On the Design of Low Delay Nearly-PR and PR FIR Cosine Modulated Filter Banks having approximate cosine-roll transition band

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ABSTRACT

An efficient method for designing a class of nearly perfect reconstruction (NPR) or perfect reconstruction (PR) cosine modulated filter banks (CMFBs) with prototype filters having an approximate cosine-rolloff transition band is presented. The NPR CMFBs in [1,2] is first generalized to include low-delay CMFBs. It is shown that a prototype filter with cosine-rolloff transition band will automatically satisfy the flatness condition required by an NPR CMFB. This allows us to formulate the design problem as a convex minimax or least squares problem, and its optimal solution can be solved by the Remez exchange algorithm or more sophisticated semidefinite programming (SDP) method. By using the NPR CMFB so obtained as the initial guess to the nonlinear constrained optimization algorithm such as the Fmincon function of MATLAB, high quality low-delay PR CMFBs can readily be obtained. CMFBs with arbitrary-length linear-phase or low-delay prototype filters can also be obtained readily by this method. Design examples and comparison with existing methods are provided to illustrate the effectiveness of the proposed method.

1. INTRODUCTION

Perfect reconstruction filter banks (PR FBs) have important application in speech, audio, image and array processing. One very efficient FB is the cosine modulation FB (CMFB) [1,2], which has very low design and implementation complexities and excellent frequency selectivity. The design of PR or nearly PR CMFBs is usually obtained by iterative procedures involving unconstrained/constrained nonlinear optimization [1,3,4,10]. When the number of variables and constraints increases, the optimization procedure is rather sensitive to the initial guess of the prototype filter, especially for low-delay CMFBs. This has motivated a number of researchers to study the problem of choosing effective initial guess of the prototype filters in order to get faster convergence speed and better performance [6,7]. Most of them are focused on CMFBs with linear-phase prototype filters [1,5,6,7,8,10].

In this paper, we first generalize the NPR CMFBs in [1,2], which is focused on linear-phase prototype filter, to include low-delay NPR CMFBs. Then, we show that the passband flatness condition of these low-delay NPR CMFBs is equivalent to imposing an approximate cosine-rolloff transition band on the prototype filter. This allows us to formulate the design problem as a convex minimax or least squares problem. Thus, its optimal solution can be solved by the conventional Remez exchange algorithm or the semidefinite programming (SDP) method [9]. Compared to PR systems, NPR systems have larger stopband attenuation but higher reconstruction errors, because the PR constraints are only approximately satisfied. Apart from being able to design NPR CMFB with low system delays, the design complexity of the proposed method is considerably lower than the conventional NPR CMFB with linear-phase prototype [1], which is based on nonlinear optimization. The NPR CMFBs obtained by the proposed method can also be used as the initial guess to nonlinear constrained optimization algorithms, such as the Fmincon function in the MATLAB OPTIM toolbox, to solve for PR CMFBs. Design results show that high quality PR CMFBs with low system delay can be obtained with considerably lower design complexity as compared to an initial guess using a low-delay lowpass prototype filter which is optimized for minimum stopband ripples [3]. The violation of PR constraints of the PR CMFB so obtained is about 10^{-15}, at the expense of higher stopband attenuation. The proposed approach is also applicable to the design of NPR CMFBs with arbitrary-length linear-phase or low-delay prototype filters. Several examples on NPR and PR CMFBs are provided to illustrate the effectiveness of the proposed method. The rest of the paper is organized as follows: In Section 2, the principle of NPR low delay CMFBs derived from a cosine rolloff prototype filter is described. The design method and several design examples are given in Sections 3 and 4, respectively. Finally, conclusions are drawn in Section 5.

2. NPR LOW-DELAY CMFBs

Recall that the M analysis filters of the NPR CMFB proposed in [1,2] are obtained by modulating a linear-phase prototype filter $P_0(z)$ as follows,

$$H_k(z) = a_k U_k(z) + a_k^* V_k(z)\ ,\ 0 \le k \le M-1\ , \tag{1}$$

where $U_k(z) = c_k P_0(zW^{k+0.5})$, $V_k(z) = c_k^* P_0(zW^{-(k+0.5)})$, a_k and c_k are unit-magnitude constants, and $W_{2M}^k = e^{-2k\pi/(2M)}$. The cutoff frequency of the prototype filter is $\pi/(2M)$. The main aliasing components in the reconstruction can be cancelled, if a_k and c_k are chosen as

$$a_k = e^{j\theta_k}, \theta_k = (-1)^k(\pi/4) \text{ and } c_k = W^{(k+0.5)N/2}. \tag{2}$$

The resulting analysis and synthesis filters are given by

$$h_k(k) = 2h(n)\cos\left[\frac{\pi}{M}(k+\tfrac{1}{2})(n-\tfrac{D}{2}) + (-1)^k\frac{\pi}{4}\right],$$
$$f_k(k) = 2h(n)\cos\left[\frac{\pi}{M}(k+\tfrac{1}{2})(n-\tfrac{D}{2}) - (-1)^k\frac{\pi}{4}\right], \tag{3}$$

where $0 \le k \le M-1$, $0 \le n \le N-1$; $h(n)$, N, and $D/2=(N-1)/2$ are respectively the impulse response, filter length, and delay of the prototype filter. For the case of linear-phase prototype filter, the system transfer function is

$$T(e^{j\omega}) \approx \tfrac{1}{M}\Sigma_{k=0}^{M-1}[U_k^2(e^{j\omega}) + V_k^2(e^{j\omega})]\,. \tag{4}$$

Since $h(n)$ is linear-phase, $T(z)$ is also linear-phase. The magnitude distortion of the reconstruction can be minimized by making $T(e^{j\omega})$ as close to a constant as possible. Suppose that the passband and stopband ripples are sufficiently small, then $T(e^{j\omega})$ will be nearly constant in the passband of the analysis filters. Therefore, we need only to consider their transition bands. As $U_k(e^{j\omega})$ and $V_k(e^{j\omega})$ are frequency shifted versions of $P_0(z)$, we only need to impose the following constraints on

$$P_0(z):\quad |P_0(e^{j\omega})|^2 + |P_0(e^{j(\omega-(\pi/M))})|^2 = 1\,, \tag{5}$$

where $\omega \in [0 < \omega_s, \omega_p < \pi/M]$, and ω_s and ω_p are respectively the passband and stopband cutoff frequencies. If the least squares design criterion is employed, the problem can be formulated as

$$\min_{\boldsymbol{h}}(\phi_1 + \lambda\phi_2) \tag{6}$$

where $\phi_1 = \int_{\omega_s}^{\pi} |P_0(e^{j\omega})|^2 \, d\omega$ measures the stopband energy of the analysis filters, $\omega_s = (\pi/(2M)) + \varepsilon$ for some positive real number ε, $\phi_2 = \int_{\omega_p}^{\omega_s} \{|P_0(e^{j\omega})|^2 + |P_0(e^{j(\omega-(\pi/M))})|^2 - 1\} d\omega$ is the flatness measure which helps to minimize the magnitude reconstruction error, and λ is a positive weighting factor providing a tradeoff between the two components. Since ϕ_2 is not quadratic, (6) has to be solved using unconstrained nonlinear programming [1,page365]. In [5], it is observed that this problem is convex and a modified Remez exchange method was proposed to solve for (6) using the minimax criterion.

We now generalize these NPR CMFB to include low-delay prototype filters, which give a lower system delay. The $P_0(z)$ in these low-delay NPR CMFBs is not linear-phase and its frequency response can be written as $P_0(e^{j\omega}) = e^{-j\omega D/2} P_R(\omega)$, where D and $P_R(\omega)$ are respectively the desired passband delay and complex magnitude of $P_0(e^{j\omega})$. For linear-phase prototype filter $h(n)$, $P_R(\omega)$ is real-valued. For the low-delay case, $D/2$ can be smaller than $(N-1)/2$, and $P_R(\omega)$ is a complex function, which ideally should approximate the desired magnitude characteristics of the filter.

Careful examination shows that the major aliasing components can still be suppressed if a_k is chosen as (2). The choice of c_k only affects the passband phase offset of the analysis filters. For simplicity, we choose $c_k = W^{(k+0.5)D/2}$ so that this phase offset is equal to zero. The analysis and synthesis filters are then given by (3), except that $D/2$ is now the passband delay of the prototype filter. The distortion transfer function is still given by (4). Using a similar reasoning above, it can be shown that the flatness condition on $P_0(z)$ is

$$P_R^2(\omega) + P_R^2(\omega - (\pi/M)) = 1 . \qquad (7)$$

Note, (7) is different from (5) because $P_0(z)$ is not linear-phase and $P_R(\omega)$ is now a complex function. If the least squares criterion is used, we can define the following flatness measure : $\phi_2 = \int_{\omega_p}^{\omega_s} |P_R^2(\omega) + P_R^2(\omega - (\pi/M)) - 1|^2 \, d\omega$ and substituting it into (6) to solve for the NPR prototype filter. Since this is not quadratic, it will again involve the use of unconstrained nonlinear optimization. Here, we shall consider a different formulation which gives rise to a convex problem and it can be solved using the Remez exchange or more generally as a SDP, if LS or minimax design criterion are used together with additional linear and convex quadratic constraints.

Suppose that the transition band of $P_R(\omega)$, $\omega \in [\omega_s, \omega_p]$, has the form of a cosine function, hence the name cosine rolloff, then

$$P_R(\omega) = \cos(\tfrac{\pi}{2\Delta\omega}(\omega - \omega_p)), \; \omega \in [\omega_s, \omega_p], \qquad (8)$$

where $\Delta\omega = \omega_s - \omega_p$ is the transition bandwidth. Thus, the flatness condition in (7) is automatically satisfied if

$$\omega_p = (\tfrac{\pi}{2M} - \tfrac{\Delta\omega}{2}) \text{ and } \omega_s = (\tfrac{\pi}{2M} + \tfrac{\Delta\omega}{2}) . \qquad (9)$$

In other words, the passband and stopband cutoff frequencies are symmetric around $\frac{\pi}{2M}$. This follows from the fact that

$$\begin{aligned} &P_R^2(\omega) + P_R^2(\omega - (\pi/M)) \\ &= \cos^2(\tfrac{\pi}{2\Delta\omega}(\omega - \omega_p)) + \sin^2(\tfrac{\pi}{2\Delta\omega}(\omega - \omega_p)) = 1 . \end{aligned} \qquad (10)$$

Hence, the flatness condition can be structurally imposed and we end up with a nonlinear-phase lowpass FIR filter design problem having a cosine-shaped transition band as shown in (8). As a consequence, if the least squares criterion is used, it can be solved as a system of linear equation. Alternatively, for the minimax criterion, it is a complex Chebyshev approximation problem, which can be solved by the Cremez algorithm in MATLAB. Further, if linear or quadratic convex constraints are imposed, then it can be solved using SDP. As there is no assumption on the length of $P_0(z)$, the proposed formulation is also applicable to the design of NPR CMFBs with arbitrary length. A similar cosine-roll prototype filter was considered in [8] for the linear-phase case. It differs from (8) in that the whole passband and stopband is obtained by transforming a cosine function using polynomials In this case, it is rather difficult to define the cutoff frequencies and ripples of the various bands. Also, it uses the least squares criterion, and the problem is solved using the eigenfilter method. The low-delay case was not discussed.

3. THE PROPOSED SDP DESIGN METHOD

As mentioned earlier, the design problem can be performed by LS, Cremez, and SDP, depending on the objective functions and constraints imposed. Due to the generality of SDP, we shall formulate our problem using this framework.

In SDP, a linear function is minimized subject to the positive semidefinite constraint of a set of symmetry matrices, which are affined functions of the variable (called linear matrix inequalities (LMI)) [9]. Without loss of generality, we consider the more difficult minimax criterion, the LS design can be derived similarly. First of all, we rewrite the design problem as follows

Minimize δ

Subject to $W^2(\omega) |H(e^{j\omega}) - H_d(e^{j\omega})|^2 \le \delta$, (11)

where $-\pi \le \omega \le \pi$ and $W(\omega)$ is a positive weighting function. $H_d(z)$ is the desired response of the filter which can be written as:

$$H_d(\omega) = \begin{cases} e^{-j\omega D/2} & \omega \in [0, \omega_p] \\ \cos(\tfrac{\pi}{2\Delta\omega}(\omega - \omega_p)) \cdot e^{-j\omega D/2} & \omega \in [\omega_p, \omega_s] \\ 0 & \omega \in [\omega_s, \pi] \end{cases} . \qquad (12)$$

The desired response in $\omega \in [-\pi, 0]$ is $H_d(-\omega)$. To simplify notation, the frequency response of the prototype is denoted by $H(\omega)$ and it can be written as:

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-jn\omega} = \boldsymbol{h}^T \{\boldsymbol{c}(\omega) - j\boldsymbol{s}(\omega)\}, \qquad (13)$$

where $\boldsymbol{h} = [h(0), ..., h(N-1)]^T$, $\boldsymbol{c}(\omega) = [1, \cos(\omega), ..., \cos((N-1)\omega)]^T$, and $\boldsymbol{s}(\omega) = [1, \sin(\omega), ..., \sin((N-1)\omega)]^T$. Let $H_{R\_d}(\omega)$ and $H_{I\_d}(\omega)$ be the real and imaginary parts of $H_d(\omega)$, then (8) can be further rewritten as:

Minimize δ

Subject to $\delta - \{a_R^2(\omega) - a_I^2(\omega)\} \ge 0$, (14)

where $-\pi \le \omega \le \pi$, $a_R^2(\omega) = W^2(\omega)\{\boldsymbol{h}^T \boldsymbol{c}(\omega) - H_{R\_d}(\omega)\}^2$, and $a_I^2(\omega) = W^2(\omega)\{\boldsymbol{h}^T \boldsymbol{s}(\omega) - H_{I\_d}(\omega)\}^2$. Digitizing the frequency variable ω over a dense set of frequencies $\{\omega_i, 1 \le i \le m\}$ on the frequency interval of interests and stacking them together in the diagonal entries of matrix $\Gamma(\boldsymbol{x})$, one gets the following standard SDP problem:

Minimize $\boldsymbol{c}^T \boldsymbol{x}$

Subject to $\Gamma(\boldsymbol{x}) \ge 0$, (15)

where $\boldsymbol{c} = [1, 0, ..., 0]^T$, $\Gamma(\boldsymbol{x}) = diag\{\Gamma(\boldsymbol{h}, \omega_1), ..., \Gamma(\boldsymbol{h}, \omega_m)\}$, and $\boldsymbol{x} = [\delta, \boldsymbol{h}^T]^T$ is the augmented variable. Now, the design problem is ready to be solved by the function Lmiterm in the LMI toolbox of MATLAB. Because it is a convex optimization problem, the optimality of the solution is guaranteed, if it exists. Further, additional linear and quadratic constraints can easily be incorporated into (15) by stacking them as new diagonal entries of $\Gamma(\boldsymbol{x})$. One disadvantage of SDP is its slightly long running time. For simple design problem involving only least squares (with equality constraints) or simple minimax criterion, common functions, such as Ls (eigenfilter) and Cremez, can also be used. Since NPR CMFBs so obtained are of high quality, they can serve as initial guesses to constrained nonlinear optimizer, such as DNCONF in the IMSL mathematical library, to solve for the PR CMFB with the same filter parameters. Due to page limitation, the formulation is omitted here. Interested readers are referred to [3,4] for the formulation of the PR constraints. In this work, we wish to perform the design entirely in MATLAB. Therefore, we have used the function Fmincon in the OPTIM toolbox, which solves a constrained nonlinear optimization problem in multiple variables. The standard algorithm used by Fmincon is sequential quadratic programming (SQP), which represents the state of the art in nonlinear programming methods.

4. DESIGN EXAMPLES

We now present several design examples.

Example 1: **CMFB with Linear-Phase Prototype Filter**.

In this example, the example in [1,pp.387] is used to evaluate the efficiency of our new design method. The parameters are $M = 17$, $N = 102$, $\omega_s = 0.059\pi$. A reconstruction error (Epp) of 6.760×10^{-3} and a stopband attenuation (As) of 42.81 dB are obtained (Fig.1 and table 1). Here, Epp refers to the maximum peak-to-peak ripple of the distortion function. The result showed that our approach is comparable to the conventional method in [1], which employs nonlinear optimization. It should be noted that the weighting in (11) for different frequency bands can be used to control the relative tradeoff between the reconstruction error and stopband attenuation.

	As (dB)	ω_s	Epp
[1]	40.65	0.059π	6.790×10^{-3}
Proposed method	42.81	0.059π	6.760×10^{-3}

TABLE 1. Design results of example 1.

Example 2: **NPR and PR Low-delay FIR CMFBs**.

In this example, NPR FIR CMFBs with lower system delay will be designed. The advantages of these new NPR low-delay FIR CMFBs are their low design and implementation complexities, flexibilities in trading reconstruction errors for higher performance such as stopband attenuation or lower system delay. Fig. 2 shows an 4-channel low-delay CMFB with $N = 56$, $\omega_s = 0.1875\pi$ and a system delay of 39 samples designed by the proposed method. Figure 3 shows another low-delay CMFB with 8 channels and a longer filter length of $N = 112$. $\omega_s = 0.09375\pi$ and the system delay is 79 samples. The reconstruction error is of the order 1.946×10^{-2}.

If a low-delay lowpass filter with minimax error ripples is used as the prototype filter, then ‘bump’ or ‘dip’ will appear around $\omega =$ 0 and π as shown in Fig. 2(b) and 3(b). This illustrates the effectiveness of the passband flatness constraint. Also, much faster convergence speed was observed when the NPR cosine-rolloff prototype filters were used as initial guesses to the function Fmincon in MATLAB for determining the corresponding PR CMFBs. The details are summarized in Table 2. Compared with the initial guess using general low-delay FIR filters in the approach [3] (an approach based on constrained optimization), the proposed NPR initial guess is more efficient and the stopband attenuations of the PR CMFBs so obtained are slightly better (around 0.3-0.7 dB). The results also show that it is easier to obtain a PR CMFB using the least square (LS) criterion than using the minimax criterion. However, the performance of the latter is considerably better as shown in Fig. 2(c) and (d), and from Fig. 3(c) and (d).

	LS	Min-Max
4-channel NPR CMFBs (Fig.2)		
[3]	136	596
Proposed method	79	234
8-channel NPR CMFBs (Fig. 3)		
[3]	160	512
Proposed method	84	338

Table 2. Design result for PR CMFBs (number of iterations for Fmincon). The violation in PR constraints is of the order 10^{-15}.

Example 3: **Arbitrary-Length NPR CMFBs**.

In this example, arbitrary-length NPR CMFBs with linear-phase and low-delay prototype filters are designed. Fig. 4(a) shows the NPR CMFB so designed with $M = 4$, $N = 54$ and $\omega_s = 0.225\pi$ which is derived from a linear-phase prototype filter. The reconstruction error (Epp) is 4.955×10^{-3}. Fig. 4(b) is an NPR low-delay CMFB with $M = 3$, $N = 34$, $\omega_s = 0.27778\pi$ and the total system delay is 27 samples. The reconstruction error (Epp) is 9.881×10^{-3}. Due to page limitation, the details of their PR counterparts are omitted. The stopband attenuation of the 3-channel and 6-channel PR CMFBs are 21.29 (vs. the prototype: 23.43dB) dB and 35.87 (vs. the prototype: 38.06dB) dB, respectively. Their PR constraint violations are of the order 10^{-15} and they can be traded for higher stopband attenuation.

5. CONCLUSION

The theory and design of a class of NPR CMFBs with cosine-roll transition band and low system delay are presented. Unlike conventional method, which is based on nonlinear optimization, the new design method employs traditional filter design methods such as the complex remez, least squares or the SDP algorithms. The FBs so obtained have good stopband attenuation and low system delay with a reasonably low reconstruction error. They can also be used as initial guesses to simplify the design of PR CMFBs using nonlinear constrained optimization. Design examples show that the proposed method is very effective and efficient in designing the linear-phase and low-delay cosine-rolloff prototype filters for both NPR and PR CMFBs. Examples for designing arbitrary-length CMFBs are also presented.

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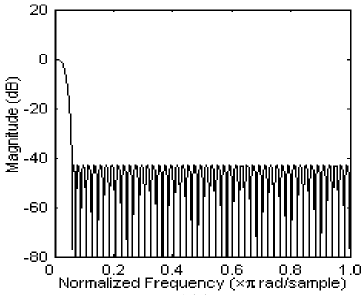
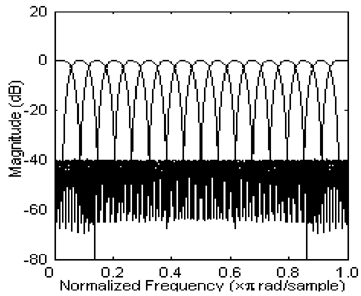
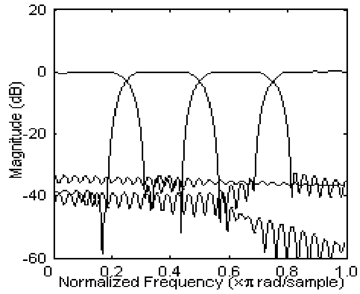


1(a)

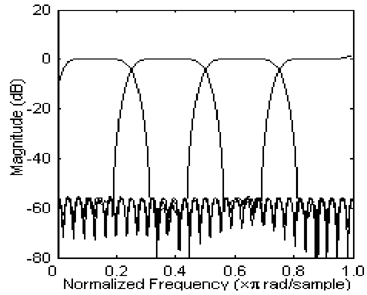


1(b)

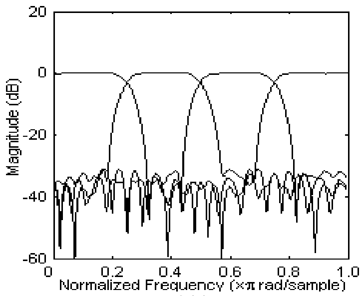
Fig.1 (a) The linear-phase cosine rolloff prototype filter ($M=17$, $N=102$); (b) its analysis filters.



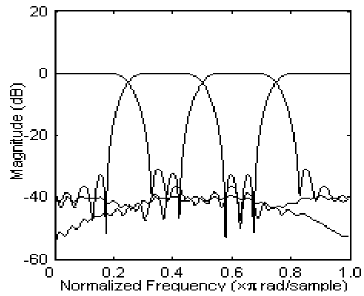
2(a)



2(b)

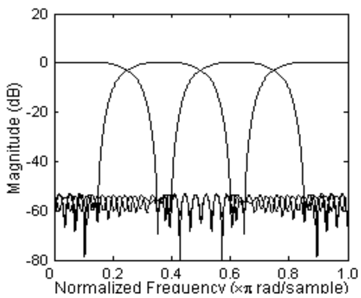


2(c)

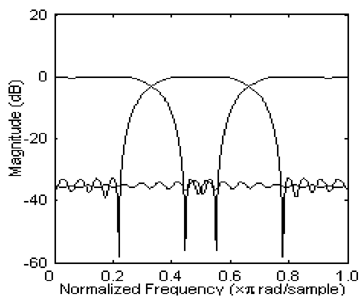


2(d)

Fig 2. Analysis filters of the 4-channel CMFBs designed ($N=56$, $D=39$). (a) NPR CMFB with the cosine rolloff prototype; (b) CMFB using a low-delay prototype low-pass filter (note, the bumps at $\omega=0$ and π ; (c) PR CMFB based on (a)'s as initial guess using minimax criterion; (d) PR CMFB based on (a)'s as initial guess using least square criterion.

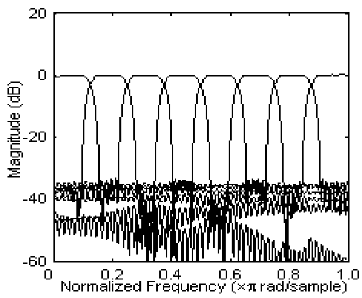


4(a)

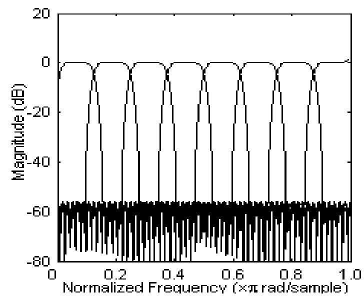


4(b)

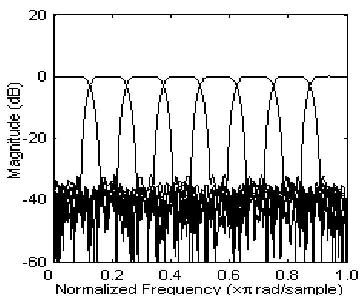
Fig. 4. Analysis filters of the arbitrary-length NPR CMFBs designed. (a) $M=4$, $N=54$, $D=53$; (b) $M=3$, $N=34$, $D=27$.



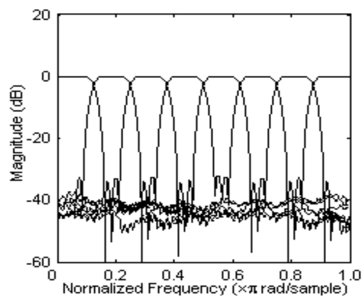
3(a)



3(b)



3(c)



3(d)

Fig 3. Analysis filters of the 8-channel CMFBs designed ($N=112$, $D=79$). (a) NPR CMFB with cosine rolloff prototype; (b) CMFB using a low-delay prototype low-pass filter (note, the bumps at $\omega=0$ and π ; (c) PR CMFB using (a) as initial guess using minimax criterion; (d) PR CMFB using (a) as initial guess using least square criterion.