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Adjoint-based Optimization for Diffractive Beam-Splitters

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Abstract: Computational optimization of diffractive elements is often limited by a large number of design parameters. Adjoint method allows the gradient of target function respect to all design variables to be calculated with only two electromagnetic simulations. We show that detail results for optimized two-dimensional fan-out gratings.

1. Introduction

Diffractive optical elements permit the generation of the spatial distribution of light beams by using a single element. They are being used for various applications because of their high design flexibility, compact size, and mass productivity [1]. At the same time, they require having complex optical functionalities such as a wide diffraction angle, which is beyond the limits of scalar paraxial diffraction domain. Thus, optical elements in the non-paraxial domain, i.e. the elements with wavelength-scale features, simulate electromagnetic properties using rigorous diffraction theory, such as rigorous coupled-wave analysis (RCWA) [2].

To design and optimize optical structures via numerical simulations, we often use the gradient of some figure of merit with respect to our design variables. With an adjoint variable method, one can calculate the gradient using only two simulations, no matter how many variables are present [3]. This capability allows for efficient, largescale, gradient-based optimization of optical structures. Therefore, we combine adjoint method and RCWA algorithm to optimize diffractive optical elements.

2. Optimization method

For the optimization, we assume the gratings consisting of pixels of relative permittivities corresponding to air ϵ_{air} and dielectric material $\epsilon_{\text{dielectric}}$. This permittivity distribution $\epsilon(\mathbf{r})$ in our grating area are design variables of the optimization. we define the Figure of Merit (FoM), often referred to as target function such as diffraction efficiency or uniformity error (UE). The UE is expressed by

$$UE = \frac{\eta_{\max} - \eta_{\min}}{\eta_{\max} + \eta_{\min}} \quad (1)$$

the difference between the maximal diffraction efficiency $\eta_{\max}$ and the weakest one $\eta_{\min}$. The gradient of FoM with respect to permittivity distribution is crucial in determining the search direction to optima points. To obtain the gradient using the adjoint method, Maxwell's equation is solved a first time for a given illumination by RCWA solver. And one has to solve the adjoint simulation which is a kind of influence function regarding the figure of merit using dipole representation and Lorentz Reciprocity. These gradients are used with the Limited-

memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) algorithm to obtain fast and stable convergence [4]. Constraints are added to the optimization to ensure that the final optimized design converges to a binary grating comprising only two relative permittivities, which are air and dielectric material.

3. Results and Discussion

We define the FoM that represents the difference between the calculated diffraction efficiencies $\eta_{m,n}$ in orders and the average of efficiencies $\bar{\eta}$ for minimizing of UE.

Fig. 1. Comparison between diffraction efficiencies of 5×5 beam-splitter with grating period of $5 \times 5 \mu\text{m}$ and depth of $1.05 \mu\text{m}$ before (a) and after optimization (b). The results are simulated by RCWA using the incident light wavelength of 940 nm and TE polarization.

To apply our optimization method, we prepared two-dimensional 5×5 beam-splitter, which is designed by the iterative Fourier transform algorithm (IFTA) which is well-established design algorithms in scalar paraxial diffraction domain [5]. This grating with binary surface profiles has a depth of $1.05 \mu\text{m}$ and grating period of $5 \times 5 \mu\text{m}$. The target diffraction pattern is a 5×5 spots matrix with equal spot intensity and spacing, and the maximum diffraction angle is 32° from the $[0, 0]^{th}$ to the $[2, 2]^{th}$ diffraction orders.

This diffraction pattern of initial design is plotted in Fig. 1(a). The initial grating has a UE of 49.43 % and UE in off-axis, i.e. excluding $[0, 0]^{th}$ order, of 36.08 %. The diffraction image of the structure resulting from our optimization process is shown in Fig. 1(b). This final structure has a UE of 13.73 % and UE in off-axis of 11.68 % at the same depth of $1.05 \mu\text{m}$. We can observe that there is a considerable improvement of uniformity over optimization. During the optimization, the theoretical efficiencies were evaluated with open-source RCWA code RETICOLO [6].

4. Conclusions

We propose a general method for optimizing diffractive optical elements using adjoint variable method. Due to the adjoint method, we can compute the gradient of the figure of merit with respect to the design parameters efficiently, independently of the number of design parameters. we show that adjoint-based optimization can be applied the inverse design of two-dimensional beam splitter and finally we find the optimized results with considerable improvement of uniformity.

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