Homogenization of Voxel Models using Material Mixing Formulas

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Abstract—In this paper, we develop a procedure for simplifying highly inhomogeneous numerical phantoms based on dielectric mixing formulas. Numerical phantoms are extremely important in designing microwave imaging systems and algorithms. However, most of the realistic phantoms, typically obtained from magnetic resonance imaging (MRI) or computerized tomography (CT) scans are unsuitable for real-time analysis due to unlikely requirements for computational power and long processing time. Hence, it is of great importance to simplify such models without sacrificing the accuracy of the electromagnetic analysis. Here, we obtain simplified models by replacing a group of voxels by an effective permittivity computed by means of Looyenga and Lichtenecker methods. To assess the accuracy of the homogenized models with different resolutions, we compare their radar cross sections as well as transmissions between the antennas placed in their vicinity.

Index Terms—Antenna, Microwave Imaging, Breast Phantom, Lichtenecker, Looyenga, WIPL-D.

I. INTRODUCTION

Over the last decade, there has been a significant interest in using the microwaves for breast-cancer detection [1]. Soon after, the list of potential applications of microwave imaging in medical diagnostics included bone imaging, stroke detection, etc. [2]. The main advantages of microwave imaging systems are cost-effectiveness and portability, compared to the conventional technologies such as magnetic resonance imaging (MRI) and computed tomography (CT). Although MRI and CT are still considered as gold standards in clinical diagnosis, they have several drawbacks such as high cost, large size, and ionizing radiation.

Phantoms play an important part in the design of microwave imaging systems [3]. They are equally important in hardware design, i.e., when optimizing the performance of the antennas, as well as in software design, i.e., when testing the imaging algorithms. In order to study the interaction between the human tissues and electromagnetic waves, many phantoms have been developed, e.g., [3]–[5].

In contrast to physical phantoms, numerical phantoms can readily capture the complex heterogeneous interior of human tissues. In this paper, we use the realistic breast phantom derived from the series of T1-weighted Magnetic Resonance Images (MRI), available at the online repository [6]. The proposed numerical phantom captures detail about structural complexities and dispersive dielectric distribution of tissues inside the breast. With its 0.5 mm resolution, voxel model is

valuable due to its high-fidelity; however, it is difficult to use it in real time applications due to the large number of voxels. Namely, each voxel has a different permittivity value so the numerical implementation of such model requires a large number of unknowns.

In this paper, we propose a homogenization method, based on Lichtenecker and Looyenga mixing formulas [7]–[10] to decrease the number of unknown coefficients in the numerical analysis of such a model. By homogenization, we mean assigning some average permittivity to a group of heterogeneous voxels, while keeping the electromagnetic response almost intact. To validate the proposed method, we compute the scattering from the homogenized model as well as the transmission between antennas placed in the vicinity of the homogenized model. In this paper, we conduct uniform homogenization.

The paper is organized as follows. After the Introduction, in Section II we give a short description of the utilized voxel model. In Section III, we describe the homogenization method and the mixing formulas. Section IV presents a brief analysis of the obtained numerical results and demonstrates the effectiveness of the homogenization method in reducing the number of unknowns without sacrificing the accuracy. Finally Section V concludes the paper.

II. VOXEL MODEL OF BREAST

In this paper, we use a breast voxel model from the online repository that comprises the database of anatomically realistic numerical breast phantoms [6]. The phantom was derived from an MRI image of a patient. It is a three-dimensional (3D) grid of voxels where each voxel is a cube of the size 0.5 mm x 0.5 mm x 0.5 mm. Besides breast tissues, the model also contains a skin-layer of the thickness 1.5 mm, subcutaneous fat-layer of the thickness 1.5 cm, and a muscle chest wall of the thickness 0.5 mm. There are three different repository text-files in the database, named as “breastInfo.txt”, “mtype.txt”, and “pval.txt”. The 3D numerical model is computed with the help of those three files. The first file “breastInfo.txt” gives the basic information about the grid size in the 3D geometry, as s_1 , s_2 , and s_3 are the number of the voxels along the x , y , and z -axis, respectively. The second file “mtype.txt” provides information about the tissue type for each voxel in the grid. As shown in Table I, one value is assigned for each tissue. The third file “pval.txt” describes the

variation of the electrical properties of the same type of tissues.

TABLE I. TISSUE TYPES AND CORRESPONDING MEDIA NUMBER

Tissue Type	Media Number
Immersion Medium	-1
Skin	-2
Muscle	-4
Fibroconnective/glandular – 1	1.1
Fibroconnective/glandular – 2	1.2
Fibroconnective/glandular – 3	1.3
Transitional	2
Fatty – 1	3.1
Fatty – 2	3.2
Fatty – 3	3.3

The assigned permittivities were obtained from the large-scale study of ultrawideband microwave dielectric properties of normal breast [11] and were analytically described by means of single pole Cole-Cole formula.

To illustrate the level of heterogeneity of tissues within the considered breast phantom, we show a permittivity distribution within a small cube of the size 8x8x8 in voxels (i.e., 4 mm x 4 mm x 4mm). The red color indicates higher values of dielectric constant and the blue color indicates the lower values of the dielectric constant. (In general, the relative permittivity in the model ranges from less than five for fatty tissue up to sixty for fibro-glandular tissue.) Hence, computing the electromagnetic response of the whole model requires high computational power and long processing time due to the large variability of tissue properties. Therefore, a homogenization process is required to simplify the model and make it suitable for numerical analysis.

Fig.1. The permittivity distribution with a cube of the size 8x8x8 (in voxels)

III. HOMOGENIZATION METHOD

In this paper, we propose a homogenization method for highly inhomogeneous tissues, which provides an effective dielectric constant for groups of voxels. We assume that the voxels are grouped together in large cubes. The size of the large cube is defined by the integer n , which represents the number of the original voxels along each axis of the cube. For

example, $n = 8$, means that the total number of the voxels in the cube is 512. The size of the large cube is same for the whole phantom. In Fig. 2, we show the equivalent breast model obtained after the homogenization.

Fig. 2. The homogenized breast model with the size of the cube $n = 8$.

Initially, the effective permittivity of large cubes was determined by the standard averaging procedure

$$\epsilon_{\text{eff}} = \frac{1}{N} \sum_{i=1}^N \epsilon_i \quad (1)$$

where N is the total number of the voxels in one large cube and ϵ_i is the permittivity of the i^{th} voxel. However, the effective permittivity obtained in this way did not provide optimal results. Namely, even for small number of N scattering from the homogenized model significantly differed from the original model. In order to optimize the effective permittivity, different mixing formulas were studied [7]-[10]. The best results were obtained by means of Looyenga and Lichtenecker equations described below.

A. Lichtenecker

The effective permittivity of a dielectric mixture given by Lichtenecker formula [9] is

$$\ln(\epsilon_{\text{eff}}) = \frac{1}{N} \sum_{i=1}^N \ln(\epsilon_i), \quad (2)$$

where N is the total number of different dielectrics and ϵ_i is the permittivity of the i^{th} dielectric. Here, N is the total number of voxels in one large cube and is the permittivity of the i^{th} voxel. The Lichtenecker formula has been used to describe a wide range of biological materials such as human blood [9]. Typically, the number of the components in a mixture was two or three. However, the number of the components in the considered problem was significantly larger, e.g., for $n = 8$, the total number of components was

512. Hence, we had to adapt the formula for such a large number of components.

B. Looyenga

The effective permittivity of a mixture, by means Looyenga formula [10], [11] is given by

$$\epsilon_{\text{eff}}^{1/M} = \frac{1}{N} \sum_{i=1}^N \epsilon_i^{1/M}, \quad (3)$$

where $M = 1, 2, 3$ is a dimensionless factor [11]. In this work, we used $M = 3$.

IV. NUMERICAL RESULTS

To assess the impact of the homogenization, we computed the radar cross section (RCS) of the breast models obtained for different values of the parameter n . The simulations were performed at 1 GHz, using the WIPL-D software [12]. Fig. 3 and Fig. 4 show the RCS of the homogenized breast models obtained by means of Lichtenecker and Looyenga mixing formula, respectively. In the simulations, the smallest value of the parameter n was 12. The corresponding side length of the large cube was 6 mm. The largest considered value was $n = 32$, corresponding to the cube of the side length 16 mm.

Fig. 3. Normalized RCS of the homogenized breast model obtained by means of Lichtenecker formula with $n = 12$, $n = 16$, $n = 24$ and $n = 32$, computed at 1 GHz.

Both mixing formulas yielded similar results for the RCS. The results obtained for $n = 12$ and $n = 16$ were almost the same. Discrepancy occurs for angles close to $\phi = 180^\circ$, when $n = 24$ and $n = 32$.

Fig. 4. Normalized RCS of the homogenized breast model obtained by means of Looyenga formula with $n = 12$, $n = 16$, $n = 24$ and $n = 32$, computed at 1 GHz.

To further explore the effect of the homogenization, we computed the transmission coefficient between a pair of dipole antennas placed at the opposite sides of the breast model, as shown in Fig. 5. In Fig. 6, we show the results for the transmission coefficient (s_{21}) obtained using the Looyenga mixing formula for $n = 12, 16, 24$, and 32 . The corresponding frequency range was from 0.9 GHz to 1.1 GHz. As in the case of RCS, there was an excellent agreement between the results obtained for $n = 12$ and $n = 16$. However, the results obtained for $n = 32$ significantly differed from the results obtained for $n = 12, 16, 24$, in particular in the lower part of the selected frequency bandwidth. Similar results were obtained using the Lichtenecker formula.

Fig. 5. Two dipole antennas placed at opposite sides of the breast model obtained for $n = 16$.

Fig.6. Transmission coefficient between two dipoles placed at the opposite sides of the homogenized breast model obtained by means of Looyenga formula with $n = 12$, $n = 16$, $n = 24$ and $n = 32$, computed at 1 GHz.

The purpose of the homogenization is to reduce the computational time. Hence in Tab. II, we show the number of the unknowns for each model.

TABLE II NUMBER OF UNKNOWN OF HOMOGENIZED MODELS WITH DIFFERENT RESOLUTIONS

nn	Homogenized cube side length	Number of unknowns in the WIPL-D model
12	6 mm	26682
16	8 mm	13812
24	12 mm	5212
32	16 mm	2708

V. CONCLUSION

In this paper, we proposed the method for the reduction of the level of complexity of breast phantoms using Lichtenecker and Looyenga mixing formulas. By computing the RCS and transmission between two antennas, we estimated what is optimal trade-off between the numerical efficiency and the accuracy of the electromagnetic model. At 1 GHz, the significant reduction of quality appeared for cubes larger than 12 mm.

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