical approaches can be ascribed by the choice of mesh size in case of OpenFOAM and inter particle distance in case of DualSPHysics. This can be optimized by using advanced hardware tools with the compromise of high computational cost and high data storage capacity. In spite of the possibility to generate irregular waves on the chosen software tool, the investigations were done only for regular waves on both numerical models. This is due to the same fact of being computationally expensive to simulate a statistically useful number of waves (commonly 1000 waves). The future research work is aimed to investigate the influence of curvatures on wave overtopping numerically due to the advancement in DualSPHysics which made possible for measuring overtopping using a measuring tool called FlowTool. Only initial investigations were made on the dike flanks within this study, and therefore the influence of dike flanks will be further analyzed.

Author Contributions: H.S., T.S. and N.B.K. conceived the research design and enhanced structure of manuscript. S.P.S. and B.S. developed the theory and performed the numerical study. B.S. investigated the numerical model using OpenFOAM and S.P.S. investigated the numerical model using DualSPHysics. M.S. conducted the hydraulic model tests. M.S., S.L., T.S. and N.B.K. analyzed and verified the results from physical model tests. S.L. and T.S. verified the analytical methods. H.S. encouraged S.P.S. to investigate a comparison for the findings from the numerical study with the findings of the physical model tests and supervised the findings of this work. C.A. guided and validated the DualSPHysics model. All authors discussed the results and contributed to the final manuscript.

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