Novel non-linear relationship to evaluate the critical plane orientation

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ABSTRACT

Damage in offshore structures is mainly due to fatigue multiaxial stress state at structural joints, and fatigue strength assessment of such joints through the critical-plane criterion is well-accepted by the scientific community. A novel relationship to estimate the critical plane orientation is proposed in the present paper. Such a relationship is implemented in the stress-based critical-plane criterion by Carpinteri et al., which is applicable to any metallic material under multiaxial constant amplitude fatigue loading. Fatigue life of specimens made of different metallic materials is herein

evaluated by using such a criterion. The experimental fatigue life of each specimen examined is compared with the analytical one. The relationship above seems to be a useful tool for estimations of fatigue life under multiaxial loading.

KEYWORDS: critical plane, fatigue life, fatigue strength ratio, multiaxial loading.

NOMENCLATURE

a, b, c [-]	parameters of the relationship proposed in Eq. (7)
C_a, C_m [MPa]	amplitude and mean of the shear component of stress vector
E_{RMS} [-]	root mean square logarithmic error
k [cycle/MPa]	inverse slope of S-N curve under fully reversed normal stress
k^* [cycle/MPa]	inverse slope of S-N curve under fully reversed shear stress
N_a, N_m [MPa]	amplitude and mean of the normal component of stress vector
N_{cal} [cycles]	calculated number of loading cycles to failure
N_{exp} [cycles]	experimental number of loading cycles to failure
$N_{eq,a}$ [MPa]	equivalent normal stress amplitude
N_0 [cycles]	number of loading cycles corresponding to $\sigma_{af,-1}$ stress level
N_0^* [cycles]	number of loading cycles corresponding to $\tau_{af,-1}$ stress level
$W(t)$ [-]	weight function
t [s]	time instant
T [s]	period of the cyclic loading
T_{RMS} [-]	mean square error
β [$^\circ$]	angle between $\hat{\mathbf{i}}$ -direction and normal to the critical plane as per Eq. (7)
δ [$^\circ$]	original formulation of angle between $\hat{\mathbf{i}}$ -direction and normal to the critical plane as per Eq. (3)

ϕ, θ, ψ [°]	principal Euler angles
$\hat{\phi}, \hat{\theta}, \hat{\psi}$ [°]	averaged values of principal Euler angles
σ_u [MPa]	ultimate tensile strength
$\sigma_{af,-1}$ [MPa]	fully-reversed normal stress fatigue strength
$\sigma_1, \sigma_2, \sigma_3$ [MPa]	instantaneous principal stresses
$\tau_{af,-1}$ [MPa]	fully-reversed shear stress fatigue strength

1. INTRODUCTION

Fatigue is a major cause of failure in offshore structures: almost 25% of all structural damages requiring repair is classified as fatigue damage [1-3]. Under service loading conditions, the joints of such structures are subjected to multiaxial stress state. The joint safety can be assessed through criteria based on nominal stresses, hot-spot stresses, and notch stresses [2]. Then, the detail category-based S-N curves in design standards are used for fatigue life estimation of such structural details, as in the case of tubular joints. These criteria are also based on the stress concentration factors (SCFs).

However, the fatigue damage due to multiaxial stress state is not exactly the same as that evaluated by employing an equivalent uniaxial stress together with SCFs. Moreover, the above criteria cannot be used for multiplanar complex joints or stiffened joints [4].

Therefore, more accurate and easier criteria to design the above structures are needed, by representing the actual stress state more precisely. Several criteria proposed over the past decades to capture the multiaxial stress state can be broadly classified into four categories: (a) stress-based criteria, (b) strain-based criteria, (c) energy-based criteria, and (d) fracture mechanics-based criteria [5]. Note that, while the stress-based criteria are suitable for high-cycle fatigue regime (where the major part of the lifetime is spent during the crack initiation stage) and strain-based criteria for low-cycle fatigue regime (where crack propagation takes a much larger amount of the overall fatigue lifetime), the energy-based criteria can be adopted for both these regimes [6-9]. Even though the strain energy

is a scalar quantity, such a damage parameter can be used to evaluate the orientation of crack initiation and propagation, as is proved in Ref.[9]. However, energy criteria based on SWT (Smith-Watson-Topper) parameter can lead to some uncertainty since the cycles are extracted from the shear direction. This might result in scattering of results as is reported in Ref.[9].

In the context of the above criteria, the critical plane-based approach is often recommended, providing high accuracy in the fatigue assessment of engineering components [10]. Strain energy-based criteria employing the critical plane concept have recently been proposed by some researchers [11,12]. Towards the end of the 20th century, mesoscopic scale (grain scale) fatigue damage theories were developed, wherein some grains undergo local plasticity while the rest of the matrix behaves elastically [13-15]. Mesoscopic scale criteria were used in conjunction with the critical plane-based approach and energy criteria as well [16,17]. Though these criteria yield good results, their application by the designer community is still a challenge, since determination of the model parameters depends on complex processes of error optimization.

The stress-based criteria can be further categorised based on the type of stress: empirical equivalent stress, stress invariants, average stress, and critical plane stress [18]. The earliest works included classical yield theories of Lamé and Tresca as well as the von Mises criterion, based on equivalent stress formulations [19]. Thereafter, several other formulations of equivalent stress were proposed, followed by criteria based on stress-invariants and averaged

stress [20-24]. Though these approaches give us unsatisfactory results especially under non-proportional loading, the von Mises criterion is still widely used due to its simplicity of application.

Towards the beginning of 21st Century, Carpinteri and co-workers proposed a multiaxial fatigue criterion based on the critical plane approach, for assessment of metallic structural components under multiaxial constant amplitude cyclic loadings [25-28]. This criterion requires the determination of a critical plane orientation, followed by an equivalent stress evaluation performed on such a plane. Note that the critical plane may or may not be coincident with the fatigue fracture plane.

In more detail, the evaluation of the critical plane orientation according to the Carpinteri et al. criterion is performed in two stages: first, averaged principle stress directions at the material verification point are computed; then, the normal to the critical plane, defined by means of an off-angle is estimated. Such an off-angle is a material characteristic depending only on the fatigue strength ratio $\sigma_{af,-1}/\tau_{af,-1}$ of the material, being $\sigma_{af,-1}$ and $\tau_{af,-1}$ the fatigue strengths under fully-reversed normal stress and shear stress, respectively.

Several off-angle expressions have been proposed in the literature, the original one being a non-linear expression by Carpinteri et al. [25]. This expression can only be employed for materials with a fatigue strength ratio in the range from $1/\sqrt{3}$ to 1. Subsequently, some relationships have been presented by Walat et al. in 2014 [29].

Recently, Carpinteri et al. in 2017 have proposed another off-angle expression [30]. Although such an expression provides more accurate fatigue lifetime results than those determined by using the original one, there are still slight deviations from the experimental results. Moreover, the expression does not exactly meet the boundary conditions for both elastic-brittle and elastic-plastic materials.

In the present paper, a novel relationship to compute the off-angle is proposed in order to improve fatigue life estimation deduced through the Carpinteri et al. criterion. Several metallic material tests under biaxial loading are analysed, and some conclusions are drawn. The results in terms of fatigue lifetime are compared with those derived by employing the original expression of the off-angle.

2. THE CARPINTERI ET AL. CRITERION

The flowchart of the Carpinteri et al. criterion [25–28] is shown in **Figure 1**. Let us consider a body subjected to constant amplitude multiaxial cyclic loading. The material is characterised by the tensile ultimate strength σ_u and the following fatigue properties: fatigue strength under fully reversed normal stress $\sigma_{af,-1}$ evaluated at N_0 loading cycles, inverse slope of the S-N curve under fully reversed normal stress, k ($\sigma^k N = C_\sigma$), fatigue strength under fully reversed shear stress $\tau_{af,-1}$ evaluated at N_0^* loading cycles, and inverse slope of the S-N curve under fully reversed shear stress, k^* ($\tau^{k^*} N = C_\tau$).

Figure 1.

The time-varying stress state at the verification material point P (that is, the point where fatigue assessment is performed), expressed with respect to the fixed frame XYZ , can be described by means of the stress tensor $\boldsymbol{\sigma}$. The instantaneous principal stress directions 1,2 and 3 (being $\sigma_1 \geq \sigma_2 \geq \sigma_3$) at point P can be worked out from $\boldsymbol{\sigma}$. The instantaneous reference frame $P123$ can be defined through the principal Euler angles ϕ, θ and ψ . Being such angles time-varying, the averaged values $\hat{\phi}, \hat{\theta}$ and $\hat{\psi}$ and consequently the averaged reference frame $P\hat{1}\hat{2}\hat{3}$ can be determined as follows (**Figure 2**):

$$\hat{\phi} = \int_0^T \phi(t) W(t) dt \quad \hat{\theta} = \int_0^T \theta(t) W(t) dt \quad \hat{\psi} = \int_0^T \psi(t) W(t) dt \quad (1)$$

being $W(t)$ a weight function given by

$$W(t) = \begin{cases} 0, & \sigma_1(t) < \sigma_{1,\max} \\ 1, & \sigma_1(t) = \sigma_{1,\max} \end{cases} \quad (2)$$

where t is the time, T is the period of the cyclic loading, and $\sigma_{1,\max}$ is the maximum value of σ_1 during T .

The normal w to the critical plane is assumed to be correlated to the above $\hat{1}$ -direction through the original expression of the off-angle δ (rotation to be performed in the $\hat{1}\hat{3}$ plane, **Figure 2**):

$$\delta = \frac{3}{2} \left[1 - \left(\frac{\tau_{af,-1}}{\sigma_{af,-1}} \right)^2 \right] 45^\circ \quad (3)$$

Let us consider the stress vector related to the above verification point P on the critical plane. The time-history of the normal component of the stress vector can be defined by means of a scalar period function, characterised by the mean value N_m and the amplitude N_a , the normal component direction being fixed with respect to time. As far as the shear component of the stress vector is concerned, the vector field has to be defined, the shear component direction being time-varying. More precisely, such a field may be described by a vector, representing the mean value $\mathbf{C}_m$, and a scalar quantity, representing the amplitude C_a . The procedure proposed by Araujo et al. [31] is here implemented to compute $\mathbf{C}_m$ and C_a .

Finally, according to the Carpinteri criterion, the number of loading cycles to failure, N_{cal} , can be obtained from the solution of the following expression:

$$\sqrt{N_{eq,a}^2 + \left(\sigma_{af,-1} / \tau_{af,-1}\right)^2 (N_{cal} / N_0)^{-2/k} \left(N_0^* / N_{cal}\right)^{-2/k*} C_a^2} = \sigma_{af,-1} (N_{cal} / N_0)^{-1/k} \quad (4)$$

where $N_{eq,a}$ (representing the amplitude of an equivalent normal stress) is given by

$$N_{a,eq} = N_a + \sigma_{af,-1} \left(\frac{N_m}{\sigma_u} \right) \quad (5)$$

taking into account the diagram of Goodman.

3. A NOVEL RELATIONSHIP TO EVALUATE THE OFF-ANGLE

3.1 Formulation of the logistic curve (S-curve)

Now a novel expression is proposed to compute the above off-angle, named β in the following in order to distinguish it from the original expression (δ in Eq.3).

The novel relationship is based on a logistic curve, also named S-curve, whose general expression is given by [32]:

$$f(x) = \frac{1}{1 + e^{-\alpha(x-x_0)}} \quad (6)$$

where x is the variable, and α and x_0 are the slope and inflection point of the curve, respectively. According to Eq.(6), the following relationship is proposed:

$$\beta \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}} \right) = \frac{a}{1 + e^{-b[(\sigma_{af,-1}/\tau_{af,-1}) - c]}} \quad (7)$$

which represents an advancement of Eq.(6), due to an additional parameter a instead of 1 at the numerator of Eq.(6). Such an optimization allows to govern the peak value of the curve. The unit of β is degree, and the variable is the fatigue strength ratio $\sigma_{af,-1}/\tau_{af,-1}$. The dimensionless parameters a, b, c define peak value, slope and inflection point of the curve, respectively.

3.2 Parametric study of the proposed logistic curve (S-curve)

A parametric study is here performed to analyse the influence of the parameters a, b, c on the shape of the proposed logistic curve (Eq.(7)).

Parameter a determines the peak value of the curve, as is shown in **Figure 3**, where different curves are plotted by varying a from 30 to 50 and taking b and c as constants.

The slope of the curve is governed by parameter b , as is shown in **Figure 4**, where different curves are plotted by varying b from 15 to 35 and taking a and c as constants.

Parameter c determines the inflection point of the curve, as is shown in **Figure 5**, where different curves are plotted by varying c from 1.0 to 2.0 and taking b and c as constants. It can also be said that this parameter governs the shifting of the curve along the abscissa axis.

Figure 3.

Figure 4.

Figure 5.

3.3 S-curve calibration

In order to calibrate the S-curve described by Eq.(7), some fatigue test data available in the literature are examined. They are related to the following metallic materials: five steels (mild steel D30 and hard steel 982FA [33], SM45C [34], SUS304 [35], 10HNAP [36]), two cast irons (GGG40 and GTS45 [36]), and CuZn40Pb2 brass [37]. The specimens are smooth and subjected to combined in-phase normal loading (tension or bending) and torsion loading, with loading ratio equal to -1.0. The fatigue strength under normal loading and that under torsion

loading are listed in **Table 1**, together with the corresponding loading cycle numbers N_0 and N_0^* , respectively.

Table 1.

First of all, for each experimental data related to the above metallic materials, the estimated fatigue life N_{cal} is computed from Eq.(4) by varying β from 0° to 45° with a step equal to 1° . Note that the angle β varies in such a range, being equal to 0° for very hard metals ($\sigma_{af,-1}/\tau_{af,-1}=1$) and equal to 45° for mild/hard metals ($\sigma_{af,-1}/\tau_{af,-1}=\sqrt{3}$). The series of planes considered by varying β from 0° to 45° are candidate planes where to perform the fatigue assessment, that is to say, one of them is adopted as the critical plane. The critical plane is the verification plane not experimentally measurable; it is herein assumed to be different from the experimentally measurable fatigue fracture plane.

Then, for each of the above materials, the value of the root mean square logarithmic error is computed for 45 times, corresponding each one to a different value of β ranging from 0° to 45° :

$$E_{RMS} = \sqrt{\frac{\sum_{i=1}^n \log^2 \frac{N_{exp}}{N_{cal}}}{n}} \quad (8)$$

where n is the total number of the experimental data for a given metallic material, N_{exp} is the experimental multiaxial fatigue life, and N_{cal} is the estimated life. The corresponding mean square error T_{RMS} is computed as follows:

$$T_{RMS} = 10^{E_{RMS}} \quad (9)$$

Finally, for each metallic material examined, the off-angle $\beta_{T_{RMS},min}$ corresponding to the minimum value $T_{RMS,min}$ of the mean square error is registered. The experimental values of $\beta_{T_{RMS},min}$ against $\sigma_{af,-1}/\tau_{af,-1}$ are represented in **Figure 6** by using symbols, whereas the curve plotted is determined as is described below.

Figure 6.

The experimental data shown in **Figure 6** are the starting point of the procedure of the S-curve calibration. As a matter of fact, a regression analysis is performed on such data, where the parameters a, b, c are made to vary in order to minimise the regression coefficient. The results obtained are $a=45$, $b=18$, and $c=1.37$, with a regression coefficient equal to 0.97, and the proposed S-curve (Eq.(7)) is plotted with a continuous line. Note that such a curve satisfies with a good accuracy the aforementioned experimental observations, that is to say: $\beta = 0.056^\circ$ when $\sigma_{af,-1}/\tau_{af,-1} = 1$, whereas $\beta = 44.934^\circ$ when $\sigma_{af,-1}/\tau_{af,-1} = \sqrt{3}$.

3.4 Comparison between the novel curve and other literature relationships

By comparing the proposed curve (Eq.(7) with $a=45$, $b=18$, and $c=1.37$) with the original one (Eq.(3)), it can be observed that there is significant deviation between such two curves (see **Figure 7**).

Figure 7.

In the same Figure, a slight deviation can be observed between the herein proposed relationship and that presented in Ref.[30], especially for very hard metals ($\sigma_{af,-1}/\tau_{af,-1} \rightarrow 1$) because the expression in Ref.[30] does not exact fulfil the zero boundary condition when $\sigma_{af,-1}/\tau_{af,-1}=1$. For hard and mild/hard metallic materials, instead, such two curves show a good agreement with an average deviation of about 1%. Note that the curve presented in Ref.[30] can be considered as a particular case of that described by Eq.(7) taking the S-curve parameters equal to $a=43.5, b=20$, and $c=1.36$.

The orientation of the fracture plane is connected to the averaged principal stress directions, and does not depend on the β angle. Therefore, to verify the accuracy of the proposed expression for β angle, comparing the estimated fatigue life with the experimental fatigue life seems to be more appropriate than comparing the estimated fracture plane with the experimental one. The comparison related to the fracture plane could be meaningful for materials characterised by small values of β angle, because the fracture plane in such cases tends to coincide with the critical plane (verification plane). However, to the best knowledge of the authors, no data related to the fracture plane orientation are available in the literature for GGG40, CuZn40Pb2 and GTS.

4. VERIFICATION OF THE NOVEL RELATIONSHIP

Let us consider the fatigue test data used for S-curve calibration [33-37] and other experimental data available in the literature, related to the following metallic materials: steel (30CrNiMo8 [38]), cast iron (IC2 [33]), and aluminium alloy (PA4 or 6082-T6 [39]). The specimens are smooth and subjected to the same type of loading as that used for tests in Refs [33-37]. The fatigue strengths under both normal and torsion loading are listed in **Table 1**, together with the corresponding loading cycle numbers N_0 and N_0^* , respectively.

Now, by employing Eq.(7) with $a=45$, $b=18$, $c=1.37$ instead of Eq.(3), the Carpinteri et al. criterion is applied to such experimental data [33-39] in order to compute the number of loading cycles to failure, N_{cal} . The values of the mean square error T_{RMS} are plotted in **Figure 8**. A quite good accuracy is deduced in terms of fatigue life estimation since T_{RMS} is lower than 3.5 for all examined materials.

Figure 8.

Note that the curve in **Figure 6** is calibrated by taking into account eight (8) materials, but it is used to determine the critical plane orientation for eleven (11) materials, that is, the effect of the proposed curve on fatigue life is assessed by examining three (3) further materials (IC2, 982FA and 10HNAP) which are not considered for the curve calibration.

Then, the Carpinteri et al. criterion is applied according to its original formulation (Eq.(3)). The computed values of the mean square

error are shown in **Figure 9**. Such results are worse than the previous ones (in **Figure 8**), being T_{RMS} much greater than 3.0 for many materials.

Figure 9.

In **Figure 10**, experimental data are compared with theoretical evaluations. The solid line indicates $N_{cal} = N_{exp}$ (all results above such a line are conservative), the dashed lines correspond to N_{cal}/N_{exp} equal to 0.5 and 2, respectively, and the dash-dot lines correspond to N_{cal}/N_{exp} equal to 1/3 and 3, respectively. In more detail, the fatigue life estimations in **Figure 10(a)** are determined through the novel relationship (Eq.(7) with $a=45, b=18, c=1.37$), whereas those in **Figure 10(b)** are deduced through the original formulation (Eq.(3)). It can be observed that, for the materials being examined, there is significant difference between the two expressions in medium-cycle fatigue regime ($10^4 \leq N_{exp} \leq 10^6$) compared to the high-cycle fatigue regime. In the case of high-cycle fatigue regime, the values are different but not very scattered.

Figure 10.

For the materials and fatigue loadings being examined, it can be observed that 83% of the results fall within the 3x band and 67% within the 2x band by employing the novel relationship, whereas 84% of the results fall within the 3x band and 65% within the 2x band by employing the original formulation. According to such percentages,

the two relationships above seem to produce results with a very similar accuracy.

To better estimate the reliability of such two relationships, the mean square error (T_{RMS}) values have to be examined because they take into account how far the results are from the solid line $N_{cal} = N_{exp}$. Since the average of T_{RMS} is equal to 2.18 by using the novel relationship (average computed on the 11 T_{RMS} values in **Figure 8**) against 3.10 by using the original expression (average computed on the 11 T_{RMS} values in **Figure 9**), the accuracy of the novel relationship results is greater than that of the original expression ones.

Note that the fatigue life N_{cal} is computed in Ref.[30] by applying a criterion different from that here employed as far as both equivalent stress amplitude evaluation and fatigue life computation are concerned. As a matter of fact, the criterion used in Ref.[30] consists in determining the amplitude of an equivalent stress (acting on the critical plane) evaluated by linearly combining the amplitude of the normal stress vector component with the amplitude of the shear stress vector component (both related to the critical plane), the latter being calculated as is reported in Ref.[39]. The fatigue life is then estimated by using the S-N curve.

Instead, the criterion here employed (see Refs [25–28]) consists in determining the amplitude of an equivalent stress (acting on the critical plane) evaluated by a non-linear combination (see Eq.(4)) of the amplitude of an equivalent normal stress vector component (normal to the critical plane, see Eq.(5)) with the amplitude of the shear

stress vector component acting on the same plane, the latter being computed as is reported in Ref.[31]. The fatigue life is then estimated by exploiting the Basquin expression and obtaining Eq.(4).

5. CONCLUSIONS

A novel off-angle relationship to evaluate the critical plane orientation is proposed in the present paper. Such a relationship is used in conjunction with Carpinteri et al. criterion in order to improve the accuracy in fatigue life estimation of any metallic material under multiaxial constant amplitude fatigue loading.

Several tests on metallic materials under biaxial loading are available in the literature. The fatigue life for each test examined has been computed by employing both the novel relationship and the original off-angle expression, and then such results have been compared with the experimental fatigue life.

A good agreement has been observed between experimental data and theoretical life estimations when the novel relationship is used since the mean square error is equal to about 2, while such an error is equal to about 3 when the original expression is adopted.

The novel relationship used together with the Carpinteri et al. criterion seems to be a useful tool to obtain life estimations for any metallic material under multiaxial loading. Such a relationship should be further assessed in the case of both different materials and different loadings. As a matter of fact, it can be employed to estimate the critical plane orientation even in the case of metallic

specimens under variable amplitude loading. However, for such a type of loading, the present criterion cannot be used to evaluate the fatigue life, but other criteria formulated in time- or in frequency-domain have to be used, as is detailed in Ref.[28].

However, for such a type of loading, the present criterion

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Novel non-linear relationship to evaluate the critical plane orientation

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FIGURES AND TABLES

Figure 1. Flowchart of the Carpinteri et al. criterion.

Figure 2. Frame $\hat{P}\hat{1}\hat{2}\hat{3}$ of the averaged principal stresses, off-angle in the $\hat{1}\hat{3}$ plane, and normal w to the critical plane.

Figure 3. S-curve shape, by varying the value of the parameter a ($b=25$ and $c=1.5$).

Figure 4. S-curve shape by varying the value of the parameter b ($a=45$ and $c=1.5$).

Figure 5. S-curve shape by varying the value of the parameter c ($a=45$ and $b=25$).

Table 1. Fatigue properties of the examined metallic materials.

MATERIAL	Ref.	$\sigma_{af,-1}$ [MPa]	N_0 [cycles]	k	$\tau_{af,-1}$ [MPa]	N_0^* [cycles]	k^*
GGG40	[36]	257.067	10^6	10.95	237.413	10^6	12.41
CuZn40Pb2	[37]	243.990	10^6	5.86	194.480	10^6	17.17
GTS45	[36]	284.259	$2.5(10)^5$	19.40	224.762	$2.5(10)^5$	12.80
SUS304	[35]	213.717	$2.5(10)^3$	7.04	156.906	$2.5(10)^3$	8.70
SM45C	[34]	341.964	10^5	10.30	243.835	10^5	18.60
D30	[33]	178.265	$2(10)^6$	10.75	119.117	$2(10)^6$	9.20
982FA	[33]	338.007	10^6	12.10	218.127	10^6	18.60
10HNAP	[36]	386.598	$2(10)^6$	9.50	206.304	$2(10)^6$	8.20
IC2	[33]	102.651	10^6	8.80	88.862	10^6	19.50
30CrNiMo8	[38]	630.957	10^5	8.05	419.043	10^5	24.62
PA4 (6082-T6)	[39]	153.947	$2(10)^6$	8.00	91.391	$2(10)^6$	7.70

Figure 6. Proposed S-curve: data employed to perform the regression analysis, and plot of the curve characterized by $a=45$, $b=18$, and $c=1.37$.

Figure 7. Comparison between off-angle relationships: present study (Eq.(7) with $a=45$, $b=18$, and $c=1.37$), original curve (Eq.(3)), and expression in Ref.[30].

Figure 8. Mean square error T_{RMS} by employing the novel off-angle relationship (Eq.(7) with $a=45, b=18, c=1.37$). Results obtained from experimental data not used in the regression analysis are shown with diagonal hatching.

Figure 9. Mean square error, T_{RMS} , by employing the original off-angle relationship (Eq.(3)).

Figure 10. Experimental fatigue life vs predicted fatigue life for each experimental data set analysed, by applying: (a) the novel relationship, (b) the original relationship. All results above the solid line $N_{cal} = N_{exp}$ are conservative.