

k -th Best Device Selection for Scheduling in Wireless Powered Communication Networks

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Abstract—This paper studies the performance of a wireless powered communication network consisting of a finite number of batteryless devices that harvest radio frequency energy. We consider novel selection/scheduling schemes, where the k -th best device is selected for information transmission. The proposed schemes correspond to different complexity and are based on: a) the end-to-end (e2e) signal-to-noise ratio (SNR), b) the energy harvested at the devices, and c) the conventional channel-based max-min selection policy. By considering a non-linear energy harvesting (EH) model, we derive analytical expressions for the outage probability of each selection scheme by using high order statistics. We also consider an asymptotic scenario, where the number of devices increases and analyze the behavior of the system by applying extreme value theory. Due to the saturation effects of the non-linear EH model, the performance of all the proposed schemes converges to an error floor. Our results show that the scheme based on the e2e SNR achieves the best performance and the one based on the EH the worst. The derived analytical framework provides useful insights on the design of such networks.

Index Terms— k -th best selection, order statistics, wireless power transfer, extreme value theory.

I. INTRODUCTION

The rapid evolution of the Internet of Things leads inevitably to the large-scale deployment of sensor nodes and a huge amount of information flow, and thus recharging and controlling these devices regularly becomes inconvenient. Undoubtedly, during the last few years, the revolutionary contribution of wireless power transfer (WPT) in the field of wireless communications is visible through many sectors of our lives. This technique constitutes a promising solution for extending the lifetime of power-constrained devices because of the ability of powering devices remotely by harvesting energy obtained from dedicated radio frequency (RF) signals [1].

However, as it is known, wireless communication channels suffer from various factors such as path-loss, shadowing and fading. In order to mitigate the fading effect and improve the performance of those channels, diversity techniques can be implemented [2]. Particularly, in multiuser wireless systems, multiuser selection diversity scheme exploits the existence of the differently faded replicas of the same signal, by scheduling the user with the best channel conditions for transmission [3].

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In addition, in cooperative relaying networks, the selection of the best relay is vital for the system performance optimization [4]. Several relay selection protocols have been studied in the literature [5], [6].

In recent works, multiuser selection diversity has been implemented in wireless powered communication systems. In the context of simultaneous wireless information and power transfer (SWIPT), the outage performance of relay selection in an amplify-and-forward relay network considering as relay selection criterion the highest end-to-end (e2e) signal-to-noise ratio (SNR), is studied in [7]. In order to prolong the lifetime of a SWIPT network with multiple energy-preserving decode-and-forward relays, the relay selection scheme that is proposed in [8] is based on the maximal remaining energy of the relays. In wireless powered communication networks (WPCN) context, the problem of relay selection in wireless powered cooperative networks is studied in [9], analyzing the outage performance of the system for different relay selection protocols.

The theory of order statistics has found application in the performance analysis of such diversity techniques, since it is known for its simple principle "select the best" [3]. However, the best user may not be available due to scheduling or load-balancing conditions [10]. As a result, the k -th best user selection scheme constitutes an interesting practical solution. In [10], the authors analyze the performance of cooperative-diversity networks with the N -th best relay selection scheme, deriving closed form expressions in terms of different performance metrics over identical and non-identical Rayleigh fading channels. An extreme value theory (EVT) approach is proposed in [12], for the asymptotic analysis of effective, average throughput and average bit error probability of the k -th best link over different fading channels. Closed form asymptotic expressions for the average and effective throughput of the k -th best secondary user in noise-limited and interference-limited secondary multiuser network of underlay cognitive radio systems are derived in [13] and [14], respectively.

In our paper, we examine the k -th best selection problem in a WPCN. We study a WPCN scenario, where a finite number of batteryless devices harvest RF energy from a dedicated energy transmitter and the k -th best device is selected for information transmission to the receiver. The selection mechanism consists of three novel selection/scheduling schemes corresponding to different complexity based on: a) the e2e SNR, b) the energy harvested at the devices, and c) the

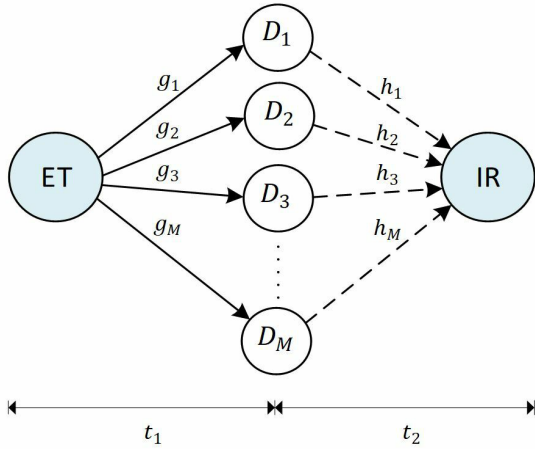


Fig. 1. A WPCN topology consisting of an ET that transmits power to M batteryless devices and an IR that receives the information from the k -th best device.

conventional channel-based max-min selection policy. In contrast to previous works, which use a linear energy harvesting (EH) model, we assume a non-linear EH model that captures practical limitations of the energy harvester, i.e., the saturation effect (the output power remains constant above a power level) [11]. Taking into consideration the complicated form of the analytical expressions due to the non-linearities of the model, we simplify the results and study the asymptotic behavior of the system as the number of the devices increases by using EVT tools. We prove that our model converge to the Gumbel distribution and by using the CDF of the best we generalize for the k -th best device. We derive analytical expressions for both the asymptotic and non-asymptotic performance of the system in terms of outage probability for the proposed selection schemes and provide useful insights on the design of the network.

II. SYSTEM MODEL

A. Topology/Channel Model

We consider a WPCN network consisting of an energy transmitter (ET), M independent and identically distributed (i.i.d.) devices D_i , $i = 1, \dots, M$ and an information receiver (IR); all the nodes are equipped with single antennas. Time is slotted and the time slot duration is equal to T (time units). During the harvest phase with duration $t_1 T$, the ET transmits an RF energy signal with power P_t to the devices which harvest energy based on the received RF signal. During the communication phase with duration $t_2 = (1 - t_1)T$, the k -th best device (determined by the selection mechanism) transmits information to IR by using all the stored energy. We consider Rayleigh fading and denote by g_i , h_i the channel coefficient for energy and information transmission, respectively. They are both complex Gaussian distributed with zero mean and unit variance. We consider symmetric additive white Gaussian noise with variance σ_n^2 for all the wireless links. Finally, the channel state information is perfectly known. Fig. 1 shows the considered system model.

B. Energy harvesting model

The devices of the network have WPT capabilities, and through their rectenna harvest RF power and convert it to direct current power in order to be active. The rectification process is based on diode circuits and is a non-linear function. In order to capture the non-linear characteristics of EH circuits, several practical EH models have been proposed. In our analysis, we are using a non-linear EH model that was proposed in [11]. This EH model belongs to high energy harvester input power regime group that captures practical limitations of the energy harvester i.e., the saturation effect. The parameters of the model are determined through curve fitting tools in order to achieve a more realistic approximation of the energy harvester behavior. It is worth noting that this model is utilized for both saturation and non-saturation regimes and it is characterized by high accuracy. Compared to other models is more mathematically tractable and as a result simplifies our analysis. Assuming $T = 1$ (time units) for simplification, the energy E_i harvested by the i -th device during the harvest phase t_1 is described by [11]

$$E_i = t_1 \left(\frac{aP_t |g_i|^2 + b}{P_t |g_i|^2 + c} - \frac{b}{c} \right), \quad (1)$$

where $|g_i|^2$ is the channel gain between the ET and the i -th device and a , b , c the parameters determined by the rectification circuit through curve fitting.

C. Information transfer and selection

Since all the low power devices harvest energy and become active, the k -th best device is selected to access the channel and transmits its own data by using its stored energy. The output SNR for the i -th device is given by

$$X_i = \frac{|h_i|^2}{t_2 \sigma_n^2} E_i, \quad (2)$$

where $|h_i|^2$ is the channel gain between the i -th device and the IR. The k -th best selection mechanism is based on the principles of each selection scheme and the details of the selection policies are described in the following section.

D. Order statistics and extreme value theory

According to order statistics theory [3], we assume the ordering: $\gamma_1 \leq \gamma_2 \leq \dots \leq \gamma_M$, where γ_i , $i \in \{1, \dots, M\}$ are i.i.d. random variables which are defined based on the specific selection scheme. In other words, γ_1 corresponds to the worst device and γ_M to the best. Assuming that i^* denotes the k -th best device's index for each selection scheme, the PDF of γ_{i^*} is given by [12]

$$f_{\gamma_{i^*}}(x) = k \binom{M}{k} f_{\gamma_i}(x) F_{\gamma_i}(x)^{M-k} (1 - F_{\gamma_i}(x))^{k-1}, \quad (3)$$

where $F_{\gamma_i}(x)$ and $f_{\gamma_i}(x)$ are the cumulative distribution function (CDF) and probability density function (PDF) of a random device, respectively. For scenarios, where the selection is over a large number of devices, $M \rightarrow \infty$, we analyze the system behavior through an EVT based approach. In asymptotic

analysis of order statistics, the limiting distribution is a useful tool to approximate the system behavior when the number of the samples is too large. Specifically, the limiting distribution of the k -th best device can be approximated through the limiting distribution of the best device ($k = 1$) which has been established to be one of the three types: Fréchet, Weibull and Gumbel distribution [3]. In particular, it is proved that the limiting distribution of the best device is of the Gumbel type with CDF given by

$$G(x) = \exp(-\exp(-x)), \quad -\infty < x < \infty, \quad (4)$$

satisfying the following criterion for all the selection schemes

$$\lim_{x \rightarrow \infty} \frac{1 - F_{\gamma_i}(x)}{f_{\gamma_i}(x)} = \lambda, \quad \lambda > 0, \quad (5)$$

The normalizing constants η and ξ satisfy the following condition $\lim_{M \rightarrow \infty} F_{\gamma_M}(\xi x + \eta) = G(x)$, where $F_{\gamma_M}(\cdot)$ is the CDF of the best device. These constants can be obtained by solving the following equations [3]

$$1 - F_{\gamma_i}(\eta) = \frac{1}{M}, \quad 1 - F_{\gamma_i}(\eta + \xi) = \frac{1}{eM}, \quad (6)$$

where e is Euler's number. The CDF of the limiting distribution of the k -th best device for fixed k and $M \rightarrow \infty$ is described in [12] by

$$G^{(k)}(x) = G(x) \sum_{j=0}^{k-1} \frac{[-\log(G(x))]^j}{j!}. \quad (7)$$

where $G(x)$ is given by (4).

III. SELECTION SCHEMES AND PERFORMANCE ANALYSIS

In this section, we present the proposed k -th best selection schemes and analyze their performance in terms of outage probability by deriving analytical expressions. The outage probability is defined as the probability that the information rate falls below the required threshold level. The general expression of the outage probability for the k -th best selection schemes in the considered system model is given by

$$\Pi^{(k)}(x) = \mathbb{P}\{t_2 \log_2(1 + X_{i^*}) \leq Q\} = \mathbb{P}\{X_{i^*} \leq x\}, \quad (8)$$

where X_{i^*} is the output SNR of the k -th best device given by (2), Q is the required threshold level and $x = 2^{\frac{Q}{t_2}} - 1$. Below the three proposed selection schemes are described in details.

A. SNR-based selection scheme

According to the SNR-based selection (SBS) scheme, the k -th best device is the one that achieves the k -th highest output SNR. Assuming the ordering for the output SNRs of M devices as $X_1 \leq X_2 \leq \dots \leq X_M$, the output SNR of the k -th best device that is selected for information transmission to the IR, is denoted by X_{i^*} , with

$$i^* = \arg \max_{i \in \{1, \dots, M\}}^{(k)} \{X_1, \dots, X_M\}. \quad (9)$$

In order to derive the outage probability, it is required to find the CDF of the k -th best device's output SNR, which is given by the following proposition.

Proposition 1. *The outage probability for the SBS scheme where the k -th best is selected, is defined by*

$$\Pi_{\text{SBS}}^{(k)}(x) = I_{F_{X_i}(x)}(M - k + 1, k), \quad (10)$$

where $I_\psi(p, q) = \frac{1}{B(p, q)} \int_0^\psi t^{p-1} (1-t)^{q-1} dt$ denotes the incomplete beta function and

$$F_{X_i}(x) = 1 - \exp(-\zeta) 2\sqrt{\theta} K_1(2\sqrt{\theta}), \quad (11)$$

describes the CDF of the i -th device's output SNR; $K(\cdot)$ is the modified Bessel function of the second kind of the first order, $\theta = \frac{\zeta c}{P_t}$ and $\zeta = \frac{\sigma_n^2 c t_2 x}{t_1(a c - b)}$.

Proof. See Appendix A. ■

Remark 1. *For the special case where the best device is selected ($k = 1$), the CDF of the k -th best device's output SNR is simplified and the outage probability for the SBS scheme is given by*

$$\Pi_{\text{SBS}}^{(1)}(x) = \left(1 - \exp(-\zeta) 2\sqrt{\theta} K_1(2\sqrt{\theta})\right)^M. \quad (12)$$

Remark 2. *For the special case with $P_t \rightarrow \infty$, the asymptotic outage probability for the SBS scheme where the k -th best is selected, is given by*

$$\Pi_{\text{SBS}}^\infty(x) = I_{F_{X_i}^\infty(x)}(M - k + 1, k), \quad (13)$$

where $F_{X_i}^\infty(x) = 1 - \exp(-\zeta)$.

B. Energy-based selection scheme

According to the energy-based selection (EBS) scheme, the k -th best device is the device that harvests the k -th most energy. Assuming the ordering for the harvested energy of M devices as $E_1 \leq E_2 \leq \dots \leq E_M$, the output SNR of the k -th best device which is selected for information transmission to the IR, is denoted by X_{i^*} , with

$$i^* = \arg \max_{i \in \{1, \dots, M\}}^{(k)} \{E_1, \dots, E_M\}. \quad (14)$$

Proposition 2. *The outage probability for the EBS scheme where the k -th best is selected, can be written as*

$$\Pi_{\text{EBS}}^{(k)}(x) = k \binom{M}{k} \left(B(k, M - k + 1) - \Phi(m) \right), \quad (15)$$

where $B(p, q) = \int_0^1 t^{p-1} (1-t)^{q-1} dt$ is the beta function and

$$\Phi(m) = 2 \exp(-\zeta) \sum_{m=0}^{M-k} (-1)^m \binom{M-k}{m} \sqrt{\frac{\theta}{k+m}} \times K_1 \left(2\sqrt{\theta(k+m)} \right). \quad (16)$$

Proof. See Appendix B. ■

Remark 3. *For the special case where the best device is selected ($k = 1$), the outage probability for the EBS scheme is given by*

$$\Pi_{\text{EBS}}^{(1)}(x) = 1 - 2M \exp(-\zeta) \sum_{m=0}^{M-1} (-1)^m \binom{M-1}{m} \times \sqrt{\frac{\theta}{m+1}} K_1 \left(2\sqrt{\theta(m+1)} \right). \quad (17)$$

Remark 4. For the special case with $P_t \rightarrow \infty$, the asymptotic outage probability for the EBS scheme where the k -th best is selected, is given by

$$\Pi_{\text{EBS}}^\infty(x) = 1 - \exp(-\zeta). \quad (18)$$

C. Max-min selection scheme

According to the max-min selection (MMS) scheme, the worst link of each device pair is determined and then the device pair with the strongest worst link is selected [15]. Assuming that the worst link of each device is denoted by $\rho_i = \min\{|g_i|^2, |h_i|^2\}$, $i \in \{1, \dots, M\}$ and $\rho_1 \leq \rho_2 \leq \dots \leq \rho_M$, the output SNR of the k -th best device which is selected for information transmission to the IR, is denoted by X_{i^*} , where

$$i^* = \arg \max_{i \in \{1, \dots, M\}}^{(k)} \{\rho_1, \dots, \rho_M\}. \quad (19)$$

Proposition 3. The outage probability for the MMS scheme where the k -th best is selected, can be expressed as

$$\begin{aligned} \Pi_{\text{MMS}}^{(k)}(x) &= k \binom{M}{k} \sum_{m=0}^{M-k} (-1)^m \binom{M-k}{m} \\ &\times \left(\frac{1 - \exp(-2s(k+m))}{k+m} \right. \\ &\quad - \int_0^s \exp\left(-\zeta - \frac{\theta}{y} - y(2k+2m-1)\right) dy \\ &\quad \left. - \int_r^s \exp\left(\frac{\theta}{\zeta-z} - z(2k+2m-1)\right) dz \right), \end{aligned} \quad (20)$$

where $s = \frac{1}{2}(\sqrt{\zeta^2 + 4\theta} + \zeta)$, $\theta = \frac{\zeta c}{P_t}$ and $r = \zeta = \frac{\sigma_n^2 c t_2 x}{t_1(ac-b)}$.

Proof. See Appendix C. ■

Remark 5. For the special case where the best device is selected ($k = 1$), the outage probability for the MMS scheme is given by

$$\begin{aligned} \Pi_{\text{MMS}}^{(1)}(x) &= M \sum_{m=0}^{M-1} (-1)^m \binom{M-1}{m} \left(\frac{1 - \exp(-2s(m+1))}{m+1} \right. \\ &\quad - \exp(-\zeta) \int_0^s \exp\left(-\frac{\theta}{y} - y(2m+1)\right) dy \\ &\quad \left. - \int_r^s \exp\left(\frac{\theta}{\zeta-z} - z(2m+1)\right) dz \right). \end{aligned} \quad (21)$$

Remark 6. For the special case with $P_t \rightarrow \infty$, the asymptotic outage probability for the MMS scheme where the k -th best is selected, is given by

$$\begin{aligned} \Pi_{\text{MMS}}^\infty(x) &= k \binom{M}{k} \sum_{m=0}^{M-k} (-1)^m \binom{M-k}{m} \\ &\times \left(\frac{1 - \exp(-2\zeta(k+m))}{k+m} - \frac{\exp(-\zeta) - \exp(-2\zeta(k+m-1))}{2k+2m-1} \right). \end{aligned} \quad (22)$$

IV. EVT-BASED PERFORMANCE ANALYSIS

In this section, we examine the asymptotic behavior of the system, when the number of the devices increases. We analyze the performance of the proposed k -th best selection schemes by applying an EVT approach. Through this methodology, we observe that the analytical expressions for the outage probability are simplified. The outage probability of the k -th best can be approximated as in [12]

$$\Pi^{(k)}(x) \approx \mathbb{P} \left\{ X_{i^*} \leq \frac{x - \eta}{\xi} \right\} = G^{(k)} \left(\frac{x - \eta}{\xi} \right), \quad (23)$$

where $G^{(k)}$ is given by (7). Below, we derive the asymptotic analytical expressions in terms of outage probability for the proposed selection schemes.

A. SBS scheme

By using the CDF of a random device's output SNR described in (11) and substituting in (6), we obtain the normalizing constants η^{SBS} and ξ^{SBS} . Substituting those constants in (23), we obtain the asymptotic outage probability of the k -th best for the SBS scheme as it is shown below.

$$\begin{aligned} \Pi_{\text{SBS}}^{(k)}(x) &= \sum_{j=0}^{k-1} \frac{1}{j!} \exp\left(-j \frac{x - \eta^{\text{SBS}}}{\xi^{\text{SBS}}}\right) \\ &\quad \times \exp\left(-\exp\left(-\frac{x - \eta^{\text{SBS}}}{\xi^{\text{SBS}}}\right)\right). \end{aligned} \quad (24)$$

B. EBS scheme

By using the CDF of a random device's channel gain denoted by $F(y) = 1 - \exp(-y)$ and substituting in (6), we evaluate $\eta^{\text{EBS}} = \log(M)$ and $\xi^{\text{EBS}} = 1$. We substitute those values to (23) and by differentiating and substituting in (30), we obtain the asymptotic outage probability of the k -th best for the EBS scheme,

$$\Pi_{\text{EBS}}^{(k)}(x) = \int_0^\infty \frac{dG^{(k)}(y - \log(M))}{dy} \left(1 - \exp\left(-\zeta - \frac{\theta}{y}\right) \right) dy. \quad (25)$$

C. MMS scheme

The EVT is implemented over the worst links from each device pair. Assuming that the CDF of a random device's worst channel gain is denoted by $F(y) = 1 - \exp(-2y)$ or $F(z) = 1 - \exp(-2z)$, for the cases $|g_{i^*}|^2 < |h_{i^*}|^2$ and $|g_{i^*}|^2 > |h_{i^*}|^2$, respectively, we calculate the normalizing constants as $\eta^{\text{MMS}} = \frac{1}{2} \log(M)$ and $\xi^{\text{MMS}} = \frac{1}{2}$. We substitute those values into the (23) and by differentiating and substituting in (33), we find the asymptotic outage probability of the k -th best for the MMS scheme,

$$\begin{aligned} \Pi_{\text{MMS}}^{(k)}(x) &= \frac{1}{2} \int_0^s \int_y^w f_{h_i}(z) \frac{dG^{(k)}(2y - \log(M))}{dy} dz dy \\ &\quad + \frac{1}{2} \left(\int_0^r f_{h_{i^*}}(z) dz + \int_r^s \int_z^v f_{g_i}(y) \frac{dG^{(k)}(2z - \log(M))}{dz} dy dz \right). \end{aligned} \quad (26)$$

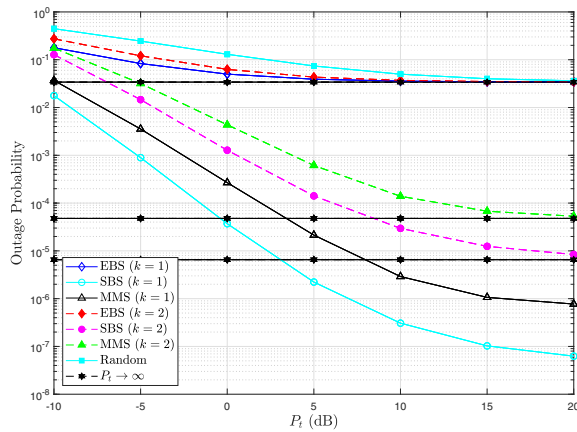


Fig. 2. Outage probability versus transmit power P_t for $k = 1, 2$.

V. NUMERICAL RESULTS

In this section, we validate the derived analytical expressions with simulations. The selection of the k -th best device occurs from a set of $M = 5$ devices. The normalizing constants for the EH model determined by the rectification circuit are set as $a = 2.463$, $b = 1.635$, $c = 0.826$, through curve fitting [12]. The harvest phase is set with duration $t_1 = 0.3$ (time units), the required threshold level is set as $Q = -10$ dB and $\sigma_n^2 = -10$ dB. The random selection where a device is randomly selected is used as a benchmark.

Fig. 2 plots the outage probability performance of the proposed selection schemes in terms of the transmit power P_t . As expected, the performance of the proposed selection schemes improves as the power transmit increases. The SBS outperforms the other two selection schemes; the MMS scheme is the second best and the EBS scheme has the worst performance. It is clear that the EBS scheme converges to the random selection asymptotically, as $P_t \rightarrow \infty$. In addition, due to the fact that the EH model considered, belongs to the group of saturation non-linear models, it can be noticed the convergence of the selection schemes to an error floor, given in Remarks 2, 4, 6 for SBS, EBS and MMS scheme, respectively, for high values of P_t .

The impact of time fraction can be observed in Fig. 3 with $P_t = 0$ dB, $Q = -5.23$ dB, which plots the outage probability versus the duration of the harvest phase t_1 . A trade off between the outage probability and t_1 can be noticed for the proposed selection schemes. The outage performance improves as t_1 increases, as more energy can be harvested by the devices until a cross point ($t^* = 0.7$). A further increase of t_1 , i.e. $t_1 > t^*$, degrades the performance due to the fact that the time duration of communication phase (information transmission) t_2 , decreases significantly. This indicates that the time fraction is very critical for the design of the network and should be optimized.

Finally, the asymptotic performance of the network which is obtained through EVT for different values of the number of devices is illustrated in Fig. 4, with $\sigma_n^2 = 0$ dB and $P_t = 10$ dB. Our theoretical analysis provides a good approximation to the simulation results which converge to the limiting distri-

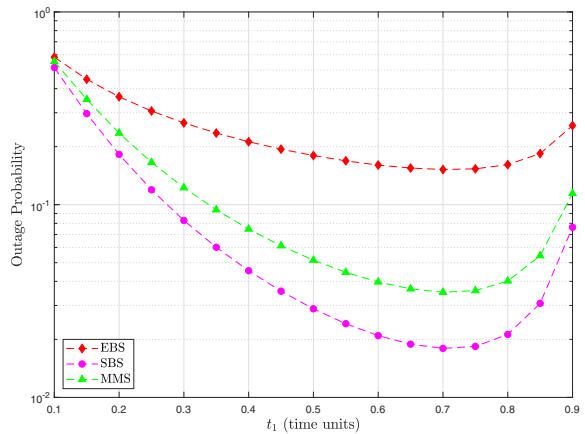


Fig. 3. Outage probability versus the duration of the harvest phase t_1 for $k = 2$.

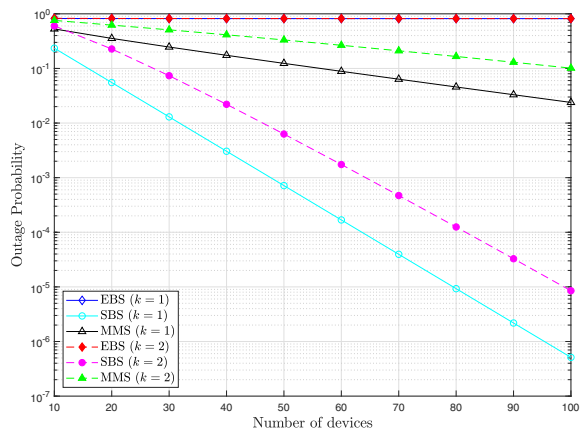


Fig. 4. Asymptotic outage probability versus number of devices.

bution of the network. We observe that the asymptotic outage performance improves as the number of devices increases and the SBS scheme provides the best performance among the three proposed selection schemes which is in line with the non-asymptotic case. Theoretical curves match with our simulation results and validate our analytical framework.

VI. CONCLUSIONS

In this paper, we studied the problem of the k -th selection in a WPCN. We considered novel selection schemes corresponding to different complexity and derived analytical expressions for their outage probability performance, assuming a non-linear EH model. We also considered an asymptotic scenario, where the number of devices increases and analyzed the behavior of the system in terms of outage probability by applying extreme value theory. It was shown that the SBS scheme provides the best performance among the three proposed selection schemes for both the asymptotic and non-asymptotic case. The joint selection of the k -th and j -th best device as well as the impact of the correlation between the channels on the outage performance of the system will be investigated as a part of future work.

A. Proof of Proposition 1

The CDF of a random device's output SNR is expressed as

$$\begin{aligned} F_{X_i}(x) &= \mathbb{P} \left\{ t_1 \left(\frac{aP_t |g_i|^2 + b}{P_t |g_i|^2 + c} - \frac{b}{c} \right) \frac{|h_i|^2}{t_2 \sigma_n^2} \leq x \right\} \\ &= \mathbb{P} \left\{ |h_i|^2 \leq \frac{\sigma_n^2 c t_2 x (P_t |g_i|^2 + c)}{P_t t_1 |g_i|^2 (ac - b)} \right\} \\ &= \mathbb{E} \left[1 - \exp \left(- \frac{\sigma_n^2 c t_2 x (P_t |g_i|^2 + c)}{P_t t_1 |g_i|^2 (ac - b)} \right) \right], \quad (27) \end{aligned}$$

which follows from the CDF of $|h_i|^2$ which is an exponential random variable. Thus, we have

$$F_{X_i}(x) = \int_0^\infty \exp(-y) \left(1 - \exp \left(- \frac{\sigma_n^2 c t_2 x (P_t y + c)}{P_t t_1 y (ac - b)} \right) \right) dy. \quad (28)$$

The final expression is derived by setting $\theta = \frac{\zeta c}{P_t}$, $\zeta = \frac{\sigma_n^2 c t_2 x}{t_1 (ac - b)}$ and by using [16, 3.324-1], simplifies to (11).

Now, we derive the CDF of the k -th best device's output SNR by making use of the PDF given by (3)

$$F_{X_{i^*}}(x) = \int_0^x f_{X_{i^*}}(y) dy, \quad (29)$$

which simplifies to (10).

B. Proof of Proposition 2

We derive the PDF for the k -th best device's channel gain $|g_{i^*}|^2$. By making use of (27), we have

$$\Pi_{\text{EBS}}^{(k)}(x) = \int_0^\infty \left(1 - \exp \left(-\zeta - \frac{\theta}{y} \right) \right) f_{g_{i^*}}(y) dy, \quad (30)$$

where $f_{g_{i^*}}(y)$ is given by (3) and denotes the PDF of the k -th best channel gain $|g_{i^*}|^2$ as $|g_i|^2$ follows an exponential distribution. After some algebraic manipulations, we have

$$\begin{aligned} \Pi_{\text{EBS}}^{(k)}(x) &= k \binom{M}{k} \int_0^\infty \left(\exp(-y)^k (1 - \exp(-y))^{M-k} \right. \\ &\quad \left. - \exp(-y)^k (1 - \exp(-y))^{M-k} \exp \left(-\zeta - \frac{\theta}{y} \right) \right) dy \\ &= k \binom{M}{k} \left(\int_0^\infty \left(\frac{(1 - \exp(-y))^{M-k}}{\exp(y)^k} dy - \exp(-\zeta) \right. \right. \\ &\quad \left. \left. \times \int_0^\infty \sum_{m=0}^{M-k} (-1)^m \binom{M-k}{m} \exp \left(-(k+m)y - \frac{\theta}{y} \right) dy \right), \quad (31) \end{aligned}$$

where the first term follows by [16, 3.312.1] and the second term follows by [16, 3.324-1] and the binomial theorem $(x + y)^n = \sum_{m=0}^n \binom{n}{m} x^{n-m} y^m$.

C. Proof of Proposition 3

The outage probability based on MMS can be defined as

$$\begin{aligned} \Pi_{\text{MMS}}^{(k)}(x) &= \mathbb{P}\{X \leq x, |g_{i^*}|^2 < |h_{i^*}|^2\} + \mathbb{P}\{X \leq x, |g_{i^*}|^2 > |h_{i^*}|^2\} \\ &= \frac{1}{2} \mathbb{P}\left\{X \leq x \mid |g_{i^*}|^2 < |h_{i^*}|^2\right\} + \frac{1}{2} \mathbb{P}\left\{X \leq x \mid |g_{i^*}|^2 > |h_{i^*}|^2\right\}. \quad (32) \end{aligned}$$

Thus, the outage probability of the k -th best device is

$$\begin{aligned} \Pi_{\text{MMS}}^{(k)}(x) &= \frac{1}{2} \int_{y=0}^s \int_{z=y}^w \underbrace{f_{h_i}(z) f_{g_{i^*}}(y) dz dy}_{|g_{i^*}|^2 < |h_{i^*}|^2} \\ &\quad + \frac{1}{2} \left(\int_{z=0}^r \underbrace{f_{h_{i^*}}(z) dz}_{|g_{i^*}|^2 > |h_{i^*}|^2} + \int_{z=r}^s \int_{y=z}^v f_{g_i}(y) f_{h_{i^*}}(z) dy dz \right), \quad (33) \end{aligned}$$

where $f_{g_{i^*}}(y)$ and $f_{h_{i^*}}(z)$ are given by (3), where $f_{g_i}(y)$ and $f_{h_i}(z)$ are the PDFs of the minimum channel gain g_i and h_i , respectively and $v = -\frac{\theta}{\zeta - z}$, $w = \zeta + \frac{\theta}{y}$, $s = \frac{1}{2}(\sqrt{\zeta^2 + 4\theta} + \zeta)$, $r = \zeta$.

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