BLIND EQUALIZATION BASED ON FOURTH-ORDER CUMULANTS

Kamel Abderrahim, Ridha. Ben Abdennour*, Faouzi. Msahli*, Mekki. Ksouri**, Gerard Favier****

(*) : Ecole Nationale d'Ingénieurs de Gabès, Route de Medenine, 6029, Gabès, Tunisia.
Tel 00 216 5 392 100, Fax 00 216 5 392 190, e-mail : {Boulbaba.Ghoudi, Ridha.BenAbdennour}@enig.rnu.tn
(**) : Ecole Nationale d'Ingénieurs de Tunis, B.P. 47, Le Belvédère, 1002, Tunis, Tunisia.
e-mail: Mekki.Ksouri@isetr.rnu.tn
(***) : Laboratoire I3S, UNSA/CNRS, 2000 Route des Lucioles, F-06410, Biot, France. e-mail :Gerard.FAVIER@i3s.unice.fr

ABSTRACT

This paper addresses the problem of blind equalization based on Higher-Order statistics. In fact, we propose a new method for blind identification which uses only fourth order cumulants. The Channel impulse response coefficients are obtained by using a least squares method. Next, we exploit this method in the context of blind equalization. Numerical examples are also presented in order to illustrate the performances of the proposed method.

1 INTRODUCTION

In this work, we focus on the problem of channel estimation in digital communication systems. This problem is a particular form of linear system identification [2]. It consists in removing intersysmbol interference from the received signal which is corrupted by linear channel distortions and white additive noise [6, 11]. This is called equalization which can be insured either by the estimation of the channel impulse response or by the determination of the inverse channel. The last approach can fail in the case of channel zeros are located near the unit circle in the z-domain. For this reason, the use of direct methods of blind system identification without exploiting the inverse channel is the desirable goal. The first methods using this approach solve this problem by means of initial training period where a known data sequence is transmitted. The receiver uses the known sequence response's to estimate the channel [6]. But, the transmission of the training sequences reduce the efficiency of the digital communications [6, 11]. In fact, it is preferred to identify the channel response without using training sequences. This approach is called blind system identification or yet blind equalization. The implementation of this approach must consider the following problems:

- The channels used in digital communication systems have a nonminimum phase.
- The distribution of the channel input sequence are non-Gaussian.
- The received signal is generally contaminated by additive Gaussian noise.

Higher-Order Statistics (HOS) allow to solve all of the last problems. So, HOS based methods are very useful to deal with non-Gaussian and/or nonminimum phase linear channels, as well as non linear channels [13].
Recently, several approaches for the blind system identification using cumulants have been proposed in the literature which can be classified in three categories of solutions [14]: optimization solutions, closed form solutions and linear algebra solutions. The methods based on optimization solutions consist in minimizing a non-linear cost function between the cumulants estimated from real data and those obtained from analytical expressions [15]. These methods are computationally expensive, heavily dependent upon the initial guess of the parameters and also there is no guarantee that the algorithm converges to the global minimum. The recursive form solutions are based on explicit relations between the impulse response coefficients and the cumulants. These methods are interesting from a theoretical point of view because they demonstrate the possibility of estimating the impulse response coefficients from just output measurements [13]. But, they are not recommended in numerical calculation because they do not smooth out the affect of measurement noise [13]. Finally, the methods based on linear algebra solutions consist in constructing a system of equations obtained from explicit relations and solving this system using either the least squares or the total least squares approach. These methods have been found to be more robust to measurement noise and have a much lower variance when compared with the recursive form solutions [14, 16]. Even though the linear algebra solutions do not perform as well as the non-linear optimization solutions, they are computationally attractive and can also be used as a good initial guess for the non linear optimization methods [14].

In this paper, we address the application of linear algebra methods based on HOS in the context of blind equalization. However, that third order cumulants cannot be used, due to the even distribution of digital communication signals [6]. Consequently, only the methods based on even order cumulants are considered. The first method is proposed by Giannakis & Mendel [9]. It uses the second order cumulants (autocorrelation function) in addition to the fourth order cumulants and consequently their performance degrades in the presence of additive Gaussian noise. This problem can be solved by obtaining methods which do not use second order statistics. This solution is exploited by many authors. The first one is that of Comon [4] which is based on the utilization of one slice of the fourth order cumulants. But, this slice is characterized by cumulants whose arguments are away to the origin. In fact, they need very long data length in order to reduce their variance [15]. Another interesting method is proposed by Stogioglou & McLaughlin [16] which exploits nearly all the available fourth order statistics. But, it leads to great systems of equations when the impulsion response order of the system increases.

Two another linear approaches are presented in the literature. The first one is proposed by Fonollosa & Vidal [7]. This approach is based on the idea of expressing the impulse response coefficients as a linear combination of the cumulant slices with the advantage that the autocorrelation sequence need not be used. The second approach is that of Dembele [5, 6] which based on a Cholesky type a matrix of fourth order cumulants. The use of this requires all impulse response coefficients to be non zero.

In this paper, we propose a new blind identification method based on fourth order cumulants. The channel impulse coefficients are obtained using a least squares method. The application of this method in the context of blind equalization is also considered.

2 MODEL AND ASSUMPTIONS

Consider a Linear Time Invariant (LTI) system as shown in figure 1.

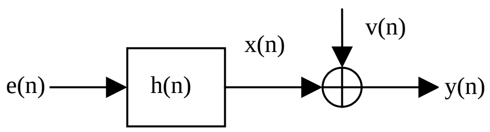

Figure 1. The system

We associate to this system the following relations:

$$x(n) = \sum_{i=0}^{q} h(i) e(n-i) \qquad (1)$$

$$y(n) = x(n) + v(n) \qquad (2)$$

where {e(n)} is the input sequence, {h(i)} is the impulse response coefficients, q is the order of the system, {x(n)} is the non measurable output sequence, {v(n)} is the noise sequence and {y(n)} is the noisy output sequence.

The following assumptions are assumed to be held:

A1. The sequence {e(n)} is an independent and identically distributed (i.i.d.) zero mean, non Gaussian, stationary process.

A2. The sequence {v(n)} is a zero mean Gaussian sequence with unknown variance and is statistically independent of {e(n)}.

A3. The system is assumed to be exponentially stable and has h(0)=1.

Problem Statement. Identify the parameters of the system $\{h(i)\}_{i=1,\cdots,q}$, using the cumulants of the measured output process y(n).

3 THE PROPOSED METHOD

The proposed method is derived from the general relationship which relates different slices of cumulant of the same order m to the MA model parameters [16]. In the special case when m=4, we have :

$$\sum_{i=s_1}^{s_2} h(i) h(i+\alpha_1) h(i+\alpha_2) C_{4,x}(\tau_1, \tau_2, i+\beta_3) = \sum_{i=s_3}^{s_4} h(i) h(i+\tau_1) h(i+\tau_2) C_{4,x}(\alpha_1, \alpha_2, i+\beta_3) \qquad (3)$$

where

$$s_1 = \max[0, -\alpha_1, -\alpha_2],\ s_2 = \min[q, q-\alpha_1, q-\alpha_2]$$
$$s_3 = \max[0, -\tau_1, -\tau_2] \text{ and } s_4 = \min[q, q-\tau_1, q-\tau_2]$$

if we take $\tau_1 = \alpha_1 = \alpha_2 = 0$ in (3), we have

$$\sum_{i=0}^{q} h^3(i) C_{4,x}(0, \tau_2, i+\beta_3) = \sum_{i=s_3}^{s_4} h^2(i) h(i+\tau_2) C_{4,x}(0,0,i+\beta_3) \qquad (4)$$

where $s_3 = \max[0, -\tau_2]$ and $s_4 = \min[q, q-\tau_2]$

Concatenating (4) for β_3 and τ_2 given by the following range:

$$\begin{cases} -q \le \tau_2 \le -1 \\ -2q \le \beta_3 \le q+\tau_2 \end{cases} \quad \text{and} \quad \begin{cases} 1 \le \tau_2 \le q \\ -2q+\tau_2 \le \beta_3 \le q \end{cases}$$

we obtain the following system of equations:

$$A\theta = b \qquad (5)$$

where

$$\theta = [h^3(1) \ \ldots \ h^3(q) \ \ h^2(1) \ \cdots \ h^2(q) \ \ h(1)h^2(2) \ \ h(1)h^2(3) \ \cdots \ h(1)h^2(q) \ \cdots \ h(q-1)h^2(q) \ \ h(1) \ \ldots \ h(q) \ \ h(2)h^2(1) \ \ldots \ h(q)h^2(q-1)]^T$$

is a vector of $(q^2 + 2q)$ elements.

A is a matrix of size $(5q^2 + q, q^2 + 2q)$ and b is a vector of $(5q^2 + q)$. elements.

The contents of A and b are determined according to equation (4).

The matrix A is full rank, hence the system (5) has a unique least squares solution defined by:

$$\theta = \left(A^T A\right)^{-1} A^T b \qquad (6)$$

Remarks.

- A recursive algorithm is also developed to claim the uniqueness of the least squares solution which is not presented here.
- The parameters of the vector θ are treated to be independent as in [16], which obviously are not. There might be some problems as described in [11]. But, this approach seems to lead to reasonably good results as well as being a simple method to implement [13].
- In the presence of Gaussian measurement noise, we can still use (4) by replacing the third and fourth order cumulants of x(n) by those of y(n) because $C_{4,y}(\tau_1,\tau_2,\tau_3) = C_{4,x}(\tau_1,\tau_2,\tau_3)$. This is an advantage of the proposed method because the algorithms using the second order cumulants may be affected significantly by Gaussian noise because $C_{2,y}(\tau_1) = C_{2,x}(\tau_1) + C_{2,v}(\tau_1)$.
- The proposed method uses cumulants whose arguments are close to the origin. Consequently, it requires a lower data length to estimate the cumulants by comparison to the other methods [4, 16].

Solving (6), the estimates of $\{h(i)\}_{i=1,\ldots,q}$ can be extracted directly from θ. This would be the end of the matter when there are no measurement noise and estimation errors. If this is not the case, we propose to use the same approach presented in [16] which exploit nearly all the available information provided by the vector θ.

4 BLIND EQUALIZATION BASED ON HIGHER-ORDER CUMULANTS

The equalizer system is presented in figure 2 :

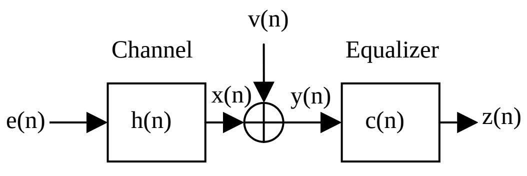

Figure 2. Equalizer system

where $\{e(n)\}$ is the input sequence which is assumed to be non Gaussian, $\{h(i)\}_{i=0,\cdots,q}$ are the impulse response of the channel, $\{v(n)\}$ is the Gaussian noise sequence, $\{y(n)\}$ is the noisy output sequence, $\{c(i)\}_{i=0,\cdots,L}$ are the equalizer coefficients (the Equalizer is assumed to be a tap-delay line of finite length L), $\{z(n)\}$ is the Equalizer output sequence .

We define also d as a transmission delay and $\{s(i)\}_{i=0,\cdots,M}$ as a the impulse response of the global system (channel + equalizer) where M=L+q. All coefficients of this response are equal to zero except the (d+1) coefficient which is equal to one.

Problem Statement. Estimate the Equalizer coefficients $\{c(i)\}_{i=0,\cdots,L}$ in order to reconstruct the input sequence $\{z(n)\}$ from the noisy observations $\{y(n)\}$.

This estimation can be achieved using the following steps:

1. Estimate $\{h(i)\}_{i=0,\cdots,q}$ using a blind system identification method based on HOS such as the proposed method.
2. Estimate $\{c(i)\}_{i=0,\cdots,L}$ minimizing the mean square error (MSE) defined by :

$$J_{MSE} = E\left\{\left|z(n+d) - e(n)\right|^2\right\} \qquad (7)$$

For the minimum-square error Equalizer the impulse response of the global system is given by [3]:

$$s = \left(H^H H + \frac{\gamma_{2,v}}{\gamma_{2,e}} I\right) H^H c \qquad (8)$$

where $\gamma_{2,e}$ and $\gamma_{2,v}$ are the variances of the input and the noise which can be calculated from the estimates of $\{h(i)\}_{i=0,\cdots,q}$ using:

$$\gamma_{2,e} = \frac{1}{q}\sum_{\tau=1}^{q} \frac{C_{2,y}(\tau)}{\sum_{k=0}^{q} h^*(k)h(k+\tau)} \qquad (9)$$

$$\gamma_{2,v} = C_{2,y}(0) - \gamma_{2,e}\sum_{i=0}^{q}\left|h(k)\right|^2 \qquad (10)$$

where $h^*(k)$ is the conjugate of h(k) and H^H is the conjugate transpose of the matrix H defined by:

$$H = \begin{pmatrix} h(0) & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ h(1) & h(0) & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ h(q) & h(q-1) & \cdots & h(0) & 0 & \cdots & 0 & 0 \\ 0 & h(q) & \cdots & h(1) & h(0) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & h(q) & h(q-1) \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & h(q) \end{pmatrix}$$

3. Reconstruct z(n) using $z(n) = \sum_{i=0}^{L} c(i)y(n-i)$

5. SIMULATIONS

The objective of the simulations is to compare the performance of the proposed method (P4G) with the methods of Comon (Com92) [4] and of Stogioglou & McLaughlin (STM96) [16].

The simulations are performed under the following conditions:

- The input sequence is a PSK4 {±1±j} with equal probability.
- The Signal to noise ratio is equal to 40 dB.
- The results were obtained from 20 Monte Carlo runs where4096 data points were used to estimate the fourth order cumulants.
- The equalizer length L is 31 and d=15.
- To evaluate the performance, we are used the symbol constellation and intersysmbol interference Rapport defined by:

$$\mathrm{ISI(dB)} = 10\log\left(\frac{\sum_{i=0,i\neq d}^{M} |s(i)|^2}{|s(d)|^2}\right) \quad (11)$$

We consider the following model which can be considered as an approximation of GSM channel [6]:

$$\begin{aligned} x(n) = e(n) + (0.9+0.6j)e(n-1) + (0.1250+0.8975j)e(n-2) \\ + (0.3261-0.7026)e(n-3) + (-0.7162+0.2851j) \\ + e(n-4) + (0.4422-0.6021j)e(n-5) \end{aligned} \quad (12)$$

The results are presented in table 1 and figure 3.

Method	**Com92**	**STM96**	**P4G**
ISI(db)	-44.3908	-45.8098	-61.7741

Table 1. ISI rapport

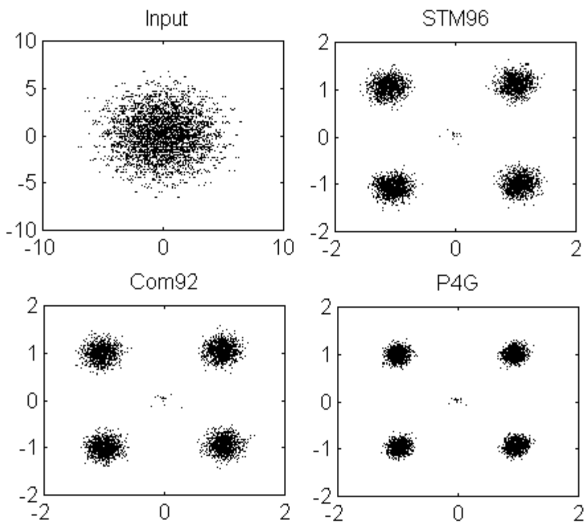

Figure 3. The symbol constellation (a) before, and (b) after equalization

We can observe from figure 3 that the proposed method gives the best constellation. Moreover, it gives the lower ISI (table 1).

6. CONCLUSION

In this paper, we have considered the problem of blind identification based on HOS. This problem can be solved by the estimation of the channel impulse response. In fact, we have proposed a new method for blind identification. This method uses only fourth order cumaulnts and consequently yields consistent estimates in the presence Gaussian noise. It exploits cumulants whose arguments are near to the origin which improve the estimates accuracy. Numerical simulation results were presented to demonstrate the performance of the proposed method in the context of blind equalization.

7. REFERENCES

[1] K. Abderrahim, R. Ben Abdennour, F. Msahli, M. Ksouri, G. Favier, *Identification of NMP FIR systems using the fourth order cumulants*, Third European Robotics, Intelligent Systems & Control Conference (EURISCON'98), 22-25 Juin 1998, Athène, Grèce.

[2] D. Boss, K. D. Kammeyer, T. Petermann, *Is blind channel estimation feasible in mobile communication systems? A study based on GSM*, IEEE Journal on selected areas in communications, 16(8), pp: 1479-1492, October, 1998.

[3] A. P. Clark, equalizers for digital modems, London, Pentech, 1985.

[4] P. Comon, *MA identification using fourth order cumulants*, Signal Processing, 26(3) pp:381-388, March 1992.

[5] D. Dembélé and G. Favier, *Une nouvelle méthode d'identification de modèles RIF à l'aide des cumulants d'ordre quatr*e, application à l'égalisation de canal, GRETSI, Juan-les-Pins, France, September 1995.

[6] D. Dembélé, *Identification de modèles ARMA linéaires à l'aide de statistiques d'ordre élevé: application à l'égalisation aveugle*, Thèse de Doctorat, Université de Nice Sophia Antipolis, 1995.

[7] J. A. Fonollosa, J. Vidal, *System identification using a linear combination of cumulants slices*, IEEE SP, 41(7) pp: 2405-2412, July 1993.

[8] D. Godard, Self recovering equalization and carrier tracking in two dimensional data communication, IEEE Tras. On Communications, COM-28 no. 11, pp. 1867-1875, 1980.

[9] G.B. Giannakis, J.M. Mendel, *Identification of nonminimum phase systems using higher order statistics*, IEEE SP 37(3) pp: 360-377, March 1989.

[10] G. H. Golub, C.F. Van Loan, *Matrix computations*, second edition, The Johns Hopkins University press, Baltimore and London, 1989.

[11] B. Jelonnek and K.D. Kammeyer, *Improved methods for the blind system identification using higher order statistics*, IEEE SP 40(12) pp: 2947-2960, December, 1992.

[12] J. L. Lacoume, P. O. Amblard, P. Comon, *Statistiques d'ordre supérieur pour le traitement du signal*, Masson, 1997.

[13] J. M. Mendel, *Tutorial on higher-order statistics (spectra) in signal processing and system theory: Theoretical results and some applications*, Proc. IEEE, vol. 79, pp. 278-305, 1991.

[14] C. L. Nikias, A. P. Petropulu, *Higher-Order spectra analysis*, PTR Prentice Hall, n Englewood Cliffs, New Jersey, 07632, 1993.

[15] L. Srinivas, *FIR system identification using higher order cumulants - A generalized approach*, Master Thésis, Indian Institute of Science, Bangalore, 560012, India, July 1994.

[16] A. G. Stogioglou, S. McLaughlin, *MA parameter estimation and cumulant enhancement*, IEEE Trans. Signal Processing, vol. 44, no. 7, pp. 1704-1718, 1996.