Exploiting Spatial-temporal Joint Sparsity for Underwater Acoustic Multiple-input-multiple-output Communications

Yuehai Zhou, Feng Tong, Aijun Song, and Roe Diamant

Abstract

Multiple-input-multiple-output (MIMO) system offers a promising way for high data rate communication over bandwidth limited underwater acoustic channels. However, MIMO communication not only suffers from inter-symbol interference, but also introduces the additional co-channel interference, which brings challenge for underwater acoustic MIMO channel estimation and for channel equalization. In this paper, we propose novel interference cancellation (IC) methods for handling this co-channel interference problem in the design of both channel estimation and channel equalization. Our method for channel estimation utilizes the spatial joint sparsity and the temporal joint sparsity in the multipath structure to estimate sparse channels with common delays under distributed compressed sensing (DCS) framework. In this way, we enhance channel estimates with common delays, thus suppress co-channel interference. Meanwhile, to address the case of multipath arrivals with different delays, which are estimated as noise under simultaneous orthogonal matching pursuit (SOMP) algorithm, we introduce forward-reverse strategy to SOMP algorithm, which is referred to as FRSOMP algorithm. Our proposed FRSOMP algorithm performs SOMP algorithm to achieve the initial channel estimates, performs forward-add process which attempts to add promising candidates into support-sets, and performs reverse-fetch process to check if the candidates in the support-set are retained or removed. The purpose of channel estimation is to directly calculate the filter coefficients for channel estimation based decision feedback equalization (CE-DFE). In this paper, we also propose a novel CE-DFE receiver with IC component. We design IC filters based on the traditional CE-DFE, and we derive the coefficients of the feedforward filters, feedback filters and IC filters based on the channel estimate metric obtained by FRSOMP algorithm, so the co-channel interference will be suppressed both in channel estimation and channel equalization. We demonstrate the performance of our approach by numerical simulation, lake experiment, and sea experiment. Results are provided to demonstrate the effectiveness of the proposed methods, results show that the proposed methods obtain higher output signal-to-noise ratio (SNR), lower bit error rate (BER), and more separated constellations compared with traditional compressed sensing channel estimation method and traditional CE-DFE method.

Index Terms

Distributed compressed sensing, forward-reverse strategy, channel estimation based decision feedback equalization, interference cancellation, underwater acoustic MIMO communication, channel estimation and channel equalization.

I. INTRODUCTION

Multiple-input-multiple-output (MIMO) system has the potential to improve the channel capacity significantly in limited bandwidth underwater acoustic communications. Specifically, since the channel spatial diversity is high, the capacity increase for even small MIMO configuration is significant. Compared with wireless channel, underwater acoustic channel is much more challenging for MIMO technology. Typically, underwater acoustic channel is characterized by long time delay spread which causes inter-symbol interference (ISI), and by time-variations. Moreover, multiple transmitters tremendously

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improve the data rate, but they lead to severe co-channel interference (CoI), which introduces large noise in the per-channel demodulation process. All of these pose great difficulties for underwater acoustic MIMO channel estimation and channel equalization.

In this paper, we suggest to combat CoI by utilizing the temporal and spatial joint sparsity of the underwater acoustic MIMO channels. Because of the sea surface and sea bottom reflection, usually the underwater acoustic channel only has a couple of significant taps over a large delay spread, i.e., the underwater acoustic channel is typically sparse. This sparsity can be exploited by employing compressed sensing (CS) method to improve underwater acoustic channel estimation performance [1]–[11]. However, the performance of the traditional CS channel estimation algorithms such as matching pursuit (MP) and orthogonal matching pursuit (OMP) in the existing literatures listed above, will be decreased for underwater acoustic MIMO communication, because of the strong CoI.

In contrast to conventional CS theory that reconstructs single sparse signal alone, distributed compressed sensing (DCS) method exploits the sparsity correlation among multiple observed signals to enhance sparse data recovery [12]. DCS based channel estimation has been successfully used in radio wireless communication [13]–[16]. For example, in [16], DCS-based channel estimation was adopted in MIMO-OFDM systems. Simulation results showed that DCS-based MIMO-OFDM could improve both bandwidth efficiency and bit error rate (BER) performance. In [14], multi-user massive MIMO systems exhibited a joint sparsity structure in the user channel matrices, a joint orthogonal matching pursuit recovery algorithm was proposed to perform the channel-state-information-at-transmitter estimation.

Limited investigations of the DCS technique have been reported for underwater acoustic channel estimation. In [2], DCS based channel estimation algorithm was utilized to suppress CoI. Adjacent data blocks in time domain were combined to simultaneously recover underwater acoustic channels using simultaneous orthogonal matching pursuit (SOMP) algorithm. In [3], DCS was proposed to reduce the number of pilot subcarriers in underwater acoustic OFDM channels. Two adjacent data blocks in the frequency domain were combined to obtain joint sparsity channel estimation using SOMP algorithm. In [4], DCS was used for underwater acoustic multiband transmissions. The traditional SOMP algorithm was used to enhance channel estimates that have common delays, and the proposed multiple selection strategy based SOMP algorithm was utilized to reconstruct channel taps that have different delays. So far, the DCS method has not been extensively investigated in underwater acoustic MIMO channel estimation.

In MIMO communication systems, since the transmitting and receiving antennas are collocated, the propagation delay is roughly the same for all transmit-receive antenna pairs. As a result, the locations of the significant arrivals in all channels are nearly identical [16]. This identity can be utilized through a DCS channel estimation scheme. For a typical underwater MIMO communication system, since the communication range is far larger than the receiver aperture, the channels from different transmitters to a receiver may exhibit correlation, namely, some channel taps have same channel delays, but with different coefficients. Furthermore, if underwater acoustic channels are time invariant or slow time-varying, channels from adjacent data blocks also show strong correlation. DCS based channel estimation method is able to utilize such correlation to enhance underwater acoustic MIMO channel estimation performance. In other words, channel estimates that have common delays are enhanced which result in suppressing the CoI. SOMP algorithm was directly utilized to solve DCS problems [2], [3], [13]–[16]. However, channels that come from different transmitters to a receiver may still have different arrival structures, the different arrival structures will be estimated as noise when SOMP algorithm is used. In this paper, we use DCS method to estimate underwater acoustic MIMO channel, it has the potential to suppress the CoI, we not only consider to enhance the channel estimates that have common delays, but also consider to reconstruct the channel estimates that have different delays, so that the performance of underwater acoustic MIMO channel estimation will be improved. Different with in [2] and [3] which only used SOMP algorithm to temporal joint sparsity, and different with in [4] which used SOMP and multiple selection strategy based SOMP algorithm to temporal and sub-band joint sparsity in multiband communication system, in this paper, the proposed channel estimation method explores both temporal and spatial joint sparsity. Channel taps that have different delays in [2] and [3] are treated as noise, while in [4], multiple selection strategy

SOMP is proposed to address them, in this paper, FRSOMP algorithm is proposed to deal with the channel taps that have different delays.

Equalization including time domain equalization and frequency domain equalization is intensively investigated to eliminate the ISI and frequency-selective distortion. Frequency domain channel equalization usually has lower computational complexity than time domain equalization, thus, frequency domain orthogonal frequency division multiplexing (OFDM) approach can be adopted for MIMO underwater acoustic communication purposes [8], [17], [18]. However, because of the high peak average power ratio, OFDM systems are generally not preferable from an amplifier efficiency point of view. Moreover, the OFDM systems are very sensitive to Doppler shift and phase noise.

For single carrier underwater acoustic communications, the common channel equalization approaches in time domain are the direct adaptive decision feedback equalization (DA-DFE) [5], [19]–[23] and the channel estimation based decision feedback equalization (CE-DFE) [6], [24]–[27]. The DA-DFE approach uses adaptive algorithm (such as recursive least square algorithm) to equalize received symbols. The merit of DA-DFE is not to estimate the channel explicitly, but DA-DFE approach requires long training sequences to achieve convergence. In contrast, the CE-DFE approach directly measures the channel via shorter training sequences and calculates the channel equalizer's filters based on channel estimates. DA-DFE was intensively used to equalize underwater acoustic SIMO or MIMO channels. In [5], [19]–[21], a DA-DFE was followed by time reversal operation to further eliminate the ISI for underwater acoustic SIMO communication. In [22] and [23], DA-DFE was utilized for underwater MIMO equalization. In [22], two stage time reversal based DA-DFE was utilized. At first stage, time reversal based DA-DFE was applied to get initial demodulation results; at second stage, time reversal based DA-DFE was applied again after suppressing parallel interference cancellation (IC). In [23], DA-DFE combined with IC was applied to underwater acoustic MIMO communications. The CE-DFE also has been used in underwater acoustic SIMO communication, in [6], [24]–[27], CE-DFE was utilized for underwater acoustic SIMO equalization, the feedforward filters and feedback filter were derived based on channel estimates. The traditional CE-DFE in [6], [24]–[27] is limited for underwater acoustic MIMO communication because of the serious CoI. So far, limited lectures investigated CE-DFE with IC in underwater acoustic MIMO communications.

Another type equalization technology is turbo equalization. In [11], [23], [28], [29], turbo equalization method is used for underwater MIMO communication with soft IC. In [23], an iterative MIMO decision feedback equalizer with successive interference cancellation for both space-time trellis codes and layered space-time codes was proposed in turbo system. In [11], an enhanced linear minimum mean square error turbo equalization scheme for a spatially multiplexed MIMO system with frequency-selective fading was introduced. In [28], turbo block decision feedback equalization, where the soft-decision equalizer performed successive soft interference cancellation of both ISI and CoI was proposed for high data rate single carrier MIMO underwater acoustic communications. In [29], a low-complexity frequency-domain turbo equalizer combined with phase rotation compensation and soft successive interference cancellation was adopted for single carrier MIMO underwater acoustic communication. Different with classic channel equalizer, the turbo equalizer needs to iteratively perform equalization and decoding, which unfortunately costs significant computational complexity and decreases the raw data rate. Moreover, the structure of the turbo equalizer is complicated.

Because DCS method needs less observation data to recover signals, it is successfully used in wireless network [13]–[16], but it has not been widely used in underwater acoustic channel estimation. In previous research, DCS is used to improve the channel estimation performance based on the JSM2, the taps that have common delays are improved. However, the different delays are not considered which will degrade the channel estimation performance. Moreover, the traditional channel estimation methods such as least square and OMP algorithms suffer from strong CoI when they are used in underwater acoustic MIMO communication systems. Traditional CE-DFE requires less training sequences to achieve the coefficients of feedforward filters and feedback filter, and the traditional CE-DFE needs less controlling parameters compared with DA-DFE, it is convenient for underwater acoustic SIMO communication [6], [24]–[27], but

the traditional CE-DFE is limited for underwater acoustic MIMO communication because of the critical CoI.

In this paper, our solution heavily relies on the concept of interference cancellation both in underwater acoustic MIMO channel estimation and channel equalization. We use the concept of DCS to enhance the accuracy of the channel estimation for underwater acoustic MIMO systems. In this process, we utilize both the spatial joint sparsity and the temporal joint sparsity to reduce CoI for channel taps with common delays. In order to address the channel taps with different delays, we design a novel method that combines forward-reverse strategy with SOMP algorithm, thus, the different arrivals have potential to be reconstructed correctly. The derived algorithm is referred to as forward-reverse SOMP (FRSOMP). To further suppress the CoI in the channel equalizer, we propose a CE-DFE receiver which contains IC component for MIMO channel equalization. The coefficients of feedforward filters, feedback filters, and IC filters are calculated based on channel estimates. Our proposed CE-DFE with IC needs less training symbols to achieve convergence, and directly eliminates the CoI using IC filters. Finally, numerical simulation, lake experiment, and sea experiment demonstrate the effectiveness of the proposed methods.

The contribution of this work is twofold:

- 1) We propose FRSOMP strategy to SOMP channel estimation algorithm, so that, channel estimates with common delays have the capability to suppress the CoI, and channel estimates with different delays are reconstructed correctly. While in tradition SOMP channel estimation, the channel estimates with different delays are treated as noise. Thus, the performance of MIMO channel estimation is improved.
- 2) We propose channel estimation based decision feedback equalization with IC component for underwater acoustic MIMO equalization, thus the CoI is suppressed further. We derive the coefficients for feedforward filters, feedback filters, and IC filters based on channel estimates. So, the performance of channel equalization is improved.

The remaining of this paper is organized as follows. Section II describes the channel model for underwater acoustic MIMO system. Section III outlines DCS based channel estimation and our proposed FRSOMP channel estimation for underwater acoustic MIMO communications. Section IV derives our proposed CE-DFE receiver that contains IC component. Numerical simulation results, lake and sea experimental results are described and analyzed in section V. Finally, some concluding remarks are drawn in section VI.

The following notations are used in this paper. Bold upper case and lower case letters denote matrices and column vectors, respectively. Superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^\dagger$ denote the conjugation, transpose, Hermitian transpose, and pseudo inversion operation, respectively. Notations $\mathbf{0}_{P \times L}$ and $\mathbf{I}$ denote zero matrix with P rows and L columns, and the identity matrix, respectively. Notations $\|\cdot\|_0$ and $\|\cdot\|_2$ denote l_0 norm and l_2 norm. Matrix $\mathbf{A}[\mathbf{j}]$ denotes a sub-matrix obtained from $\mathbf{j}$ -th columns of $\mathbf{A}$. Notation $\mathbf{a} \odot \mathbf{b}$ denotes the dot product of $\mathbf{a}$ and $\mathbf{b}$. Notation $\langle \mathbf{a}, \mathbf{b} \rangle$ denotes inner product of $\mathbf{a}$ and $\mathbf{b}$. Notation $\mathbb{E}(\cdot)$ denotes the expectation operation. Notations $\cup$ and $\emptyset$ denote the union operation and the empty set.

II. CHANNEL MODEL FOR UNDERWATER ACOUSTIC MIMO SYSTEM

We consider a multiple-input-multiple-output acoustic communication system that has number of M transmitters and number of N receivers, the equation of discrete baseband receiver at the n -th hydrophone k -th data block can be written as:

$$y_n(k, i) = \sum_{m=1}^M \sum_{l=0}^{L-1} x_m(k, i-l) h_{m,n}(k, l) + w_n(k, i), \quad (1)$$

where $y_n(k, i)$ and $w_n(k, i)$ are the received signal and ambient noise at n -th hydrophone k -th data block respectively. Notation $x_m(k, i)$ is the transmitted symbols from m -th transmitter k -th data block. Notation $h_{m,n}(k, i)$ is the channel impulse response from m -th transmitter to n -th receiver at k -th data block which

is defined by

$$h_{m,n}(k, \tau) = \sum_{l=0}^{L-1} \alpha_{m,n}(k, l) \delta(\tau - \tau_{m,n}(k, l)), \quad (2)$$

where $\alpha_{m,n}(k, l)$ and $\tau_{m,n}(k, l)$ are denoted as amplitude and delay for l -th path respectively. Notation L and i are the length of channel impulse response and time index for observation time respectively. It is assumed that the acoustic communication channel is time-invariant within a data block (noted as P samples), equation (1) can be expressed as:

$$\mathbf{y}_n(k) = \sum_{m=1}^M \mathbf{A}_m(k) \mathbf{h}_{m,n}(k) + \mathbf{w}_n(k), \quad (3)$$

where $\mathbf{A}_m(k)$ is a Toeplitz type matrix defined as:

$$\mathbf{A}_m(k) = \begin{pmatrix} x_m(k, L-1) & x_m(k, L-2) & \cdots & x_m(k, 0) \\ x_m(k, L) & x_m(k, L-1) & \cdots & x_m(k, 1) \\ \vdots & \vdots & \ddots & \vdots \\ x_m(k, L+P-2) & x_m(k, L+P-3) & \cdots & x_m(k, P-1) \end{pmatrix}. \quad (4)$$

Notations $\mathbf{y}_n(k)$, $\mathbf{h}_{m,n}(k)$, and $\mathbf{w}_n(k)$ are defined as:

$$\mathbf{y}_n(k) = (y_n(k, L-1) \ y_n(k, L) \ \cdots \ y_n(k, L+P-2))^T, \quad (5)$$

$$\mathbf{h}_{m,n}(k) = (h_{m,n}(k, 0) \ h_{m,n}(k, 1) \ \cdots \ h_{m,n}(k, L-1))^T, \quad (6)$$

$$\mathbf{w}_n(k) = (w_n(k, L-1) \ w_n(k, L) \ \cdots \ w_n(k, L+P-2))^T. \quad (7)$$

The MIMO channel impulse response $\mathbf{h}_{m,n}(k)$ can be estimated by least square algorithm based on (3). Underwater acoustic channel typically exhibits sparse, and the Toeplitz type matrix $\mathbf{A}_m(k)$ satisfies restricted isometry property [30], so the compressed sensing method can be directly utilized to estimate the underwater acoustic sparse channel. Because of strong CoI, the performance of the traditional CS based channel estimation algorithm for underwater acoustic MIMO channels will be decreased.

III. DISTRIBUTED COMPRESSED SENSING FOR UNDERWATER ACOUSTIC MIMO CHANNEL ESTIMATION

In this section, we introduce the SOMP based distributed compressed sensing, and then derive our proposed FRSOMP algorithm.

A. Distributed compressed sensing

For underwater acoustic MIMO systems, if the channels from different transmitters to a receiver and channels from adjacent data blocks exhibit correlation, they can be modelled by joint sparsity model 2 (JSM2) in DCS theory [12]. In JSM2, all the channels share common support-set, namely, all the channels have the same tap delays, but different coefficients. Figure 1(a) shows an example of joint sparsity, the two channel impulse responses share a common support-set, but the taps are different. The SOMP algorithm can directly solve the JSM2 problem [2], [3], [13]–[16].

The way that utilizes multiple data blocks to find the same channel delays is referred to as joint sparsity estimation. Figure 1(b) illustrates spatial joint sparsity and temporal joint sparsity channel estimation. The sparse channel delays can be estimated by multiple data blocks that come from different transmitters and come from adjacent data blocks shown in Fig. 1(b), the channel coefficients are measured individually

Fig. 1. The diagram of joint sparsity model 2 and diagram of spatial joint sparsity and temporal joint sparsity channel estimation. (a) The diagram of joint sparsity model 2. The two channels share a common support-set, but the taps are different. (b) Diagram of spatial joint sparsity and temporal joint sparsity channel estimation. The figure shows the diagram of received signals and channels. The x-axis is the index of data blocks and its corresponding channels, the y-axis is the data blocks and their corresponding channels from m -th ($m = 1, \dots, M$) transmitter to n -th receiver. The received signal $\mathbf{y}_n(q)$ consists of signals from different transmitters, e.g. $\mathbf{y}_n(q) = \sum_{m=1}^M \mathbf{y}_{m,n}(q)$. In this paper, we use the signals from different transmitters and from adjacent data blocks to simultaneously recover the underwater acoustic MIMO channels.

216 based on the channel delays. This is the core principle of SOMP algorithm. If channel delays are estimated
 217 by the data blocks that come from spatial domain, which marked with dotted rectangle in Fig. 1(b), we
 218 denote it as spatial joint sparsity simultaneous orthogonal matching pursuit (S-SOMP) channel estimation.
 219 If channel delays are estimated by both spatial domain and temporal domain, which marked with dotted
 220 rectangle and dotted ellipse, we denote it as spatial joint sparsity and temporal joint sparsity simultaneous
 221 orthogonal matching pursuit (ST-SOMP) channel estimation.

222 Consider multiple received data blocks, $\mathbf{y}_n(1), \dots, \mathbf{y}_n(Q)$, and their corresponding measurement matrices,
 223 $\mathbf{A}_1(1), \dots, \mathbf{A}_1(Q), \dots, \mathbf{A}_M(1), \dots, \mathbf{A}_M(Q)$, the purpose of DCS method is to reconstruct number
 224 of MQ channel impulse responses simultaneously. Based on the DCS theory, we establish the following
 225 model [2], [16]:

$$\begin{pmatrix} \mathbf{y}_{1,n}(1) \\ \vdots \\ \mathbf{y}_{1,n}(Q) \\ \vdots \\ \mathbf{y}_{M,n}(1) \\ \vdots \\ \mathbf{y}_{M,n}(Q) \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1(1) & \cdots & \mathbf{0}_{P \times L} & \cdots & \mathbf{0}_{P \times L} & \cdots & \mathbf{0}_{P \times L} \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \mathbf{0}_{P \times L} & \cdots & \mathbf{A}_1(Q) & \cdots & \mathbf{0}_{P \times L} & \cdots & \mathbf{0}_{P \times L} \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \mathbf{0}_{P \times L} & \cdots & \mathbf{0}_{P \times L} & \cdots & \mathbf{A}_M(1) & \cdots & \mathbf{0}_{P \times L} \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \mathbf{0}_{P \times L} & \cdots & \mathbf{0}_{P \times L} & \cdots & \mathbf{0}_{P \times L} & \cdots & \mathbf{A}_M(Q) \end{pmatrix} \begin{pmatrix} \mathbf{h}_{1,n}(1) \\ \vdots \\ \mathbf{h}_{1,n}(Q) \\ \vdots \\ \mathbf{h}_{M,n}(1) \\ \vdots \\ \mathbf{h}_{M,n}(Q) \end{pmatrix} + \begin{pmatrix} \boldsymbol{\omega}_{1,n}(1) \\ \vdots \\ \boldsymbol{\omega}_{1,n}(Q) \\ \vdots \\ \boldsymbol{\omega}_{M,n}(1) \\ \vdots \\ \boldsymbol{\omega}_{M,n}(Q) \end{pmatrix}, \quad (8a)$$

$$\begin{pmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_k \\ \vdots \\ \mathbf{y}_{MQ} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1 & \cdots & \mathbf{0}_{P \times L} & \cdots & \mathbf{0}_{P \times L} \\ \ddots & \ddots & \ddots & \ddots & \ddots \\ \mathbf{0}_{P \times L} & \cdots & \mathbf{A}_k & \cdots & \mathbf{0}_{P \times L} \\ \ddots & \ddots & \ddots & \ddots & \ddots \\ \mathbf{0}_{P \times L} & \cdots & \mathbf{0}_{P \times L} & \cdots & \mathbf{A}_{MQ} \end{pmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_k \\ \vdots \\ \mathbf{h}_{MQ} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\omega}_1 \\ \vdots \\ \boldsymbol{\omega}_k \\ \vdots \\ \boldsymbol{\omega}_{MQ} \end{pmatrix}, \quad (8b)$$

$$\tilde{\mathbf{y}} = \tilde{\mathbf{A}}\tilde{\mathbf{h}} + \tilde{\omega}. \quad (8c)$$

In (8a), notation $\mathbf{y}_{m,n}$ is the received signal from m -th transmitter to n -th receiver, similarly, $\omega_{m,n}$ is the noise from m -th transmitter to n -th receiver. To simplify the equation, we eliminate the receiving hydrophone index n , and we adopt one subscript instead of two subscripts in notations. Equation (8b) is the mapping of (8a). In fact, $\mathbf{y}_{m,n}$ and $\omega_{m,n}$ can not be obtained directly, they are replaced by $\mathbf{y}_n$ and ω_n respectively for all $m = 1, \dots, M$. In (8a), a comprehensive temporal joint sparsity and spatial joint sparsity with a dimension of MQ are considered. The advantage of (8a) is that multiple data blocks can be used to improve detection for the common tap delays, while in the OMP algorithm, only one data block is used. So the DCS has more potential to improve the underwater acoustic MIMO channel estimation performance. Based on the DCS theory, underwater acoustic MIMO channel estimation can be formulated as following optimization problem [4], [31], [32]:

$$\min \|\tilde{\mathbf{y}} - \tilde{\mathbf{A}}\tilde{\mathbf{h}}\|_2^2 \quad s.t. \quad \|\mathbf{h}_1 \odot, \dots, \odot \mathbf{h}_m \odot, \dots, \odot \mathbf{h}_{MQ}\|_0 = S. \quad (9)$$

The constraint condition can be described that, all the channels contain S multipath arrivals with common delays, while the sparsity of each channel may be different. The SOMP algorithm can be directly utilized to solve (9). The key idea of the SOMP algorithm is shown in (10), the candidate is measured via multiple data blocks, while in the traditional OMP algorithm, the candidate is measured by only one data block. Because of the strong CoI in MIMO channel estimation, the correlation between received signal $\mathbf{y}_k$ and its corresponding measurement matrix $\mathbf{A}_k$ will be degraded, thus the capability of searching candidates will decrease. However, if channel taps have the same delays, the term $\sum_{k=1}^{MQ} |\langle \mathbf{A}_k[l], \mathbf{u}_k^s \rangle|$ will have larger amplitude, thus the CoI is suppressed, so that the tap delays are easy and correct to be selected. If the number of data block $k = 1$, the SOMP algorithm will reduce to traditional OMP algorithm. The SOMP algorithm is described in Algorithm 1 [32].

$$\lambda = \arg \max_{l=1, \dots, L} \sum_{k=1}^{MQ} |\langle \mathbf{A}_k[l], \mathbf{u}_k^s \rangle|. \quad (10)$$

Algorithm 1 The SOMP channel estimation algorithm

Input: $\mathbf{y}_k, \mathbf{A}_k, k = 1, \dots, MQ$. The sparsity S .

- 1: **Initialization:** $\Gamma_{SOMP} = \emptyset, \mathbf{u}_k^0 = \mathbf{y}_k, k = 1, \dots, MQ$.
- 2: **for** $s = 1 : S$ **do**
- 3: Select the candidate λ in (10).
- 4: Add the candidate to the support-set $\Gamma_{SOMP} = \Gamma_{SOMP} \cup \lambda$.
- 5: Measure the individual channel coefficients $\mathbf{h}_k$.
- 6: Measure the individual residual $\mathbf{u}_k^s$.
- end for**

return $\tilde{\mathbf{h}}_k[\Gamma_{SOMP}] = \mathbf{h}_k$.

B. Forward-reverse based simultaneous orthogonal matching pursuit algorithm

We introduce forward-reverse based simultaneous orthogonal matching pursuit (FRSOMP) in this subsection. It is assumed that all the channel taps have the same delays in JSM2. In fact, channel taps that come from different transmitters to a receiver may contain different delays. Under JSM2, the different delays will be estimated as noise, which leads to worse channel equalization performance. In [33], a new mixed support-set signal model (MSSM) with a shared structure where the signal vector consists two

parts was introduced. The channels contain two parts:

$$\mathbf{h}_k = \mathbf{h}_k^{(c)} + \mathbf{h}_k^{(d)}. \quad (11)$$

In (11), the superscripts (c) and (d) denote the common part and the different part. Under JSM2, the common part will be enhanced by SOMP channel estimation algorithm, so it has the potential to suppress the CoI. On the other hand, the different part will be estimated as noise. Previous work [2], [3], [13]–[16] was investigated based on JSM2, in this paper, the MSSM is considered, so that, the different part will be reconstructed correctly.

In traditional OMP and SOMP algorithms, the candidates are carefully selected index-by-index; on the other hand, the selected candidates in support-set will remain forever, which means that if the candidates are not correctly chosen, the error will propagate to next iteration. In this paper, we propose forward-reverse based simultaneous orthogonal matching pursuit algorithm for underwater acoustic MIMO channel estimation. The forward-reverse strategy includes forward-add and reverse-fetch process. The serial add strategy of including a potential candidate into the support-sets is referred as forward-add. After the forward-add strategy is performed, it is natural to include a reliability testing strategy. The fetch-reverse is a testing scheme where the s most prominent candidates are chosen from $(s+1)$ candidates in support-sets. Since the initial channel delays are determined by SOMP algorithm, the channel taps that have common delays are enhanced. By reverse-fetch testing scheme, the incorrect different delays are removed, and the potential different delays are added to the support-set and correctly reconstructed. The FRSOMP algorithm has the following steps:

- 1) The SOMP algorithm is performed to achieve the initial channel estimates $\tilde{\mathbf{h}}_k$ by Algorithm 1, thus, estimation for common delays are enhanced which can suppress the CoI. Meanwhile, the different delays are estimated as noise. We use forward-reverse strategy to remove the erroneous tap delays and introduce correct tap delays in the following steps.
- 2) Generate the corresponding residual vectors for the reliability testing. The initial support-sets $\tilde{\Gamma}_k$ and the initial residual $\mathbf{u}_k^s$ are measured based on the initial channel estimates shown in (12). The superscript s is the iteration index, where $s = 1, \dots, S$, and the subscript k is the data block index, where $k = 1, \dots, MQ$. In this step, number of $S \times MQ$ support-sets and number of $S \times MQ$ residual are stored ($s = 1, \dots, S$). The purpose of the residual is to provide the initial residual which is used to compare with in step 4.

$$\tilde{\Gamma}_k = \arg \max_{|\tilde{\Gamma}_k|=s} |\tilde{\mathbf{h}}_k|, \quad (12a)$$

$$\mathbf{u}_k^s = \mathbf{y}_k - \mathbf{A}_k[\tilde{\Gamma}_k] \mathbf{A}_k^\dagger[\tilde{\Gamma}_k] \mathbf{y}_k. \quad (12b)$$

In (12a), we select the first s maximum coefficients, and save their corresponding positions into $\tilde{\Gamma}_k$. Note that in (12a), notation $|\tilde{\Gamma}_k| = s$ represents that the cardinality of $\tilde{\Gamma}_k$ is s , in other words, there are number of s non-zero elements in $\tilde{\Gamma}_k$.

- 3) The forward-add process. A promising candidate is chosen by DCS method in (13a), and attempts to add into the support-sets in (13b), so the cardinality in Γ_k^{s+1} is larger than in Γ_k^s by one, which vividly describes the forward-add process. It shows that, in (13a) the candidate λ_{max} is determined by multiple data blocks, and the residual is measured individually based on the selected support-set in (13c). If the added promising candidate is the desired candidate, it will remain in the support-sets, otherwise, it will be directly removed by reverse-fetch strategy in step 4.

$$\lambda_{max} = \arg \max_{l=1, \dots, L} \sum_{k=1}^{MQ} |\langle \mathbf{A}_k[l], \mathbf{u}_k^s \rangle|, \quad (13a)$$

$$\Gamma_k^{s+1} = \Gamma_k^s \cup \lambda_{max}, \quad (13b)$$

$$\mathbf{u}_k^{s+1} = \mathbf{y}_k - \mathbf{A}_k[\Gamma_k^{s+1}] \mathbf{A}_k^\dagger[\Gamma_k^{s+1}] \mathbf{y}_k. \quad (13c)$$

- 4) The reverse-fetch process. The reverse-fetch strategy is a testing method which is used to test whether the previous added candidates are available. Based on the support-sets in the forward-add process, the new channel coefficients $\mathbf{h}_k$ are measured shown in (14a). For clarity, we denote $\mathbf{u}_k^s$, $\mathbf{u}_k^{s+1}$, and $\tilde{\mathbf{u}}_k$ as current residual, intermediate residual, and temporary residual respectively. Analogous to residual, we denote Γ_k^s , Γ_k^{s+1} , and $\tilde{\Gamma}_k$ as the current support-sets, intermediate support-sets, and temporary support-sets. Temporary support-sets $\tilde{\Gamma}_k$ are formed through the tap delays that have first s maximum coefficients shown in (14b), and temporary residual is measured by the temporary support-sets shown in (14c). Note that, there are number $(s+1)$ non-zero elements in $\mathbf{h}_k$ in (14a).

$$\mathbf{h}_k = \mathbf{A}_k^\dagger[\Gamma_k^{s+1}] \mathbf{y}_k, \quad (14a)$$

$$\tilde{\Gamma}_k = \arg \max_{|\tilde{\Gamma}_k|=s} |\mathbf{h}_k|, \quad (14b)$$

$$\tilde{\mathbf{u}}_k = \mathbf{y}_k - \mathbf{A}_k[\tilde{\Gamma}_k] \mathbf{A}_k^\dagger[\tilde{\Gamma}_k] \mathbf{y}_k. \quad (14c)$$

Considering the residual norm as the model fit measure, for each k , where $k = 1, \dots, MQ$, a comparison between current residual norm $\|\mathbf{u}_k^s\|_2$ and temporary residual norm $\|\tilde{\mathbf{u}}_k\|_2$ is performed. For the comparison, if the temporary residual norm $\|\tilde{\mathbf{u}}_k\|_2$ is smaller than the current residual norm $\|\mathbf{u}_k^s\|_2$, which means that there exists incorrect candidates in the current support-sets Γ_k^s , then the temporary support-sets $\tilde{\Gamma}_k$ acts as the new current support-sets Γ_k^s , and the current residual $\mathbf{u}_k^s$ is replaced by temporary residual $\tilde{\mathbf{u}}_k$. After comparing the all residual, the iteration counter is decreased by one and continues the reverse operation of refining the support-sets, this is why it is called reverse-fetch process. If the current residual norm is smaller than the temporary norm, the reverse operation is not performed. By the reverse-fetch process, the channel taps with common delays will remain all the time, the channel taps with different delays which are incorrectly estimated by SOMP will be replaced by correct taps gradually.

- 5) Finally, the channel estimates are measured based on the final support-sets Γ_k^s where $s = S$,

$$\hat{\mathbf{h}}_k = \mathbf{A}_k^\dagger[\Gamma_k^s] \mathbf{y}_k. \quad (15)$$

The most difference between traditional sparse channel estimation methods and the proposed FRsOMP method is that, the proposed FRsOMP has the capability to suppress the CoI. The most difference between SOMP algorithm and our proposed FRsOMP algorithm is that, the proposed FRsOMP algorithm attempts to add promising candidates into support-sets by forward-add process, and to remove the incorrect candidates by reverse-fetch process. While in SOMP algorithm, the incorrect candidates remain in the support-set forever. The channel estimates with common delays are enhanced which have the capability to suppress the CoI, and the channel estimates with different delays are recovered correctly under our proposed FRsOMP algorithm. The pseudo-code for our proposed FRsOMP is shown in Algorithm 2.

If spatial joint sparsity is used by FRsOMP, the channel estimation algorithm is referred to as S-FRsOMP, if both spatial joint sparsity and temporal joint sparsity are used by FRsOMP, the channel estimation algorithm is referred to as ST-FRsOMP.

C. Computational complexity analysis

In this subsection, we roughly compare the computational complexities for the traditional OMP, SOMP, and our proposed FRsOMP algorithm. The complexity of OMP algorithm at s -th iteration is $\mathcal{O}(PL +$

Algorithm 2 The FRSOMP channel estimation algorithm**Input:** $\mathbf{y}_k, \mathbf{A}_k, S, k = 1, \dots, MQ$.1: **Initialization:**2: perform SOMP algorithm to obtain $\tilde{\mathbf{h}}_k$.3: **for** $s = 1 : S$ **do**4: Measure the first s maximum candidates in $\tilde{\mathbf{h}}_k$, and store their positions $\tilde{\Gamma}_k$ via (12a).5: Measure the corresponding residual $\mathbf{u}_k^s$ based on $\tilde{\Gamma}_k$ via (12b). **end for**6: $s = S, \Gamma_k^S = \tilde{\Gamma}_k$ ($\tilde{\Gamma}_k$ here is the output from (12a) at S -th iteration).7: **repeat**

8: perform (13a) - (13c). (Step 8 is called forward-add process.)

9: **repeat**

10: perform (14a) - (14c).

11: **if** $(\|\tilde{\mathbf{u}}_k\|_2) \leq \|\mathbf{u}_k^s\|_2$ **then**12: $\mathbf{u}_k^s = \tilde{\mathbf{u}}_k, \Gamma_k^s = \tilde{\lambda}_k,$ 13: $s = s - 1.$ 14: **else**

15: break.

end if16: **until** $(s = 0)$ (Step 9 - 16 is called reverse-fetch process.)17: $s = s + 1$ 18: **until** $(s = S + 1)$ 19: **Output:**20: $\hat{\mathbf{h}}_k = \mathbf{A}_k^\dagger[\Gamma_k^S]\mathbf{y}_k.$

346 $Ps + Ps^2 + s^3$) [34]. Consider that the sparsity of underwater acoustic channel is limited, namely the
 347 maximum value of s is quite small, so $PL \gg (Ps + Ps^2 + s^3)$, thus the dominating term PL is chosen
 348 for analysis convenience [35]. The total computational complexity of OMP algorithm is $\mathcal{O}(SPL)$, where
 S is the sparsity.

TABLE I. Computational complexity

Algorithmic description	computational complexitiy
OMP	$\mathcal{O}(SPL)$
SOMP	$\mathcal{O}(SPL)$
FRSOMP	$\mathcal{O}((S + 1)PL) - \mathcal{O}(3(S + 1)PL)$

349 Table I shows the computational complexities of OMP, SOMP, and FRSOMP for each channel estimate.
 350 The SOMP algorithm has the same computational complexity as OMP. For the FRSOMP algorithm,
 351 because the total iteration times can not be exactly determined, we simply provide the lower bound
 352 and upper bound. If all the channels have common tap delays, the minimum computational complexity
 353 is $\mathcal{O}((S + 1)PL)$, if all the channels do not have common tap delays, the maximum computational
 354 complexity is $\mathcal{O}(3(S + 1)PL)$. From Table I, we can see that, the proposed FRSOMP obtains the better
 355 channel estimation performance at the cost of computational complexity.
 356

IV. CHANNEL ESTIMATION BASED DECISION FEEDBACK EQUALIZATION WITH INTERFERENCE CANCELLATION

In this section, we derive the channel estimation based decision feedback equalization with IC component. Figure 2 shows the structure of CE-DFE with IC which has a very similar structure to the self-iterative equalization solution in [23], the most difference between our receiver and the self-iterative equalization receiver is that, the proposed receiver utilizes channel estimate to measure the filter coefficients directly shown in Fig. 2, while self-iterative equalization receiver uses adaptive algorithm to calculate the filter coefficients iteratively. The equalization process is performed symbol-wise process, in the following we omit the block index. We consider the underwater acoustic MIMO channel model that has number of M transmitters and number of N receivers. At n -th receiver, the received baseband signal is given as

$$\mathbf{y}_n = \sum_{k=1}^M \mathbf{H}_{k,n} \mathbf{x}_k + \boldsymbol{\omega}_n, \quad (16)$$

where

$$\mathbf{y}_n = \left(y_n(i + N_c) \quad \cdots \quad y_n(i) \quad \cdots \quad y_n(i - N_a + 1) \right)^T, \quad (17)$$

$$\mathbf{x}_k = \left(x_k(i + N_c) \quad \cdots \quad x_k(i) \quad \cdots \quad x_k(i - N_a - L + 1) \right)^T, \quad (18)$$

$$\boldsymbol{\omega}_n = \left(\omega_n(i + N_c) \quad \cdots \quad \omega_n(i) \quad \cdots \quad \omega_n(i - N_a + 1) \right)^T, \quad (19)$$

$$\mathbf{H}_{k,n} = \left(\begin{array}{cc|cc} h_{k,n}(0) & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & 0 \\ 0 & \cdots & h_{k,n}(L-1) & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & h_{k,n}(0) & \cdots & h_{k,n}(L-1) \end{array} \right), \quad (20)$$

$$\mathbf{H}_{k,n} = (\mathbf{B}_k | \mathbf{q}_k | \mathbf{C}_k). \quad (21)$$

In (17) - (19), N_a and N_c denote the number of acausal and causal taps respectively.

Base on the model in Fig. 2, the soft decision can be depicted as

$$x_{s_m}(i) = \mathbf{p}_m^H \mathbf{v}_m, \quad (22)$$

where $\mathbf{p}_m$ and $\mathbf{v}_m$ are the follows,

$$\mathbf{p}_m = (\mathbf{f}_m^H, \mathbf{r}_{m,1}^H, \cdots, \mathbf{r}_{m,m-1}^H, \mathbf{b}_m^H, \mathbf{r}_{m,m+1}^H, \cdots, \mathbf{r}_{m,M}^H)^H, \quad (23a)$$

$$\mathbf{v}_m = (\mathbf{y}_n^H, \mathbf{x}_{b_1}^H, \cdots, \mathbf{x}_{b_{m-1}}^H, \mathbf{x}_{b_m}^H, \mathbf{x}_{b_{m+1}}^H, \cdots, \mathbf{x}_{b_M}^H)^H. \quad (23b)$$

In Fig. 2, the feedforward filters and feedback filters are denoted as $\mathbf{f}_m$ and $\mathbf{b}_m$, the IC filters which are used to mitigate the signal from k -th transmitter are denoted as $\mathbf{r}_{m,k}$, where $k \neq m$. The lengths of the feedforward filters, feedback filters, and IC filters are denoted as N_f , N_b , and N_r respectively. Compared with traditional CE-DFE in [25], our proposed CE-DFE with IC in (22) contains additional terms $\mathbf{r}_{m,k}^H \mathbf{x}_{b_k}$, whose purpose is to mitigate the CoI caused by received signal from k -th transmitter.

Similar to [36], we partition the transmitted symbols $\mathbf{x}_m$ into three groups: $\mathbf{x}_{b_m} = (x_m(i+N_b), \cdots, x_m(i+1))^T$, $x(i)$, and $\mathbf{x}_{0_m} = (x_m(i-1), \cdots, x_m(i-N_a-L+1))^T$. Similarly, we partition the channel matrix

Fig. 2. Diagram of structure for CE-DFE with IC. The feedforward filters, feedback filters, and IC filters are measured by channel estimates.

388 $\mathbf{H}_{m,n}$ in three parts $\mathbf{B}_m$, $\mathbf{q}_m$, and $\mathbf{C}_m$, shown in (21). So (16) can be written as

389
$$\mathbf{y}_n = \mathbf{q}_m x_m(i) + \mathbf{C}_m \mathbf{x}_{b_m} + \sum_{k=1, k \neq m}^M \mathbf{q}_k x_k(i) + \sum_{k=1, k \neq m}^M \mathbf{C}_k \mathbf{x}_{b_k} + \sum_{k=1}^M (\mathbf{u}_k + \mathbf{B}_k \mathbf{x}_{0_m}). \quad (24)$$

390 The first term in (24) contains the transmitted symbol $x_m(i)$, which needs to be recovered $x_m(i)$. The
 391 second can be cancelled by feedback filters. The third term which contains transmitted symbols from other
 392 transmitters is treated as CoI which should be cancelled by feedforward filters. The feedforward filters
 393 try to mitigate the last term which is treated as observation noise. In order to eliminate the transmitter
 394 symbols from other transmitters, we design a channel based decision feedback equalizer with IC. Based
 395 on the traditional CE-DFE, we add IC filters shown in Fig. 2, so the fourth term in (24) can be cancelled
 396 by the IC filters, while in traditional CE-DFE, the third term and the fourth term in (24) are ignored.

397 Consider the transmitted symbols $\mathbf{x}_m$ to be zero mean, white sequences with variance of $\sigma \mathbf{I}$, the
 398 transmitted symbols are independent with channel impulse response and received noise $\mathbf{u}_m$. Also consider
 399 that the received noise $\mathbf{u}_m$ to be zero-mean with covariance $\mathbf{D}_{u_m}$ which is independent with channel
 400 impulse response. Then we obtain the effective observation noise correlation matrix $\mathbf{R}_m$ as

401
$$\mathbf{R}_m = \mathbb{E}(|\mathbf{u}_m + \mathbf{B}_m \mathbf{x}_{0_m}|^2) = \mathbf{D}_{u_m} + \sigma \mathbf{B}_m \mathbf{B}_m^H, \quad (25)$$

402 The optimal equalization coefficients are the solution to

403
$$\hat{\mathbf{p}}_m = \arg \min_{\mathbf{p}_m} \mathbb{E} \left[|\mathbf{p}_m^H \mathbf{v}_m - x_m(i)|^2 \right]. \quad (26)$$

404 With the model defined in (26), a number of approaches can be measured the optimal filter coefficients
 405 ($\mathbf{f}_m$, $\mathbf{b}_m$, and $\mathbf{r}_{m,k}$). In this paper, we adopt the linear minimum mean square error method to calculate
 406 the filter coefficients. The expressions of these optimal filter coefficients are (see the APPENDIX A for
 407 further details)

408
$$\mathbf{f}_m = \left(\sum_{k=1}^M (\sigma \mathbf{q}_k \mathbf{q}_k^H + \mathbf{R}_k) \right)^{-1} \sigma \mathbf{q}_m, \quad (27a)$$

$$\mathbf{b}_m = -\mathbf{C}_m^H \sigma \mathbf{q}_m = -\mathbf{C}_m^H \left(\sum_{k=1}^M (\sigma \mathbf{q}_k \mathbf{q}_k^H + \mathbf{R}_k) \right)^{-1} \sigma \mathbf{q}_m \quad (27b)$$

$$\mathbf{r}_{m,k} = -\mathbf{C}_k^H \sigma \mathbf{q}_m = -\mathbf{C}_k^H \left(\sum_{k=1}^M (\sigma \mathbf{q}_k \mathbf{q}_k^H + \mathbf{R}_k) \right)^{-1} \sigma \mathbf{q}_m, k \neq m. \quad (27c)$$

Compared with turbo equalizers in [11], [23], [28], [29], the proposed CE-DFE with IC uses IC filters to directly remove the CoI, while in turbo equalizer, the CoI is eliminated iteratively which unfortunately increases the computation complexity. Also, the proposed CE-DFE with IC has more simple structure shown in Fig. 2. Compared with DA-DFE, the proposed CE-DFE with IC directly utilizes channel estimates to calculate the filter coefficients, which needs less training symbols to achieve convergence, so that the bandwidth efficiency is improved. Also, the proposed CE-DFE with IC needs less controlling parameters, thus, the CE-DFE with IC is more robust than DA-DFE.

V. EXPERIMENTAL ANALYSIS AND RESULTS

We now present results from numerical simulation, lake experiment, and sea experiment. The performance is compared between our proposed CE-DFE (noted as CE-DFE with IC) and traditional CE-DFE (noted as CE-DFE without IC). The filter coefficients of the two kinds of CE-DFE are measured by channel estimates which are obtained by traditional channel estimation algorithms and our proposed FRSOMP channel estimation algorithms. In the experiments, we use the following channel estimation algorithms:

- Least square QR-factorization (LSQR): least square channel estimation which is non-sparse channel estimation algorithm.
- OMP: the conventional compressed sensing channel estimation algorithm.
- S-SOMP: only spatial joint sparsity is used to estimate underwater acoustic MIMO channels, which is traditional SOMP algorithm.
- S-FRSOMP: our proposed FRSOMP channel estimation algorithm. Only spatial joint sparsity is used in FRSOMP algorithm.
- ST-SOMP: both spatial joint sparsity and the temporal joint sparsity are used to estimate underwater acoustic MIMO channels, which is traditional SOMP algorithm.
- ST-FRSOMP: our proposed FRSOMP channel estimation algorithm. Both spatial joint sparsity and temporal joint sparsity are used in FRSOMP algorithm.

We use output SNR, bit error rate, and constellation to evaluate the performance of our proposed FRSOMP channel estimation algorithms and our CE-DFE with IC methods. Define the output SNR as:

$$\gamma_{OSNR} = 10 \log_{10} \left(\frac{\|\mathbf{x}_m\|_2^2}{\|\mathbf{x}_m - \mathbf{x}_{s_m}\|_2^2} \right), \quad (28)$$

where $\mathbf{x}_m$ is the transmitted symbols from m -th transmitter, $\mathbf{x}_{s_m}$ is the soft output from receiver. For description convenience, we denote the m -th transmitter, n -th receiver as Txm and Rxn respectively, from surface to bottom, channel which comes from m -th transmitter to n -th receiver is denoted as Txm-Rxn. In simulation experiment, lake experiment, and sea experiment, quadratic phase shift keying (QPSK) mapping method is utilized.

A. Numerical simulation

In this subsection, we use numerical simulation to demonstrate our channel estimation and channel equalization methods. In the simulation experiment, the underwater acoustic MIMO system contains three transmitters and eight receivers, with a range of 2000 m. The depth of sea is 20 m, the depths of the

transmitters are 5 m, 10 m, and 15 m, the depths of the receivers cover from 2 m - 16 m with uniform distribution. We use Bellhop model [37] to generate channels. For channels from Tx2 and Tx3, we replace the tap coefficients manually but keep the tap delays. The tap coefficients are generated manually and show exponential decay with decay factors from 0.012 to 0.019 randomly, the phases of tap coefficients are randomly distribution. To make sure that the channels which come from different transmitters to a receiver have 3 - 4 common tap delays, one or two taps are moved by hand. We select the first 10 maximum amplitudes as channel multipath.

Figure 3 shows the channels from Tx1-Rx6, Tx2-Rx6, and Tx3-Rx6. In the figure, it shows that, the multipath delays reach about 35 ms, and show typical sparse. Moreover, some channel taps which come from different transmitters have the common delays, their estimation performance will be improved by distributed compressed sensing method.

Fig. 3. The channel impulse responses.

We investigate our channel estimation and our channel equalization methods in terms of output SNR. The SNR for each channel is set 20 dB. The sparsity for OMP, S-SOMP, S-FRSOMP, ST-SOMP, and ST-FRSOMP is 10, in ST-SOMP and ST-FRSOMP algorithms, the adjacent data blocks is set $Q = 2$. The symbol rate for each transmitter is 6000 symbols/s. The channel length is set 36.7 ms which corresponds to 220 symbols. The lengths of feedforward filters, feedback filters, and IC filters are 440 symbols, 219 symbols, and 219 symbols respectively.

In the simulation, the CE-DFE with IC and CE-DFE without IC work in decision-directed mode. The length of signal frame is 10 s. At the beginning of signal frame, a preamble which is known to receiver is designed for obtaining the initial channel estimates. Because the channels in the simulation are time-invariant, channel estimates are not updated periodically.

Figure 4 shows the output SNR obtained from CE-DFE without IC. From the sub-figures, we can see that, when the observation length is less than 91.7 ms, the output SNR increases with observation length, because longer observation length improves the channel estimation performance. Also, we can observe that our ST-FRSOMP achieves the highest output SNR among the three sub-figures, because our ST-FRSOMP not only enhances the channel estimation which have common delays, but also reconstructs the taps which have different delays. Because of the strong CoI, the output SNR which is obtained by CE-DFE without IC converges at 5 dB.

Figure 5 shows the output SNR obtained by CE-DFE with IC. Different with in Fig. 4, the output SNR increases with observation length from 18.3 ms to 110 ms. Because the CE-DFE with IC not only mitigate the second term and the last term in (24), but also mitigate the third term and the fourth term in (24) which are denoted as CoI, so the CE-DFE with IC obtains much higher output SNR than that obtained by CE-DFE without IC. It can observed that, our ST-FRSOMP channel estimation combined with our CE-DFE with IC achieve the highest output SNR, the reason is that, both channel estimation and channel equalization have the ability to eliminate CoI.

Fig. 4. Output SNR versus observation length. (a) Output SNR of Tx1 from CE-DFE without IC. (b) Output SNR of Tx2 from CE-DFE without IC. (c) Output SNR of Tx3 from CE-DFE without IC.

Fig. 5. Output SNR versus observation length. (a) Output SNR of Tx1 from CE-DFE with IC. (b) Output SNR of Tx2 from CE-DFE with IC. (c) Output SNR of Tx3 from CE-DFE with IC.

B. Lake experiment

The lake experiment was conducted at Black Warrior River, where is in the vicinity of University of Alabama, USA. The depth of the lake is around 10 m. We tested an underwater acoustic MIMO system of 2 transmitters and 6 receivers. The transmitting array and the receiving array were mounted from a harbor dock, shown in Fig. 6(a). The six-element receiving array covered from 2 m - 7 m with an element spacing of 1 m. The transmitter elements were deployed at 2.5 m and 3.5 m respectively. The distance between transmitting array and receiving array was roughly 110 m. Figure 6(b) shows the sound speed profile. We observe a constant sound speed across depth of roughly 1484 m/s.

Fig. 6. Diagram of lake experiment and sound speed. (a) Experimental diagram. (b) Sound speed profile.

For transmissions, a data frame of 6 s at rate of 4250 symbol/s and a carrier frequency of 85 kHz was used. The average SNR among the 6 receivers was measured to be 35.6 dB. Examples of the channel impulse responses are given in Fig. 7(a) and Fig. 7(b) for Tx1-Rx3 and Tx2-Rx3 respectively. The channel length is set at 33.33 ms, the channel estimation algorithm is LSQR, the observation length (100 ms) is three times as channel impulse response. From the 2 sub-figures, we observe a delay spread of roughly 10 ms, we can adopt the compressed sensing channel estimation to improve the underwater acoustic channel estimation. We can also observe that, the multipath from the 2 sub-figures contains rich common delays and different delays, which motivates the usage of our FRSOMP distributed compressed sensing algorithm.

Fig. 7. Channel estimation. (a) Tx1-Rx3 channel estimation. (b) Tx2-Rx3 channel estimation.

TABLE II. Parameters setting in lake trial

Parameters	Description	Value
R	Symbol rate	4250 symbols/s
N_T	Number of transducers	2
N_R	Number of hydrophones	6
K_{os}	Over sampling factor	1
$T_{preamble}$	Duration of preamble	0.25 s
T_{ch}	Channel impulse response duration	33.33 ms
L	Length of discrete channel	$L = T_{ch} * R$
N_f	Feedforward filter span	$2L$
N_b	Feedback filter span	$L - 1$
N_r	Interference cancellation filter span	$L - 1$

The parameters of the receiver are listed in TABLE II. The over sampling factor K_{os} is set at 1, the length of channel impulse response T_{ch} is set at 33.33 ms, which corresponds to $L = 200$ symbols. The lengths of feedforward filters, feedback filters, and IC filters are set at $N_f = 2L$ symbols, $N_b = L - 1$ symbols, and $N_r = L - 1$ symbols respectively. In the OMP, S-SOMP, S-FRSOMP, ST-SOMP, and ST-FRSOMP algorithms, the sparsity is set at 30. In the ST-SOMP and ST-FRSOMP algorithms, the number of adjacent data blocks is set at $Q = 2$.

In the training mode, channel estimators assume perfect knowledge of transmitted symbols. Figure 8 shows the output SNR in training mode for both Tx1 and Tx2. As expected, the output SNR increases with observation length. Obviously, the LSQR channel estimation algorithm results in the worst performance, since LSQR is non-sparsity channel estimation algorithm. Compared with OMP, S-SOMP, S-FRSOMP, ST-SOMP, and ST-FRSOMP channel estimation algorithms, we conclude that our proposed S-FRSOMP and ST-FRSOMP algorithms achieve higher output SNR than S-SOMP and ST-SOMP algorithms respectively, because the taps with common delays are enhanced, so that they suffer from less CoI, and the taps with

Fig. 8. Output SNR versus observation length in lake experiment. (a) Output SNR of Tx1 from CE-DFE without IC. (b) Output SNR of Tx2 from CE-DFE without IC. (c) Output SNR of Tx1 from CE-DFE with IC. (d) Output SNR of Tx2 from CE-DFE with IC. The filter coefficients of CE-DFE without IC and CE-DFE with IC are measured by channel estimates which are obtained by ST-FRSOMP, ST-SOMP, S-FRSOMP, S-SOMP, OMP, and LSQR algorithms.

different delays are correctly reconstructed based on our proposed FRSOMP algorithm. For example, in Fig. 8(a), under the observation length of 28.23 ms, the S-FRSOMP and S-SOMP algorithms achieve 4.79 dB and 4.49 dB respectively, the ST-FRSOMP and ST-SOMP algorithms achieve 5.68 dB and 5.30 dB respectively, in Fig. 8(b), the results are 4.38 dB and 4.06 dB, 5.29 dB and 4.86 dB. In general, the distributed compressed sensing algorithms achieve higher output SNR than OMP in Fig. 8, because the DCS channel estimators suppress the CoI by combining multiple data blocks to estimate taps with common delays.

Compared with Fig. 8(a) and Fig. 8(c), it can be observed that, in Fig. 8(c), the output SNR is not converged under long observation length, it still leads to slight performance improvement with observation length, while in Fig. 8(a), the output SNR is converged. The gain obtained by proposed ST-FRSOMP and S-FRSOMP over OMP is higher in Fig. 8(c) compared with in Fig. 8(a). For example, in Fig. 8(c), the gain obtained by proposed ST-FRSOMP and S-FRSOMP estimators over OMP estimator is 1.63 dB, 0.98 dB under 37.65 ms, while in Fig. 8(a) it is 0.87 dB and 0.51 dB. It can also be observed that, all the sparse channel estimators obtain higher output SNR in Fig. 8(c) than that in Fig. 8(a), when the observation length is long. The reason is that, in Fig. 8(c) and Fig. 8(d), the CE-DFE with IC we proposed has the capability to mitigate the CoI. It shows that our proposed ST-FRSOMP channel estimation algorithm combined with our proposed CE-DFE with IC achieve the best OSNR in Fig. 8(c) and Fig. 8(d), because the CoI is suppressed both in channel estimation and channel equalization. While under short observation length (less than 40 ms), all the sparse channel estimators obtain lower output SNR in Fig. 8(c) than that in Fig. 8(a). The reason is that short observation length obtains bad channel estimates which lead to inaccurate equalizer coefficients ($\mathbf{f}_m$, $\mathbf{b}_m$, and $\mathbf{r}_{m,k}$). The same results can be achieved in Fig. 8(b) and Fig. 8(d).

In the decision-directed mode, the previous detected symbols are used in both channel estimation and

Fig. 9. Output SNR in decision-directed mode in lake experiment. (a) Output SNR of Tx1 from CE-DFE without IC. (b) Output SNR of Tx2 from CE-DFE without IC. (c) Output SNR of Tx1 from CE-DFE with IC. (d) Output SNR of Tx2 from CE-DFE with IC. The filter coefficients of CE-DFE without IC and CE-DFE with IC are measured by channel estimates which are obtained by different channel estimation algorithms.

Doppler estimation. Because the Doppler scale is small, we use narrow band Doppler estimation and compensation [38]. A 0.25 s preamble is used to estimate the initial channel impulse responses and Doppler estimation. Channel estimates obtained by five channel estimators (ST-FRSOMP, ST-SOMP, S-FRSOMP, S-SOMP, and OMP) are used to measure the filter coefficients of CE-DFE without IC and our CE-DFE with IC. The parameters are the same as in training mode listed in TABLE II. The observation length is set at 47.1 ms. Periodical training sequences are utilized to prevent error propagation. The periodical training symbols accounted for 25% of the transmitted symbols are used to adjust the channel estimates and Doppler estimates.

The ST-FRSOMP, ST-SOMP, S-FRSOMP, S-SOMP, and OMP estimators achieve the output SNR of 5.21 dB, 4.98 dB, 4.72 dB, 4.42 dB, and 4.24 dB respectively in Fig. 9(a), 4.48 dB, 4.28 dB, 4.07 dB, 3.82 dB, and 3.57 dB in Fig. 9(b), 6.67 dB, 6.20 dB, 5.85 dB, 5.40 dB, and 5.04 dB in Fig. 9(c), 6.33 dB, 5.84 dB, 5.44 dB, 4.97 dB, and 4.62 dB in Fig. 9(d). Our proposed ST-FRSOMP channel estimator combined with our proposed CE-DFE with IC achieve the highest output SNR in both Fig. 9(c) and Fig. 9(d). The comparison results in decision-directed mode are consistent with those in training mode. It can be observed that, in Fig. 9(a) and Fig. 9(b), the output SNR shows an oscillation feature, because of the error propagation which is caused by strong CoI.

Next, low density parity check code (LDPC) channel coding technology is utilized [39] with a coding rate at 2/3. The final BER in the decision-directed mode is provided in TABLE III and TABLE IV. The results show that, the ST-FRSOMP algorithm achieves the lowest BER compared with other estimators both in TABLE III and TABLE IV. Because of the high raw BER in TABLE III, the BER after LDPC decoding does not exhibit much improvement or even worse. While in TABLE IV, the BER is reduced significantly after LDPC channel coding, for example, the raw BER obtained by ST-FRSOMP is 5.33% and 6.47% for Tx1 and Tx2, after channel coding, the BER is 0 and 0.81% respectively. The results

TABLE III. Lake BER obtained from CE-DFE without IC

	OMP	S-SOMP	S-FRSOMP	ST-SOMP	ST-FRSOMP
TX1					
Uncoded BER (%)	20.83	19.45	17.32	15.24	13.67
Coded BER (%)	21.16	19.50	17.06	14.84	12.99
TX2					
Uncoded BER (%)	32.11	31.15	29.76	28.35	26.86
Coded BER (%)	32.07	31.21	30.37	29.07	27.22

TABLE IV. Lake BER obtained from CE-DFE with IC

	OMP	S-SOMP	S-FRSOMP	ST-SOMP	ST-FRSOMP
TX1					
Uncoded BER (%)	10.85	9.44	7.85	6.49	5.33
Coded BER (%)	9.42	7.29	2.47	0.44	0
TX2					
Uncoded BER (%)	13.09	11.44	9.59	7.89	6.47
Coded BER (%)	12.41	10.38	7.65	4.98	0.81

in TABLE IV show that our proposed ST-FRSOMP channel estimation algorithm combined with our proposed CE-DFE with IC achieve significant low BER.

C. Sea experiment

To further verify the performance of the proposed channel estimation and channel equalization methods, we conducted the sea trial in Wuyuanbay, Xiamen, China. The depth at the test site was about 10 m. A 2×8 MIMO system was used, the two transmitters were suspended to the depths of 4 m and 6 m, and an eight-element vertical array covered the depth range of 1 m to 8 m, with a spacing of 1 m. The range was roughly 1000 m. The QPSK symbols with a symbol rate of 3200 symbol/s and a carrier frequency of 15500 Hz were transmitted in a duration of 8 s. The average received SNR was 33.5 dB. The sparsity S is set 20, the number of adjacent data blocks Q is set 2. The parameters for MIMO system are shown in Table V.

TABLE V. Parameters setting in sea trial

Parameters	Description	Value
R	Symbol rate	3200 symbols/s
N_T	Number of transducers	2
N_R	Number of hydrophones	8
K_{os}	Over sampling factor	1
$T_{preamble}$	Duration of preamble	0.25 s
T_{ch}	Channel impulse response duration	62.5 ms
L	Length of discrete channel	$L = T_{ch} * R$
N_f	Feedforward filter span	$2L$
N_b	Feedback filter span	$L - 1$
N_r	Interference cancellation filter span	$L - 1$

Fig. 10. Channel estimation for sea trial. (a) Tx1-Rx1 channel estimation. (b) Tx2-Rx1 channel estimation.

Figure 10 shows examples of the sea trial channel estimates. The channel length is set at 62.5 ms corresponding to 200 symbols. To obtain more exact channel estimates, the observation length (187.5 ms) is set at three times of channel length. The channel estimation algorithm is LSQR. From the two sub-figures, it is obvious that the multipath structures exhibit sparse. Moreover, it is evident that there are more than two multipath taps that have the common delays. Meanwhile, multipath taps that have different delays can be easily found. We utilize forward-reverse strategy to improve the estimation of such taps that have different delays.

Fig. 11. Output SNR versus observation length in sea experiment. (a) Output SNR of Tx1 from CE-DFE without IC. (b) Output SNR of Tx2 from CE-DFE without IC. (c) Output SNR of Tx1 from CE-DFE with IC. (d) Output SNR of Tx2 from CE-DFE with IC. The filter coefficients of CE-DFE without IC and CE-DFE with IC are measured by channel estimates which are obtained by different channel estimation algorithms.

Figure 11 shows the output SNR versus observation length in the training mode. As in the lake experimental results, our ST-FRSOMP estimator achieves the best output SNR. For example, the output SNR obtained by ST-FRSOMP, ST-SOMP, S-FRSOMP, S-SOMP, and OMP is 4.78 dB, 4.61 dB, 4.38 dB,

4.23 dB, and 4.06 dB in Fig. 11(a), and 6.32 dB, 5.96 dB, 5.59 dB, 5.31 dB, and 4.88 dB in Fig. 11(c), under the observation length of 37.5 ms. The ST-FRSOMP estimator results in 0.72 dB and 1.44 dB gain over OMP estimator in Fig. 11(a) and Fig. 11(c). It can be observed that, the output SNR obtained by the DCS estimators (ST-FRSOMP, ST-SOMP, S-FRSOMP, S-SOMP) is slightly low than OMP estimator obtained in Fig. 11(a) and Fig. 11(b). While in Fig. 11(c) and Fig. 11(d), the output SNR obtained by DCS estimators is still higher than OMP obtained when the observation length is long. For example, in Fig. 11(c), the output SNR obtained by ST-FRSOMP, ST-SOMP, S-FRSOMP, S-SOMP, and OMP is 8.08 dB, 8.03 dB, 7.90 dB, 7.72 dB, and 7.74 dB under the observation length of 93.75 ms. In Fig. 11, our proposed ST-FRSOMP channel estimation algorithm combined with our proposed CE-DFE with IC achieve the best output OSNR.

Fig. 12. Output SNR in decision-directed mode in sea experiment. (a) Output SNR of Tx1 from CE-DFE without IC. (b) Output SNR of Tx2 from CE-DFE without IC. (c) Output SNR of Tx1 from CE-DFE with IC. (d) Output SNR of Tx2 from CE-DFE with IC. The filter coefficients of CE-DFE without IC and CE-DFE with IC are measured by channel estimates which are obtained by different channel estimation algorithms.

Figure 12 is the output SNR in decision-directed mode. The training length is 93.8 ms. To prevent error propagation, the training symbols accounted for 20% of the transmitted symbols are used for adjusting the channel estimation and Doppler estimation. The average output SNR obtained by ST-FRSOMP, ST-SOMP, S-FRSOMP, S-SOMP, and OMP is 4.16 dB, 4.00 dB, 4.05 dB, 3.95 dB, and 3.82 dB in Fig. 12(a), 6.37 dB, 6.30 dB, 6.26 dB, 6.19 dB, and 6.16 dB in Fig. 12(b), 7.24 dB, 7.08 dB, 7.04 dB, 6.93 dB, and 6.65 dB in Fig. 12(c), 8.50 dB, 8.34 dB, 8.28 dB, 8.13 dB and 7.93 dB Fig. 12(d). Our proposed ST-FRSOMP algorithm combined with our proposed CE-DFE with IC still achieve the highest output SNR. Noted that, in Fig. 12(b) and Fig. 12(d), it can be seen that, the output SNR varies with time because of the Doppler variant.

In sea experiment, there is no channel coding method. Table VI and Table VII provide the BER, we can see that, our ST-FRSOMP estimator achieves the lowest BER in both two tables. The BER decreases significantly when our proposed FRSOMP algorithm and proposed CE-DFE with IC are used, for example, in Table VI, the BER obtained by ST-FRSOMP is 19.55 % and 8.55 %, in Table VII, the BER obtained

605 by ST-FRSOMP is 4.34 % and 1.14 %. The BER reduces by 15.21 % and 7.41 % respectively.

TABLE VI. Sea trial BER obtained from CE-DFE without IC

	OMP	S-SOMP	S-FRSOMP	ST-SOMP	ST-FRSOMP
TX1					
RAW BER (%)	22.19	21.10	20.44	20.61	19.55
TX2					
RAW BER (%)	9.95	9.63	9.29	9.02	8.55

TABLE VII. Sea trial BER obtained from CE-DFE with IC

	OMP	S-SOMP	S-FRSOMP	ST-SOMP	ST-FRSOMP
TX1					
RAW BER (%)	6.32	5.25	5.00	4.80	4.34
TX2					
RAW BER (%)	2.06	1.66	1.52	1.34	1.14

Fig. 13. Constellations. (a) Tx1 output constellation from CE-DFE without IC, OMP channel estimator was used. (b) Tx1 output constellation from CE-DFE with IC, ST-FRSOMP channel estimator was used. (c) Tx2 output constellation from CE-DFE without IC, OMP channel estimator was used. (d) Tx2 output constellation from CE-DFE with IC, ST-FRSOMP channel estimator was used.

606 Finally, some constellations obtained from sea trial are provided in Fig. 13. Our proposed ST-FRSOMP
 607 channel estimation algorithm combined with our proposed DFE with IC methods obtain the most separated
 608 constellation in Fig. 13(b) and Fig. 13(d) compared with traditional channel estimation algorithm combined
 609 with traditional CE-DFE without IC.

610 VI. CONCLUSION

611 For underwater acoustic MIMO communications, the channels that come from different transmitters to
 612 a receiver, and channels that come from adjacent data blocks may exhibit strong correlation. DCS method
 613 is able to use such correlation to improve channel estimation performance. In this paper, we used SOMP
 614 algorithm to enhance channel estimates that had common delays under distributed compressed sensing
 615 framework, in this way, the CoI was suppressed. To address the different delays, we introduced forward-
 616 reverse method to SOMP, which was referred to as FRSOMP algorithm. The FRSOMP performed SOMP
 617 algorithm to get initial channel estimates, performed forward-add process to add promising candidates into

the support-sets based on distributed compressed method, and performed reverse-fetch process to check if the candidates in the support-set were available. Thus, it was possible to remove the incorrect candidates. We also proposed a CE-DFE structure that contained IC component for MIMO equalization. We derived coefficients for feedforward filters, feedback filters, and IC filters based on DCS channel estimates.

Numerical simulation results, lake experimental and sea experimental results showed that, the output SNR obtained by the proposed methods (our ST-FRSOMP combined with our CE-DFE with IC) obtained higher output SNR, lower BER, and more separated constellations compared with traditional OMP algorithm combined with traditional CE-DFE without IC component. It should be noted that, the better performance achieved by the proposed methods is at cost of higher computational complexity.

APPENDIX A

DERIVATION OF FILTER COEFFICIENTS FOR CE-DFE WITH IC

We assume that the channel estimates and the past decoding symbol are accurate. To obtain the solution of equalization coefficients, we perform a partial derivative as following

$$\frac{\partial \mathbb{E} \left[|\mathbf{p}_m^H \mathbf{v}_m - x_m(i)|^2 \right]}{\partial \mathbf{p}_m} = \frac{\partial \mathbb{E} \left[x_m^2(i) - 2x_m(i) \mathbf{p}_m^H \mathbf{v}_m + \mathbf{p}_m^H \mathbf{v}_m \mathbf{v}_m^H \mathbf{p}_m \right]}{\partial \mathbf{p}_m} = 0. \quad (29)$$

Thus, the optimal coefficients are obtained as

$$\hat{\mathbf{p}}_m = \mathbf{G}_{\mathbf{v}_m, \mathbf{v}_m}^{-1} \mathbf{g}_{x_m, \mathbf{v}_m}, \quad (30)$$

where

$$\mathbf{G}_{\mathbf{v}_m, \mathbf{v}_m} = \mathbb{E} (\mathbf{v}_m \mathbf{v}_m^H), \quad (31a)$$

$$\mathbf{g}_{x_m, \mathbf{v}_m} = \mathbb{E} (x_m^*(i) \mathbf{v}_m). \quad (31b)$$

As we assumed that $\mathbf{x}_m$ is with variance of $\sigma \mathbf{I}$, and independence of noise and channel estimates. Then substituting (23b) into (31), we yield

$$\mathbf{g}_{x_m, \mathbf{v}_m} = (\sigma \mathbf{q}_m^H, \mathbf{0}, \dots, \mathbf{0})^H, \quad (32)$$

and

$$\mathbf{G}_{\mathbf{v}_m, \mathbf{v}_m} = \begin{pmatrix} \mathbf{Y} & \mathbf{C}_1 & \dots & \mathbf{C}_m & \dots & \mathbf{C}_M \\ \mathbf{C}_1^H & \mathbf{I} & \dots & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{C}_m^H & \mathbf{0} & \dots & \mathbf{I} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{C}_M^H & \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{I} \end{pmatrix}, \quad (33)$$

where

$$\mathbf{Y} = \mathbb{E}(\mathbf{y}_n \mathbf{y}_n^H) = \sum_{k=1}^M (\sigma \mathbf{q}_k \mathbf{q}_k^H + \sigma \mathbf{C}_k \mathbf{C}_k^H + \mathbf{R}_k). \quad (34)$$

Partition $\mathbf{R}_{\mathbf{u}_m, \mathbf{u}_m}^{-1}$ as following,

$$\mathbf{R}_{\mathbf{u}_m, \mathbf{u}_m}^{-1} = \begin{pmatrix} \tilde{\mathbf{Z}}_{00} & \cdots & \tilde{\mathbf{Z}}_{0m} & \cdots & \tilde{\mathbf{Z}}_{0M} \\ \tilde{\mathbf{Z}}_{10} & \cdots & \tilde{\mathbf{Z}}_{1m} & \cdots & \tilde{\mathbf{Z}}_{1M} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{Z}}_{m0} & \cdots & \tilde{\mathbf{Z}}_{mm} & \cdots & \tilde{\mathbf{Z}}_{mM} \\ \vdots & \vdots & \ddots & \vdots & \ddots \\ \tilde{\mathbf{Z}}_{M0} & \cdots & \tilde{\mathbf{Z}}_{Mm} & \cdots & \tilde{\mathbf{Z}}_{MM} \end{pmatrix}, \quad (35)$$

Substituting (32) and (35) into (30), then we yield

$$\mathbf{f}_m = \tilde{\mathbf{Z}}_{00} \sigma \mathbf{q}_m, \quad (36a)$$

$$\mathbf{b}_m = \tilde{\mathbf{Z}}_{m0} \sigma \mathbf{q}_m, \quad (36b)$$

$$\mathbf{r}_{m,k} = \tilde{\mathbf{Z}}_{k0} \sigma \mathbf{q}_k, k \neq m. \quad (36c)$$

Based on $\mathbf{G}_{\mathbf{v}_m, \mathbf{v}_m} \mathbf{G}_{\mathbf{v}_m, \mathbf{v}_m}^{-1} = \mathbf{I}$, we get

$$\tilde{\mathbf{Z}}_{00} = \left(\sum_{k=1}^M (\sigma \mathbf{q}_k \mathbf{q}_k^H + \mathbf{R}_k) \right)^{-1}, \quad (37a)$$

$$\tilde{\mathbf{Z}}_{m0} = -\mathbf{C}_m^H \tilde{\mathbf{Z}}_{00} = -\mathbf{C}_m^H \left(\sum_{k=1}^M (\sigma \mathbf{q}_k \mathbf{q}_k^H + \mathbf{R}_k) \right)^{-1}. \quad (37b)$$

Substituting (37) into (36) results in (27).

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