DETERMINISTIC PARTICLE FILTERING FOR GPS NAVIGATION IN THE PRESENCE OF MULTIPATH

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ABSTRACT

In GPS navigation, distortion of the delay and phase of the received signal due to multipath propagation can degrade seriously the position estimation. This paper proposes deterministic particle filtering for joint multipath detection and navigation parameter estimation. Numerical simulations show the performances of the proposed method.

1. INTRODUCTION

The Global Positioning System (GPS) is a navigation system which estimates a user's position and velocity from the delay and Doppler of direct-sequence code-division multiple-access (DS-CDMA) signals sent by satellites [1]. It is well known that in the presence of multipath propagation, the estimated delay does no longer correspond to the line-of-sight (LOS) component, causing a bias of up to several tens of meters in the position estimates [1]. Several techniques have been proposed so far to mitigate this phenomenon including subspace-based timing estimators [2], interference cancellation [5], the narrow correlator [3] and beamforming [4]. Recently, bayesian Monte-Carlo filtering has been proposed to jointly estimate the LOS and multipath delays along with the amplitudes of the received signal using Dirac particles [6]. Even if multipath is compensated for, the navigation solution will experience either a bias or higher mean square error (MSE) when compared to LOS conditions. In applications such as aviation navigation systems, it is of high importance to be able to detect the occurrence of multipath in order to monitor the integrity of positioning [7]. Although the aforementioned particle based method has very good performances, the drawback is that the presence or absence of multipath components must be detected via an external module.

In this paper, we model the presence/absence of multipath as a discrete random variable subsequently referred to as multipath status. We propose joint estimation of the multipath status, navigation message, amplitude, phase, Doppler and delays using particle filtering. It is well known that the required number of particles increases with the dimension of the state space. Therefore, in order to keep the complexity at a reasonable level, extended Kalman filtering (EKF) is used to estimate as many parameters as possible. In particular, we show that using Dirac particles for the multipath status, the navigation message and the relative delay between multipath and LOS components is sufficient.

The remainder of the paper is organized as follows. In Sec. 2, we present particle filtering methods for joint estimation of discrete and continuous parameters. Sec. 3 introduces the GPS system model. Finally, Sec. 4 presents the performances of the proposed scheme.

2. PARTICLE FILTERING

We consider a discrete-time dynamical system of the form

$$\{\mathbf{d}_{k+1}, \boldsymbol{\theta}_{k+1}\} = f_k\left(\mathbf{d}_k, \boldsymbol{\theta}_k, \mathbf{u}_k\right)$$
$$\mathbf{x}_{k+1} = g_k(\mathbf{d}_k, \mathbf{x}_k) + \mathbf{v}_k \qquad (1)$$
$$\mathbf{y}_k = h_k(\mathbf{d}_k, \boldsymbol{\theta}_k, \mathbf{x}_k) + \mathbf{n}_k.$$

The first equation is a *process* equation. At instant k, the discrete part of the state $\mathbf{d}_k$ takes discrete values in a finite alphabet $\mathscr{A} = (\mathbf{a}_1, \ldots, \mathbf{a}_Q)$. The continuous part of the state is denoted by $\boldsymbol{\theta}_k$ and the process noise by $\mathbf{u}_k$. The second equation is a *process* equation conditioned on $\mathbf{d}_k$ involving a continuous state $\mathbf{x}_k$ and an associated Gaussian process noise $\mathbf{v}_k$ with covariance matrix $\mathbf{Q}_k$. The third equation is the *measurement* equation, where the state dependent observations $\mathbf{y}_k$ are corrupted by white Gaussian measurement noise $\mathbf{n}_k$, with covariance matrix $\mathbf{R}_k$. The functions f_k, g_k and h_k are in general nonlinear. It is also assumed that $\mathbf{u}_k$, $\mathbf{v}_k$ and $\mathbf{n}_k$ are uncorrelated.

Particle filtering approximates the posterior density $p(\mathbf{d}_{1:k}, \boldsymbol{\theta}_{0:k}|\mathbf{y}_{1:k})$ by a set of N weighted Dirac functions

$$\hat{p}(\mathbf{d}_{1:k}, \boldsymbol{\theta}_{0:k}|\mathbf{y}_{1:k}) = \sum_{i=1}^{N} w_k^{(i)} \delta\left(\{\mathbf{d}_{1:k}, \boldsymbol{\theta}_{0:k}\} - \{\mathbf{d}_{1:k}^{(i)}, \boldsymbol{\theta}_{0:k}^{(i)}\}\right),$$
$$(2)$$

where $\{\mathbf{d}_{1:k}^{(i)}, \boldsymbol{\theta}_{0:k}^{(i)}\}$ is the i-th discrete particle and $w_k^{(i)}$ the corresponding weight at instant k.

2.1 Monte-Carlo particle filtering

Particle filtering was originally introduced as a sequential importance sampling technique [8]. The desired posterior density is estimated using a Monte Carlo approximation and the particles are therefore random samples obtained by simulation:

- Prediction: Draw

$$\{\mathbf{d}_k^{(i)}, \boldsymbol{\theta}_k^{(i)}\} \sim q\left(\mathbf{d}_k, \boldsymbol{\theta}_k|\mathbf{d}_{k-1}^{(i)}, \boldsymbol{\theta}_{k-1}^{(i)}\right),$$

where we choose the importance function q to be the prior

$$q\left(\mathbf{d}_k, \boldsymbol{\theta}_k|\mathbf{d}_{k-1}^{(i)}, \boldsymbol{\theta}_{k-1}^{(i)}\right) = p\left(\mathbf{d}_k|\mathbf{d}_{k-1}^{(i)}\right) p\left(\boldsymbol{\theta}_k|\boldsymbol{\theta}_{k-1}^{(i)}, \mathbf{d}_{k-1}^{(i)}, \mathbf{d}_k\right)$$

- Correction: Update the weights according to

$$w_k^{(i)} \propto w_{k-1}^{(i)} p\left(\mathbf{y}_k|\mathbf{d}_{1:k}^{(i)}, \boldsymbol{\theta}_{0:k}^{(i)}, \mathbf{y}_{1:k-1}\right).$$

- Resample the particles from the discrete density $\{w_k^{(i)}\}_{i=1,\ldots,N}$ (if needed).

Assuming that g_k and h_k in (1) can be locally linearized, we obtain

$$
\begin{aligned}
&p\left(\mathbf{y}_k | \mathbf{d}_{1:k}^{(i)}, \theta_{0:k}^{(i)}, \mathbf{y}_{1:k-1}\right) \\
&\approx \mathcal{N}\left(h_k(\mathbf{d}_k^{(i)}, \theta_k^{(i)}, \hat{\mathbf{x}}_{k|k-1}^{(i)}), \mathbf{H}_k^{(i)} \mathbf{P}_{k|k-1}^{(i)} \mathbf{H}_k^{(i)^T} + \mathbf{R}_k\right)
\end{aligned}
$$

where $\hat{\mathbf{x}}_{k|k-1}^{(i)}$ and $\mathbf{P}_{k|k-1}^{(i)}$ are the predicted estimate and error covariance matrix of $\mathbf{x}_k$ obtained by EKF [11] conditioned on $\{\mathbf{d}_{1:k}^{(i)}, \theta_{0:k}^{(i)}\}$ and

$$
\mathbf{H}_k^{(i)} = \frac{\partial h_k(\mathbf{d}_k, \mathbf{x}_k)}{\partial \mathbf{x}_k}\Big|_{\mathbf{d}_k = \mathbf{d}_k^{(i)}, \mathbf{x}_k = \hat{\mathbf{x}}_{k|k-1}^{(i)}}.
$$

2.2 Deterministic particle filtering

As mentioned in [9], drawing randomly the variable $\mathbf{d}_k^{(i)}$ from a discrete distribution during the prediction stage is inefficient since it introduces an unwanted approximation error. Therefore, for each particle retained at instant $k-1$, the Q hypotheses corresponding to the choice $\mathbf{d}_k = \mathbf{a}_j$, $j = 1, \ldots, Q$ are explored deterministically. We summarize deterministic particle filtering by the following algorithm:

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1. Prediction: For each particle $\{\mathbf{d}_{0:k-1}^{(i)}, \theta_{0:k-1}^{(i)}\}$ form the Q extensions $\{\mathbf{d}_{0:k-1}^{(i)}, \mathbf{d}_k = \mathbf{a}_j, \theta_{0:k-1}^{(i)}, \theta_k^{(i)}\}$, $j = 1, \ldots, Q$ where

$$
\theta_k^{(i)} \sim p\left(\theta_k | \theta_{k-1}^{(i)}, \mathbf{d}_{k-1}^{(i)}, \mathbf{d}_k = \mathbf{a}_j\right).
$$

2. Correction: Compute the weight of each extension as

$$
\begin{aligned}
w_k^{(i,j)} \propto w_{k-1}^{(i)} &\, p\left(\mathbf{d}_k = \mathbf{a}_j\right) \times \\
&p\left(\mathbf{y}_k | \mathbf{d}_{1:k-1}^{(i)}, \mathbf{d}_k = \mathbf{a}_j, \theta_{0:k}^{(i)}, \mathbf{y}_{1:k-1}\right)
\end{aligned}
$$

3. Resample N particles from the discrete density $\{w_k^{(i,j)}\}_{\substack{i=1,\ldots,N \\ j=1,\ldots,Q}}$

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3. SYSTEM MODEL

Each satellite of the GPS constellation transmits periodically a Gold sequence of length 1023 chips with chip duration $T_c = 1\mu s$ [1]. The period of the Gold sequence is therefore $T \approx 1ms$. Rectangular pulse shaping is used. Each Gold sequence transmits a bit from the navigation message using BPSK (binary phase shift keying). Since each bit is repeated $R = 20$ times, the transmission of one bit corresponds to 20 Gold sequences (or approximately 20 ms). Assuming that two consecutive Gold sequences do not interfere, the output of the matched filter for time interval $lT + \tau - T/2 \le t < lT + \tau + T/2$ in LOS conditions can be written as

$$
y(t) = b(t)e^{j\phi(t)}A(t)h(t - lT - \tau(t)) + n(t), \quad (3)
$$

where $h(t)$ is a triangular pulse at the matched filter output and $n(t)$ is an additive white Gaussian noise with variance N_0. The modulated bit at instant t is denoted by $b(t)$. The parameters of the LOS component are the following: $A(t)$ represents the amplitude, $\phi(t)$ the phase and $\tau(t)$ is the propagation delay from the satellite to the GPS receiver. In case of multipath propagation, we assume for simplicity that the received signal consists of the LOS component and one specular reflected signal:

$$
\begin{aligned}
y(t) = b(t)e^{j\phi(t)}\Big[&A(t)h(t - lT - \tau(t)) + \\
&a(t)h(t - lT - \tau(t) - \theta(t))\Big] + n(t).
\end{aligned} \quad (4)
$$

The parameters of the specular component are the following: $a(t)$ represents the complex amplitude and $\tau(t) + \theta(t)$ is the propagation delay from the satellite to the GPS receiver with $\theta(t) > 0$. All these parameters can fluctuate from one Gold sequence to the next.

At the receiver side, a sampling clock $\hat{\tau}_l$ is generated using a classical noncoherent early-late timing loop [10] of the form

$$
\begin{aligned}
\hat{\tau}_{l+1} = \hat{\tau}_l + \gamma \mathrm{Re}\Big\{&y(lT + \hat{\tau}_l)^* \times \\
&[y(lT + T_c/2 + \hat{\tau}_l) - y(lT - T_c/2 + \hat{\tau}_l)]/2\Big\},
\end{aligned}
$$

where γ denotes the loop gain.

The received signal corresponding to the l-th Gold sequence is then sampled at instants $t = lT + \hat{\tau}_l + qT_c/2$, for $q = -1, 0, 1, 2$ to ensure observability of the specular component. Let the real observation vector

$$
\tilde{\mathbf{y}}_l = \begin{bmatrix} \mathrm{Re}(y(lT - T_c/2 + \hat{\tau}_l)) \\ \mathrm{Im}(y(lT - T_c/2 + \hat{\tau}_l)) \\ \vdots \\ \mathrm{Re}(y(lT + T_c + \hat{\tau}_l)) \\ \mathrm{Im}(y(lT + T_c + \hat{\tau}_l)) \end{bmatrix}, \quad (5)
$$

denote the collection of those samples. Assuming that we wish to evaluate the position and velocity of the user every R Gold sequence (i.e. at instant $t = kRT$, $k = 1, 2, \ldots$), we also consider the corresponding stacked vector of observations

$$
\mathbf{y}_k = [\tilde{\mathbf{y}}_{(k-1)R+1}^T, \ldots, \tilde{\mathbf{y}}_{kR}^T]^T.
$$

Fig. 1 illustrates the observation vectors $\tilde{\mathbf{y}}_l$ and the stacked vectors $\mathbf{y}_k$ for time index $k = 1$.

We now model the transmission system corresponding to each satellite as a discrete-time nonlinear dynamical system. We use the notation x_k to denote the value of a continuous-time process x(t) at instant $t = kRT$.

Let us begin with the discrete variables. Let $b_k \in \{-1, +1\}$ denote the navigation bit corresponding to the stacked observation vector $\mathbf{y}_k$. By construction, b_k is constant during R consecutive Gold sequences. Let $S_k \in \{0, 1\}$ denote the random variable corresponding to the multipath status for the stacked observation vector $\mathbf{y}_k$. $S_k = 0$ (resp. $S_k = 1$) corresponds to the absence (resp. presence) of a specular reflection. Implicitly, we assume that the multipath status is constant during R consecutive Gold sequences. This model seems sensible because for physical reasons, a specular reflection cannot appear or disappear instantaneously.

We model b_k (resp. S_k) as a discrete Markov chain whose state transition diagram is given by Fig. 2 a) (resp. Fig. 2 b)). With the notations introduced in Sec. 2.2, we have $\mathbf{d}_k = [b_k, S_k]^T$.
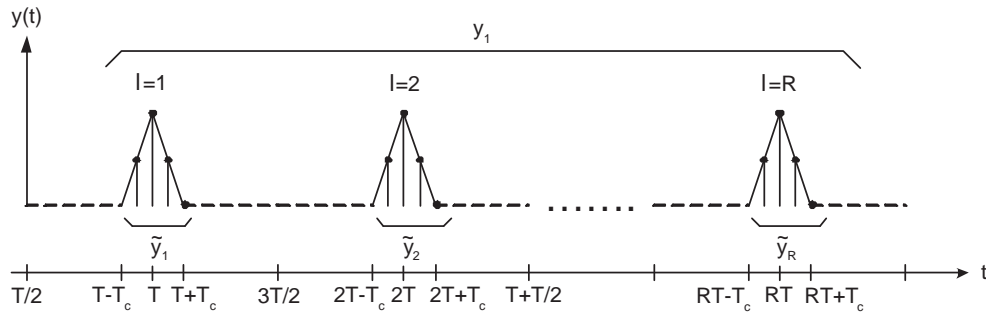
Figure 1: Samples corresponding to time index $k = 1$ assuming $\tau = 0$.
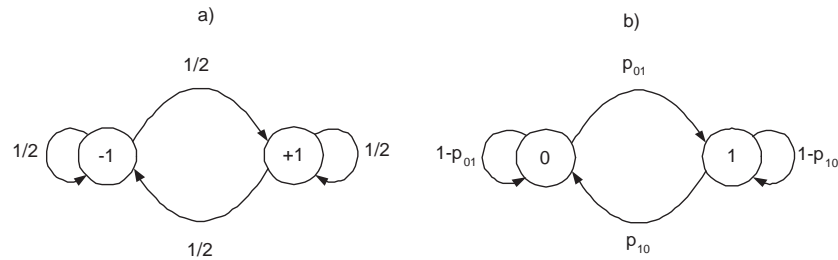
Figure 2: a) State transition diagram of b_k - b) State transition diagram of S_k.

Now consider the continuous variables. Let θ_k be the relative delay between the LOS component and the specular component (if present) for the stacked observation vector $\mathbf{y}_k$. We propose the following model:

$$(\theta_k | \theta_{k-1}, S_k = 0, S_{k-1} = 0) \sim \delta(\theta_k)$$
$$(\theta_k | \theta_{k-1}, S_k = 1, S_{k-1} = 0) \sim U([0.1T_c, 2T_c])$$
$$(\theta_k | \theta_{k-1}, S_k = 1, S_{k-1} = 1) \sim \mathcal{N}(\theta_{k-1}, 0.05^2)$$
$$(\theta_k | \theta_{k-1}, S_k = 0, S_{k-1} = 1) \sim \delta(\theta_k).$$

If $S_k = 0$, the rest of the continuous variables are given by $\mathbf{x}_k = [A_k, \phi_k, \tau_k, \Delta f_k]^T$, where A_k, ϕ_k, τ_k and Δf_k represent the received amplitude, phase, propagation delay and Doppler shift of the LOS component. If $S_k = 1$, the rest of the continuous variables are given by $\mathbf{x}_k = [A_k, \phi_k, \tau_k, \Delta f_k, \mathrm{Re}(a_k), \mathrm{Im}(a_k), \mathrm{Re}(a_{k-1}), \mathrm{Im}(a_{k-1})]^T$, where a_k represents the complex amplitude of the specular reflection. The dynamics of $\mathbf{x}_k$ are given by

$$
\begin{cases}
A_k = A_{k-1} + w_k^a, & w_k^a \sim \mathcal{N}(0, 0.001^2) \\
\phi_k = \phi_{k-1} + 2\pi \Delta f_{k-1} T + w_k^\phi, & w_k^\phi \sim \mathcal{N}(0, 0.001^2) \\
\tau_k = \tau_{k-1} - \dfrac{\Delta f_{k-1}}{f_0} T + w_k^\tau, & w_k^\tau \sim \mathcal{N}(0, 0.05^2) \\
\Delta f_k = \Delta f_{k-1} + w_k^{\Delta f}, & w_k^{\Delta f} \sim \mathcal{N}(0, 0.001^2) \\
\mathrm{Re}(a_k) = -c_1 \mathrm{Re}(a_{k-1}) - c_2 \mathrm{Re}(a_{k-2}) + v_k^1, & v_k^1 \sim \mathcal{N}(0, \sigma^2) \\
\mathrm{Im}(a_k) = -c_1 \mathrm{Im}(a_{k-1}) - c_2 \mathrm{Im}(a_{k-2}) + v_k^2, & v_k^2 \sim \mathcal{N}(0, \sigma^2)
\end{cases}
$$

where $f_0 = 1575$ MHz is the carrier frequency. Note that the correlated Rayleigh fading amplitude of the specular component is modeled by a second-order autoregressive (AR) process defined by the constants c_1, c_2 and σ^2.

Finally, the observation function h_k relating $\mathbf{y}_k$ to the defined state space is given by (3) when $S_k = 0$ and by (4) when $S_k = 1$, at time $t = kRT$.

4. SIMULATION RESULTS

In Sec. 3, we have described the GPS transmission system for one satellite as a nonlinear dynamical system of the form (1). We use the satellite configuration considered in [12]. We will use satellite SV1, SV5, SV6 and SV9 with a time reference $t = 0$ corresponding to GPS receiver time 247080 s. The received signal is bandlimited to 2 MHz and the signal to noise ratio, A_k^2/N_0, is fixed to 10 dB. Since the two last satellites have the lowest elevation angle, we assume that SV6 (resp. SV9) is subject to multipath for $1000 \leq t \leq 6000$ ms (resp. $2000 \leq t \leq 7000$ ms). For these satellites, the direct-to-multipath ratio (DMR), $A_k^2/E(a_k^2)$, is fixed to 5 dB and the relative delay between the LOS and the specular reflection is fixed to $T_c/4$. The amplitude of the specular component a is simulated as a time-varying complex Rayleigh fading process with maximum Doppler of 5 Hz. The parameter p_{01} (resp. p_{10}) in Fig. 2 is fixed to 10^{-4} (resp. 5.10^{-4}). The user position (m) and clock bias (m) in Earth Centered Earth Fixed Cartesian Coordinates (ECEFC XYZ) [1] are simulated as

$$
\begin{cases}
x(t) = x(0) + v^x t \\
y(t) = y(0) + v^y t \\
z(t) = z(0) + v^z t \\
b(t) = b(0) + v^b t
\end{cases}
$$

with

$$\begin{cases} x(0) = 3894216.581 \quad m, \quad v^x = 2.0 \quad m/s \\ y(0) = 318933.001 \quad m, \quad v^y = 3.0 \quad m/s \\ z(0) = 5024282.536 \quad m, \quad v^z = 0.8 \quad m/s \\ b(0) = 3.0 \quad m, \quad v^b = 0.03 \quad m/s. \end{cases}$$

Deterministic particle filtering (see Sec. 2.2) is applied for each satellite. The maximum-likelihood estimate of the navigation bit at $t = kRT$ is obtained as

$$p_{-1} = \sum_{i=1}^{N} w_k^{(i)} \delta(b_k^{(i)} - (-1))$$

$$p_{+1} = \sum_{i=1}^{N} w_k^{(i)} \delta(b_k^{(i)} - (+1))$$

$$\hat{b}_k = \begin{cases} +1 & \text{if} \quad p_{+1} > p_{-1} \\ -1 & \text{otherwise} \end{cases}$$

and for the multipath status estimate at $t = kRT$ we have

$$p_0 = \sum_{i=1}^{N} w_k^{(i)} \delta(S_k^{(i)} - 0)$$

$$p_1 = \sum_{i=1}^{N} w_k^{(i)} \delta(S_k^{(i)} - 1)$$

$$\hat{S}_k = \begin{cases} 1 & \text{if} \quad p_1 > p_0 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the minimum mean square error (MMSE) estimates of the propagation delay and the Doppler frequency at $t = kRT$ are obtained as

$$\hat{\tau}_k = \sum_{i=1}^{N} w_k^{(i)} \tau_k^{(i)}$$

$$\hat{\Delta f}_k = \sum_{i=1}^{N} w_k^{(i)} \Delta f_k^{(i)}.$$

The discrete-time navigation equation [1] is solved using a properly initialized extended Kalman filter taking the values of $c\hat{\tau}_k$ and $\hat{\Delta f}_k$ at the output of the particle filter corresponding to each satellite as noisy pseudorange and Doppler observations. The resulting position (resp. velocity) estimate at instant $t = kRT$ is denoted by $(\hat{x}_k, \hat{y}_k, \hat{z}_k, \hat{b}_k)$ (resp. $(\hat{v}_k^x, \hat{v}_k^y, \hat{v}_k^z, \hat{v}_k^b)$).

We compare the performances of two deterministic particle filters averaged over 20 runs with different noise trajectories. First consider the filter ignoring the presence of multipath by removing the multipath status S_k and the multipath component parameters a_k, θ_k from the state space. Our experiments showed that in this case, $N = 1$ particle is sufficient. Intuitively, this corresponds to tracking the LOS component's parameters $(A_k, \phi_k, \tau_k, \Delta f_k)$ with two EKF, one corresponding to each navigation bit hypothesis. The particle filter for SV6 and SV9 ignore the presence of interference produced by the specular component. The resampling stage then keeps the best hypothesis. Assuming that the first transmitted bit is known by the receiver, for SV1 and SV5 no bit error was found, while for SV6 and SV9, approximately 25

percent of the navigation bits are erroneous. This is because the phase estimates of the LOS become jittery when a specular component is present leading to wrong decisions during the resampling stage. This technique requires $8N$ EKF steps per ms considering that there are four satellites and two possible values of b_k to test.

Now, consider the complete filter estimating both the LOS and the multipath component, if present. We found that $N = 100$ particles for each SV is enough to obtain a good precision for the propagation delay and Doppler estimates. All the navigation bits were correctly recovered even for the satellites affected by multipath. A quantity important in the context of positioning with integrity monitoring [7] is the *time-to-alarm* defined as the elapsed time between a change of S_k from 0 to 1 and the consecutive change of $\hat{S}_k$ from 0 to 1. In our experiments, the time-to-alarm is typically a few tens of ms for SV6 and SV9. This technique requires $16N$ EKF steps per ms considering that there are four satellites, two possible values of b_k and two possible values of S_k to test.

Fig. 3 and 4 illustrate the average MSE for the propagation delay and Doppler frequency for satellite SV9. Note that for the particle filter ignoring the multipath, the MSE increases by two orders of magnitude during the occurrence of a specular component (i.e. $2000 \le t \le 7000$ ms). This result is not surprising since it is well known that in the presence of multipath the propagation delay estimate of the LOS experiences a bias [1] and the Doppler estimate of the LOS becomes unstable.

Fig. 5 (resp. Fig. 6) illustrates the average MSE for the first coordinate of the position (resp. velocity) estimate obtained by solving the navigation equations [1], based on the propagation delay and Doppler estimates of the four satellite. For $1000 \le t \le 7000$ ms, since at least one satellite is affected by multipath, the accuracy of the position and velocity estimates becomes very poor if the particle filters corresponding to each SV ignore the presence of multipath.

5. CONCLUSIONS

In this paper, we considered joint multipath detection and navigation parameter estimation. A deterministic particle filtering receiver adapted to the mixed discrete/continuous nature of the state space was proposed. Numerical simulations showed that when integrity monitoring is of concern, it is possible to detect the occurrence of multipath within a few tens of milliseconds. Moreover, when multipath is present the position and velocity errors are mitigated compared to receivers ignoring the presence of multipath.

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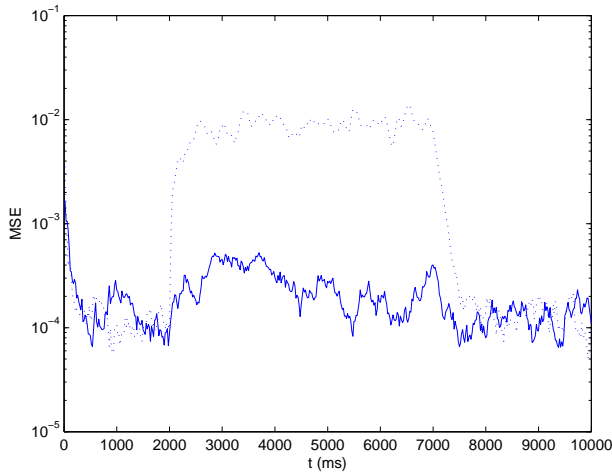
Figure 3: $E[(\tau - \hat{\tau})^2]$ for SV9 with deterministic particle filter estimation: receiver estimating multipath (solid) and ignoring multipath (dotted).
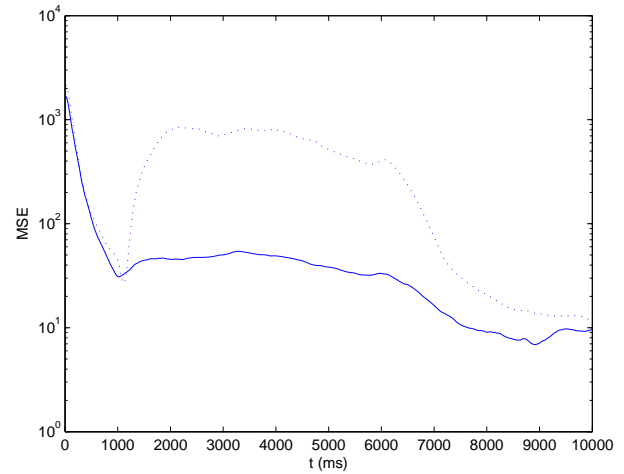

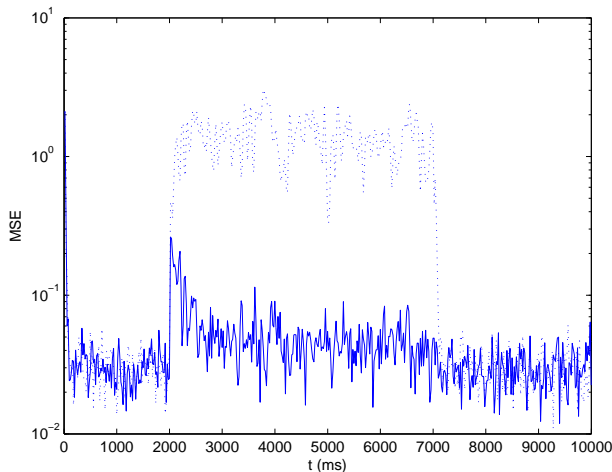Figure 5: $E[(x - \hat{x})^2]$ when the four SV estimate multipath (solid) and ignore multipath (dotted).

Figure 4: $E[(\Delta f - \hat{\Delta f})^2]$ for SV9 with deterministic particle filter estimation: receiver estimating multipath (solid) and ignoring multipath (dotted).
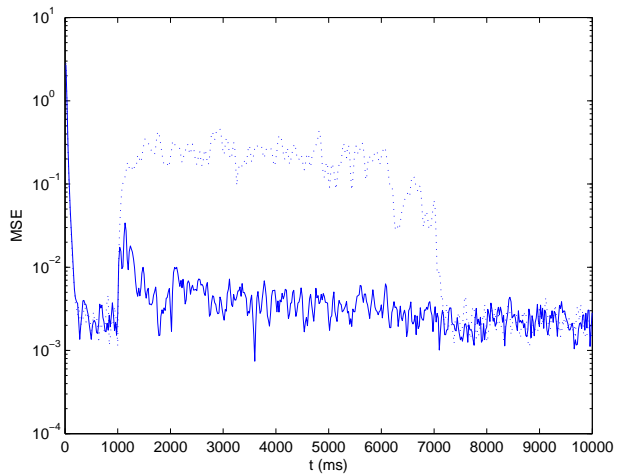

Figure 6: $E[(v^x - \hat{v}^x)^2]$ when the four SV estimate multipath (solid) and ignore multipath (dotted).

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