WAVELET BASED NEURAL NETWORK ARCHITECTURE FOR ECG SIGNAL COMPRESSION

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ABSTRACT

This paper addresses the problem of compressing Electrocardiogram (ECG) signals using the concept of adaptive sampling. The concept of adaptive sampling relates to optimum estimation of wavelet parameters that best represents a given signal. These wavelet parameters are estimated by minimizing the least mean square error between the original and approximated signal. Such an optimization approach is implemented within the frame work of neural networks by using wavelet non-linear functions in its neurons. We apply this technique for the compression of ECG signals. The experimental details of ECG compression are provided. For these experiments, the standard ECG database that was created by the American Heart Association (AHA) is used.

1. INTRODUCTION

The basic concept of wavelets is that they can be used as a set of basis functions and any given signal can be expanded as the weighted linear combinations of this set of basis functions. That is, the signal $x(t)$ can be represented as:

$$\hat{x}(t) = \sum_{k=1}^{K} w_k h\left(\frac{t-b_k}{a_k}\right), \qquad (1)$$

where $h(t)$ is the mother wavelet. A set of basis functions from this wavelet function can be obtained by choosing the shift parameter (time) b_k and the scale parameter a_k of the mother wavelet. Generally, these parameters are selected by using a fixed (uniform) sampling scheme. For example, $a_k = 2^k$ and $b_k = b_0 2^k$ where b_0 is some constant. Such a fixed sampling scheme results in a set of fixed shape wavelet basis functions which is input data independent. The authors in [1], have developed a method to select a data dependent mother wavelet from a library of wavelet functions. Even though such a method can lead to fast implementation; however, from the point of view of achieving higher compression rate or better representation of a given data, it has been shown in [2, 3] that it is advantageous to adaptively compute the wavelet functions that best describe the data rather than choosing from a fixed library. Therefore, we introduced an adaptive sampling scheme in [7, 8] which relates to adaptively estimating the wavelet parameters. Using such a sampling scheme, the shape of the originally chosen mother wavelet can be changed adaptively. This can be achieved by creating another mother wavelet which is a linear combination of one or more wavelets whose parameters were sampled adaptively in the previous iteration. Such an adaptive sampling scheme can be implemented efficiently by using wavelet non-linear functions in the neurons of a neural network and then by using any learning or optimization algorithms that are popular in the neural network literature. The usage of wavelet non-linear functions in the neurons of a neural network is similar in concept to the Radial Basis function based neural network [4]. However, the wavelet function in each of the neurons are related to the other by affine scale transformation.

In this paper, we apply the adptive sampling scheme developed by us in [7, 8] for ECG signal compression. The ECG signal is a recording of heart's electrical activity of humans that gives information about the functionality of the heart. Hence, it is used to diagnose heart problems by physicians. A typical ECG signal of one cardiac cycle consists of events such as: (i) P wave: de-polarization of the atria, (ii) QRS complex: de-polarization of the ventricles, (iii) T wave: re-polarization of the ventricles and (iv) U wave: further re-polarization of ventricles. Generally, U wave is not predominant in the actual ECG signal. With the widespread deployment of electronic instruments in medical fields, digital ECG recorders are being widely used in clinical practices. In addition, the ECG data is generally transmitted over telecommunication network for consultation. The telecommunication channel can not be efficiently utilized if the data is sent as is. In order to overcome the limitations of memory requirements of these recorders and to transmit ECG data efficiently over the telecommunication networks there is need for an efficient ECG coder. In this paper, we describe such a coder which can achieve compression ratio of 24.5 which is five times better than the previously reported coder based on the extrema of the WT [5]. In the next section, overview of the adaptive sampling scheme is provided.

2. SAMPLING THE TIME-SCALE PLANE

Consider the completely redundant continuous time wavelet representation with respect to the basis $\Psi_{a,b}(t)$ which are obtained by shifting and scaling $\Psi(t)$.

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\Psi\left(\frac{t-b}{a}\right)$$

where $a, b \in R$. The necessary condition for Ψ_{ab} to yield a numerically stable reconstruction of x from $(<\Psi_{ab} x>)$

is that Ψ_{ab} should constitute a frame for the space of functions x that we are interested in. To obtain Ψ_{ab} which satisfies this necessary condition in the case of *adaptive sampling*, we sought a middle ground between non-redundant orthonormal bases which are known to yield unique stable reconstructions, and extremely redundant continuous wavelet representations where the scale and time parameters are continuous. Here, we apply such a scheme for ECG signal compression since, the most suitable set of input data dependent basis functions can be obtained. In the next section, this sampling scheme is briefly described.

2.1. Adaptive sampling

The *adaptive sampling* scheme refers to a scheme where the sampling points a_k for the scale axis and b_k for the time axis are chosen according to the input signal, to minimize the error of reconstruction of $x(t)$, if the transform was sampled at (a_k, b_k).

Let us assume that there exists a particular wavelet $h(t)$ and a numerical algorithm to determine a_k, b_k and w_k so that for a given N (the number of wavelets), given ϵ (arbitrary constant), the following holds:

$$\| \sum_{k=1}^{N} w_k h(\frac{t-a_k}{b_k}) - x(t)\|^2 \leq \epsilon \tag{2}$$

Then we can prove that

$$\Psi_{a_k,b_k} = h(\frac{t-a_k}{b_k}) \tag{3}$$

constitutes a frame. The definition of a frame is given below; however, for the statement of the theorem and proof refer to [7]. Hence, under certain mild conditions, a numerically stable method of reconstructing the signal x from the coefficients related to these basis functions exists.

Definition 1 *The family $\Psi_{a,b}$ constitutes a* **frame** *of some space $\mathcal{H}$ if $\forall\ x \in \mathcal{H}$*

$$A\|x\|^2 \leq \sum_{a,b} | < \Psi_{a,b}, x > |^2 \leq B\|x\|^2$$

is satisfied for some $A, B \in R$ and $0 < A \leq B < \infty$.

It can be shown that the constants $A = \frac{1}{2\max|w_k|}$ and $B = N\max_k \|h_k\|^2$ [7] depend on the function x since the sampling points a_k and b_k are chosen based on x. Therefore, for every function x, these sampling points need to be specified. This justifies the terminology *adaptive sampling*. Using this adaptive sampling scheme in the framework of neural network two problems can be formulated – representation and classification. In the following section, the representation problem formulation is given. For the classification formulation, refer to [8].

2.2. Adaptive sampling – Representation

Eq. (1) implies that any given signal can be approximated (represented) as a weighted linear combination of wavelet basis functions. From the discussion in the previous sections, it is clear that to represent a given signal with minimum reconstruction error, the optimum adaptive wavelet parameters need to be determined along with the weights w_ks. These parameters can be determined by using the wavelet non-linearities instead of the standard sigmoidal non-linearities in the neurons of the neural network architecture that is shown in Figure 1. The conjugate gradient optimization (learning) algorithm [6] that minimizes the error function

$$E = \frac{1}{2}\| \sum_{k=1}^{N} w_k h(\frac{t-b_k}{a_k}) - x(t)\|^2 \tag{4}$$

is used in conjunction with the proposed neural network architecture. Let $\hat{x}(t) = \sum_{k=1}^{N} w_k h(\frac{t-b_k}{a_k})$ and the mother wavelet be Morlet which is defined as:

$$h(t) = cos(1.75t)exp(-\frac{t^2}{2}) \tag{5}$$

The gradients for the iterative conjugate gradient algorithm are obtained by differentiating E with respect to w_k, a_k & b_k, respectively. These gradients are then used in the two step procedure of this algorithm. During the first step, this algorithm finds the search direction at the i^{th} iteration and during the second step, new weight is computed using a variable step size α. At each iteration, for each of the adaptive wavelet parameters and w_k, these two steps are computed. The convergence of this algorithm yields the optimum adaptive wavelet parameters and thus adaptive sampling of the transform is achieved. These are then used to reconstruct the signal. The reconstruction error can be made arbitrarily small by choosing N such that it satisfies the conditions that are mentioned in theorem 1 of [7].

Note that the choice of Morlet wavelet is for mathematical convenience and works well for biological signals such as speech (voiced sounds) and ECG. In addition, for Morlet wavelet, the conjugate gradient algorithm almost always converges for the signals mentioned above. However, the choice of the mother wavelet depends on the application. Depending on the choice of the mother wavelet, a set of gradient equations needed in the conjugate gradient algorithm has to be derived.

2.3. ECG signal compression

In this section, we discuss the ECG signal compression. In the last section, we showed that the adaptive sampling scheme leads to representation problem. In principle, this representation formulation provides a method to estimate the optimum wavelet parameters that best represent a given signal. This implies that the input signal can be optimally represented by a set of adaptive wavelet parameters whose dimension is much less than the signal. Therefore, the input signal can be compressed by encoding the optimum adaptive wavelet parameters. We have used this concept in developing a low bit rate ECG coder that is described below.

The adaptive wavelet parameters corresponding to one segment of the ECG signal are obtained using the algorithm described in the previous section. We used Morlet wavelet that is defined in Eq. (5) as the mother wavelet in the neurons of the neural network architecture of figure 1. The conjugate gradient optimization algorithm that is described in the previous section is applied to estimate the optimum adaptive wavelet parameters ($a_k s, b_k s$) and weights (w_ks). These parameters are then encoded by rounding them off to nearest integer and using the simple scalar quantization technique. Thus compression of one segment of the ECG signal is achieved. This compressed data in the form of encoded wavelet parameters and weights is either stored in memory or transmitted over the telecommunication channel. The encoded parameters from the memory or received at the receiver are decoded and used to re-synthesize the ECG data by applying Eq. (1) to recover the original data.

This compression technique was applied to ECG signals that were obtained from the AHA database. In the following section, experimental details of the compression experiment are given. In addition, the computation of compression ratio is also described.

2.3.1. Experimental details:

The ECG data was segmented such that each segment contains one cardiac cycle. The optimum adaptive wavelet parameters corresponding to each segment were then obtained by initializing N, a_k, b_k and w_k as follows:
1. N was set to 4 since there are 4 types of waves in one cardiac cycle of the ECG signal
2. $a_0 = a_3 = 12$, $a_1 = a_2 = 4$
3. w_k were set to 0
4. b_0, b_1, b_3 and b_4 were set to approximate locations of the P, R, S and T waves of the ECG signal, respectively. These locations were found empirically with *a priori* knowledge about the signal.

The conjugate gradient algorithm was then executed by setting step size $\alpha_{w_k} = 0.0001$ and $\alpha_{a_k} = \alpha_{b_k} = 0.0000001$, respectively, and till $\epsilon = 0.0001$ or number of iterations = 100. The adaptive wavelet parameters and the weights that are obtained when the algorithm stopped were considered as the optimum parameters. These were then rounded off to nearest integers. It was observed that the dynamic range of b_ks was 0 to 256, a_ks was 0 to 16 and w_ks was -512 to 512. Therefore, we need 8 bits for b_ks, 4 bits for a_ks and 10 bits for w_ks to uniquely represent them using simple scalar quantization technique. Since we use four wavelets with three parameters associated with each of them, we need $22 \times 4 = 88$ bits to represent one segment of an ECG signal. In Figures 2, 3 and 4 ten segments of ECG signal with resynthesized signal overlaid on top of the original signal are plotted for 3 different types of ECG data. Note that these ECG signals are not normal signals i.e., contains arrythmias that correspond to some heart problems. From these figures, it can be seen that the resynthesized signal from the optimum adaptive wavelet parameters using Eq. (1) is very close to the original and preserve all the clinically important features which correspond to preserving all the ECG waves and their locations. In some cases ST elevations are mis-represented. However, this can be rectified by considering a separate wavelet function for ST portion of the ECG signal. From these figures also, it can be seen that each segment consists of about 180 samples. Let each sample be quantized and be represented by 12 bits. From the above description, we know that by using the proposed compression algorithm we would need 88 bits to compress 180 samples. This implies that we need 0.49 bits/sample. Therefore, we can achieve a CR of 24.49. This CR is approximately 5 times better than the previous study [5]. This demonstrates the applicability of the proposed compression algorithm to develop a low bit rate ECG coder. Note that by using Vector Quantization (VQ) based techniques we can further reduce the bit rate.

3. CONCLUSION

In this paper, we have demonstrated the applicability of adaptive sampling scheme in conjunction with neural networks for the compression of biological signals such as ECG. Preliminary results are promising and indicate the potential of the proposed methodology for this application. Future work warrants the application of the proposed methodology to larger database and further reduction of bit rate by applying VQ based techniques.

REFERENCES

[1] R. Coifman and M. Wickerhauser, "Entropy based algorithms for best basis selection," *IEEE Trans. on Inf. theory*, Vol. 38, pp. 713-718, March 1992.

[2] H. Szu, B. Telfer and S. Kadambe, "Neural network adaptive wavelets for signal representation and classification," *Journal of Optical Eng.*, Vol. 31, pp. 1907-1916, September 1992.

[3] S. Kadambe and P. Srinivasan, "Applications of adaptive wavelets for speech," *Journal of Optical engg.*, Vol. 33, No. 7, pp. 2204-2211, July 1994.

[4] D. Broomhead and D. Lowe, "Multi-variable functional interpolation and adaptive networks," *Complex Syst.*, Vol. 2, pp. 321, 1988.

[5] A. E. Cetin, A. W. Tewfik and Y. Yardimci, "Coding of ECG signals by wavelet transform extrema," *Proc. of the IEEE-SP Intnl. symp. on Time-frequency and time-scale analysis*, pp. 544-547, October 1994.

[6] R. Fletcher, "Practical methods of optimization," *John Wiley and Sons, New York*, 1987.

[7] P. Srinivasan and S. Kadambe, "Wavelet based neural network architecture for signal classification and compression," Submitted for *IEEE Trans. on SP*, 1997.

[8] S. Kadambe and P. Srinivasan, "Adaptive wavelet phoneme recognition," *Proc. of the 40th midwest symposium on Circuits and systems*, pp. 720-723, August 1997.

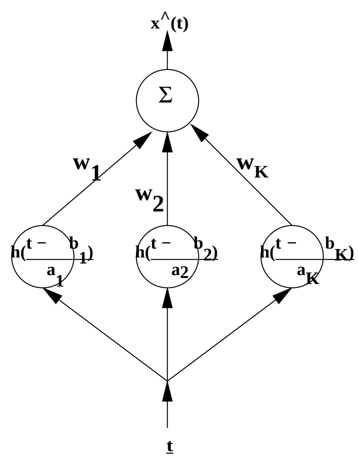

Figure 1. The neural network architecture for the adaptive sampling scheme (representation).

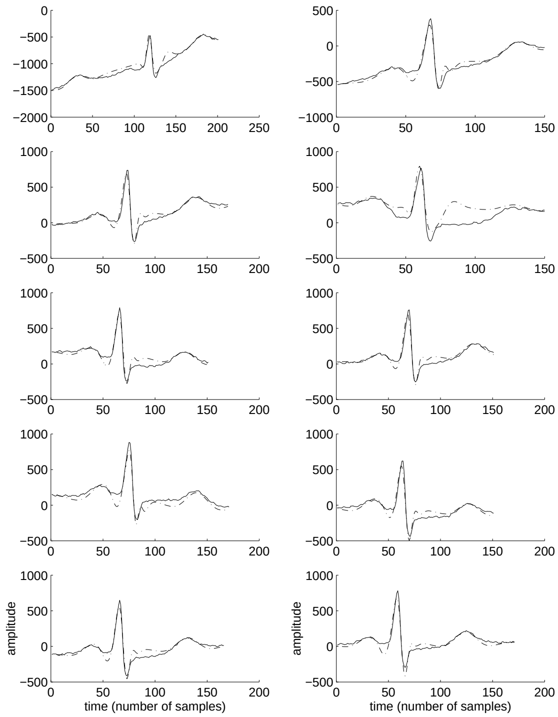

Figure 2. The original (solid line) and the resynthesized (dashed line) segments of ECG signal from a1204 tape of the AHA ECG database

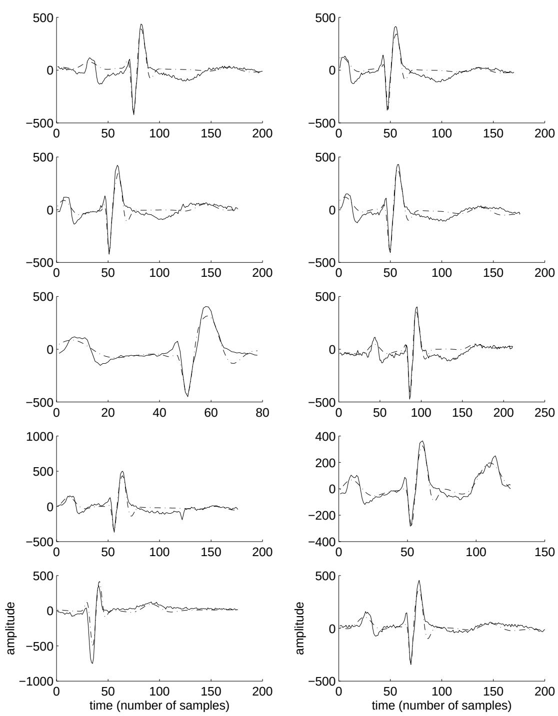

Figure 3. The original (solid line) and the resynthesized (dashed line) segments of ECG signal from a1205 tape of the AHA ECG database

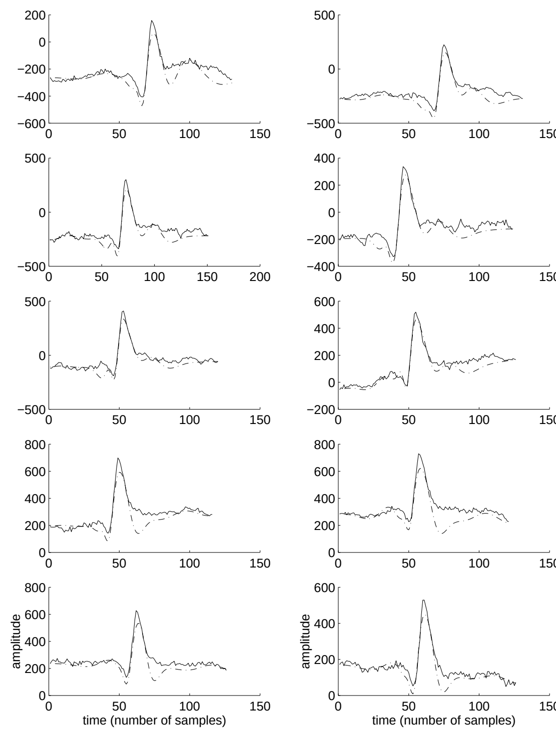

Figure 4. The original (solid line) and the resynthesized (dashed line) segments of ECG signal from a1209 tape of the AHA ECG database