Majoritarian and Consensual Democracies: The Seat Product Connection

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Abstract: This study connects two apparently disparate fields of inquiry in a specific quantitative way. In Lijphart's *Patterns of Democracy* (1999, 2012) the majoritarian-consensual "executives-parties" score (L) combines five indices. Four of these largely or entirely derive from factors that Shugart and Taagepera, *Votes from Seats* (2017), logically deduced from the product of two numbers: the number of seats in an average electoral district (M) and in the first or only chamber of representative assembly (S). Hence L connects to the purely institutional Seat Product MS (logged) along a simple logistic pattern. It can be predicted from $L = \frac{4}{1+35\,(MS)^{-0.56}} - 2$, with R^2 = 0.59. Thus, surprisingly, the majoritarian-consensual typology largely stems from the number of seats available.

Keywords: electoral systems; majoritarian democracies; consensual democracies; Seat Product; connections among connections

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Introduction

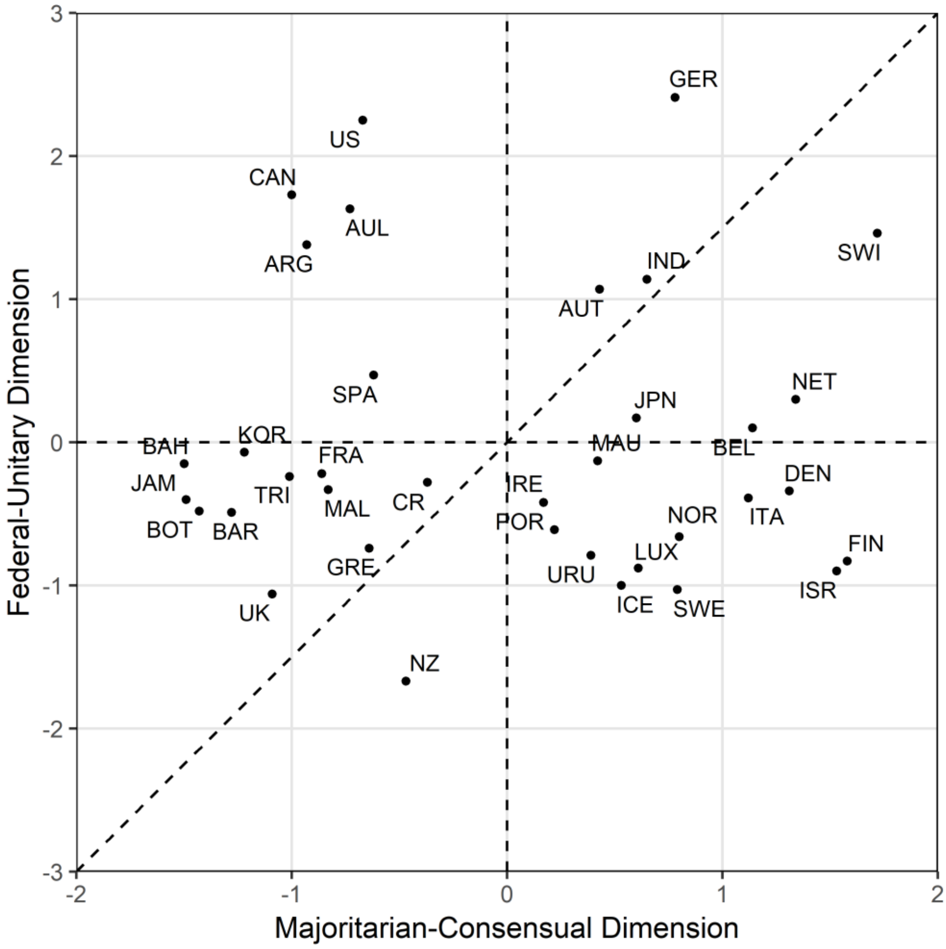
This study connects in a specific quantitative way two apparently disparate fields of inquiry: Lijphart's (1984, 1999, 2012) typology of democratic behavior, and deducing the number and size of parties from electoral rules (Shugart and Taagepera, 2017). Science aims at an ever-widening range of predictable connections, and this is a step in that direction.

In his momentous book on *Democracies* (1984) Arend Lijphart came up with a continuum of democratic behaviors and institutions. It is anchored by two different views on how democracy should be run: majority or consensus? Lijphart elaborated on this typology in a much-expanded *Patterns of Democracy* (1999, 2012), graphing countries in two dimensions, "executives-parties" and "federal-unitary", as shown in Figure 1. Leaving aside the federal-unitary, we focus on what institutional factors lie behind the "executives-parties dimension", which we'll call the majoritarian-consensual dimension.[1]

The focus of *Votes from Seats* by Shugart and Taagepera (2017) looks quite different. It aims at predicting the number and size of parties, depending heavily on a single institutional input, the "Seat Product" (MS). The latter is the product of the number of seats in an average electoral district (M) and in the first or only chamber of the representative assembly (S). This seems to have little to do with Lijphart's work. Indeed, Lijphart is mentioned only three times in Shugart and Taagepera (2017: 29, 66, 308), but never directly connected with the majoritarian-consensual typology.

[1] Lijphart expects a philosophical connection between unitary and majoritarian features on the one hand and federal and consensual democracies on the other, as if only federal countries could be truly consensual and only unitary countries truly majoritarian. Yet Figure 1 also shows a highly consensual-unitary Israel and a highly majoritarian-federal Canada. Any combination seems possible.

Figure 1. Lijphart's two-dimensional map of democracy, based on numbers in Lijphart (2012: 305–306). Lijphart's own graph (2012: 244) reverses their signs.



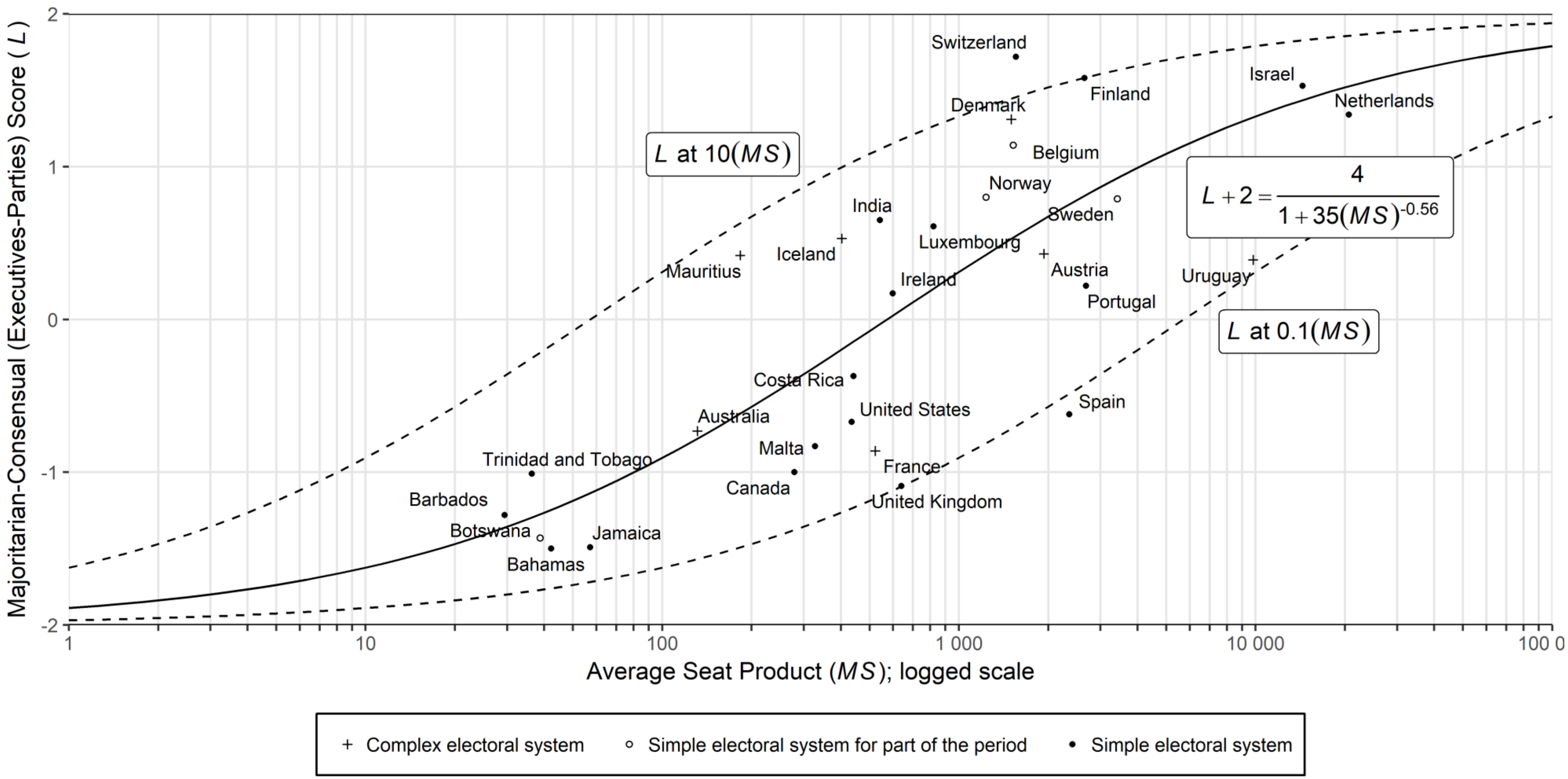
Yet, we find a surprisingly strong connection between Lijphart's (2012) scores on the majoritarian-consensual ("executives-parties") dimension (L) and the Seat Product (MS), the main workhorse in Shugart and Taagepera (2017). Figure 2 shows this connection. The central curve in Figure 2 shows the simple logistic fit between L and $\log(MS)$.[2] The dashed curves show the full curve shifted right or left by 1 unit on the logarithmic MS scale, meaning multiplying or dividing the actual MS by 10. This is a visual measure of range of random scatter. It can be seen that all countries fit into the zone delineated by the dashed curves, apart

[2] See Appendix A for data and B for the format of the equation shown.

from Switzerland and Spain. We acknowledge that an extensive work with mathematical expressions (such as ours) is not a conventional style in political science, despite being common in other disciplines (Colomer, 2007). Nonetheless, we call for its assessment based on the results, not the style.

The details of Figure 2 will be discussed in due time. For the moment, this graph serves to present our core research question: How come that a broad score to characterize the type of democracy (L) is connected to a narrowly institutional number (MS)? By answering this question, we establish a logical and quantitative connection between two major fields of study in political science: electoral institutions and the nature of representative democracy.

Figure 2. How Lijphart's (2012: 305–306) majoritarian-consensual (executives-parties) score (L) relates to Seat Product (MS). Data from Appendix A.



It may be understandable that the number of seats available affects the number of parties. But how could it connect with Lijphart's score L, which combines a number of apparently distinct indices to characterize the nature of democracies?

The brief answer is that, out of the five components of score L, two (effective number of assembly parties, and Gallagher's (1991) index of disproportionality) connect in a logical and quantitatively specific way to Seat Product MS. Another two (frequency of one-party cabinets, and executive dominance) also depend largely on quantities logically connected to MS. This leaves only one component, Siaroff's (1999) index of interest group pluralism, not explicitly connected to MS. Unexpectedly, even this component correlates with MS to some degree.

Our work has several major implications for political science. *First*, it establishes a clear connection between electoral institutions and nature of representative democracies. *Second*, our mathematical expression reveals the internal dynamics of this connection in a more fine-grained manner. The formula presented allows one to review the impact of minor changes in electoral system or in measures conjointly representing the majoritarian or consensual nature of democracies. That goes beyond the scope of directional approach which is based mostly on description of vague tendencies. Therefore, our product is especially useful for the purposes of constitutional engineering. *Third*, for scholars interested in expanding Lijphart's typology to additional cases, we offer a tool that can meaningfully substitute for values of some indices for which relevant data are unavailable (provided that a given country allows for unambiguous computation of average district magnitude M).

We first summarize the key features of Lijphart's typology, along with the basic indices he employs. We then summarize the relevant parts of Shugart and Taagepera (2017). The subsequent section develops connections between Lijphart's components and Seat Product; we show that the latter largely determines the former. We then discuss wider implications of this connection for Lijphart's conceptual framework, followed by conclusions and remaining issues.

Five characteristics of majoritarian and consensual democracies

By Lijphart's count (2012: 9–29), a purely majoritarian democracy includes five major features.

- It has only two parties in its representative assembly.
- The disproportionality is large between seat and vote shares of parties, especially when third parties try to run.
- One-party cabinets form, and they are "minimal winning" by definition.
- The executive branch of government dominates the legislative.
- Many separate interest groups exist.

In sum, a single party runs the show, based on a rather narrow majority in the assembly. Its power may be based on a minority of popular votes. This implies that, if opinions are split in multiple ways, it is felt that the largest minority group that can form a united team should be entitled to govern single-handedly. While Great Britain was the original home of the majoritarian idea, New Zealand used to be the purest case (Lijphart, 1987: 87), until this very excess of majoritarian features may have caused New Zealand to flip in 1996 (see Denemark, 2003).

In contrast, a purely consensual democracy tries to govern based on popular support as wide as possible (Lijphart, 2012: 30–45). Switzerland and Finland come closest to this pattern. It offers the following contrasts to majoritarian democracy.

- It has electoral rules that allow many parties to win seats in the legislative assembly.

- The seat shares of parties are proportional to their vote shares.
- Oversized cabinets are formed. These are not "minimal winning" because they include parties not needed to achieve majority support in the assembly. Minority cabinets also are tolerated.
- The legislative branch dominates the executive, as the legislative can change cabinets and often does.
- Interest groups are fused into two major peak organizations that represent the employers and the labor, respectively. This is often called a corporative setup.

Measures used by Lijphart

How is one to measure the degree to which these conceptual extremes occur in practice? Lijphart combines five measures.[3] See summary in Table 2.

The central criterion is the number parties in the first or only chamber of the assembly. For this, Lijphart uses the well-known Laakso-Taagepera effective number of assembly parties (N_s) (Laakso and Taagepera, 1979; Lijphart, 2012: 66):

$$N_S = \frac{S^2}{\sum S_i^2}$$

where S is the number of seats in the assembly and S_i is the number of seats for the i-th party. For 100 seats distributed as 50–40–10, as is typical for majoritarian democracies, N_S = 2.38. For 30–25–20–15–5–2–2–1, as is typical for highly consensual democracies, N_S = 4.58.

For electoral disproportionality Lijphart uses the well-known Gallagher's (1991) square index of disproportionality (D_2):

[3] Majoritarian democracies may differ from consensual ones in their sociopolitical outputs. Lijphart's first book (1999) to claim such differences raised a heated debate, which is reviewed in the second (2012: 256–272). Some differences seem fairly clear (see e.g., Gallagher, 2014). This issue does not enter the present study.

$$D_2 = \sqrt{\frac{1}{2}\sum_{i=1}^{n}(v_i - s_i)^2}$$

It takes the square root of half the sum of the squared absolute differences between votes shares (v_i) and seat shares (s_i) of parties ($i = 1 \dots n$) in a given system.[4] The subscript 2 reminds us that this index involves squaring, in contrast to D_1 for the alternative Loosemore-Hanby index (1971), which sums up the differences between vote shares and seat shares of the parties, a measure Lijphart does not use. Gallagher's index effectively weights the deviations by their own values and creates a responsive index theoretically bounded by 0 and 1 (or 0 and 100, when percentages are used). As long-term average disproportionality, Lijphart's data included in Appendix A indicates that the most majoritarian countries reach up to 22%, while the most consensual ones reach down to 1%.

Lijphart's third measure, frequency of "minimal winning, one-party cabinets", is somewhat complex. Lijphart (2012: 87) distinguishes five types of cabinets: one-party minimal winning ($MW1$); minimal winning coalition (MWc); minority, one-party ($m1$); minority coalition (mc); and oversized (O).[5] Lijphart (2012: 99–100) lists three *partly overlapping* frequencies (in percent): all minimal winning cabinets ($MW1 + MWc$); all one-party cabinets ($MW1 + m1$); and their mean (our notation f_{MW}), which is the measure he uses thereafter. This mean frequency boils down to $f_{MW} = MW1 + \frac{(MWc+m1)}{2}$. The complement of f_{MW} would represent the frequency of consensual cabinets: $1 - f_{MW} = O + mc + \frac{(MWc+m1)}{2}$. Thus, one-party minimal winning cabinets are considered fully majoritarian; minority coalition and

[4] We systematically use upper case for the number of seats and lower case for relative share of seats. Thus, S stands for the total number of seats in the assembly, S_1 is the *number* of seats for the largest party, and s_1 is the largest seat *share*. Hence $s_1 = S_1/S$.

[5] The notation used in parentheses is our own.

oversized cabinets are considered fully non-majoritarian (consensual); while one-party minority and minimal winning coalition cabinets are counted as semi-majoritarian (and semi-consensual). Lijphart's shorthand "minimal winning, one-party" cabinets should be seen in the light of this operational content. In practice, f_{MW} ranges from 4% for Switzerland to 100% for Botswana and 4 small island countries: Bahamas, Barbados, Jamaica and Malta.

Lijphart's (2012: 105–129) measure for executive dominance (our designation E_D) is also complex. For 28 parliamentary countries it is simply the average cabinet duration (in years) by Dodd (1976) criteria. It ranges from 1.46 years for Israel to 9.64 years for Jamaica. For the 6 presidential regimes Lijphart (2012: 119) introduces adjustments that make them look as if they had average cabinet durations ranging from 3 years (Costa Rica) to 8 years (Korea). Switzerland is assigned 1.0 year, because the cabinet does not depend on legislative confidence and its head changes yearly. Botswana, where the same party has been in power for 45 years, is whittled down to 9.9 years. Thus cabinet durations range from 1.46 to 45 years, but the executive domination index values range from 1.00 to an arbitrarily set upper limit 9.90 (Lijphart, 2012: 120–121).

Finally, Lijphart makes use of Siaroff's (1999) index of interest group pluralism (designated here as I_P). In theory, this index runs from 0 to 4; however, the actual values range from 0.35 in consensual Sweden to 3.25 in majoritarian Canada (Lijphart, 2012: 125–126). Countries with few parties have many separate interest groups, and vice versa. This may look counterintuitive. But this seems to be a balancing act at the topmost level of interest aggregation. When parties are few, then interest groups can afford to be splintered. But when parties are too many to handle interest aggregation, then interest groups must do the job – and they must consolidate for this purpose.

Table 1. Summary of Lijpart's indices used as components of majoritarian-consensual score (L)

Component	**Measure**	**Notation**	**Formula**	**Theoretical range**	**Empirical range** (based on Appendix A)
Number of parties	Effective number of parties	N_s	$N_S = \frac{S^2}{\sum S_i^2}$	1 and up	1.31–5.20
Electoral disproportionality	Gallagher's index	D_2	$D_2 = \sqrt{\frac{1}{2}\sum_{i=1}^{n}(v_i - s_i)^2}$	0–1	0.01–0.22
Minimal winning, one-party cabinets	Frequency of minimal winning, one-party cabinets	f_{MW}	$f_{MW} = MW1 + \frac{(MWc + m1)}{2}$	0–100	4–100
Executive dominance	(Mostly) average cabinet duration	E_D	Adjusted average cabinet duration (Lijphart, 2012)	1.00–9.90	1.00–9.90
Interest group pluralism	Siaroff's index	I_P	See Siaroff (1999)	0–4	0.35–3.25

Majoritarian-consensual score

Having these 5 measures for each country, Lijphart combines them (in a way outlined in Appendix A) into a score (designated here as L) that ranges from close to -2 (-1.50 for Bahamas) to close to +2 (1.72 for Switzerland). This is what Figure 1 shows on its horizontal axis. Most democracies are rather halfway between purely majoritarian and purely consensual.

Countries have different profiles regarding individual components. The effective number of parties and the prevalence of one-party and other minimal winning cabinets relate fairly linearly, with moderate scatter (R^2 = 0.72) – see graph and R values in Lijphart (2012). These are the core measures. In contrast, the combinations of executive dominance and interest group pluralism are all over the place (their R^2 being down to 0.25).

What lies behind these commonalities, strong or weak? One might guess that philosophical preferences are driving them all. Qualitatively, one could see how a consensual attitude could lead to (a) many parties, (b) low electoral disproportionality, (c) few one-party cabinets, (d) assembly dominance and (e) possibly even few separate interest groups – and vice versa for majoritarian attitudes. But how is one to measure quantitatively the degree of predilection for consensual or majoritarian philosophy? We will show that something much more mundane ties logically together at least 4 of the 5 components, in a quantitative way.

Distinction of representative democracies along the majoritarian-consensual line was recognized as one of the most innovative contributions in comparative political research (Mainwaring, 2001: 171). Nevertheless, Lijphart's work has not been spared from criticism directed towards various building blocks of the conceptual framework (Armingeon, 2002; see e.g., Bogaards, 2000; Grofman, 2000; Kaiser, 1997; Roller, 2005; Tsebelis, 2002; Vatter, 2009; Vatter and Bernauer, 2009) or measurement issues (Flinders, 2005; see e.g., Kaiser et al., 2002; Keman, 2000; Lijphart, 2003). However, the most fundamental critique is tied with the deficiency in the number of recognized dimensions (Shikano, 2006: 76–77), namely the negligence of direct democracy (Grofman, 2000; Vatter, 2000, 2009; Vatter and Bernauer, 2009), and even Lijphart (2012: 222) admits that the use of referendums “should probably be seen as a separate third dimension”. To the best of our knowledge, there has been no work that would meaningfully challenge Lijphart’s majoritarian-consensual dimension of representative democracies or propose a relevant adjustment to the included measures that would significantly increase its construct validity. The only exception is perhaps Coppedge (2018) who uses Lijphart’s concepts, however, searches for their completely new operationalization in the Varieties of Democracy (V-Dem) dataset (Coppedge et al., 2018) and arrives to results that are far away from Lijphart’s original work. Therefore, we speak directly to the concept as originally presented by Lijphart (1999, 2012).

The Seat Product Model

This model makes specific quantitative predictions on the basis of simple logical considerations. First outlined in Taagepera and Shugart (1993) for the number and size of parties in legislative assemblies, the Seat Product Model is expanded in Shugart and Taagepera, *Votes from Seats* (2017) to include the number and size of electoral parties, and deviation from proportionality. The basic “Duvergerian” idea behind it is that more seats available offer more opportunities to more parties.

It has long been known that countries tend to have two-party systems when they use one-seat electoral districts with plurality allocation rule (First-Past-The-Post, FPTP), such as practiced in UK and the US. In contrast, when countries use districts with many seats, and parties obtain seats proportional to their votes, multiparty systems tend to form (Duverger, 1954). This tendency was empirically confirmed in numerous works (see e.g., Gallagher and Mitchell, 2008; Powell, 2000; Rae, 1967).

What has been less appreciated is that the total size of the assembly also plays a role (Li and Shugart, 2016: 24). At the same district magnitude, a larger assembly has room for more parties. Indeed, the true institutional base for predicting the number of parties in a country is found to be the product of two numbers. One is the number of seats in the average electoral district – district magnitude, M. The second one is the number of seats in the assembly (or its first chamber) – assembly size, S. This Seat Product MS is the crucial building block, leading to various testable predictions. The set of models-presented below will make it apparent that we intentionally build our research on the product combining M and S – i.e., seat product MS – because it was logically derived by Shugart and Taagepera (2017) as expected basis for predicting the value of various indicators (e.g., effective number of parties, proportionality, cabinet duration) which overlap with many components of Lijphart's majoritarian-consensual score L.

When a proportional representation (PR) allocation formula is used, the most likely number of parties to obtain seats in a single district of M seats is the square root of M.[6] The logical foundation of this basic building block is outlined in Online Appendix C. Extending similar reasoning to the country level introduces seat product MS instead of just M: The most likely number of parties to obtain seats in an assembly of S seats allocated in districts of M seats is the fourth root of MS (see graph in Shugart and Taagepera, 2017: 104).[7] Applying such

[6] This model includes FPTP as a limiting case. Indeed, as district magnitude is reduced to M = 1, all usual PR allocation formulas are in fact reduced to FPTP.

[7] Space constrains do not allow us to comprehensively discuss how the number of seats available sets a logical ground for making a prediction about the number of assembly parties. However, the logic is also effectively summarized by Li and Shugart (2016: 26–27) or Shugart and Taagepera (2017: 101–124, 2018).

logical considerations several times over, Shugart and Taagepera (2017: 101–113) make and test the following two predictions that will concern us regarding Lijphart's majoritarian-consensual scores.

First, the most likely seat share of the largest party (s_1) in an elected assembly is the inverse of the 8th root of MS:

$$s_1 = (MS)^{-1/8}$$

Worldwide data for individual elections agree. Shugart and Taagepera (2017: 112) find R^2 = 0.54 between the logarithms of s_1 and MS, and the statistical best fit line comes remarkably close to the prediction.

Second, the most likely effective number of parties in an elected assembly (N_S) is the 6th root of seat product:

$$N_S = (MS)^{1/6}$$

Data again agree. Shugart and Taagepera (2017: 112) find R^2 = 0.61 between the logarithms of N_S and MS, and the statistical best fit line comes very close to the prediction. Thus, the seat product model accounts for more than one-half of the variation in the largest seat share and the effective number of parties.[8] Societal cleavages and institutional factors beyond MS may account for some of the rest, but this is beyond the scope of the present study.

Given the logical basis, empirical validity, and predictive ability of these models, Shugart and Taagepera (2017: 139–140) claim these models qualify as laws of socio-political nature in the strongest sense of the term – they are the Basic Laws of Party Seats. Note that the so-called Duverger's law (Duverger, 1954) cannot be written as an equation that expressly

[8] At a given M, D'Hondt seat allocation formula may reduce the number of parties to a barely detectable degree, compared to other usual PR formulas (Shugart and Taagepera, 2017: 124).

predicts some output from some input – it just expresses a vague tendency (Shugart and Taagepera, 2017: 117–120). The basic laws of party seats offer quantitative predictions.

From these two laws for party seats, two analogous laws follow for the largest vote share and the effective number parties based on votes. These basic laws of party votes do not concern us directly when it comes to Lijphart's majoritarian-consensual scores. But they contribute to a logical prediction for deviation from proportionality, which does concern us, because this deviation belongs among Lijphart's measures presented above. The most likely value for Gallagher's (1991) measure of deviation from proportionality is (Shugart and Taagepera, 2017: 141–146)

$$D_2 = 0.5(MS)^{-1/3}$$

This model does not yet fully qualify as a law of socio-political nature, given some remaining logical issues. Still, R^2 = 0.58 between the logarithms of D_2 and MS (Shugart and Taagepera, 2017: 152) is fairly high, so its predictive power is considerable.

Another quantity of interest is average cabinet duration (C). A logical model connects it to the effective number of assembly parties and hence, indirectly, to MS. The best guess for cabinet duration is a constant divided by the cube root of the seat product. Empirically, the constant is found to be 42 years (Shugart and Taagepera, 2017: 108–109). Then

$$C = \frac{42\ yrs.}{(MS)^{1/3}}$$

This set of models makes it apparent that both M and S act through the product MS, and this may create an impression that their impact is equal. This is not so, due to the differences between the largest and the smallest values of M and S empirically observed. While the largest observed M (450) exceeds the smallest M (1) 450-fold, the largest observed S (around 650) exceeds the smallest (about ten) only sixty-five-fold (Shugart and Taagepera, 2017: 114). Therefore, M has a bigger impact on MS than has S.

How the Seat Product Model connects the separate majoritarian-consensual components

We now proceed to connect the components of Lijphart's majoritarian-consensual score to the seat product. For this purpose, we use the score values as given in Lijphart (2012: 305–307). As noted earlier, the way he reaches these results has been subject to considerable criticism, and we have added our own reservations. But here we are not out to improve on Lijphart. The point here is to investigate how his numbers, if taken at face value, relate to the seat product. We are using those 29 countries, out of the 36 in Lijphart (2012), that had sufficiently simple and stable electoral systems so that an average district magnitude could be determined.[9]

The seat product – product of average district magnitude M and assembly size S – will constitute the main element in establishing the quantitative connection between electoral systems and individual measures combined into the majoritarian-consensual score which describe the nature of representative democracies. For the countries and time spans used by Lijphart (2012), the seat product covers a huge range. It was 24 for some elections in Barbados (in 1971 and 1976) – an assembly of 24 seats elected in one-seat districts. It is 22,500 in The Netherlands since 1956, where an assembly of 150 seats is elected in a single 150-seat district that includes the entire country.

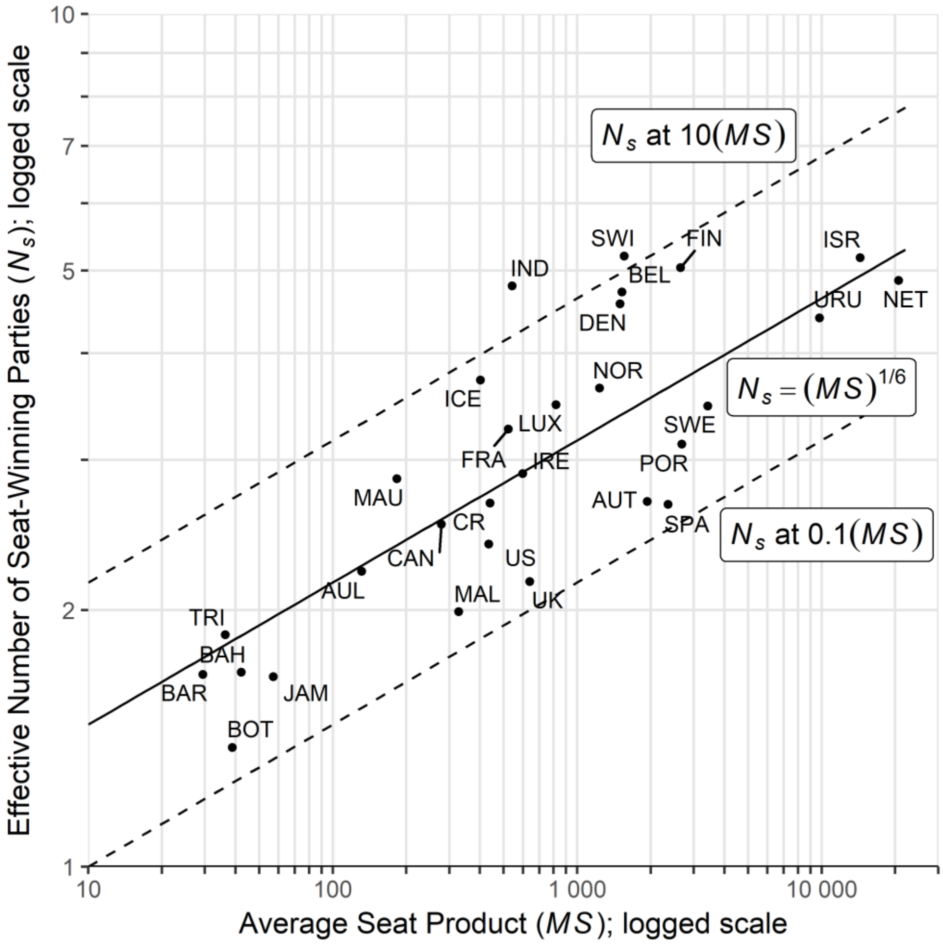
The *effective number of parties* enters Lijphart's majoritarian-consensual score directly, as its central measure. Once the model $N_S = (MS)^{1/6}$ was established, it was obvious that seat product affects the majoritarian-consensual score through this component. Our Figure 3 merely confirms the validity of this model in the case of our data set. In this Figure 3 both N_S and MS are on logarithmic scales, which turn the equation above into a straight line. Note that the central line shown is NOT a statistical data fit but a prediction based on logical grounds, with

[9] The seven omitted countries are Argentina, Germany, Greece, Italy, Japan, New Zealand, and South Korea. See Appendix A for data and criteria for inclusion.

no data input. The statistical best fit line, not shown, is almost indistinguishable from the prediction. Indeed, while statistical best fit accounts for 66.8% of the variation in $\log N_S$ with varying $\log(MS)$, the model $N_S = (MS)^{1/6}$ does almost as well, accounting for 66.2% [$R^2_{best\ fit}$ = 0.668 vs. $R^2_{log.model}$ = 0.662].

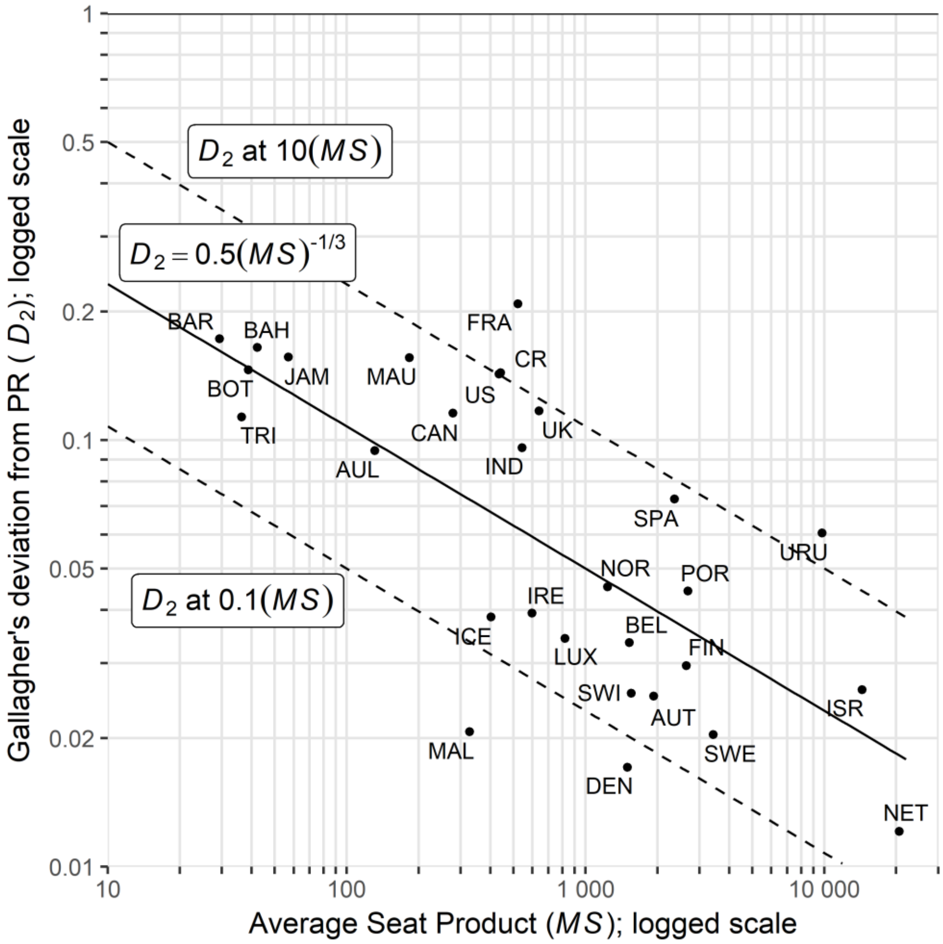
The dashed lines show the full line shifted right or left by 1 unit on logarithmic scale, meaning multiplying or dividing the actual MS by 10. This visual measure of range of random scatter shows that almost all countries fit into the zone delineated by the dashed lines; only Switzerland and India deviate more. We use analogous dashed curves in other graphs where the x-axis represents $\log(MS)$.

Figure 3. How the nationwide effective number of seat-winning parties (N_S) relates to seat product (MS), both on logarithmic scales. Data from Appendix A.



Lijphart's second measure, *deviation from proportionality*, can also be logically predicted on the basis of seat product, through $D_2 = 0.5(MS)^{-1/3}$. Figure 4, again on logarithmic scales, shows the degree of fit for 29 of Lijphart's 36 countries. With R^2 = 0.518, the fit with logical prediction is less sharp than for the effective number of parties, but still appreciable. (The statistical best fit does hardly better, with R^2 = 0.520.) The dashed lines again show the full line shifted right or left by 1 unit on logarithmic scale. This visual measure of range of random scatter shows that most countries fit into the zone delineated by the dashed lines; only Malta, Denmark, France and Uruguay deviate more. Thus, seat product also affects the majoritarian-consensual score through this component. This connection could not have been made before the model $D_2 = 0.5(MS)^{-1/3}$ was established in Shugart and Taagepera (2017). Our Figure 4 broadly agrees with the one presented by Shugart and Taagepera (2017: 146).

Figure 4. How Gallagher's measure of deviation from PR (D_2) relates to seat product (MS). Data from Appendix A.



Now consider Lijphart's third measure, the *frequency of "minimal winning, one-party cabinets"*, designated here as f_{MW}. This measure depends appreciably on the seat share of the largest party (s_1) (R^2 = 0.68 for linear fit). This is so because a largest seat share exceeding 0.5 (50%) automatically enables formation of a minimal winning, one-party cabinet. In contrast, oversized or multiparty minority cabinets are most likely when all parties are so small that no one predominates.[10]

[10] An even better fit than linear can be obtained with a "drawn-out S" curve that respects logical upper and lower limits on f_{MW} (0 and 1, respectively).

In turn, the largest seat share can be logically predicted on the basis of seat product, through $s_1 = (MS)^{-1/8}$. With MS largely determining the largest seat share and the latter largely determining Lijphart's measure for frequency of minimal winning, one-party cabinets (f_{MW}), this measure is bound to connect to MS. Figure 5 graphs this relationship, with f_{MW} on regular scale and MS on logarithmic scale. While $\log(MS)$ can range from 0 to very large, f_{MW} can range only from 0 to 1, suggesting a "drawn-out S" shape of the simple logistic model.[11] The best Logit fit to $\log(MS)$ implies (see Appendix B)

$$f_{MW} = \frac{1}{[1 + \ 0.00012\ (MS)^{1.19}]}$$

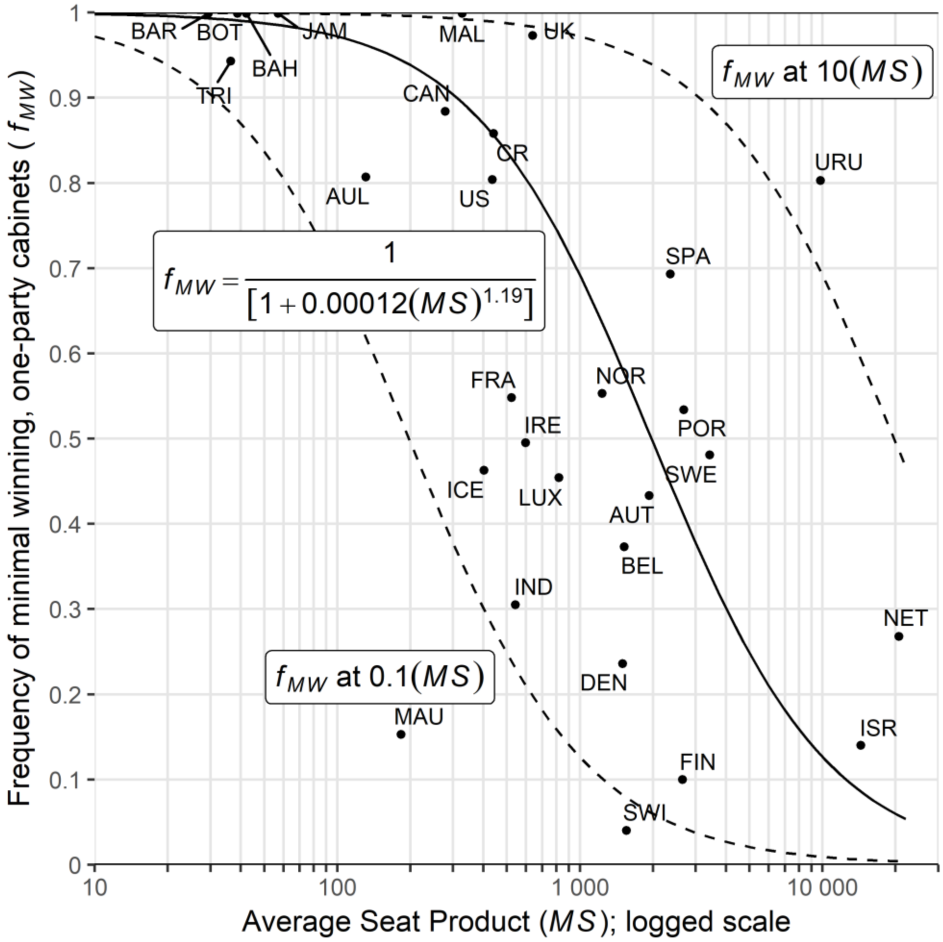
with R^2 = 0.49.

Figure 5 shows this equation as a full curve. The dashed curves again show the full curve shifted right or left by 1 unit on logarithmic scale, meaning multiplying or dividing the actual MS by 10. It can be seen that almost all countries fit into the zone delineated by the dashed curves; only Mauritius, Uruguay and Switzerland deviate more.

While the connection of the largest seat share to the seat product through $s_1 = (MS)^{-1/8}$ was known earlier, the connection of f_{MW} to the largest share is novel in the present study. This is what allows us to connect Lijphart's frequency of "minimal winning, one-party" cabinets (f_{MW}) to the seat product (MS). While MS leads logically to s_1, the subsequent connection to f_{MW} involves soft reasoning, empirically confirmed. Thus, the simple logistic format used in Figure 5 emerges from both logical and empirical considerations.

[11] Five of the included cases (Bahamas, Barbados, Botswana, Jamaica and Malta) have f_{MW} = 1. Since these values would blow the logistic pattern $(1 - y)/y$ to infinity, the operations were executed with f_{MW} =1 recomputed into f_{MW} = 0.999. Omitting these cases, as is a default strategy in some statistical software, would underestimate the possibility of having f_{MW} = 1. That would not be correct, because f_{MW} = 1 does occur. To check the impact of the transformation used, we conducted the same set of operations with f_{MW} = 1 recomputed into f_{MW} = 0.998 and the differences were minimal.

Figure 5. How Lijphart's frequency of "minimal winning, one-party" cabinets (f_{MW}) relates to the seat product (MS). Data from Appendix A.



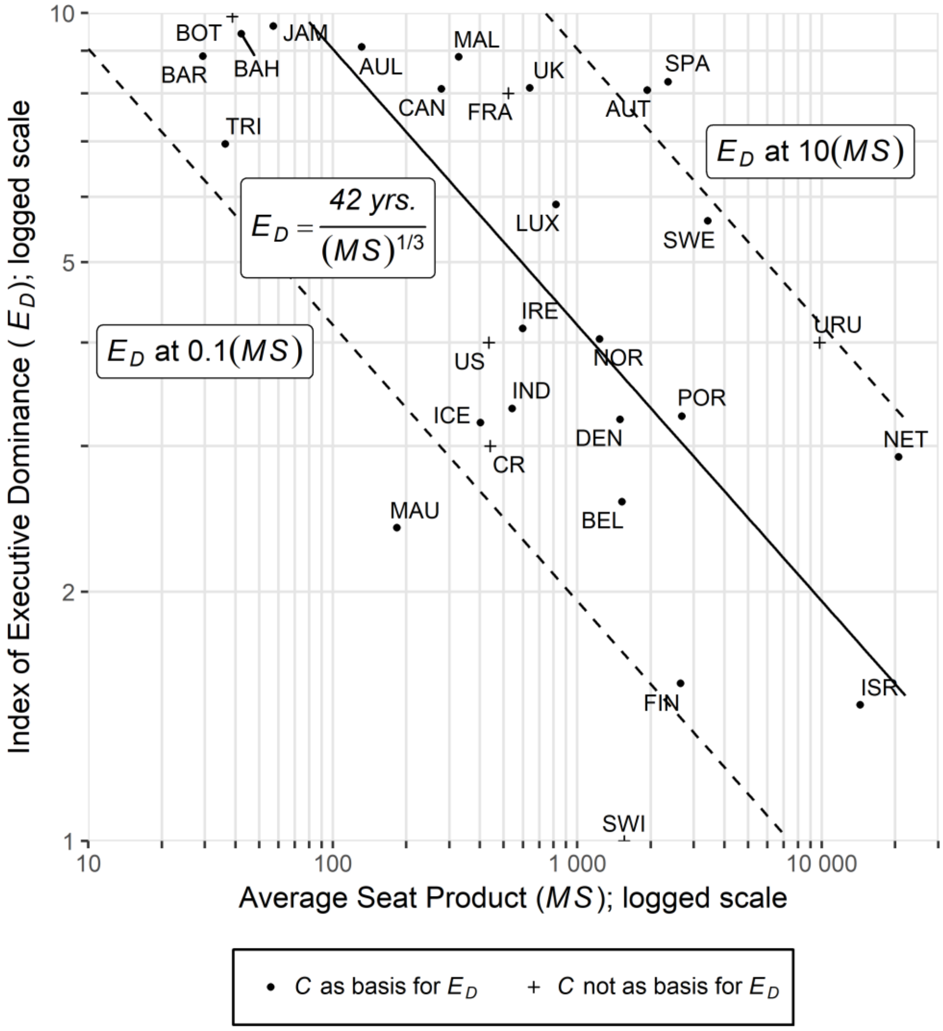
Lijphart's fourth measure, *executive dominance*, designated here as E_D, is a mixed one. For most countries, Lijphart uses simply the average duration of cabinets (C), in years, but he makes heavy adjustments for presidential regimes as well as for Switzerland and Botswana. Recall the logical model $C = \frac{42\ yrs.}{(MS)^{1/3}}$. With MS affecting cabinet duration and the latter largely representing Lijphart's measure for executive dominance, E_D might relate to MS approximately as $E_D = \frac{42\ yrs.}{(MS)^{1/3}}$, if Lijphart's adjustments do not offset it too heavily.

Figure 6 graphs this relationship as $\log E_D$ vs. $\log(MS)$. Here R^2 = 0.33 for logical model $E_D = \frac{42\ yrs.}{(MS)^{1/3}}$; the best linear fit does not improve on it.[12] The dashed lines again show the full line shifted right or left by 1 unit on logarithmic scale. This visual measure of range of random scatter shows that most countries fit into the zone delineated by the dashed lines; only Switzerland, Mauritius, Spain and Austria deviate more.

In his previous work Lijphart (1999) used the average of two ways to measure cabinet duration, only one of which (Dodd, 1976) could logically relate to MS. Lijphart (in 2012) used only the one related to MS. The graph in Figure 6 could have been constructed any time after 2012. We are just the first to actually do so.

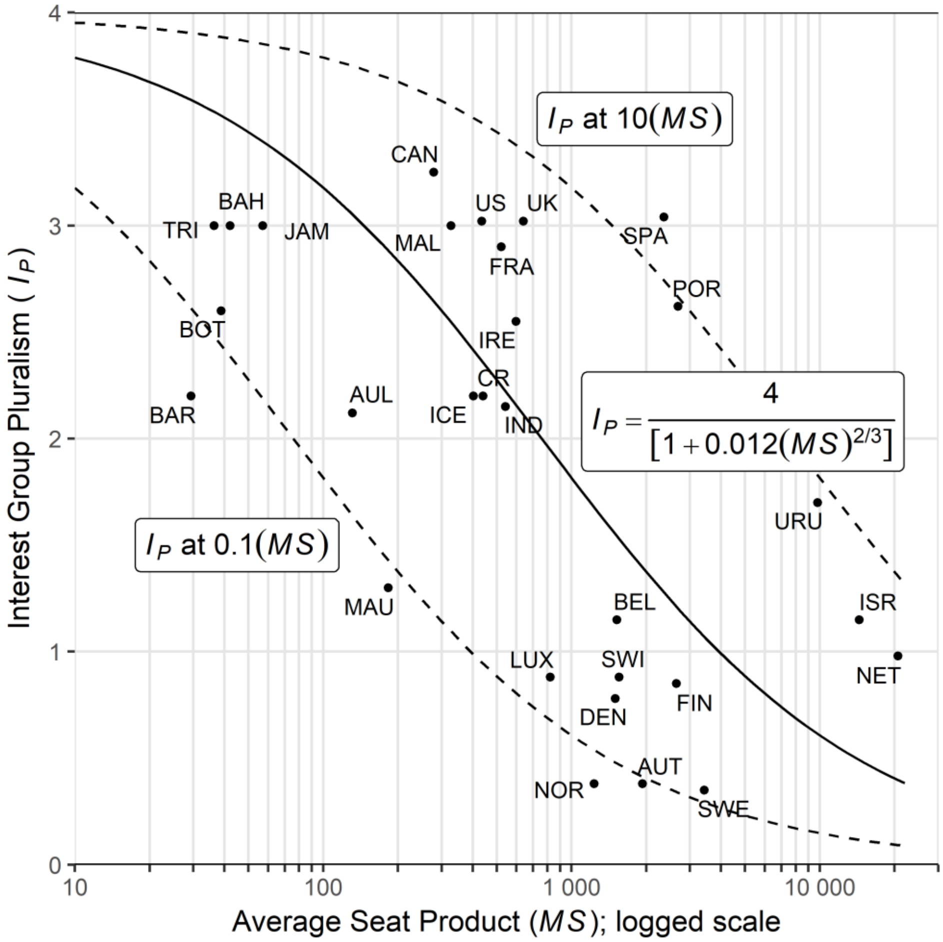
[12] We could exclude Switzerland because of its hybrid nature – see Lijphart (2012). Then our data would yield a higher R^2 = 0.39 between $\log E_D$ and $\log(MS)$. But we prefer to keep Switzerland in, even though it is an outlier.

Figure 6. How Lijphart's executive dominance (E_D) relates to the seat product (MS), on logarithmic scales. Data from Appendix A.



Thus, four out of Lijphart's five measures for majoritarian-consensual nature of democracies derive logically from the seat product, to a larger or smaller degree. Only the *index of interest group pluralism* (I_P) lacks a logical quantitative model to connect it to the seat product. We can offer only the aforementioned qualitative argument: When parties are too many to handle interest aggregation, then interest groups must consolidate for this purpose. Empirically, Lijphart (2012) finds appreciable correlation between I_P and the four previous indices. Once the latter are connected to MS, the same can be expected for interest group pluralism. Figure 7 confirms this.

Figure 7. How the index of interest group pluralism (I_P) relates to seat product (MS). Data from Appendix A.



The observations made regarding the format of index f_{MW} apply (see Appendix B). The full curve in Figure 7 shows an empirical fit with

$$I_P = \frac{4}{\left[1 + 0.012\,(MS)^{2/3}\right]}$$

The dashed curves show the full curve shifted right or left by 1 unit on logarithmic scale. The zone ranging from $0.1MS$ to $10MS$ excludes Spain, Barbados, Mauritius, Norway, and Austria. Recall that we have only indirect reasons to expect that the seat product MS would affect the index of interest group pluralism. So, the heavy scatter in Figure 7 (R^2 = 0.30) comes as no surprise. Instead, it is remarkable that any correlation exists at all. What this means is that MS faintly affects Lijphart's composite index L through interest groups, too.

This study emphasizes quantitative prediction based on logical considerations, but in the case of interest group pluralism we have to fall back on soft logical reasoning and empirical fitting as a stopgap. Establishing a logical quantitative connection between interest group pluralism and seat product is harder than for the four previous measures for the following reason. Each of the previous four could be largely reduced to a single measurable input – effective number of parties, deviation from proportionality, largest seat share, and cabinet duration, respectively. In contrast, I_P is a composite of eight aspects of the pluralism-corporatism contrast (Lijphart, 2012: 163). One would have to deal with each of them separately, and this exceeds the scope of the present article.

In sum, seat product MS contributes to all components of Lijphart's overall consensual-majoritarian scores (L) for countries where electoral rules are sufficiently simple so that the seat product can be defined. While assembly size S is always available, the average district magnitude M is unambiguous only for 29 of the 36 countries in Lijphart's set of stable democracies.

How Lijphart's overall majoritarian-consensual score relates to Seat Product

We have shown that all five components of Lijphart's majoritarian-consensual score L connect to seat product MS either logically or at least empirically. This explains why the score L is related to MS.

The functional relationships between the five factors and MS differ in form and theoretical foundation. Three of them are fixed exponent (power) relationships: $N_S = (MS)^{1/6}$, $D_2 = 0.5(MS)^{-1/3}$, and $E_D = \frac{42\ yrs.}{(MS)^{1/3}}$. Their relationship to MS has a logical basis both regarding their form and the values of exponents (1/6, -1/3 and 1/3, respectively). In contrast, the relationships of f_{MW} and I_P with MS are more empirical. These indices are of course bound to have drawn-out S relationships to $\log(MS)$, given that they are subject to conceptual lower and upper limits (0 to 1 for f_{MW}, and 0 to 4 for I_P). But these relationships need not be simple logistic (although they seem to be so) and the constant values are empirical.

Table 2 lists the degrees of correlation between the five components of Lijphart's consensus-majority score (L) with the logarithm of seat product (MS). They range from 0.30 to 0.67, with an average of 0.46. In this light, the correlation 0.59 for the composite index L itself is remarkably high. Indeed, only the effective number of parties accounts for more variation in the consensus-majority score L than does the seat product. This looks almost as if variations in some of the components were mutually compensating.

Table 2. Degree of correlation between the five components of Lijphart's consensus-majority score (L) with the logarithm of seat product (MS).

		$\mathbf{\log(\boldsymbol{MS})}$ $(\boldsymbol{R^2})$
(Fig. 3)	Effective number of assembly parties, N_S	0.67
(Fig. 4)	Gallagher's deviation from PR, D_2	0.52
(Fig. **5**)	Frequency of minimal winning, one-party cabinets, f_{MW}	0.49
(Fig. **6**)	Executive dominance, E_D	0.33
(Fig. **7**)	Interest group pluralism, I_P	0.30
(Fig. 2)	**CONSENSUS-MAJORITY SCORE, $\boldsymbol{L}$**	**0.59**

Note: In the case of logistic patterns OLS linear regression against $\log(MS)$ was carried out for $f_{MW}/(1-f_{MW})$, $I_P/(4-I_P)$ and $(L+2)/(2-L)$, respectively. In the case of fixed exponent patterns, it was carried out for logarithms of N_S, D_2, and E_D, respectively.

As was seen in Figure 2, these five components combine into a visual pattern between L and MS close to simple logistic. It is now time to revisit this Figure 2. It graphs Lijphart's

(2012) majoritarian-consensual scores (L) for the period 1945-2010 against the logarithms of the seat product (MS).[13] The observations made regarding index f_{MW} apply, given that L is largely constrained to range from -2 to +2 (see cautionary note in Appendix B). The best Logit fit to log(MS) implies

$$L = \frac{4}{1 + 34.629\ (MS)^{-0.557}} - 2$$

Without losing much predictive ability, we could approximate it with

$$L = \frac{4}{1 + 35\ (MS)^{-0.56}} - 2$$

The symbols for data points in Figure 2 distinguish between simple electoral systems where all seats are allocated in districts with average magnitude M, and more complex systems where features like legal thresholds and adjustment outside the basic electoral districts blur the impact of M. Strictly speaking, the logical relationships of MS with factors such as N_S and D_2 would apply only to simple electoral systems, but Shugart and Taagepera (2017: 285–307) observe that they also work for complex systems, on the average, just with larger random scatter. This being so, we would expect the data points for complex systems to be more widely scattered than those for simple systems. However, this does not seem to be the case. The connection of the majoritarian-consensual score to the seat product is robust even for complex electoral systems.

The largest discrepancies tend to occur at moderately large seat products (600 to 3,000), where L is around 0, the curve is steepest, and the vertical zone between the dashed curves is the widest. Here Switzerland and Finland are more consensual than expected, as if their MS

[13] See Appendix A for details of how Lijphart constructed L and how we determined MS.

were about 10 times larger than it actually is, while Spain and UK are less consensual than expected, as if their MS were about 10 times smaller than it actually is.[14]

Small countries that use single-seat districts have the lowest MS. We would expect these countries to be the most extreme on the majoritarian side, and so they are. The most majoritarian are Bahamas, Jamaica, Botswana and Barbados, which have assemblies smaller than 100 seats.[15]

At the consensual extreme we encounter The Netherlands and Israel, both with a huge seat product due to a single nationwide district, but also Switzerland and Finland, with large but not extreme seat products. The consensual nature of the latter two is boosted by an effective number of parties that surpasses the seat product expectation. Relatedly, s_1 and hence fraction f_{MW} are low.

In sum, the consensual or majoritarian nature of a country is strongly predicted by its seat product. Within this seat product MS, M is easier to change than S. Population largely imposes an assembly size. Indeed, assembly size tends to be the cube root of the population (P), for logical reasons (Shugart and Taagepera, 2017: 29):

$$S = P^{1/3}$$

So, assembly size is hard to manipulate in a major way.[16] In contrast, district magnitude is open to choose. One might guess that originally consensual societies have tended to pick

[14] Switzerland deviates mostly by having more parties, a lower frequency of minimal winning, one-party cabinets (f_{MW}), and lower executive dominance (E_D) than expected on the basis of its seat product. Finland tilts consistently in the same direction, followed by Mauritius and Denmark. On the lower side of L, Spain has markedly higher E_D and I_P than expected. UK and Uruguay tilt consistently in the same direction.

[15] With minor exception of the 1966 elections in Barbados.

[16] A country of 100,000 typically has an assembly with 46 seats. It would be 100 seats for a million people and 460 seats for 100 million. UK and the US are among the extreme outliers, in opposite directions.

large district magnitudes, while societies with a majoritarian outlook have tended to pick single-seat districts. Once the choice was made, it would have tended to lock in and possibly reinforce the original attitudes.

The choices countries make sometimes have unintended consequences. When they chose single-seat districts, the Caribbean islands tried to emulate UK. But they neglected the differences in country populations and hence assembly sizes. So, they ended up with overly dominant governments and much weaker opposition parties than in the UK. They did not intend to be more majoritarian than UK, but became so anyway.

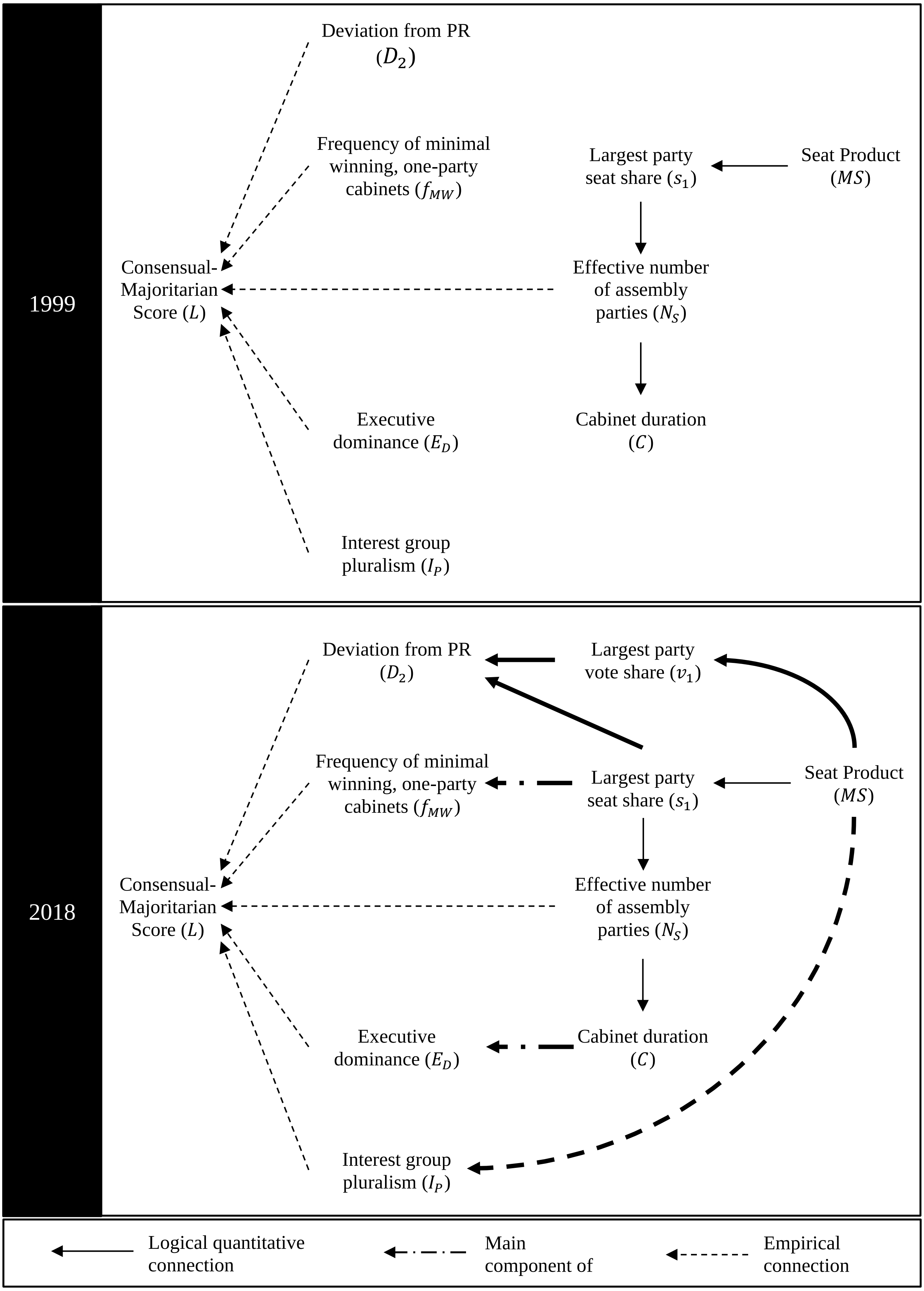
The broader framework: Connections among connections

A young science begins by establishing ways to measure phenomena – like Dodd (1976) for duration of cabinets (C); Laakso and Taagepera (1979) for the number of parties (N_S); Gallagher (1991) for electoral deviation from proportionality (D_2); and Siaroff (1999) for interest group pluralism (I_P). This science then tries to establish quantitative connections among these factors – like Lijphart (1994: 27) connected "effective threshold" to district magnitude M, and Taagepera and Shugart (1989) C to N_S.

Later, a developing science establishes connections among such factor-to- factor connections. In this way broader networks of connections take shape – like Lijphart (1999) developing a majoritarian-consensual index based on N_S, D_2, I_P, fraction of majoritarian cabinets (f_{MW}), and executive dominance (E_D). Or Taagepera and Shugart (1993) connecting the seat product MS in a logical quantitative way to largest seat share s_1 and then to N_S – and hence to C. Science aims at predictions, and connections among connections expand our predictive power.

As shown schematically in the top part of Figure 8, the two bundles of connections developed by Lijphart (1999) and Taagepera and Shugart (1993) remained largely separate as of 1999. Only N_S was a factor connecting them. The connection of E_D to C was blurred because Lijphart (1999) used Dodd's (1976) C conjointly with a different measure of cabinet duration. It could have been noticed even in 1999 that Lijphart's factor f_{MW} largely depended on s_1, but this connection was not made.

Figure 8. Expansion of connections among connections since Lijphart (1999) and Taagepera and Shugart (1993). Bold arrows show connections established later on, by Lijphart (2012), Shugart and Taagepera (2017), and the present study.



Since 1999, Lijphart (2012) shifted to basing E_D only on Dodd's C (with adjustments for just a few countries). Shugart and Taagepera (2017: 125–151) extended their Seat Product Model to the largest vote share (v_1) and through it to D_2. Thus, three of Lijphart's components for L were traced back to the seat product – N_S, D_2 and largely also E_D. Yet this extensive commonality wasn't yet noticed.

Making use of these decades-long findings by many researchers, it remained for us to join them and also to notice the connection of f_{MW} to s_1. This success further motivated us to graph the remaining component, I, against MS. We observed some connection there too. Bold arrows in the lower part of Figure 8 show the advances made by 2018. Lijphart (2012) and Shugart and Taagepera (2017) were already offering extensive groupings of connected factors. The juncture of these two groupings multiplies their explanatory and predictive power.

Conclusions

As much as this is possible, science aims at quantitatively predictable connections among separate factors, and then connections among such connections. This study has established a connection between two separate aspects of comparative politics -- Lijphart's (2012) scores on the majoritarian-consensual ("executives-parties") dimension (L), and the seat product (MS), which is the central workhorse in Shugart and Taagepera (2017) for predicting the number of parties and deviation from proportional representation. Most components of Lijphart's score are found to connect to MS in a logical quantitative way, and the remainder does so empirically. Their connections to the composite score (L) are complex. The relationship between L and logMS can logically be expected to follow a simple logistic format that corresponds to

$$L = \frac{4}{1 + a\,(MS)^{-b}} - 2$$

and a fair empirical fit (R^2 = 0.59) is obtained with

$$L = \frac{4}{1 + 35\,(MS)^{-0.56}} - 2.$$

Surprisingly, the majoritarian-consensual typology largely stems from the number of seats available in the assembly and in the electoral district. This does not mean that the number of seats “causes” the location of a country on the majoritarian-consensual scale. It’s rather the political culture, majoritarian or consensual, that affects the choice of district magnitude. But once this choice is made, district magnitude keeps affecting politics – in conjunction with assembly size. The outcomes may be unexpected. The small Caribbean countries did not intend to become more majoritarian than UK, but they did, because they forgot about their small assembly size. In the opposite direction, Protestant Europe tends to be more consensual than their seat product would seem to call for.

Instead of two separate islands of knowledge, one for broad patterns of democracy, the other for electoral outputs, we now have a joint one, grounded in a narrow institutional input. We consider such junctures significant advances in scientific inquiry. As all successful models, this one also raises new questions and opens further avenues of research.

First, this study highlights an empirical correlation between interest group pluralism and the seat product. By what process do they interact?

Second, how does the type of democracy connect with various socioeconomic outputs? Lijphart has investigated this issue for his two dimensions. Other studies have investigated the impact of district magnitude, but the results have been ambiguous, possibly because they overlooked the impact of assembly size. These studies could be repeated, replacing district magnitude by seat product. However, the effective number of parties might impact socioeconomic outputs more directly than does MS. So, this relationship should also be investigated more thoroughly.

Third, seat product can account for 59% of variation in Lijphart’s majority-consensus index L. That leaves 41% of variation in L to be accounted for by other factors and random noise. As noted earlier, originally consensual societies may have tended to pick large district magnitudes, while those with a majoritarian outlook may have tended to pick single-seat districts. Thus, along with population, political culture may have had a hand in determining seat product MS. When also considering the impact of political culture, appreciably more than 59% of variation in Lijphart's majority-consensus index L could be accounted for. Therefore, future research should examine the impact of cultural features in order to explain the remaining variation unaccounted for by seat product.

From a science of politics consisting of disparate pieces of knowledge, we are making headway toward a more unified field. In this field factors should be connected not only by broad qualitative concepts but preferably also by a network of quantitatively predictive equations. The present study is part of this advance.

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ONLINE APPENDIX A:

Seat Product (MS) and Lijphart's majoritarian-consensual scores (L) for 1945-2010

For the 36 countries Lijphart (2012) analyzes, Table A.1 shows the number of seats in the first or only chamber of the assembly (S) and in the average basic electoral district (M), and their product, MS. The countries are shown in increasing order of seat product MS. Data come from several databases: *Election Passport* (Lublin, n.d.), *Caribbean Elections Archive* (Caribbean Elections Archive, n.d.), *A guide to the electoral systems of the Americas* (Jones, 1995), and *Political Database of the Americas* (Center for Latin American Studies, n.d.).

In most countries MS largely determines the number and size of parties. When further complexities in electoral rules blur the impact of M, then M and MS are shown with *, even if the complexities apply to a limited part of the time period. Some countries included by Lijphart had electoral rules so complex or changing (Massicotte & Blais, 1999; Shugart & Wattenberg, 2003) that no unambiguous average M could be estimated. In such a case, "N/A" is displayed under M, S and MS.

Table A.1 further reproduces Lijphart's data covering the period 1945-2010 in 36 countries included in his sample (see Lijphart, 2012, pp. 305–307): the majoritarian-consensual score L, and its components – effective number of assembly parties (N_S), index of disproportionality (D_2), frequency of minimal winning, one-party cabinets (f_{MW}), executive dominance (E_D), and index of interest group pluralism (designated here as I_P).[17] Lijphart's

[17] Lijphart (2012, pp. 305–307) refers to these measure as follows: L – First (executives-parties dimension); N_S – Effective number of parliamentary parties; D_2 – Index of disproportionality (%); f_{MW} – Minimal winning, one-part cabinets (%); E_D – Index of executive dominance; I_P – Index of interest group pluralism.

values for E_D mostly represent average cabinet duration (i.e., C) in years; when this is not the case, the number is shown with ** (see Lijphart, 2012, pp. 120–121).

Table A.1 also includes the largest party seat share (s_1), which we show is the main ingredient of f_{MW}. These numbers result from our own data collection which follows Lijphart's supplementary material with detailed data (Lijphart, 2006). We additionally consulted *Global Elections Database* (Brancati, n.d.), *African Elections Database* (African Elections Database, 2012) and *Constituency-Level Elections Archive* (Kollman, Hicken, Caramani, Backer, & Lublin, 2014). For Mauritius, where official reports most often provide the seat shares of electoral alliances rather than single parties, s1 is based on private information by Lijphart. Whenever possible, we crosschecked the data; we noticed no significant discrepancies.

Lijphart (2012, p. 243) obtained the composite index L as follows. Each component was standardized so as to have a mean of 0 and a standard deviation 1 for the 36 countries. These standardized values were averaged, and this average was re-standardized to a mean of 0 and a standard deviation 1. Thus, abstract statistical numbers replaced the original "physical" values, such as number of parties or years of cabinet duration. The advantage of this procedure was that all five components affected the final score to an equal extent.

One shortcoming of such lack of absolute anchoring to a fixed scale is limited ability to compare time periods. If the entire democratic world should shift to more consensual attitudes (or vice versa), the mean L would still remain at 0, by definition. Also, if all countries should shift to a judicious mean between fully consensual and fully majoritarian extremes, L would still show them as widely dispersed as before. In this respect, Lijphart's graph (2012, p. 251) of shifts by individual countries must be taken with caution. In that graph the countries' average move is 0 by definition, and the average standard deviation remains 1, by definition. This is where anchoring L in MS becomes advantageous. If the entire set of countries were to shift toward more consensual attitudes, this most likely would result from an increase in average MS. The curve in Figure 2 might well remain the same but the countries would shift toward the right and upward.

Table A.1 Assembly size (S), district magnitude (M), seat product, Lijphart's (2012, pp. 305–307) majoritarian-consensual score L for 1945-2012, and its components. Shown are averages for the time period 1945-2010 covered by Lijphart (2012).

Country	**Abb.**	S	M	MS	L	N_S	f_{MW}	E_D	s_1	D_2	I_P
Bahamas	BAH	42.25	1.00	42.25	-1.50	1.69	1.00	9.44	0.72	0.17	3.00
Jamaica	JAM	57.10	1.00	57.10	-1.49	1.67	1.00	9.64	0.71	0.16	3.00
Barbados	BAR	27.00	1.10	29.40	-1.28	1.68	1.00	8.87	0.72	0.17	2.20
Trinidad and Tobago	TRI	36.33	1.00	36.33	-1.01	1.87	0.94	6.95	0.63	0.11	3.00
Botswana	BOT	38.80	1.00*	38.80*	-1.43	1.38	1.00	9.90**	0.83	0.15	2.60
Australia	AUL	131.31	1.00*	131.31*	-0.73	2.22	0.81	9.10	0.50	0.09	2.12
Mauritius	MAU	62.00	2.95*	183.05*	0.42	2.85	0.15	2.39	0.51	0.16	1.30
Canada	CAN	278.62	1.00	278.62	-1.00	2.52	0.88	8.10	0.54	0.12	3.25
Malta	MAL	63.70	5.14	327.62	-0.83	1.99	1.00	8.85	0.52	0.02	3.00
United States	US	435.06	1.00	435.06	-0.67	2.39	0.80	4.00**	0.58	0.14	3.02
Iceland	ICE	59.05	6.61*	402.59*	0.53	3.72	0.46	3.20	0.38	0.04	2.20
Costa Rica	CR	55.40	7.91	440.83	-0.37	2.67	0.86	3.00**	0.49	0.14	2.20
France	FRA	522.08	1.00*	522.08*	-0.86	3.26	0.55	8.00**	0.48	0.21	2.90
India	IND	542.00	1.00	542.00	0.65	4.80	0.31	3.33	0.45	0.10	2.15
United Kingdom	UK	639.44	1.00	639.44	-1.09	2.16	0.97	8.12	0.55	0.12	3.02
Ireland	IRE	155.89	3.82	597.94	0.17	2.89	0.50	4.16	0.48	0.04	2.55
Luxembourg	LUX	57.14	14.29	820.36	0.61	3.48	0.45	5.87	0.40	0.03	0.88
Norway	NOR	157.06	7.83*	1235.62*	0.80	3.64	0.55	4.04	0.44	0.05	0.38
Belgium	BEL	196.76	7.97*	1524.47*	1.14	4.72	0.37	2.57	0.30	0.03	1.15
Switzerland	SWI	198.88	7.82	1555.29	1.72	5.20	0.04	1.00**	0.26	0.03	0.88
Denmark	DEN	174.12	8.54*	1499.90*	1.31	4.57	0.24	3.23	0.35	0.02	0.78

Table A.1 *(continued)* Assembly size (S), district magnitude (M), seat product, Lijphart's (2012, pp. 305–307) majoritarian-consensual score L for 1945-2012, and its components. Shown are averages for the time period 1945-2012 covered by Lijphart (2012).

Country	**Abb.**	S	M	MS	L	N_S	f_{MW}	E_D	s_1	D_2	I_P
Austria	AUT	175.80	10.82*	1932.65*	0.43	2.68	0.43	8.07	0.45	0.03	0.38
Spain	SPA	350.00	6.73	2355.77	-0.62	2.66	0.69	8.26	0.49	0.07	3.04
Finland	FIN	200.00	13.24	2648.15	1.58	5.04	0.10	1.55	0.27	0.03	0.85
Portugal	POR	241.08	11.10	2682.52	0.22	3.13	0.53	3.26	0.48	0.04	2.62
Sweden	SWE	305.79	10.83*	3423.18*	0.79	3.47	0.48	5.61	0.45	0.02	0.35
Uruguay	URU	99.00	99.00*	9801.00*	0.39	4.40	0.80	4.00**	0.43	0.06	1.70
Israel	ISR	120.00	120.00	14400.00	1.53	5.18	0.14	1.46	0.35	0.03	1.15
Netherlands	NET	142.50	142.50	20625.00	1.34	4.87	0.27	2.91	0.30	0.01	0.98
Korea	KOR	N/A	*N/A*	*N/A*	-1.22	2.85	0.86	8.00	#N/A	0.22	2.90
New Zealand	NZ	N/A	*N/A*	*N/A*	-0.47	2.28	0.81	4.54	0.54	0.09	2.68
Argentina	ARG	N/A	*N/A*	*N/A*	-0.93	3.15	0.82	8.00**	#N/A	0.18	2.70
Germany	GER	N/A	*N/A*	*N/A*	0.78	3.09	0.38	3.80	#N/A	0.03	0.88
Greece	GRE	N/A	*N/A*	*N/A*	-0.64	2.27	0.98	4.45	#N/A	0.08	3.12
Italy	ITA	N/A	*N/A*	*N/A*	1.12	4.84	0.12	1.49	0.38	0.04	2.42
Japan	JPN	N/A	*N/A*	*N/A*	0.60	3.62	0.40	3.37	0.54	0.07	1.48

Notes: S – assembly size; M – mean district magnitude; MS – seat product; L – Lijphart's majoritarian-consensual score; N_S – effective number of assembly parties; f_{MW} – frequency of minimal winning and one-party cabinets; E_D – executive dominance; s_1 – largest party seat share; D_2 – index of disproportionality; I_P –index of interest group pluralism.

Displayed are all the countries included by Lijphart (2012). If a country implements electoral rules which are too complex to compute M, the columns S, M and MS contain "N/A".

* Electoral rules are complex, but M is still possible to compute without strong reservations.

** Lijphart's values for E_D differs from average cabinet duration in years.

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ONLINE APPENDIX B:

Simple logistic data fit of logged data

When some variable x can range almost from minus to plus infinity, while a related variable y can range from a minimum to a maximum, a "drawn-out S" shape imposes itself. Often it has the symmetric "simple logistic" shape. When the minimum is 0 and maximum is 1, its equation is

$$y = 1/[1 + ae^{-kx}].$$

The maximum slope dy/dx occurs at y = 0.5 and has the value $k/4$. This equation is equivalent to

$$\ln\left[\frac{(1-y)}{y}\right] = \ln a - kx.$$

Thus, linear regression of $\log[(1-y)/y]$ against x determines the constants a and k. The standard Logit program does so automatically.

In this study, L and two of its components, f_{MW} and I_P, approximate the simple logistic pattern when graphed against $\log(MS)$, as is done in Figures 2, 6, and 8. The equation above applies, provided that we take $x = \log(MS)$ and normalize the ranges of L and I_P to extend from 0 to 1. (In the original data L ranges from -2 to +2, and I_P from 0 to 4.)

Replacing x with $\ln(MS)$ in the equation above leads to

$$y = 1/[1 + a(MS)^{-k}].$$

This corresponds to the linear equation

$$\log[(1-y)/y] \ = \log a - k\log(MS).$$

Hence linear regression must be run as $\log[1/y - 1]$ against $\log(MS)$. The standard Logit must also use $x = \log(MS)$.

This approach tacitly implies that L is bound to the range -2 to +2, but this is not quite so, the way Lijphart constructs L. These are just the levels two standard deviations off the mean. On the federal-unitary scale in Figure 1, Germany and the US actually do surpass 2. Nonetheless, these limits offer close to the optimal logistic fit to $\log(MS)$.

ONLINE APPENDIX C:

How seat product MS determines the of number of parties and the size of the largest

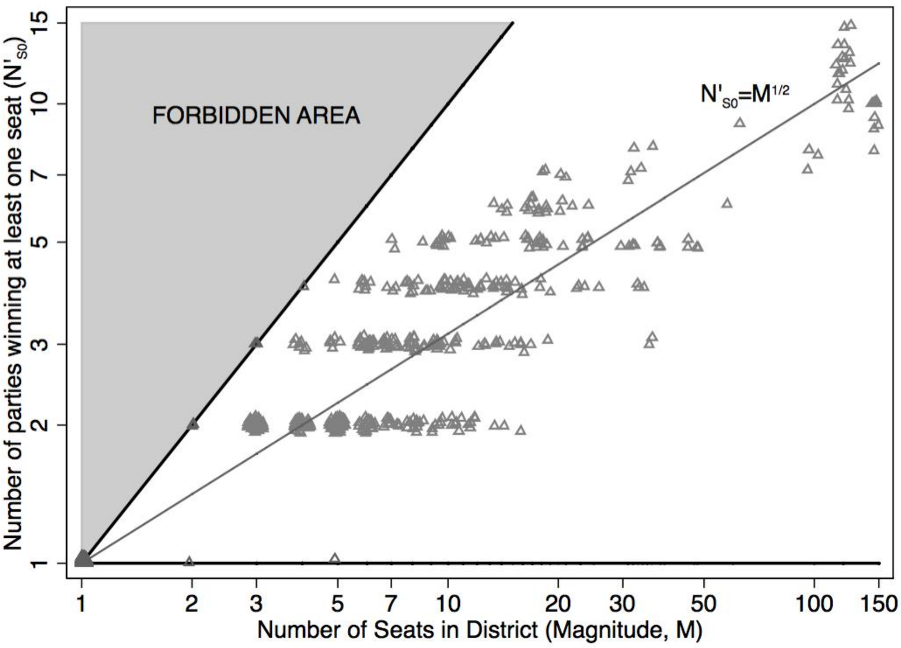
What is the logic behind laws like $s_1 = (MS)^{-1/8}$ and $N_S = (MS)^{1/6}$? Start with a simple example.

Consider a single electoral district, with 25 seats. Assume some PR allocation rule. How many parties are most likely to win seats? This is a *logically bounded situation*. At one extreme, 1 party could win all 25 seats. At the other extreme, 25 parties could win 1 seat each. The most balanced would be 5 parties winning an average of 5 seats each. This 5, the square root of 25, is the geometric mean of the *logically possible extremes*, 1 and 25. In the absence of any further information, this would be our best guess for the median outcome.

We can quickly generalize for any district magnitude. The most likely number of seat-winning parties can be expected to be the square root of district magnitude: $N = M^{1/2}$. This logical model includes First-Past-The-Post as a limiting case. Indeed, for M = 1 the model predicts N = 1, as it should. Figure C.1 shows a worldwide test of this basic logical model. Note that it includes FPTP (M = 1) data. The best linear fit is very close to the line shown, $N = M^{1/2}$. Yes, it is as simple as that! – as a first step.

Extending such reasoning by *the mean of conceptually allowed limits* to the nationwide scene introduces seat product MS instead of just M. Applying it several times over, one can calculate the most likely value for the largest seat share and effective number of parties -- see Shugart and Taagepera (2017, pp. 101-109). Note that this approach uses logical thinking – not data fitting. Data and statistics enter only when testing the logical model (Shugart and Taagepera, 2017, pp. 109-113).

Figure C.1 How the number of seat-winning parties in a single district relates to district magnitude. From Shugart and Taagepera (2018).



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