Bankruptcy Problem in Network Sharing: Fundamentals, Applications and Challenges

Angelos Antonopoulos, *IEEE Senior Member*
Telecommunications Technological Centre of Catalonia (CTTC/CERCA)
Castelldefels, Spain
Email: aantonopoulos@cttc.es

***Abstract*—Network sharing has been already adopted by mobile network operators as a reliable and effective countermeasure to the constantly increasing network cost. The introduction of the fifth generation (5G) mobile communications is expected to revolutionize the telecommunications world, however it also brings new challenges to the network sharing, as new technologies (e.g., network virtualization, cloud architectures, etc.) and types of resources (e.g., computation and storage) come to the fore. Despite their inherent differences, the common characteristic of the emerging resources is that they are limited and, in many cases, insufficient to cover the rising traffic demands. In this article, we highlight the equivalence between the distribution of a limited number of resources and the bankruptcy problem, where the demands of different agents over a given commodity exceed its total quantity. In particular, i) we review the fundamentals and existing solutions of the bankruptcy problem, ii) we provide potential applications of this problem in mobile network sharing, and iii) we list challenges and open issues for the application of the bankruptcy game in the mobile communications domain. The main goal of our work is to identify new research lines that will foster the 5G network sharing.**

***Index Terms*—Bankruptcy game; Game theory; 5G; Infrastructure Sharing.**

I. Introduction

The fifth generation (5G) mobile communications is just around the corner, signaling the transformation of telecommunication networks from mere communication channels into key enablers for a variety of important vertical industries, such as e-Health, manufacturing and automotive, among others. It is expected that upcoming 5G networks will provide extremely high data rates and ultra low latency, thus being able to fulfill the strict requirements of different traffic types, e.g., augmented reality (AR) or remote surgery applications.

The high potential of 5G networks, along with the rapid evolution of mobile devices, constitute some of the main factors for the anticipated data traffic explosion. In particular, according to Cisco forecasts, the global mobile data traffic will increase seven-fold between 2017 and 2022, reaching 77.5 exabytes per month by 2022 [1]. Apparently, to cope with these unprecedented traffic demands, mobile network stakeholders (e.g., mobile operators, infrastructure providers, etc.) need to upgrade and expand their networks, something that intrinsically implies an important increase in capital and operational expenditure (CAPEX and OPEX, respectively), threatening the viability of future mobile networks.

Network sharing [2] has already become a standard part of the operating model for mobile operators, while the trend is constantly accelerating. Through sharing both active and passive equipment, operators have been able to significantly reduce the total cost of ownership, while improving network quality. Hitherto, operators have been able to achieve significant savings[1], while recent reports foresee that these savings will be even more impressive (i.e., up to 50%) in 5G, as greenfield deployment is better suited for sharing, since it avoids the cost of network consolidation [3].

Nonetheless, despite its strong potential, 5G network sharing is not a clear-cut concept and comes along with several challenges. More specifically, the evolution of the mobile networks increases their complexity and, consequently, the different types of resources that can be encountered. In particular, unlike the traditional sharing schemes that focus on the physical infrastructure and the communication resources in basic network parts (e.g., radio access or transport network), the introduction of a series of different paradigms in future networks brings new heterogeneous resources that need to be explicitly considered. For instance, the cloud radio access network (C-RAN) technology generates new network parts (i.e., fronthaul) that did not exist in previous mobile generations, the adoption of fog and multi-access edge computing (MEC) implies new computational and storage resources, while there are also hardware developments with the appearance of unmanned aerial vehicles (UAVs) that can be part of the network by carrying small base stations. On top of this, the embracement of renewable energy sources for the network power supply adds another degree of complexity that should be contemplated by the forthcoming network sharing schemes.

Despite their heterogeneity and their inherent differences, all the aforementioned resources share a common characteristic, i.e., they are limited and they become even more valuable as the data traffic grows. Therefore, as the network expansion and the inclusion of additional

[1] Please see: https://www.gsma.com/futurenetworks/wiki/infrastructure-sharing-an-overview

resources is not always feasible (e.g., due to space limitations or cost), efficient sharing approaches should be established. In this context, this challenging situation of sharing limited resources among various interested parties can be formulated as a *bankruptcy problem*, which is a distribution problem that involves the allocation of a given amount of a single commodity among a group of agents, when this amount is insufficient to satisfy all their demands [4].

Taking into account the similarities of the limited resource sharing with the bankruptcy problem and the fact that bankruptcy theory has not yet been fully exploited in mobile network sharing scenarios, the main goal of this article is to identify new research lines in order to pave the way for the application of the bankruptcy problem in network sharing. To that end, our contribution is mainly threefold. First, we present the fundamentals of the bankruptcy problem, along with a list of possible solutions and the key ideas behind these solutions. Then, we focus on the telecommunications domain and we provide potential applications in existing and future mobile networks, where the resource sharing can be modeled and solved as a bankruptcy problem. Finally, we list some intriguing open research lines and challenges for the application of the bankruptcy theory in wireless networks.

II. Bankruptcy Problem: Background

The origins of the bankruptcy problem go several centuries back, following the fundamental human need for fair division of commodities[2]. In principle, each bankruptcy problem is characterized by an entity $E \in \mathbb{R}$ that has to be divided among N agents, whose individual claims C_i, $i \in N$, add up to an amount higher than the total entity, i.e., $\sum_{i=1}^{N} C_i > E$. Apparently, as it is not possible for all claims to be satisfied, different approaches may be followed for the division. In this section, we present a toy example along with some of the most popular solutions and the outcome allocations (A_i) they yield, summarized also in Table I.

A. Bankruptcy Problem: Toy Example

Without loss of generality, we assume that a total entity of $E = 45$ units has to be allocated to three claimants, whose claims are $C_1 = 10$, $C_2 = 20$ and $C_3 = 30$, respectively. As the sum of all claims is greater than the total entity, i.e., $\sum_{i=1}^{3} C_i = 60 > 45$, it is not possible to fulfill all claims.

B. Equal Sharing

Equal sharing is the simplest form of sharing a commodity among a set of interested agents. Equal sharing completely neglects the individual claims of the involved parties and divides the commodity into equal shares that are allocated to each agent, respectively. It is worth noting that, following this approach, it is possible that one agent receives an amount higher than the one she claimed. In our example, the entity would be divided in three equal parts, i.e., $A_1 = A_2 = A_3 = 15$. Obviously, the first claimant would receive a quantity higher than the requested, while the requirements of the other two claimants would not be satisfied.

[2]The Babylonian Talmud (a record of discussions about Jewish laws and customs) includes various cases with regard to the fair distribution.

C. Water Filling

Water filling solution is another simple method that is mainly used to avoid that a claimant is awarded with a portion higher than the requested. In this case, we can think the claims as tanks of different height (i.e., proportional to each claim) that are gradually filled with part of the entity. Once the lower tank is filled, we continue only with the rest ones, maintaining always the same "water" level in the tanks to be filled. However, one important disadvantage of this approach is that the losses are not equally shared, as usually the smaller claim is completely satisfied and the losses affect mainly the high claims. Fig. 1 demonstrates the water filling allocation in four different cases. In our toy example, following this approach, all claimants would receive 10 units (until the demand of the first claimant is fulfilled) and then the remaining 15 units would be equally split to the other two claimants.

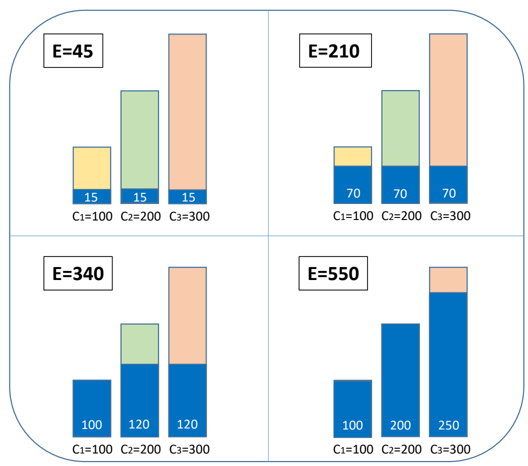

Fig. 1: Water filling allocation in the bankruptcy problem

D. Loss Sharing

To overcome the unbalanced loss issue, one possible solution would be to observe the problem from a different point of view and focus on the fair distribution of losses instead of sharing the entity. More specifically, as the sum of the claims is always higher than the entity, we may estimate their difference and then split this difference equally to the N agents. You may also notice that, in case that there are big discrepancies between the claims, this approach may induce negative values. For instance, if $E = 45$, $C_1 = 5$ and $C_2 = 60$, the loss sharing method would allocate $A_1 = -5$ and $A_2 = 50$. In this case,

TABLE I: Sharing solutions for the bankruptcy problem

Example Scenario: $E = 45$, $C_1 = 10$, $C_2 = 20$, $C_3 = 30$				
Approach	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	**Comments**
Equal Sharing	15	15	15	All claimants receive the same amount (i.e., 15 units). However, this amount is higher than the claim of the first agent, while the other two claimants receive less than what they claimed.
Water Filling	10	17.5	17.5	All players receive 10 units (which is the claim of the first agent) and, then, the 15 remaining units are equally distributed to the two other players. Using this approach, the losses are not equally shared, as the loss for player two and three is 2.5 and 12.5 units, respectively.
Loss Sharing	5	15	25	As the total amount of claims is equal to 60 units, the total loss ($Loss = \sum_{i=1}^{3} C_i - E$) is equal to 15 units. This loss is equally shared among the three players, i.e., 5 units loss for each.
Proportional Sharing	7.5	15	22.5	The three players receive amounts proportional to their claims. It can be also easily shown that the losses are also shared in a proportional way, i.e., 2.5, 5 and 7.5 units for the three players respectively.
Contested Garment Rule	5	15	25	The players receive the same amount as if they solved individual two-player bankruptcy problems. It is worth noting that this concept is identical to the nucleolus concept in cooperative games.
Shapley Value	$6^2/_3$	$14^1/_6$	$24^1/_6$	Corresponds to the average of the payoffs over all different combinations in the order of arrival.

agent 1 should subsidize agent 2 with 5 units. However, in such extreme cases, the accepted solution is to allocate the whole amount to the higher claim. Focusing in our particular example, the total loss of 15 units would be allocated equally to the three players and, as a result, each of them would lose 5 units, i.e., $A_1 = 5$, $A_2 = 15$ and $A_3 = 25$.

E. Proportional Sharing

This is another approach that explicitly takes into account the claims, as the sharing takes place proportionally to the claims. Proportional division is a quite appealing approach and it is also adopted in several real life use cases (e.g., in case of natural disasters, insurance companies pay off the losses with a fixed amount per dollar). It can be also easily proven that the proportional sharing of the losses provides the same solution, so we avoid dissatisfaction due to unbalanced losses. In our example, the second and the third claimant would receive an amount of two and three times higher, respectively, than the first player.

F. Contested Garment Rule

This method is one of the oldest sophisticated ones and named after the problem of sharing a garment. The main logic of this approach lies in the fact that the division takes place on the dispute part (i.e., the part that both agents claim), while the higher claimant receives the undisputed claim), while the higher claimant receives the undisputed part. For instance, considering the case of $E = 125$, $C_1 = 100$ and $C_2 = 200$, both claimants would receive 50 units (as the common claimed part is 100), while the rest 25 units would be allocated to the second claimant. Although beyond the scope of this article, the solution in the n-player problem[3] is not straightforward and it tries to: i) provide half of the claimed amount to each claimant and ii) equalize the losses among claimants. In our example, the solution would be $A_1 = 5$, $A_2 = 15$ and $A_3 = 25$.

G. Shapley Value

As it has been turned out that the bankruptcy problem can be formulated as a cooperative game, Shapley value can provide another alternative sophisticated solution. Shapley value was introduced in 1953 by Lloyd Shapley [5] and assigns a unique distribution (among the players) of a total surplus generated by the coalition of all players. For the computation of the Shapley value, the order of arrival of the different requests is taken into account[4], while to remove any unfairness, all possible combinations of arrival are considered. For instance, in our example, the allocation according to Shapley value would be $A_1 = 6^2/_3$, $A_2 = 14^1/_6$ and $A_3 = 24^1/_6$, as demonstrated in detail in Fig. 2.

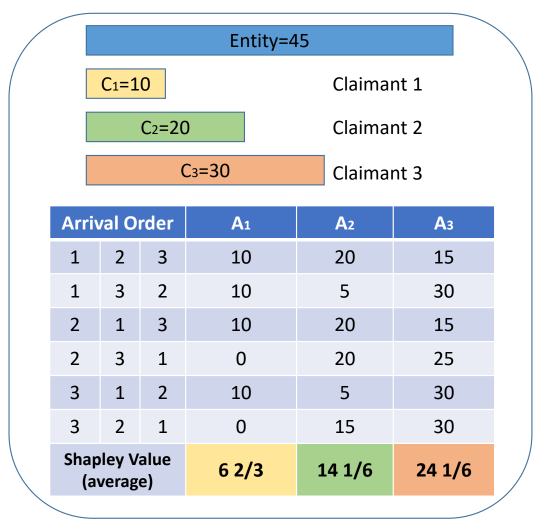

Fig. 2: Shapley value computation

[3]The interested reader may refer to [6], which provides a theoretical explanation to this solution and led Prof. Aumann to win the Nobel Memorial Prize in Economic Sciences.

[4]The requests are satisfied on their whole.

III. Potential Applications in 5G Mobile Communications

In this section, we will try to shed some light on the potential applications of the bankruptcy problem in the telecommunications domain. Apparently, we are moving through a digital era, where the user demands and requirements are constantly increasing, rendering the network resources scarce. In the following, we will present a list of possible research areas, where the sharing of the resources can be formulated as a bankruptcy problem.

A. Network Resource Management

The management of radio resources constitutes one of the most traditional problems in wireless communications and there are already some works that have treated it as a bankruptcy problem [7]. With the introduction of 5G-driven bandwidth hungry applications (e.g., AR, 4K video streaming, etc.), the radio resources become even more valuable, as it is not trivial to allocate the bandwidth in a fair and efficient manner. However, the fact that the new applications come with clear requirements and key performance indicators (KPIs) promotes the formulation of the problem as a bankruptcy case, where the application requirements correspond to the claims.

In upcoming 5G networks, it is expected that the exploitation of high frequency bands and the mmWave technology will provide a solution to this problem, as spectrum will be abundant and no longer a constrained resource. However, in such scenarios, other network parts can constitute a bottleneck, e.g., the transport network or the fronthaul link in C-RAN topologies. Hence, bankruptcy game theory could still be a valuable tool for overcoming these bottlenecks by efficiently sharing the limited network resources.

B. Infrastructure Sharing

Infrastructure sharing is a quite broad concept that has emerged during the last decade and, in its basic form, involves the sharing of passive (e.g., cooling, warehouses) and active (e.g., antennas, base stations) network equipment. The entry of new stakeholders, such as mobile virtual network operators, in the telecommunications landscape has already motivated the research community to study the efficient sharing of the underlaid network infrastructure, using advanced analytical tools from game and auction theory [2].

With the 5G evolution, the networks become more dense (hence the number of small base stations increases), however the number of interested parties that are willing to have access and control over the deployed infrastructure is also growing. In addition, the concept of a trusted 3rd party that provides the network infrastructure (also known as infrastructure provider) also gains ground, escalating further the problem of a fair and profitable allocation of the 5G small cells to the network operators. To that end, bankruptcy theory can be considered as an alternative solution to this problem, being complementary to traditional game theoretic approaches.

C. Energy Sharing

Energy commodities have always been precious in real life, as well as in various industrial domains. In particular, the information and communication technologies (ICT) sector is considered as one of the most power demanding industries, responsible for ∼2 per cent of global greenhouse gas emissions. To confront this issue, mobile operators are increasingly adopting the utilization of renewable (or clean) energy sources, such as wind, water or solar. Although clean energy sources contribute significantly to the reduction of CO2 emissions, their power supply is intermittent and cannot be accurately predicted, thus being unreliable for mobile networks. The use of batteries could partially provide a solution to this issue, however battery capacity is also limited and efficient sharing mechanisms should be put in place.

Bankruptcy theory would be an appropriate framework to model this problem in different use cases. For instance, in the context of network sharing, the mobile operators that share a base station could claim different amount of energy according to their traffic demands [8]. Another possible application would be in single-operator scenarios, where many base stations are powered by a pool of batteries. In this case, the various applications could have different claims according to their criticality and bankruptcy modeling could solve the problem.

D. Load Shedding

Load shedding is a common technique that was primarily introduced in information systems (mainly in web services) to avoid overloading the system and, thus, making it unavailable for all users. The main idea behind shedding part of the load is that some requests may be ignored so as the rest of the request can be efficiently served. In the same context, with the smart grid revolution, load shedding is required when there is an imbalance between electricity demand and electricity supply.

Apparently, efficient schemes for handling situations of this kind should be properly devised, as load shedding can be very inconvenient and irritating to the users that are rejected by the respective system (either requests by operators/end users in cloud data centers or energy requests in smart grid scenarios). To that end, bankruptcy theory can provide new insights and solutions to the specific problem by allocating the resources in a fair manner, while maximizing the number of users that can be served.

E. Cloud and Edge Computing

Cloud computing infrastructures are expected to have a leading role in upcoming 5G networks thanks to two important enablers. First, network virtualization gradually transforms the existing rigid mobile networks to flexible software platforms, suitable to accommodate sophisticated virtual network functions (VNFs) that need computational resources for their execution. In addition, the adoption of novel computing paradigms, such as fog computing or

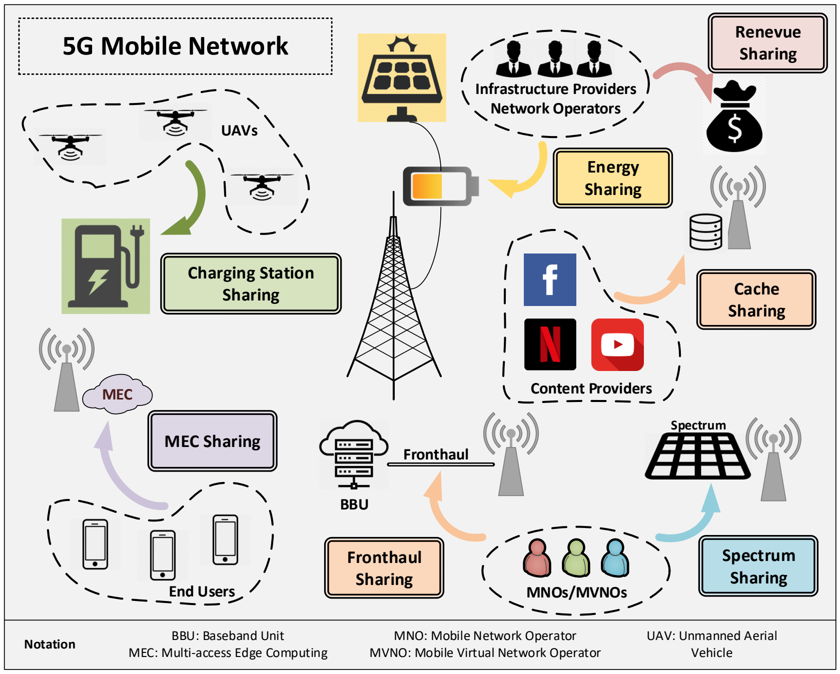

Fig. 3: 5G network sharing use cases

MEC, brings the computational resources at the edge of the network.

The main idea behind the aforementioned new paradigms is that, by running applications and performing processing tasks closer to the end user, network congestion is reduced (e.g., less signaling in the core network), while the performance of existing and new applications can be considerably improved (e.g., we are able meet the strict delay requirements of ultra low latency traffic). Nonetheless, despite the potential advantages of edge computing, there are also important cost and space limitations that do not allow their vast implementation in future networks. Hence, the limited edge resources become valuable, while the claims over these resources is expected to increase, as the user density and requirements are also increasing.

Apparently, the allocation of the insufficient edge resources to a number of interested parties can be inherently formulated as a bankruptcy problem, where the claimants could be either the different mobile operators that request computing resources by a 3rd party that acts as an infrastructure provider, or even the end users that compete for these resources in single-operator scenarios [9].

F. *Content Caching*

Content caching in mobile networks has been introduced as an effective countermeasure for the reduction of the increased backhaul data rates due to the constantly growing data traffic [10]. Caching can be applied in various scenarios and topologies, such as C-RAN, heterogeneous networks, device-to-device communications, among others. However, regardless of the use case, caching is used to bring popular and viral contents close to the end user so as to improve i) the user quality of experience with lower delays, and ii) the network utilization for the operator by avoiding the repeated transmission of the same content over the network links.

Similar to the cloud computing case, the deployment of cache servers in all network elements is not feasible, as it can be either costly or impossible due to space constraints. On the other hand, the number of over-the-top (OTT) content providers is still increasing, while they are heavily interested in obtaining access to these resources in order to enhance their quality of service. This conflicting situation, i.e., a set of OTT providers that compete over a limited set of resources to cache their contents, can be modeled as a bankruptcy problem. In a similar context, multiple virtual MNOs could have the role of the claimants, asking for caching resources from the infrastructure provider, who has also to take into account the popularity of the different contents for estimating its potential gains. In all cases, bankruptcy theory could be employed to yield efficient solutions and useful insights [11].

G. *Revenue Sharing*

As the bankruptcy framework was initially applied to economic problems, its application to financial scenarios is quite straightforward. More specifically, 5G is expected to further boost the already high revenues of mobile networks, while the coexistence of multiple claimants is very possible to generate conflicting situations where the total claimed amount will be higher than the actual profit. It is also worth noting that the issue of revenue sharing can appear in different parts of the network with different entities as players.

For instance, in the various infrastructure sharing cases, where the operators share different parts of the network, different claims may arise according to their contributions (e.g., one operator may provide the infrastructure, another the spectrum, etc.) and their subjective perspectives. Another example comes from the invasion of OTT content providers in mobile networks and the new relationship dynamics with mobile operators that are formed. In this case, the mobile operators could claim part of the OTTs revenues, as the OTTs operate freely over mobile networks thanks to network neutrality. In the same context, as the mobile operators assert that content providers exploit their deployed networks, they could even ask for compensation in order to share the network cost. In all these cases, bankruptcy theory could facilitate the solution of the problem.

H. Unmanned Aerial Vehicles (UAVs)

The flexible deployment along with the strong line-of-sight links of UAVs are expected to make them an important component of the future 5G and beyond wireless networks towards the provision of ubiquitous any-time connectivity [12]. Besides the straightforward application of the bankruptcy problem for spectrum sharing among the UAVs and the fixed network network infrastructure, the introduction of aerial networks implies additional challenges that need to be addressed.

In particular, energy supply of UAVs has been identified as one major issue in aerial networks, since, apart from the possibility of communication interruption, the risk of physical damages (as UAVs may hover over crowded areas) is also looming. To deal with this problem, the idea of strategically deploying charging stations in urban and rural areas has been recently proposed [13] in the literature. However, as UAVs gain ground in mobile communications and the number of UAV operators is expected to increase, the limited places in the charging stations should be claimed and allocated efficiently.

IV. Open Research Lines and Challenges

This section presents some challenges and possible extensions for the application of the bankruptcy framework in the telecommunications domain.

A. Computational Complexity

As we have seen in the previous sections, bankruptcy problem can have several applications in the mobile communications domain, while there are also many possible solutions for each one of these applications. Therefore, the selection of the appropriate approach to each use case is something that should be carefully decided and, to that end, computational complexity is a parameter that should be definitely taken into account. More specifically, there are some very simple possible solutions of low complexity (e.g., equal share), while the complexity of some other solutions grows exponentially and can be even NP-hard in some cases (e.g., Shapley value).

In upcoming 5G networks, the time frame for making a decision in each problem will also determine the range of the acceptable solutions. In offline methods, such as revenue distribution or energy sharing, where the decision can be made in a longer timescale, the application of more sophisticated approaches could yield fairer and more efficient results. On the other hand, in cases where decisions have to be made "on-the-fly", e.g., spectrum allocation or MEC assignment, solutions of lower computational complexity that are able to be executed fast should be adopted. Of course, the selection of the solution is not always straightforward, as there can be cases where the time frame is dynamic, e.g., in charging station sharing for UAVs, the available time to make a decision depends on the location of the UAVs and the distance they have to cover. In such cases, the study of the tradeoff between the time execution and the efficiency of the results could reveal quite interesting insights.

B. 5G End-to-End (E2E) Network Slicing

5G is characterized by the entry of various vertical industries with strict but distinct requirements. Applications such as AR and 4K video require very high data rates and bandwidth, autonomous driving or smart-grid connections need extremely low latency, while smart home deployments are based on the simultaneous connections of a plethora of sensors in the network. The aforementioned applications drive respectively the three main traffic types in 5G, i.e., enhanced Mobile Broadband (eMBB), Ultra Reliable and Low Latency Communication (URLLC) and massive Machine Type Communication (mMTC).

To fulfill the different requirements of these applications, the concept of e2e network slicing has been introduced, which defines the formation of virtual networks through resource reservation across the different network domains. For an efficient slice creation, it is possible that a series of different bankruptcy problems need to be jointly solved, as different types of resources are required. For instance, to experience an uninterrupted interactive AR service in a CRAN network, the reservation of fronthaul, spectrum and cache resources would be essential.

C. Game Theory

The bankruptcy problem is inherently connected with game theory, as it concerns strategic interaction between rational decision-makers. This is also the main reason for modeling this problem as a cooperative game, where the Shapley value can provide a fair solution. However, due to the nature of the specific problem and the conflicting interests among the claimants, non-cooperative game theoretic approaches could be also considered [14].

Non-cooperative game theory focuses on cases where there is no collaboration among the players, while the main individual goal of each player is to achieve a stable solution, where her received utility will not depend on the actions of the other players. This solution point is called Nash equilibrium and it has been proven that, in this point, none of the players can improve her payoff by unilaterally changing strategy. In particular, in bankruptcy problems, the utility functions of the different agents could correspond to their allocated amount, while the claims can be the outcome of the game, i.e., the strategies that will constitute the Nash equilibrium. Although, in general, cooperative game theory can achieve even optimal results, non-cooperative solutions have also some nice attributes (e.g., stability, user satisfaction, etc.) that make them appealing in scenarios where cooperation is hard to be achieved.

D. Machine Learning

Machine learning [15] is another emerging technology that could be exploited to provide novel alternative solutions in bankruptcy problems. Machine learning provides systems with the capability to automatically learn and improve from experience without being explicitly programmed. In particular, as the main goal of the bankruptcy problem solutions is to offer fair solutions that satisfy the requirements of the various claimants, machine learning could enable the system to learn through satisfaction measures of previous distributions so as to improve the allocation and achieve near-optimal solutions.

Supervised learning, where the system learns a function that maps an input to an output based on an existing training set (i.e., sets of past inputs-outputs), is a class of machine learning, whose application would be quite straightforward in sharing scenarios (in this case, a given allocation would be the input, while the user satisfaction or fairness indices could be potential outputs). Moreover, reinforcement learning is another area of machine learning that could be considered mainly for new applications of bankruptcy problems, where existing training sets do not yet exist. In such scenarios, the distributions of the commodities could be made so as to maximize some cumulative reward, such as the user satisfaction.

V. Conclusion

This article has shed some light on the potential applications of the bankruptcy problem in mobile communications. Taking into account the similarities of the bankruptcy problem with the sharing of limited resources in emerging 5G wireless networks, we tried to identify new research lines that will foster the 5G network sharing. To that end, first we introduced the intuition behind the bankruptcy problem along with a set of proposed solutions. Then, we focused on the telecommunications domain, quoting a list of possible applications that includes a broad range of emerging techniques and concepts, from energy and spectrum sharing to UAVs and MEC use cases. Finally, we highlighted the challenges and open issues for the smooth application of the bankruptcy problem in wireless networks, including also possible advancements based on game theoretic and machine learning tools.

Acknowledgement

This work was supported by the research projects AGAUR (2017-SGR-891), SPOT5G (TEC2017-87456-P) and 5G-SOLUTIONS (856691).

References

[1] Cisco, "Cisco Visual Networking Index: Global Mobile Data Traffic Forecast, 2017-2022", Feb. 2019.

[2] A. Antonopoulos, E. Kartsakli, A. Bousia, L. Alonso, C. Verikoukis, "Energy-efficient infrastructure sharing in multi-operator mobile networks", *IEEE Communications Magazine*, vol. 53, no. 5, pp. 242-249, May 2015.

[3] F. Grijpink, A. Menard, H. Sigurdsson, N. Vucevic, "Network sharing and 5G: A turning point for lone riders", *McKinsey&Company*, Feb. 2018.

[4] C. Herrero, A. Villar, "The three musketeers: four classical solutions to bankruptcy problems", *Mathematical Social Sciences*, vol. 39, no. 3, pp. 307-328, Nov. 2001.

[5] L. S. Shapley, "A value for n-person games", *Contributions to the Theory of Games*, vol. 28, no. 2, pp. 307-317, 1953.

[6] R.J. Aumann, M. Maschler, "Game theoretic analysis of a bankruptcy problem from the Talmud", *Journal of Economic Theory*, vol. 36, pp. 195-213, Aug. 1985.

[7] F. Fossati, S. Hoteit, S. Moretti, S. Secci, "Fair resource allocation in systems with complete information sharing", *IEEE/ACM Transactions on Networking*, vol. 26, no. 6, pp. 2801-2814, Dec. 2018.

[8] M. Oikonomakou, A. Antonopoulos, L. Alonso, C. Verikoukis, "Energy sharing and trading in multi-operator heterogeneous network deployments", *IEEE Transactions on Vehicular Technology*, vol. 68, no. 5, pp. 4975-4988, May 2019.

[9] S. F. Abedin, A. K. Bairagi, M. S. Munir, N. H. Tran, C. S. Hong, "Fog load balancing for massive Machine Type Communications: A game and transport theoretic approach", *IEEE Access*, vol. 7, pp. 4204-4218, 2019.

[10] L. Li, G. Zhao, R. S. Blum, "A Survey of Caching Techniques in Cellular Networks: Research Issues and Challenges in Content Placement and Delivery Strategies", *IEEE Communications Surveys & Tutorials*, vol. 20, no. 3, pp. 1710-1732, Thirdquarter 2018.

[11] R. Chai, Y. Li, Q. Chen, "Joint cache partitioning, content placement, and user association for D2D-enabled heterogeneous cellular networks", *IEEE Access*, vol. 7, pp. 56642-56655, 2019.

[12] B. Li, Z. Fei, Y. Zhang, "UAV communications for 5G and beyond: Recent advances and future trends", *IEEE Internet of Things Journal*, vol. 6, no. 2, pp. 2241-2263, Apr. 2019.

[13] P.-V. Mekikis, A. Antonopoulos, "Breaking the boundaries of aerial networks with charging stations", *IEEE ICC*, Shanghai, China, May 2019.

[14] I. Ashlagi, E. Karagozoglu, B. Klaus, "A non-cooperative support for equal division in estate division problems", *Mathematical Social Sciences*, vol. 63, no. 3, pp. 228-233, May 2012.

[15] C. Zhang, P. Patras, H. Haddadi, "Deep Learning in mobile and wireless networking: A survey", *IEEE Communications Surveys & Tutorials*, Early Access, 2019.

Angelos Antonopoulos is currently a Senior Researcher at CTTC/CERCA. Ha has authored over 100 papers on various topics, including 5G network architectures, radio resource management, content caching and dissemination, and green networking. He currently serves as an Associate Editor in IEEE Networking Letters, IEEE Access, Computer Networks (Elsevier) and Inventions (MDPI). He has received the best paper award in IEEE GLOBECOM 2014 and EuCNC 2016 and the 1st prize in the IEEE ComSoc Student Competition. He is an IEEE Senior Member and he is currently the Secretary of the IEEE ComSoc Technical Committee on Communication Systems Integration and Modeling (CSIM).