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ANALYSIS AND COMPARISON OF APOLLO3® AND TRIPOLI-4® NEUTRON NOISE SOLVERS

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ABSTRACT

Neutron noise analysis addresses the description of small time-dependent flux fluctuations in reactor cores, induced by small global or local perturbations of the macroscopic cross sections due to density fluctuations of the coolant, to vibrations of fuel elements, control rods, or structural materials. The general noise equations are obtained by assuming small perturbations around a steady-state neutron flux and by subsequently taking the Fourier transform in the frequency domain. Recently, new neutron noise solvers have been implemented in diffusion and transport theory in APOLLO3®, the multi-purpose deterministic transport code developed at CEA, and a new stochastic solver has been implemented for the neutron noise analysis in the frequency domain in the Monte Carlo code TRIPOLI-4®, also developed at CEA. In this paper, we compare the two solvers for the case of fuel pin oscillations in a simplified UOX fuel assembly, in view of proposing the examined configurations as a benchmark for neutron noise calculations.

KEYWORDS: Neutron noise, Frequency domain, Complex statistical weights, TRIPOLI-4®, APOLLO3®.

1. INTRODUCTION

Neutron noise analysis addresses the description of small time-dependent flux fluctuations induced by small global or local perturbations of the macroscopic cross sections [1]. These fluctuations may occur in nuclear reactors due to density fluctuations of the coolant, to vibrations of fuel elements, control rods, or any other structures in the core. In power reactors, ex-core and in-core detectors can be used to monitor neutron noise with the aim of detecting possible anomalies and taking the necessary measures for continuous safe power production.

In view of the importance of the neutron noise analysis for reactor applications, in 2016 new neutron noise solvers have been implemented in diffusion and transport theory in APOLLO3®, the multi-purpose deterministic transport code developed at CEA [2]. More recently, a new stochastic solver has been implemented for the neutron noise analysis in the frequency domain in the Monte Carlo code TRIPOLI-4®, also developed at CEA [3]. Both solvers are under active verification and validation in the framework of the CORTEX H2020 European project [4].

In this paper, we will compare for the first time the capabilities of these solvers by performing neutron noise simulations in a simplified UOX fuel assembly. This paper is organized as follows. In

Sec. 2, the general neutron noise theory will be briefly introduced, then the Monte Carlo algorithm implemented in TRIPOLI-4® and the deterministic algorithm implemented in APOLLO3® will be presented. In Sec. 3, we will compare the capabilities of the proposed Monte Carlo and deterministic solvers by computing the neutron noise propagation in a simplified UOX fuel assembly, for both Green's function solutions corresponding to noise sources located at fixed points, and more realistic cases where the noise source comes from a steady state neutron distribution. Conclusions will be drawn in Sec. 4.

2. DESCRIPTION OF APOLLO3® AND TRIPOLI-4® NEUTRON NOISE SOLVERS

2.1. Neutron noise theory

The general noise equations are obtained by assuming small perturbations $\Psi(\mathbf{r},\mathbf{\Omega},E,t)=\Psi_0(\mathbf{r},\mathbf{\Omega},E)+\delta\Psi(\mathbf{r},\mathbf{\Omega},E,t)$ (which allows for a linear theory) around a steady-state neutron flux Ψ_0 and by subsequently taking the Fourier transform in the frequency domain. The analysis is performed based on the neutron kinetic equations, including delayed neutron precursors. The outcome of the Fourier transform analysis is a fixed-source equation with complex operators for the perturbed neutron field $\delta\Psi$, which can then be solved so as to predict noise measurements at detector locations. For each frequency ω, the perturbed neutron flux is a complex function having an amplitude and a phase. Imposing a periodic perturbation of the kinetic operator, the noise equation reads [1]:

$$\begin{aligned}\left(\mathbf{\Omega}\cdot\nabla+\Sigma_0(\mathbf{r},E)+i\frac{\omega}{\upsilon}\right)\delta\Psi(\mathbf{r},\mathbf{\Omega},E,\omega)&=\iint\Sigma_{0,s}(\mathbf{r},\mathbf{\Omega}'\cdot\mathbf{\Omega},E'\rightarrow E)\delta\Psi(\mathbf{r},\mathbf{\Omega}',E',\omega)dE'd\mathbf{\Omega}'\\&+\frac{1}{k}\frac{\chi_p(E)}{4\pi}\iint\nu_p(E')\Sigma_{0,f}(\mathbf{r},E')\delta\Psi(\mathbf{r},\mathbf{\Omega}',E',\omega)dE'd\mathbf{\Omega}'\\&+\frac{1}{k}\sum_j\frac{\chi_d^j(E)}{4\pi}\iint\nu_{d,\omega}^j(E')\Sigma_{0,f}(\mathbf{r},E')\delta\Psi(\mathbf{r},\mathbf{\Omega}',E',\omega)dE'd\mathbf{\Omega}'+S(\mathbf{r},\mathbf{\Omega},E,\omega),\end{aligned}\tag{1}$$

where i is the imaginary unit, $\omega=2\pi f$ the angular frequency, and

$$\nu_{d,\omega}^j(E)=\left(\frac{\lambda_j^2-i\lambda_j\omega}{\lambda_j^2+\omega^2}\right)\nu_d^j(E),\tag{2}$$

for the precursor family j, $\nu_d^j(E)$ being the average number of delayed neutrons emitted for family j; all other notations are standard. Thus, because of the delayed neutrons, the production operator depends on the frequency ω. Equation (1) can be conceptually split into a system of two equations for the real and imaginary part of $\delta\Psi$. The two equations are formally coupled by two terms: $i\omega/\upsilon$ and the modified delayed production operator including the complex multiplicity $\nu_{d,\omega}^j(E)$. The noise source S is defined by:

$$\begin{aligned}S(\mathbf{r},\mathbf{\Omega},E,\omega)&=-\delta\Sigma(\mathbf{r},E,\omega)\Psi_0(\mathbf{r},\mathbf{\Omega},E)+\iint\delta\Sigma_s(\mathbf{r},\mathbf{\Omega}'\cdot\mathbf{\Omega},E'\rightarrow E,\omega)\Psi_0(\mathbf{r},\mathbf{\Omega}',E')dE'd\mathbf{\Omega}'\\&+\frac{1}{k}\frac{\chi_p(E)}{4\pi}\iint\nu_p(E')\delta\Sigma_f(\mathbf{r},E',\omega)\Psi_0(\mathbf{r},\mathbf{\Omega}',E')dE'd\mathbf{\Omega}'\\&+\frac{1}{k}\sum_j\frac{\chi_d^j(E)}{4\pi}\iint\nu_{d,\omega}^j(E')\delta\Sigma_f(\mathbf{r},E',\omega)\Psi_0(\mathbf{r},\mathbf{\Omega}',E')dE'd\mathbf{\Omega}',\end{aligned}\tag{3}$$

where $\delta\Sigma_x(\mathbf{r}, E, \omega)$ is the Fourier transform of the perturbed term of the macroscopic cross section $\Sigma_x(\mathbf{r}, E, t) = \Sigma_{0,x}(\mathbf{r}, E) + \delta\Sigma_x(\mathbf{r}, E, t)$, $\Sigma_{0,x}$ being the steady-state macroscopic cross section.

2.2. The deterministic algorithm implemented in APOLLO3®

In order to solve Eq. (1) with the deterministic code APOLLO3®, we apply the same iteration loops to the fission source (but the production operator is now complex) and to the scattering source as customary, and we add an iteration loop between the real and imaginary parts of the neutron noise equation to the solution of the one-group problem: details can be found in [5]. Thus, we can use the standard one-group diffusion or transport solver and consequently benefit from all numerical methods already implemented in these one-group solvers. This solution scheme has been implemented in the integro-differential transport (IDT) solver of APOLLO3® [2] *. The IDT solver is a lattice solver based on the S_n discrete ordinates method and on the method of short characteristics (MOSC). For the moment, this 2-D neutron noise solver developed in APOLLO3® can be applied to homogeneous Cartesian geometries. Future work will concern the extension to heterogeneous Cartesian geometries. For the sake of conciseness, the convergence properties of the deterministic solver, which depend on the relative weight of the contribution between the imaginary and real parts, will be discussed elsewhere.

2.3. The Monte Carlo algorithm implemented in TRIPOLI-4®

2.3.1. The general algorithm

We briefly sketch the Monte Carlo algorithm that we have chosen to implement in TRIPOLI-4®; a thorough description is given in [6,7]. Similarly as in [8,9], our method is based on the simulation of particles carrying complex statistical weights $w(\omega) = \{w_{\Re}(\omega), w_{\Im}(\omega)\}$, where the signs of the real and imaginary parts can be positive or negative. In [8], the complex cross section $\Sigma_0 + i\omega/v$ on the left-hand-side of Eq. (1) is dealt with explicitly by modifying the particle weights during flights. For our algorithm, we choose instead to work with a real cross section and we modify the collision kernel accordingly. This is achieved by adding a term $\omega/v \times \delta\Psi$ to both sides of Eq. (1), and moving the term $i\omega/v \times \delta\Psi$ to the right-hand-side. In this case, we work with a real modified total cross section $\tilde{\Sigma}_0(\mathbf{r}, E, \omega) = \Sigma_0(\mathbf{r}, E) + \Sigma_\omega(E) > 0$ where $\Sigma_\omega(E) = \omega/v > 0$. Hence, flight lengths are sampled as in standard Monte Carlo calculations, provided that $\tilde{\Sigma}_0$ is used instead of Σ_0. Correspondingly, we have two types of particle productions: regular fission with probability $\Sigma_{0,f}/\tilde{\Sigma}_0$, and a special ω-production associated to a copy operator with probability $\Sigma_\omega/\tilde{\Sigma}_0$. Implicit capture (with forced fission) and Russian roulette can be used as in standard Monte Carlo methods. Russian roulette and splitting are applied separately to the absolute value of the real and imaginary parts of the particle weight: the particle is killed only if the real and the imaginary parts are both killed [8]. The major advantage of this sampling scheme is that it introduces minimal modifications to standard Monte Carlo algorithms for particle transport and can thus be easily implemented in continuous-energy production Monte Carlo codes such as TRIPOLI-4®. Standard scores, such as

*This solution scheme has been also implemented in a 2-D nodal diffusion solver of APOLLO3® based on the classical Nodal Expansion Method, but we will not use this solver in the present paper.

flux and reaction rates over volumes and meshes, have been extended in order to decompose the complex Monte Carlo estimators into modulus and phase, or equivalently real and imaginary part. The convergence properties of the Monte Carlo solver also depend on the relative weight of the contribution between the imaginary and real parts [6], and will be more thoroughly discussed in a forthcoming paper.

2.3.2. The noise source sampling method

In the general case, the noise source S is a complex function depending on the stationary flux Ψ_0 and the sign of its real and imaginary part can thus be space and/or energy-dependent. In order to sample arbitrary noise sources by Monte Carlo methods, we implemented in TRIPOLI-4® the following method [6, 7]: a standard power iteration is first run in order to determine Ψ_0 without approximations. Convergence is achieved after a sufficient number of inactive cycles; then, each term in Eq. (3) is sampled by weighting the 'noise particles' generated by each component of S by the stationary collision density associated to Ψ_0. The strategy is as follows: a complete power iteration is performed once in the first batch: during the final cycle noise source particles are sampled so that the noise simulation can begin, while the generated fission neutrons are transferred to the second batch. This latter runs a few additional inactive cycles in order to ensure reasonable decorrelation between cycles before sampling the noise source for the second replica. The third batch gets the fission neutrons generated by the second one, and so on. In a parallel run, each processor would apply this strategy. The critical eigenvalue k that is required for the noise equations can be estimated during the first critical calculation (or can be obtained from a separate calculation).

3. COMPARISON OF APOLLO3® AND TRIPOLI-4® NEUTRON NOISE SOLVERS

Previous work has concerned the verification of the APOLLO3® and TRIPOLI-4® neutron noise solvers against reference analytical solutions in a homogeneous infinite medium and in an infinitely-long homogeneous cylinder (specifications taken from [8]) and in a one-dimensional homogeneous core surrounded by a reflector (specifications taken from [10]). In order to further illustrate the capabilities and the accuracy of the solvers, here we perform code-to-code comparisons in a simplified UOX fuel assembly, in two dimensions.

3.1. Description of the benchmark configuration

The chosen geometry is a simplified UOX fuel assembly, with reflecting boundary conditions, composed by 264 homogeneous square fuel pins [†] (0.7314×0.7314 cm) with 25 homogeneous water holes (1.26×1.26 cm). For the sake of simplicity, and in view of comparing the two solver on clean configurations where a high numerical accuracy can be achieved, all simulations (steady state and noise transport) have been performed with two energy groups (fast and thermal groups without up-scattering) and one precursors family. For the APOLLO3® transport solver, we chose an order of discrete ordinates S_{32} and a constant approximation of the volume flux with the spatial meshes, as shown in Fig. 1. For the criticality calculations, we obtain a critical k-eigenvalue equal

[†]The square configuration is required by the 2-D neutron noise solver developed in APOLLO3® being restricted to homogeneous Cartesian geometries for the time being.

to 0.99912 $\pm$ 8$\times 10^{-5}$ with TRIPOLI-4® (with 3$\times 10^{4}$ active cycles and 10^{4} neutrons per cycle) and to 0.99784 with APOLLO3® (10^{-6} tolerance on the outer iterations and no acceleration). The corresponding steady-state fluxes are shown in Fig. 2. Relative errors between APOLLO3® and TRIPOLI-4® for the steady-state fluxes are smaller than 1% on the whole assembly.

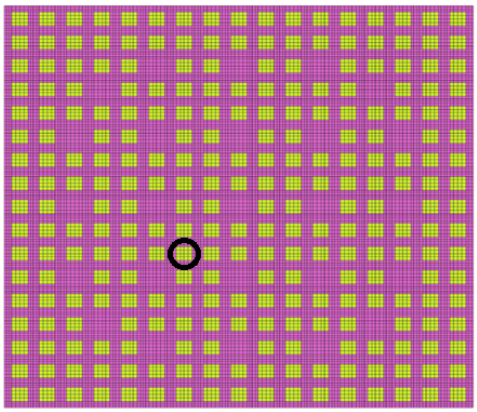
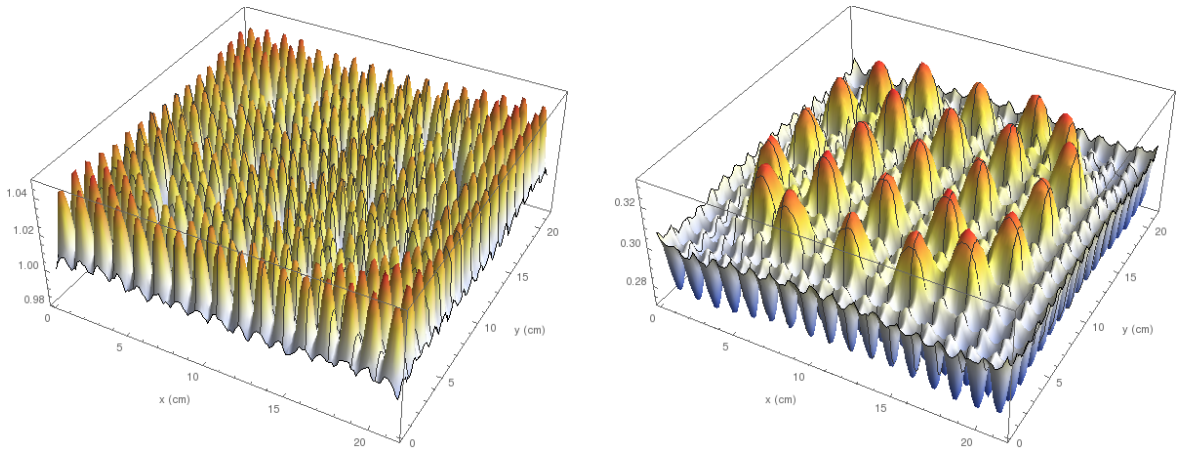


Figure 1: Simplified UOX fuel assembly: APOLLO3® 2D geometry with spatial meshes. The perturbed fuel pin is marked in black.



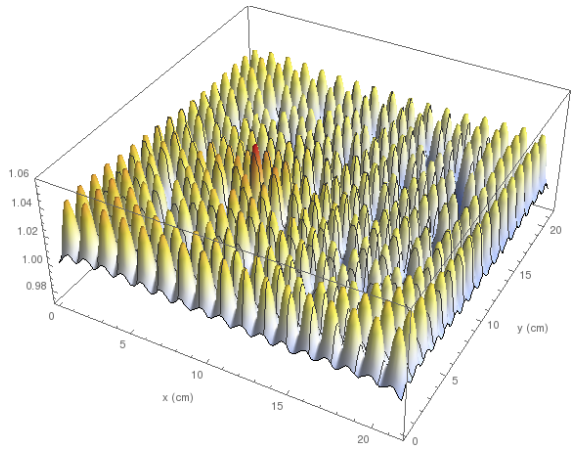
(a) Fast group (A.U.). (b) Thermal group (A.U.).

Figure 2: Steady-state flux of the reference case (scalar flux, TRIPOLI-4® results with $\sigma <$ 0.3%).

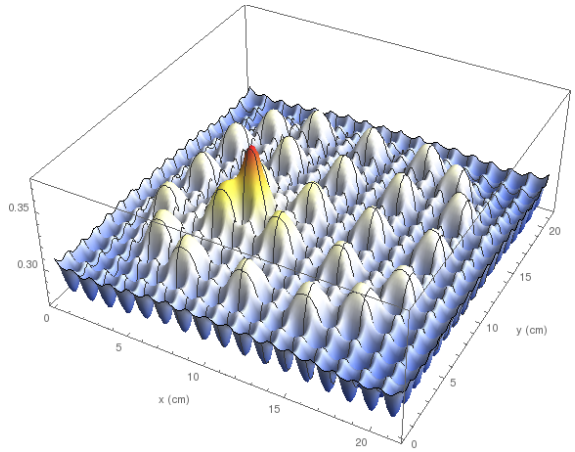
3.2. A simple noise source

As a first noise calculation for the two solvers, we impose a simple isotropic noise source $S = -1 + i$ at $\omega = 3$ Hz in a perturbed fuel pin, in the thermal group (the location of the perturbed fuel pin is detailed in Fig. 1). Simulation findings for the spatially and energy-resolved $\delta\Psi$ over the fuel assembly are shown in Fig. 3: we have converted the real and imaginary part in polar form, with modulus and phase. The phase of the neutron noise is almost constant, and is not shown. A good agreement is found between APOLLO3® and TRIPOLI-4®: the relative errors on the modulus are comparable to the relative errors on the steady-state flux, i.e. smaller than 1% over the assembly.

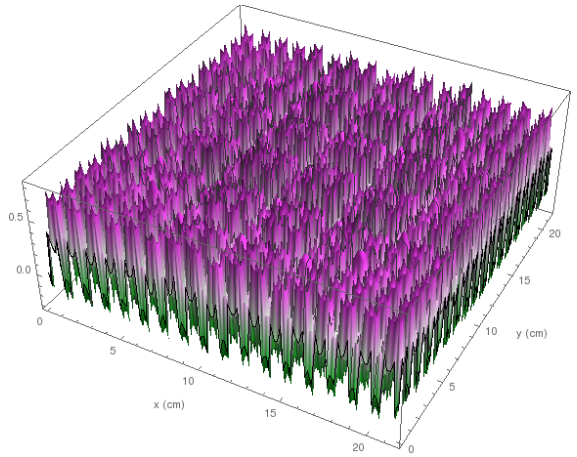
The noise calculations of TRIPOLI-4® have been performed with 3×10^3 replicas and 10^4 particles per replica.



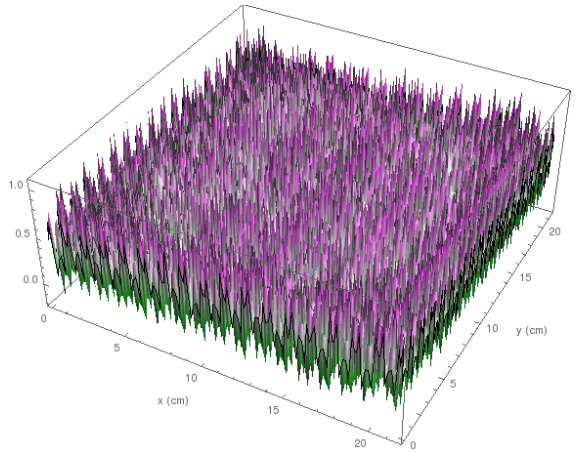
(a) Modulus of fast neutron noise (A.U., TRIPOLI-4® results with $\sigma < 0.45\%$).



(b) Modulus of thermal neutron noise (A.U., TRIPOLI-4® results with $\sigma < 0.45\%$).



(c) Relative errors on the modulus of fast neutron noise (%).



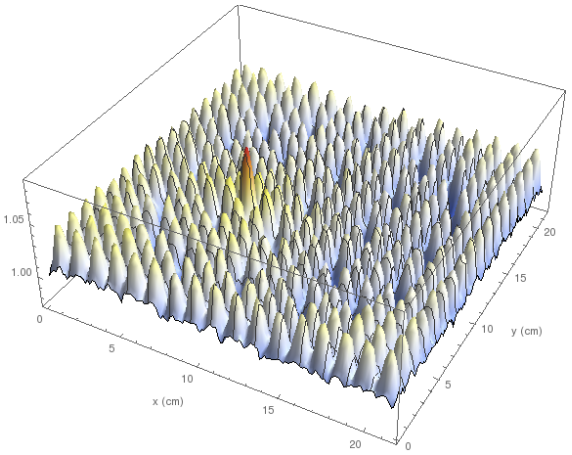
(d) Relative errors on the modulus of thermal neutron noise (%).

Figure 3: Results and relative errors (%) between TRIPOLI-4® and APOLLO3® on the modulus of $\delta\Psi(\omega)$ induced by a simple isotropic noise source at 3 Hz.

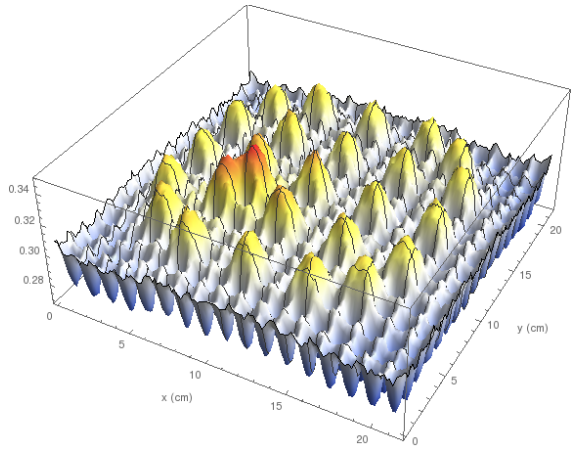
3.3. An oscillation-induced noise source

We consider next the spatially and energy-resolved modulus and phase of $\delta\Psi(\omega)$ induced by a sinusoidal *oscillation* of all macroscopic cross sections of a perturbed fuel pin at $\omega = 1$ Hz (the position of the perturbed fuel pin is detailed in Fig. 1). The perturbed term $\delta\Sigma$ is arbitrarily defined by: $\delta\Sigma(r, E, t) = \rho\Sigma_0(r, E)\cos(\omega_0 t)$ with ω_0 the angular frequency of the perturbation ($\omega_0 = 2\pi$ rad/s), $\rho = 4.1\%$ for the total cross section, $\rho = 3.4\%$ for the scattering cross section and $\rho = 2.1\%$ for the fission cross section. Contrary to the previous case, here $S(\mathbf{r}, \mathbf{\Omega}, E, \omega)$ depends on the shape of the stationary flux Ψ_0, so that we can probe the correct implementation of the noise source sampling method. Results are shown in Fig. 4 for a noise source corresponding to a critical state

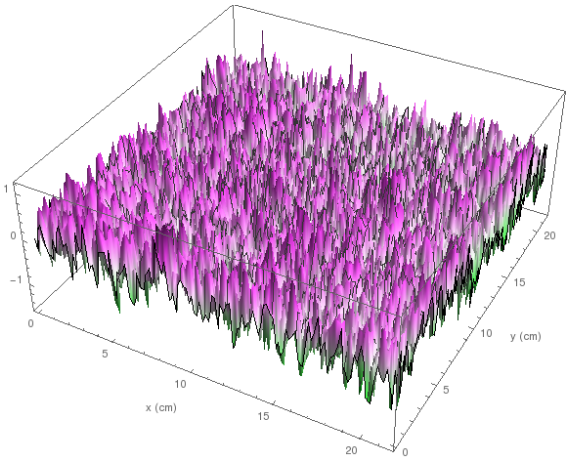
Ψ_0 computed as detailed above. The phase of the neutron noise is again almost constant, and will not be shown. Similarly as for the previous case, a good agreement is found between APOLLO3® and TRIPOLI-4® for the oscillation calculation: the relative errors on the modulus are smaller than 2%. The noise calculations of TRIPOLI-4® have been performed with 10^5 replicas and 3×10^4 particles per replica.



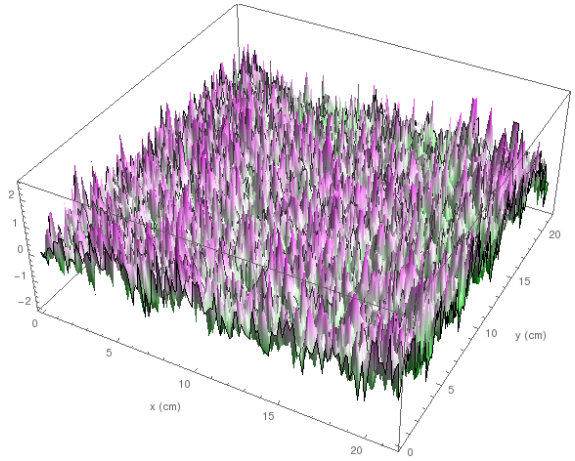
(a) Modulus of fast neutron noise (A.U., TRIPOLI-4® results with $\sigma < 4\%$).



(b) Modulus of thermal neutron noise (A.U., TRIPOLI-4® result with $\sigma < 4\%$).



(c) Relative errors on the modulus of fast neutron noise (%).



(d) Relative errors on the modulus of thermal neutron noise (%).

Figure 4: Results and relative errors (%) between TRIPOLI-4® and APOLLO3® on the modulus of $\delta\Psi(\omega)$ induced by an oscillation noise source at 1 Hz.

4. CONCLUSIONS

In this paper, we have briefly compared the deterministic APOLLO3® neutron noise solver with the new TRIPOLI-4® solver for the neutron noise analysis in the frequency domain, in view of proposing the examined configurations as benchmark cases for neutron noise calculations. Future work will concern the comparison with neutron noise simulations performed with the APOLLO3®

neutron noise solver in the time domain (currently under development and testing [11]), and the validation of the developed solvers based on the experimental campaigns carried out in the framework of the COLIBRI program at the Crocus reactor (operated by EPFL, Lausanne, Switzerland) [12].

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