A NEW SHARPENED CASCADED COMB-COSINE DECIMATION FILTER

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ABSTRACT

This paper presents a new multistage comb-cosine decimation filter with the improved magnitude response. The proposed structure consists of a comb section followed by different cascaded comb and cosine prefilter sections, each down-sampled by a specific down-sampling factor. The number of sections depends on the decimation factor of the original comb decimator, and the number of cascaded filters can be different for different stages. The first section is realized in a non-recursive form. Using the polyphase decomposition, the subfilters of the first section can be operated at the lower rate. The magnitude response is improved by using cosine prefilters which can also be moved to a lower rate. The sharpening technique is applied to all but the first comb section. The resulting structure is multiplier-free, does not have any filtering at the high input rate, and the magnitude response has a low passband droop and high stopband attenuation.

1. INTRODUCTION

A commonly used decimation filter is the Hogenauer [1] cascaded-integrator-comb (CIC) filter, which consists of two main sections, cascaded integrators and differentiators, separated by a down-sampler. This filter has very low complexity but exhibits two main problems:

1. The differentiator section operates at the lower data rate, while the integrator section works at the higher input data rate thereby resulting in higher chip area and higher power dissipation.
2. The magnitude characteristic of the CIC filter has a high droop in the desired passband and a low stopband attenuation.

Various methods have been advanced to solve these two issues. The use of a non-recursive structure of a comb filter reduces the power consumption and increases the circuit speed, [2-3]. More details on the comparison of the performances of the recursive and non-recursive structure are given in [3].

Several schemes have been proposed to improve magnitude characteristic of the comb decimation filters. The method outlined in [4] uses the sharpening technique to decrease the passband droop and to increase the stopband attenuation. The main drawback of this method is that it requires sharpening to be performed at the high input rate, thereby resulting in higher power consumption. Methods in [5-6] allow the sharpening section to operate at the lower rate. In order to attain the desired low stopband attenuation, the rotated sinc (RS) filter is introduced in [7]. The price paid for improving the stopband (the passband droop is not improved) is the introduction of two multipliers with the one multiplier working at a high rate. In [8] a new multistage comb-rotated sinc (RS) decimator with no filtering at the high input rate is introduced, thereby permitting both multipliers of the RS filter to work at the lower rate. The method in [9] uses sharpened comb filter cascaded with the RS filter to reduce the passband droop. The price paid for the improved magnitude response is that the operations are carried out at the high rate.

A new-comb-RS decimator with the sharpened magnitude response where both multipliers of the RS filter work at the lower rate and the sharpening is also performed at the lower rate is presented in [10], and a new multiplier-free CIC-cosine decimation filter with no filtering at the high input rate is introduced in [11].

The main idea of this paper is to propose the decimation filter which is multiplier-free, has no filtering at the high input rate and possesses low pass-band droop and high stop-band attenuation. The paper is organized as follows. In Section 2 the multistage cascaded comb-cosine decimation filter is presented and in Section 3 the application of sharpening technique is discussed. An efficient multistage structure is proposed in Section 4.

2. CASCADED COMB-COSINE FILTER

We use the result presented in [6] which introduces the cascaded modified comb filter. By considering the case when the down-sampling factor can be expressed as

$$M = M_1 M_2 M_3 \ldots M_N \qquad (1)$$

we can write the comb transfer function as

$$H(z) = \left[\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right]^K = \left[\prod_{i=1}^{N} H_i\left(z^{\prod_{j=0}^{i-1} M_j}\right)\right]^K, \qquad (2)$$

where

$$H_i\left(z^{\prod_{j=0}^{i-1} M_j}\right) = \frac{1}{M_i}\left(\frac{1-z^{-\prod_{j=1}^{i} M_j}}{1-z^{-\prod_{j=0}^{i-1} M_j}}\right);\ M_0 = 1. \qquad (3)$$

For example, for $M = 32$ and $N = 3$, we can select

$$M_1 = 4, M_2 = 4, M_3 = 2, \tag{4}$$

which yields

$$H_1(z) = \frac{1}{4}\left(\frac{1-z^{-4}}{1-z^{-1}}\right), H_2(z^4) = \frac{1}{4}\left(\frac{1-z^{-16}}{1-z^{-4}}\right),$$

$$H_3(z^{16}) = \frac{1}{2}\left(\frac{1-z^{-32}}{1-z^{-16}}\right). \tag{5}$$

Using Eqs. (1-3) we express the modified comb filter $H_m(z)$ as,

$$H_m(z) = H_1^{k_1}(z) \cdot H_2^{k_2}(z^{M_1}) \cdots H_N^{k_N}(z^{M_1 \cdots M_{N-1}}), \tag{6}$$

where k_i is the number of the cascaded filters H_i. Notice that the comb subfilters H_i, i=2,…N-1 can be moved to a lower rate, [12].

It follows from Eqs.(6) and (4-5) that,

$$H_m(z) = H_1^{k_1}(z) H_2^{k_2}(z^4) H_3^{k_3}(z^{16}). \tag{7}$$

To improve the magnitude characteristic of the filter (6) we use the cascaded cosine prefilter [11] introduced in [13],

$$H_{CCOS}(z) = \prod_{i=1}^{K} H_{COS}(z^{N_i}) \quad . \tag{8}$$

The corresponding magnitude response has a form

$$\left|H_{CCOS}(e^{j\omega})\right| = \frac{1}{2^K}\prod_{i=1}^{K}\left|\cos(N_i\omega) + \cos^2(N_i\omega)\right|. \tag{9}$$

The transfer function of the cascaded comb and cosine prefilters is

$$H_{mCCOS}(z) = H_m(z)\prod_{i=1}^{K} H_{COS}^{n_i}(z^{N_i}), \tag{10}$$

where n_i is the number of cascaded cosine prefilters, and from [11],

$$N_i = \frac{M}{2^{i+1}}.$$

$$N_K = M_1. \tag{11}$$

The decimator factor of the second stage, which usually follows the CIC filter, determines the frequency f_A at which the worst-case aliasing occurs, as well as the passband frequency f_c at which the worst –case aliasing occurs, [4].

For example, for the case of factor-of-8 second decimation the frequencies of interest normalized with respect to the high sampling rate Fs=1 are

$$f_c = \frac{\pi}{8M}\frac{1}{\pi} = \frac{1}{8M}.$$

$$f_A = \frac{2}{M} - \frac{1}{8M} = \frac{15}{8M} \quad . \tag{12}$$

Example 1:

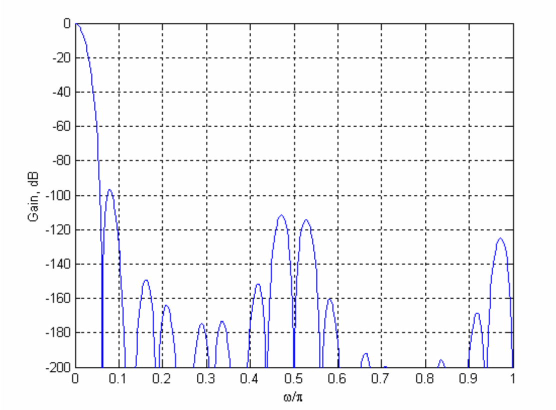
For M=32 and (11) we have

$$N_1 = 32/4 = 8; N_2 = N_K = 32/8 = 4. \tag{13}$$

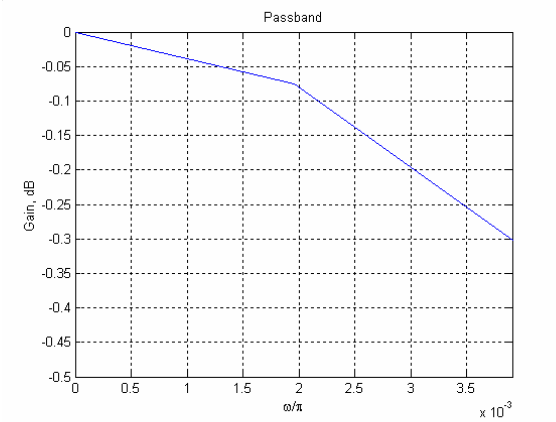
Therefore

$$H_{mCCOS}(z) = H_1^{k_1}(z) H_2^{k_2}(z^4) H_3^{k_3}(z^{16}) \times H_{COS}^{n_1}(z^8) H_{COS}^{n_2}(z^4). \tag{14}$$

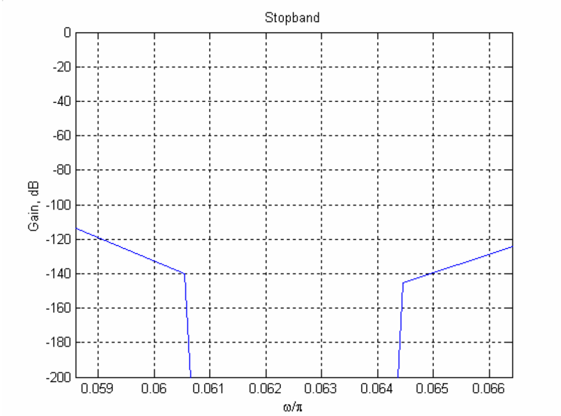
Fig. 1 shows the corresponding magnitude response for $k_1 = 4, k_2 = k_3 = 2,$ and $n_1 = 4, n_2 = 2.$ (15)



a. Overall magnitude response.



b. Passband.



c. Stopband.

Figure 1: Example 1.

3. PROPOSED DECIMATION FILTER

We now use the sharpening technique [14] to further improve the magnitude characteristic of the comb-cosine decimation filter (10). Applying the simplest sharpening method [14] given by

$$Sh\{H\} = 3H^2 - 2H^3, \tag{16}$$

to $H_i(z^{\prod_{j=0}^{i-1} M_j}), i = 2,..., N$ in (6) we arrive at the transfer function of the proposed decimation filter,

$$H_{ShCCOS}(z) = H_1^{k_1}(z)\,Sh\left\{\prod_{i=2}^{N} H_i^{k_i}\left(z^{\prod_{j=0}^{i-1} M_j}\right)\right\}\prod_{i=1}^{K} H_{COS}^{n_i}(z^{N_i}), \tag{17}$$

where Sh{ } denotes the sharpening operation. As explained in Section 4, the sharpening is not applied to H_1.

Example 2:
For M=32, choosing (4) and (13) from (17) we have

$$H_{ShCCOS}(z) = H_1^{k_1}(z)\,Sh\left\{H_2^{k_2}(z^4)\right\}Sh\left\{H_3^{k_3}(z^{16})\right\}\times H_{COS}^{n_1}(z^8)\,H_{COS}^{n_2}(z^4). \tag{18}$$

Using the values (15) we plot the magnitude characteristic of the proposed decimation filter in Fig.2. Note that the magnitude characteristic is improved in both the passband and the stopband of interest. The passband droop at fc is 0.1944 dB and the stopband attenuation at f_A is 144.5657 dB.

In the next section we consider how to obtain a more efficient structure for the decimation filter given in (17).

4. EFFICIENT STRUCTURE

The proposed decimation filter (17) and the cascade equivalence [12] are used to build an efficient structure.

We first consider an efficient structure of the filter presented in Example 2. According to (18), we can move the sharpened comb and cosine filters to a lower rate as shown in Fig.3.a.

Therefore, we have

$$H_2^{k_2}(z) = \left[\frac{1}{4}\,\frac{1-z^{-4}}{1-z^{-1}}\right]^{k_2} = \left[\frac{1}{M_2}\,\frac{1-z^{-M_2}}{1-z^{-1}}\right]^{k_2}, \tag{19}$$

and

$$H_3^{k_3}(z) = \left[\frac{1}{2}\,\frac{1-z^{-2}}{1-z^{-1}}\right]^{k_3} = \left[\frac{1}{M_3}\,\frac{1-z^{-M_3}}{1-z^{-1}}\right]^{k_3}. \tag{20}$$

The corresponding sharpened comb filters are, [4]

$$Sh\left\{H_i^{k_i}(z)\right\} = \left[H_i(z)\right]^{2k_i}\left\{3z^{-(M_i-1)k_i/2} - 2\left[H_i(z)\right]^{k_i}\right\}. \tag{21}$$

Cosine filter

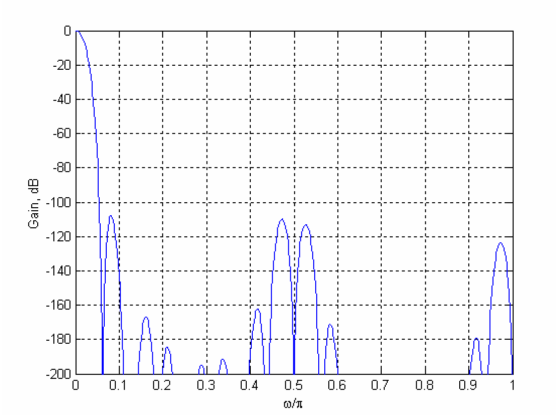
$$H_{COS}^{n_1}(z^8) = \left[0.125(1+z^{-16})(1+z^{-8})\right]^{n_1} \tag{22}$$

is moved to the second stage to become $0.125^{n_1}(1+z^{-2})^{n_1}$, and to the third stage to become $(1+z^{-1})^{n_1}$.

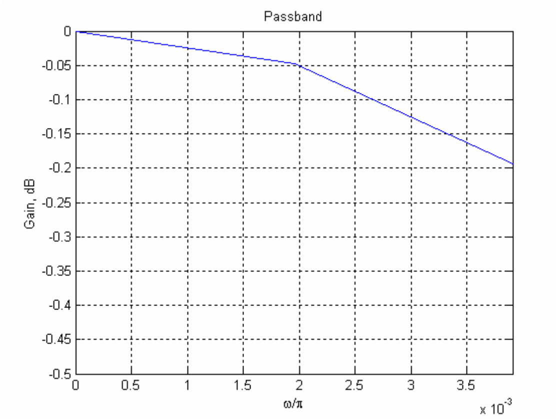
Similarly, cosine filter

$$H_{COS}^{n_2}(z^4) = \left[0.125(1+z^{-8})(1+z^{-4})\right]^{n_2} \tag{23}$$

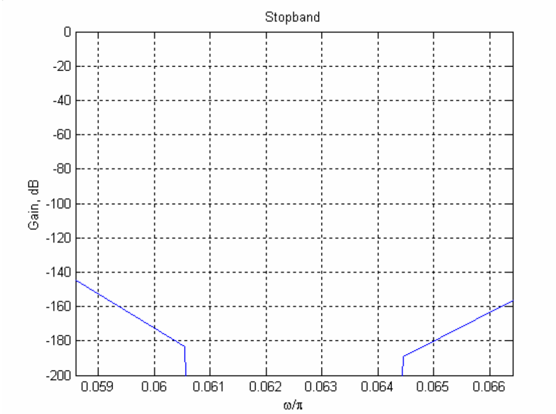
can only be moved to the second stage to become $\left[0.125(1+z^{-2})(1+z^{-1})\right]^{n_2}$.



a. Overall magnitude response.



b. Passbnd zoom.



c. Stopband zoom.

Figure 2: Example 2.

We denote cosine filters of the second and the third stage as (Fig.3.b)

$$H_{2,COS}(z) = (1+z^{-2})^{n_1+n_2}(1+z^{-1})^{n_2}. \tag{24}$$

and

$$H_{3,COS}(z) = (1+z^{-1})^{n_1}. \tag{25}$$

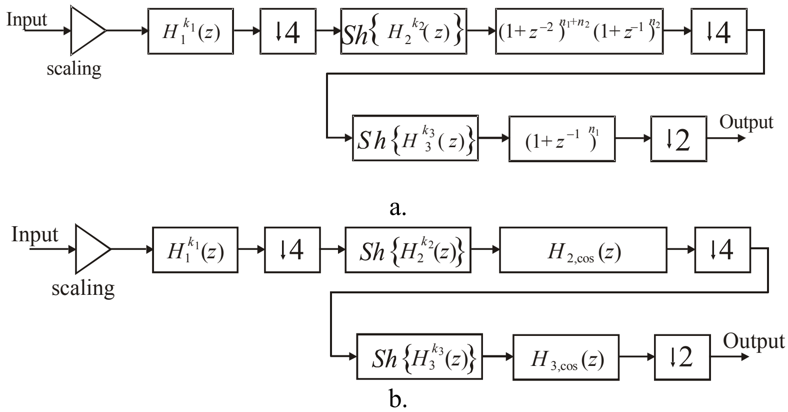


Figure 3: Efficient structure for Example 2.

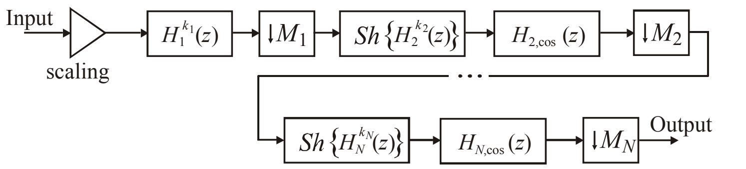


Figure 4: General structure.

Using a similar approach we can obtain the general structure shown in Fig.4. The first section is the cascade of k_1 comb filters of length M_1. The polyphase decomposition allows the polyphase filters to move to a lower rate, which is M_1 times lower than the input rate, as explained in [3].

Sharpened comb filters at all other sections can be realized as proposed in [4].

5. CONCLUSION

A new computationally efficient structure for a multistage comb decimation filter is proposed. The sharpened technique and the cosine prefilters are used to improve the magnitude characteristic of the filter.

As a result, the proposed filter has a low passband droop and high stopband attenuation at the frequencies of interest. Using different values for k_i, n_i and M_i we can manipulate the corresponding magnitude response.

The proposed structure is multiplier-free structure and has no filtering at the high input rate. The tradeoff lies in the slightly increased number of less complex comb filters in the first stage. Further research would be dedicated to the problem of systematically choosing the values of the parameters.

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