ROBUST ADAPTIVE BEAMFORMING BASED ON LOW-RANK AND CROSS-CORRELATION TECHNIQUES

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ABSTRACT

This work presents a cost-effective low-rank technique for designing robust adaptive beamforming (RAB) algorithms. The proposed technique is based on low-rank modelling of the mismatch and exploitation of the cross-correlation between the received data and the output of the beamformer. We construct a linear system of equations which computes the steering vector mismatch based on prior information about the level of mismatch, and then we employ an orthogonal Krylov subspace based method to iteratively estimate the steering vector mismatch in a reduced-dimensional subspace, resulting in the proposed orthogonal Krylov subspace projection mismatch estimation (OKSPME) method. Simulation results show excellent performance of OKSPME in terms of the beamformer output signal-to-interference-plus-noise ratio (SINR) as compared to existing RAB algorithms.

Index Terms— robust adaptive beamforming, low-rank techniques, low complexity methods.

I. INTRODUCTION

Adaptive beamforming has been one of the most important research areas in sensor array signal processing. Conventional adaptive beamformers are extremely sensitive to environmental uncertainties or steering vector mismatches which may be caused by many different factors (e.g., imprecise antenna size calibration, signal pointing errors or local scattering). In order to mitigate the effects of uncertainties on adaptive beamformers, robust adaptive beamforming (RAB) techniques have been developed. Existing approaches include worst-case optimization [2], diagonal loading [4], and eigen-subspace decomposition and projection techniques [6], [8], [11]. However, these RAB approaches have some limitations such as their ad hoc nature, high probability of subspace swap at low signal-to-noise ratio (SNR) [3] and high computational cost due to online optimization or subspace decomposition techniques. Furthermore, in the case of large sensor arrays the above mentioned RAB methods may encounter problems for their application. This is because in RAB algorithms a cubic or greater computational cost is required to compute the beamforming parameters. Therefore, rank-reduction methods ([9],[15]-[24]) have been developed to reduce the cost and improve the convergence rate.

In recent years, research efforts have been devoted to the development of robust rank-reduction techniques for RAB. The beamspace approach of [9] projects the data onto a lower dimension subspace by using a beamspace matrix, whose columns are determined by linearly independent constrained optimization problems. A more effective approach (i.e., [19]-[23]) is based on preprocessing the array received data using a Krylov subspace. However, there are different ways to generate the Krylov subspace and the choice usually depends on the cost and the performance. The Arnoldi method [12], [13], [24] and the Lanczos iterations [12], [13], [15] are typical approaches used to generate orthogonal Krylov subspaces, whereas [22] introduces a method to generate non-orthogonal ones. However, the main challenge in these techniques is the model order determination. Specifically, the model order must be properly chosen to ensure robustness against uncertainties and high performance [20].

In this work, we propose and study a novel RAB method based on low-rank and cross-correlation techniques. In the proposed method, we exploit prior knowledge that the steering vector mismatch of the desired signal is located within an assumed known angular sector. The proposed method is based on the exploitation of the cross-correlation between the array received data and the output of the beamformer, which avoids costly optimization procedures. We firstly construct a linear system involving the mismatched steering vector and the statistics of the sampled data. Then we employ an iterative full orthogonalization method (FOM) [12], [13] to compute an orthogonal Krylov subspace whose model order is determined by both the minimum sufficient rank [20], which ensures no information loss when capturing the signal of interest (SoI) with interferers, and the execute-and-stop criterion of FOM [12], [13], which automatically avoids overestimating the number of bases of the computed subspace. The estimated vector which contains the cross-correlation between the array received data and the beamformer output is projected onto the Krylov subspace in order to compute the steering vector mismatch, resulting in the proposed orthogonal Krylov subspace projection mismatch estimation (OKSPME) method. We assess the signal-to-interference-plus-noise ratio (SINR) performance of OK-

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SPME against existing algorithms via simulations.

The rest of this paper is organized as follows: The system model and problem statement are described in Section II. Section III introduces the proposed OKSPME method. Section IV provides the complexity analysis. Section V presents the simulation results. Section VI gives the conclusion.

II. SYSTEM MODEL AND PROBLEM STATEMENT

Let us consider a linear antenna array of M sensors and K narrowband signals which impinge on the array. The data received at the ith snapshot can be modeled as

$$\mathbf{x}(i) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(i) + \mathbf{n}(i), \tag{1}$$

where $\mathbf{s}(i) \in \mathbb{C}^{K\times 1}$ represents uncorrelated source signals with zero mean, $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_K]^T \in \mathbb{R}^K$ is a vector containing the directions of arrival (DoAs), $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1)+\mathbf{e}, \cdots, \mathbf{a}(\theta_K)] = [\mathbf{a}_1, \cdots, \mathbf{a}_K] \in \mathbb{C}^{M\times K}$ is the matrix which contains the steering vector for each DoA and $\mathbf{e}$ is the steering vector mismatch of the desired signal, $\mathbf{n}(i) \in \mathbb{C}^{M\times 1}$ is assumed to be complex circular Gaussian noise with zero mean and variance σ_n^2. The beamformer output is given by

$$y(i) = \mathbf{w}^H\mathbf{x}(i), \tag{2}$$

where $\mathbf{w} = [w_1, \cdots, w_M]^T \in \mathbb{C}^{M\times 1}$ is the beamformer weight vector, where $(\cdot)^H$ denotes the Hermitian transpose. The optimum beamformer is computed by maximizing the SINR which is given by

$$SINR = \frac{\sigma_1^2|\mathbf{w}^H\mathbf{a}|^2}{\mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}}. \tag{3}$$

where σ_1^2 is the desired signal power and $\mathbf{R}_{i+n}$ is the interference-plus-noise covariance (INC) matrix. The problem of maximizing the SINR in (3) can be cast as the following optimization problem:

$$\begin{aligned} &\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w} \\ &\text{subject to} \quad \mathbf{w}^H\mathbf{a} = 1, \end{aligned} \tag{4}$$

which is known as the MVDR beamformer or Capon beamformer [1], [4]. The optimum weight vector is given by $\mathbf{w}_{opt} = \frac{\mathbf{R}_{i+n}^{-1}\mathbf{a}}{\mathbf{a}^H\mathbf{R}_{i+n}^{-1}\mathbf{a}}$. Since $\mathbf{R}_{i+n}$ is usually unknown in practice, it can be estimated by the sample covariance matrix (SCM) of the received data as

$$\hat{\mathbf{R}}(i) = \frac{1}{i}\sum_{k=1}^{i}\mathbf{x}(k)\mathbf{x}^H(k). \tag{5}$$

Using the SCM for directly computing the weights will lead to the sample matrix inversion (SMI) beamformer $\mathbf{w}_{SMI} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}}{\mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a}}$. However, the SMI beamformer requires a large number of snapshots to converge and is sensitive to steering vector mismatches [2], [3]. The RAB design problem we are interested in solving includes:

- The design of cost-efficient algorithms that are robust against uncertainties in the steering vector.
- The proposed algorithms must preserve their robustness and low-complexity features for large sensor arrays.

III. PROPOSED OKSPME METHOD

In this section, the proposed OKSPME method is introduced. This method aims to construct a linear system involving only known or estimated statistics and then projects an estimated cross-correlation vector between the array received data and the beamformer output onto an orthogonal Krylov subspace, so as to update the steering vector mismatch with reduced complexity. The SCM of the array received data is estimated by (5). The cross-correlation vector between the array received data and the beamformer output can be expressed as $\mathbf{d} = E[\mathbf{x}y^*]$ or equivalently as

$$\mathbf{d} = E[(\mathbf{A}\mathbf{s}+\mathbf{n})(\mathbf{A}\mathbf{s}+\mathbf{n})^H\mathbf{w}]. \tag{6}$$

Assuming that the desired signal is statistically independent from the interferers and the noise, (6) can be rewritten as

$$\mathbf{d} = E[\mathbf{A}\mathbf{s}\mathbf{s}^H\mathbf{A}^H\mathbf{w} + \mathbf{n}\mathbf{n}^H\mathbf{w}]. \tag{7}$$

By also assuming that $|\mathbf{a}_m\mathbf{w}| \ll |\mathbf{a}_1\mathbf{w}|$ for $m = 2, \cdots, K$, the vector $\mathbf{d}$ can be rewritten as

$$\mathbf{d} = E[{\sigma_1}^2\mathbf{a}_1^H\mathbf{w}\mathbf{a}_1 + \mathbf{n}\mathbf{n}^H\mathbf{w}], \tag{8}$$

which can be estimated by the sample cross-correlation vector (SCV) given by

$$\hat{\mathbf{d}}(i) = \frac{1}{i}\sum_{k=1}^{i}\mathbf{x}(k)y^*(k). \tag{9}$$

III-A. Desired Signal Power Estimation

In this subsection, we describe an iterative method for estimation of the desired signal power (σ_1^2) based on our prior work in [10], which can be accomplished by directly using the desired signal steering vector. We need to choose an initial guess for the steering vector mismatch within the presumed angular sector, say $\hat{\mathbf{a}}_1(0)$ and set $\hat{\mathbf{a}}_1(1) = \hat{\mathbf{a}}_1(0)$. By adding the snapshot index i, we can rewrite the received data as

$$\mathbf{x}(i) = \hat{\mathbf{a}}_1(i)s_1(i) + \sum_{k=2}^{K}\mathbf{a}_k(i)s_k(i) + \mathbf{n}(i). \tag{10}$$

Pre-multiplying the above equation by $\hat{\mathbf{a}}_1^H(i)$ and assuming $\hat{\mathbf{a}}_1(i)$ is uncorrelated with the interferers, we obtain

$$\hat{\mathbf{a}}_1^H(i)\mathbf{x}(i) = \hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)s_1(i) + \hat{\mathbf{a}}_1^H(i)\mathbf{n}(i). \tag{11}$$

Taking the expectation of $|\hat{\mathbf{a}}_1^H(i)\mathbf{x}(i)|^2$, we obtain

$$\begin{aligned} E[|\hat{\mathbf{a}}_1^H(i)\mathbf{x}(i)|^2] = E[&(\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)s_1(i) + \hat{\mathbf{a}}_1^H(i)\mathbf{n}(i))^* \\ &(\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)s_1(i) + \hat{\mathbf{a}}_1^H(i)\mathbf{n}(i))]. \end{aligned} \tag{12}$$

Assuming that the noise is statistically independent from the desired signal, then we have

$$E[|\hat{\mathbf{a}}_1^H(i)\mathbf{x}(i)|^2] = |\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)|^2 E[|s_1(i)|^2] + \hat{\mathbf{a}}_1^H(i)E[\mathbf{n}(i)\mathbf{n}^H(i)]\hat{\mathbf{a}}_1(i), \quad (13)$$

where $E[\mathbf{n}(i)\mathbf{n}^H(i)]$ represents the noise covariance matrix $\mathbf{R}_n(i)$ which can be replaced by $\sigma_n^2\mathbf{I}_M$, where the noise variance σ_n^2 can be easily estimated by a specific estimation method. A proper approach is to use a Maximum Likelihood (ML) based method as in [14]. Replacing the desired signal power $E[|s_1(i)|^2]$ and the noise variance σ_n^2 by their estimates $\hat{\sigma}_1^2(i)$ and $\hat{\sigma}_n^2(i)$, respectively, we obtain

$$\hat{\sigma}_1^2(i) = \frac{|\hat{\mathbf{a}}_1^H(i)\mathbf{x}(i)|^2 - |\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)|\hat{\sigma}_n^2(i)}{|\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)|^2}. \quad (14)$$

The expression in (14) has a low cost ($\mathcal{O}(M)$) and can be directly implemented if the desired signal steering vector and the noise level are accurately estimated.

III-B. Proposed Steering Vector Mismatch Estimation

An orthogonal Krylov subspace strategy is proposed in order to estimate the mismatch with reduced cost and deal with situations in which the model order is time-varying. Our idea is based on constructing a linear system which considers the steering vector mismatch as the unknown and solving it by using an iterative Krylov subspace method. Consider a general linear system model given by

$$\mathbf{B}\mathbf{a}_1 = \mathbf{b}, \quad (15)$$

where $\mathbf{B} \in \mathbb{C}^{M\times M}$ and $\mathbf{b} \in \mathbb{C}^{M\times 1}$. Then we need to express $\mathbf{B}$ and $\mathbf{b}$ only using available information (known statistics or estimated parameters), so that we can solve the linear system with the Krylov subspace of order m ($m \ll M$) described by

$$\mathbf{K}_m = span\{\mathbf{b}, \mathbf{B}\mathbf{b}, \mathbf{B}^2\mathbf{b}, \cdots, \mathbf{B}^m\mathbf{b}\}. \quad (16)$$

Taking the complex conjugate of (11), we have

$$\mathbf{x}^H(i)\hat{\mathbf{a}}_1(i) = \hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)s_1^*(i) + \mathbf{n}^H(i)\hat{\mathbf{a}}_1(i). \quad (17)$$

Pre-multiplying both sides of (17) by the terms of (10) and simplifying, we obtain

$$\mathbf{x}(i)\mathbf{x}^H(i)\hat{\mathbf{a}}_1(i) = \hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)s_1(i)s_1^*(i) + \mathbf{n}(i)\mathbf{n}^H(i)\hat{\mathbf{a}}_1(i). \quad (18)$$

Replacing $\mathbf{x}(i)\mathbf{x}^H(i)$ by $\hat{\mathbf{R}}(i)$, $s_1(i)s_1^*(i)$ by $\hat{\sigma}_1^2(i)$ and $\mathbf{n}(i)\mathbf{n}^H(i)$ by $\hat{\sigma}_n^2(i)\mathbf{I}_M$, we obtain

$$\hat{\mathbf{R}}(i)\hat{\mathbf{a}}_1(i) = \underbrace{\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)\hat{\sigma}_1^2(i) + \hat{\sigma}_n^2(i)\hat{\mathbf{a}}_1(i)}_{\hat{\mathbf{b}}(i)}, \quad (19)$$

in which by further defining the expression on the right-hand side as $\hat{\mathbf{b}}(i)$, we can rewrite (19) as

$$\hat{\mathbf{R}}(i)\hat{\mathbf{a}}_1(i) = \hat{\mathbf{b}}(i). \quad (20)$$

Table I. Arnoldi-modified Gram-Schmidt algorithm

For $j = 1, 2, \cdots$ do:
  Compute $\mathbf{u}_j = \hat{\mathbf{R}}\mathbf{t}_j$
  For $l = 1, 2, \cdots, j$, do:
    $h_{l,j} =< \mathbf{u}_j, \mathbf{t}_l >$
    $\mathbf{u}_j = \mathbf{u}_j - h_{l,j}\mathbf{t}_l$
  End do.
  Compute $h_{j,j+1} = \|\mathbf{u}_j\|$.
  If $h_{j,j+1} = 0$ or $j \geq K + 1$,
    set $m = j$;
    break;
  Else compute $\mathbf{t}_{j+1} = \frac{\mathbf{u}_j}{h_{j,j+1}}$.
End do.

As can be seen (20) shares the same form as the linear system equation in (15) and $\hat{\mathbf{b}}(i)$ can be expressed in terms of $\hat{\mathbf{a}}_1(i)$, $\hat{\sigma}_1^2(i)$ and $\hat{\sigma}_n^2(i)$ whereas $\hat{\mathbf{R}}(i)$ can be estimated by (5). In the following step, we employ the Arnoldi-modified Gram-Schmidt algorithm from the FOM method [12], [13] associated with the minimum sufficient rank criterion discussed in [20] to compute an orthogonal Krylov subspace. We define a residue vector to represent the estimation error in the ith snapshot as

$$\hat{\mathbf{r}}(i) = \hat{\mathbf{b}}(i) - \hat{\mathbf{R}}(i)\hat{\mathbf{a}}_1(i), \quad (21)$$

and let

$$\mathbf{t}_1(i) = \frac{\hat{\mathbf{r}}(i)}{\|\hat{\mathbf{r}}(i)\|}. \quad (22)$$

Then the Krylov subspace bases can be computed using the modified Arnoldi-modified Gram-Schmidt algorithm as in Table I (the snapshot index i is omitted here for simplicity). In Table I, $<,>$ denotes the inner product and the parameters $h_{l,j}$ ($l, j = 1, 2, \cdots, m$) are real-valued coefficients, the model order is determined once if one of the following situations is satisfied:

- The execute-and-stop criterion of the original Arnoldi-modified Gram-Schmidt algorithm is satisfied (i.e., $h_{j,j+1} = 0$).
- The minimum sufficient rank for dealing with the SoI and the interferers is achieved (i.e., $j \geq K + 1$, where K is the number of signal sources), so that no more subspace components are necessary for capturing the SoI from all the existing signal sources.

Now by inserting the snapshot index, we have

$$\hat{\mathbf{T}}(i) = [\mathbf{t}_1(i), \mathbf{t}_2(i), \cdots, \mathbf{t}_m(i)], \quad (23)$$

and the Krylov subspace projection matrix is computed by

$$\hat{\mathbf{P}}(i) = \hat{\mathbf{T}}(i)\hat{\mathbf{T}}^H(i). \quad (24)$$

It should be emphasized that the Krylov subspace matrix $\hat{\mathbf{T}}(i)$ obtained here is constructed by starting with the residue vector $\hat{\mathbf{r}}(i)$. In other words, $\hat{\mathbf{T}}(i)$ is constructed with the estimation error of the steering vector. In order to extract the estimation error information and use it to update the

Table II. Proposed OKSPME method

Initialization:

$\hat{\mathbf{w}}(1) = \mathbf{1}$;

choose an initial guess $\hat{\mathbf{a}}_1(0)$ within the sector and set $\hat{\mathbf{a}}_1(1) = \hat{\mathbf{a}}_1(0)$.

For each snapshot $i = 1, 2, \cdots$:

$\hat{\mathbf{R}}(i) = \frac{1}{i}\sum_{k=1}^{i}\mathbf{x}(k)\mathbf{x}^H(k)$

$\hat{\mathbf{d}}(i) = \frac{1}{i}\sum_{k=1}^{i}\mathbf{x}(k)y^*(k)$

Compute the desired signal power

$\hat{\sigma}_1^2(i) = \frac{|\hat{\mathbf{a}}_1^H(i)\mathbf{x}(i)|^2 - |\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)|\hat{\sigma}_n^2(i)}{|\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)|^2}$

Determine the Krylov subspace

$\hat{\mathbf{b}}(i) = \hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{a}}_1(i)\hat{\sigma}_1^2(i) + \hat{\sigma}_n^2(i)\hat{\mathbf{a}}_1(i)$

$\hat{\mathbf{r}}(i) = \hat{\mathbf{b}}(i) - \hat{\mathbf{R}}(i)\hat{\mathbf{a}}_1(i)$

$\mathbf{t}_1(i) = \frac{\hat{\mathbf{r}}(i)}{\|\hat{\mathbf{r}}(i)\|}$

Apply the algorithm in Table I to determine m and $\mathbf{t}_1(i)$,$\cdots$,$\mathbf{t}_m(i)$

$\hat{\mathbf{T}}(i) = [\mathbf{t}_1(i), \mathbf{t}_2(i), \cdots, \mathbf{t}_m(i)]$

Update the steering vector

$\hat{\mathbf{P}}(i) = \hat{\mathbf{T}}(i)\hat{\mathbf{T}}^H(i)$

$\hat{\mathbf{a}}_1(i+1) = \hat{\mathbf{a}}_1(i) + \frac{\hat{\mathbf{P}}(i)\hat{\mathbf{d}}(i)}{\|\hat{\mathbf{P}}(i)\hat{\mathbf{d}}(i)\|}$

$\hat{\mathbf{a}}_1(i+1) = \hat{\mathbf{a}}_1(i+1)/\|\hat{\mathbf{a}}_1(i+1)\|$

Compute the weight vector

$\hat{\mathbf{R}}_{i+n}(i) = \hat{\mathbf{R}}(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i)$

$\hat{\mathbf{w}}(i) = \frac{\hat{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}{\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}$

End snapshot

steering vector mismatch, we can project the SCV $\hat{\mathbf{d}}(i)$ in (9) onto $\hat{\mathbf{P}}(i)$ and add the estimation error to the current estimate of $\hat{\mathbf{a}}_1(i)$ as

$$\hat{\mathbf{a}}_1(i+1) = \hat{\mathbf{a}}_1(i) + \frac{\hat{\mathbf{P}}(i)\hat{\mathbf{d}}(i)}{\|\hat{\mathbf{P}}(i)\hat{\mathbf{d}}(i)\|}. \tag{25}$$

III-C. INC Matrix and Beamformer Weight Vector Computation

Once we have estimated both the desired signal power $\hat{\sigma}_1^2(i)$ and the mismatched steering vector in the previous subsections, the INC matrix can be obtained by subtracting the desired signal covariance matrix out from the SCM as

$$\hat{\mathbf{R}}_{i+n}(i) = \hat{\mathbf{R}}(i) - \hat{\sigma}_1^2(i)\hat{\mathbf{a}}_1(i)\hat{\mathbf{a}}_1^H(i). \tag{26}$$

The beamformer weight vector is computed by

$$\hat{\mathbf{w}}(i) = \frac{\hat{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}{\hat{\mathbf{a}}_1^H(i)\hat{\mathbf{R}}_{i+n}^{-1}(i)\hat{\mathbf{a}}_1(i)}, \tag{27}$$

which has a computational costly matrix inversion $\hat{\mathbf{R}}_{i+n}^{-1}(i)$. The proposed OKSPME method is summarized in Table II.

IV. COMPLEXITY ANALYSIS

The computational complexity is discussed in this subsection. We measure the total number of additions and multiplications (i.e., flops) in terms of the number of sensors M performed for each snapshot by the proposed and the existing algorithms. Note that the SQP method in [6] and the beamspace-based approach in [9] have a highly-variable cost over different snapshots, due to the online optimization based on random choices of the presumed steering vector. The average cost of these methods is $\mathcal{O}(M^{3.5})$. The complexity of the proposed OKSPME algorithm depends on the Krylov subspace model order m, which is determined by Table I and does not exceed $K+1$. For the convenience of comparison, we eliminate all parameters except M by setting them to common values (the values of n in LCWC is set to 50, $m = K+1$, where $K = 3$) and list them in Table III.

Table III. Complexity Comparison

RAB Algorithms	Flops
LOCSME [10]	$4M^3 + 3M^2 + 20M$
RCB [4]	$2M^3 + 11M^2$
SQP [6]	$\mathcal{O}(M^{3.5})$
LOCME [7]	$2M^3 + 4M^2 + 5M$
Beamspace [9]	$\mathcal{O}(M^{3.5})$
OKSPME	$M^3 + 27M^2 + 88M$

V. SIMULATIONS

In this section, we present and discuss the simulation results of the proposed RAB algorithm by comparing it to some of the existing RAB algorithms. We consider a uniform linear array (ULA) of omnidirectional sensors with half wavelength spacing. To produce all the figures, 100 repetitions are executed to obtain each point of the curves and a maximum of $i = 300$ snapshots are observed. The desired signal is assumed to arrive at $\theta_1 = 10^\circ$ and two interferers are set to $\theta_2 = 30^\circ$ and $\theta_3 = 50^\circ$, respectively. The signal-to-interference ratio (SIR) is fixed at 0dB. As prior knowledge, the angular sector in which the desired signal is assumed to be located is chosen as $[\theta_1 - 5^\circ, \theta_1 + 5^\circ]$. The results focus on the beamformer output SINR performance versus the number of snapshots, or a variation of input SNR (-10dB to 30dB) under the coherent local scattering mismatch model, which has a time-invariant nature and the steering vector of the desired signal is modeled as

$$\mathbf{a}_1 = \mathbf{p} + \sum_{k=1}^{4} e^{j\varphi_k}\mathbf{b}(\theta_k), \tag{28}$$

where $\mathbf{p}$ corresponds to the direct path while $\mathbf{b}(\theta_k)(k = 1, 2, 3, 4)$ corresponds to the scattered paths. The angles $\theta_k(k = 1, 2, 3, 4)$ are randomly and independently drawn in each simulation run from a uniform generator with mean 10° and standard deviation 2°. The angles $\varphi_k(k = 1, 2, 3, 4)$ are independently and uniformly taken from the interval $[0, 2\pi]$ in each simulation run. Both θ_k and φ_k change from trials while remaining constant over snapshots.

We set the number of sensors to $M = 12$, the number of signal sources to $K = 3$ and illustrate the SINR versus snapshots and the SINR versus input SNR performance in Fig. 1 and Fig. 2 respectively. The results show that the proposed OKSPME method has a better performance than the LOCSME [10] and other previously reported algorithms.

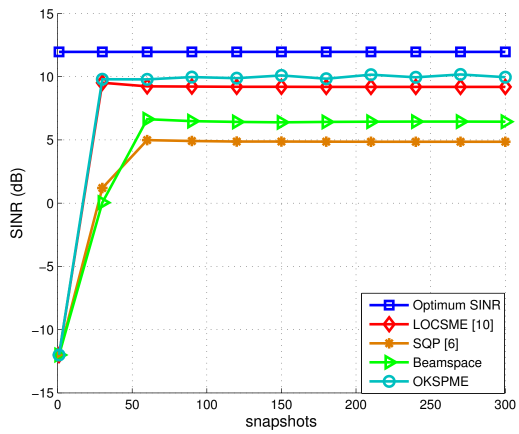

Fig. 1. SINR versus snapshots

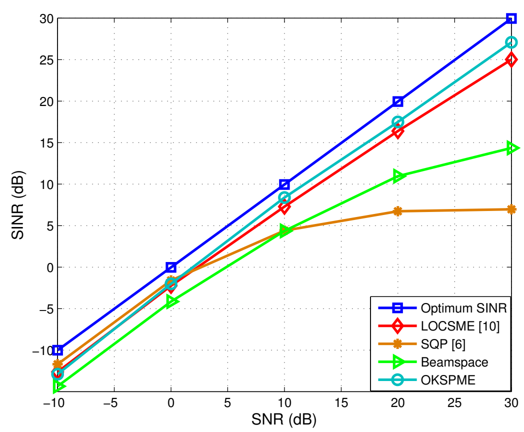

Fig. 2. SINR versus SNR

VI. CONCLUSION

We have proposed the OKSPME RAB algorithm based on the exploitation of cross-correlation mismatch estimation and the use of the orthogonal Krylov subspace. Simulation results have shown that OKSPME outperforms the prior reported methods in terms of the beamformer output SINR, while its complexity is much lower than those methods that require complex optimization algorithms and comparable to previously reported low-complexity methods.

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