A Direct Model for Triangular Neutrosophic Linear Programming

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Abstract

This paper aims to propose a new direct algorithm to solve the neutrosophic linear programming where the variables and right-hand side represented with triangular neutrosophic numbers. The effectiveness of the proposed procedure is illustrated through numerical experiments. The extracted results show that the new algorithm is straightforward and could be useful to guide the modeling and design of a wide range of neutrosophic optimization.

Keywords: Single valued neutrosophic number; Neutrosophic linear programming problem; Linear programming problem.

1. Introduction

Fuzzy set originally introduced by Zadeh [1] in 1965 is a useful tool to capture the imprecision and uncertainty in decision-making [2, 3]. It is characterized by a membership degree between zero and one, and the non-membership degree is equal to one minus the membership degree. The intuitionistic fuzzy set (IFS) theory launched by Atanassov [4], addresses the problem of uncertainty by considering a non-membership function along with the fuzzy membership function on a universal set. The membership degree of an object is complemented with a non-membership degree that gives the extent to which an object does not belong to the IFS such that the sum of the two degrees should be less than or equal to 1.

Neutrosophy has been proposed by Smarandache [5] as a new branch of philosophy, with ancient roots, dealing with "the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra". The fundamental thesis of neutrosophy is that every idea has not only a certain degree of truth, as is generally assumed in many-valued logic contexts but also a falsity and indeterminacy degrees that have to be considered independently from each other. Smarandache seems to understand such "indeterminacy" both in a subjective and an objective sense, i.e., as uncertainty as well as imprecision, vagueness, error, doubtfulness, etc. Neutrosophic set (NS) is a generalization of the fuzzy set [1] and intuitionistic fuzzy set [4] and can deal with uncertain, indeterminate and incongruous information where the indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are completely independent. It can effectively describe uncertain, incomplete and inconsistent information and overcomes some limitations of the existing methods in depicting uncertain decision information. In the neutrosophic logic, each proposition is estimated by a triplet viz, truth grade, indeterminacy grade and falsity grade. The indeterministic part of uncertain data, introduced in NS theory, plays an important role in making a proper decision which is not possible by intuitionistic fuzzy set theory. Since indeterminacy always appears in our routine activities, the NS theory can analyze the various situations smoothly. Moreover, some extensions of NSs, including interval neutrosophic set [6], bipolar neutrosophic set [7], single-valued neutrosophic set [8], multi-valued neutrosophic set [9], and neutrosophic linguistic set [10] have been proposed and applied to solve various problems [11-20].

Linear programming problem (LP) is a method for achieving the best outcome (such as maximum profit or minimum cost) in a mathematical model represented by linear relationships. Decision-making is a process of solving the problem and achieving goals under the asset of constraints, and it is very difficult in some cases due to incomplete and imprecise information. In uncertain linear programming problems, such as approaches using fuzzy and stochastic logics, interval numbers, or uncertain variables, some uncertain linear programming methods have been developed in the existing literature. For example, Bellman and Zadeh [21] introduced fuzzy optimization problems where they have stated that a fuzzy decision can be viewed as the intersection of fuzzy goals and problem constraints. Many researchers such as; Zimmermann [22], Tanaka et al.[17], Campos and Verdegay [23], Rommelfanger et al.[24], Cadenas and Verdegay [25] who were dealing with the concept of solving fuzzy optimization problems, later studied this subject. In the past few years, a growing interest has been shown in Fuzzy optimization. Buckley and Feuring [26] introduced a general class of fuzzy linear programming, called fully fuzzified linear programming (FFLP) problems, where all decision parameters and variables are fuzzy numbers. Fuzzy mathematical programming, using a unified approach, has been studied by [27]. Lodwick and Bachman [28] have studied large scale fuzzy and possible optimization problems. Buckley and Abdalla [29] have considered Monte Carlo methods in the fuzzy queuing theory. Some authors have considered fuzzy linear programming, in which not all parts of the problem were assumed to be fuzzy, e.g., only the right-hand side and the objective function coefficients were fuzzy; or only the variables were fuzzy [30-35]. The fuzzy linear programming problems in which fuzzy numbers represent all the parameters and variables are known as fully fuzzy linear programming (FFLP) problems. FFLP problem with inequality constraints studied in [36-37]. However, the main disadvantage of the solution obtained by the existing methods is that it does not satisfy the constraints exactly i.e., it is not possible to obtain the fuzzy number of the right-hand side of the constraint by putting the obtained solution in the left-hand side of the constraint. Dehghan et al. [38] proposed some practical methods to solve a fully fuzzy linear system (FFLS) that is comparable to the well-known methods. Then they extended a new method employing Linear Programming (LP) for solving square and non-square fuzzy systems. Lotfi et al. [39] applied the concept of the symmetric triangular fuzzy number, obtained a new method for solving FFLP by converting an FFLP into two corresponding LPs. Kumar et al. [40] pointed out the shortcomings of the above methods. To overcome these shortcomings, they proposed a new method for finding the fuzzy optimal solution of FFLP problems with equality constraints. Saberi Najafi and Edalatpanah [41] pointed out the method of [40] needs some corrections to make the model well in general; for other methods see[42-46]. In general, the above existing methods can be applied for the following type of FFLP problems:

i) FFLP Problem with nonnegative fuzzy coefficients and nonnegative fuzzy variables.

ii) FFLP Problem with unrestricted fuzzy coefficients and nonnegative fuzzy variables.

iii) FFLP Problem with nonnegative fuzzy coefficients and unrestricted fuzzy variables.

However, the above mentioned methods can-not deal with indeterminate optimization problems. Furthermore, the existing uncertain linear programming methods are not really meaningful indeterminate programming because these uncertain linear programming methods are generally to turn these optimization models into crisp objective programming models to find unique crisp optimal solutions rather than indeterminate solutions in uncertain situations. However, the unique crisp optimal solutions obtained by existing uncertain linear programming methods may be conservative and relatively insensitive to input uncertainty or the optimization performance may degrade significantly. From an indeterminate viewpoint, an indeterminate optimization problem should contain possible ranges of the optimal solutions (indeterminate intervals) corresponding to various indeterminate ranges to be suitable for indeterminate requirements rather than the unique crisp optimal solution under indeterminate environments. Then, Abdel-Baset et al. [47] and Pramanik [48] proposed neutrosophic linear programming methods based on the neutrosophic set (NS) concept. Also, Abdel-Baset et al. [49] introduced the neutrosophic LP models where their parameters are represented with trapezoidal neutrosophic numbers and presented a technique for solving them. However, it is observed that Abdel-Basset et al.[50] have considered several mathematical incorrect assumptions in their proposed method and hence, it is scientifically incorrect to use this method [51].

So, the main purposes of this paper are (1) to propose a new direct model, including neutrosophic variables and the right-hand side; and (2) to present a solution method for this neutrosophic LP problems. This paper organized as follows: some basic knowledge, concepts of neutrosophic set theory, an arithmetic operation are introduced in Section 2. In Section 3, we present a new algorithm to solve the neutrosophic LP. In Section 4, a numerical example is given to reveal the effectiveness of the proposed model. Finally, some conclusions are drawn in the last section.

2. Preliminaries

In this section, we present some basic definitions and arithmetic operations on neutrosophic sets.

Definition 1 [5]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x)$: $X \to [0,1]$, $I_A(x)$: $X \to [0,1]$, and $F_A(x)$: $X \to [0,1]$. Then, a Single valued neutrosophic set A is denoted by $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ which is called an SVN. Also, SVN satisfies the condition:

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

Definition 2 [5]. For SVNSs A and B, $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, and $F_A(x) \ge F_B(x)$ for every x in X.

Definition 3 [50]. A triangular neutrosophic number (TNNs) is denoted by $A^{\aleph} = \langle (a^l, a^m, a^u), (\mu, i, \omega) \rangle$ whose the three membership functions for the truth, indeterminacy, and falsity of x can be defined as follows:

$$\tau_{A^{\aleph}}(x) = \begin{cases} \dfrac{(x - a^l)}{(a^m - a^l)}\mu & a^l \le x < a^m, \\ \mu & x = a^m, \\ \dfrac{(a^u - x)}{(a^u - a^m)}\mu & a^m \le x < a^u, \\ 0 & otherwise. \end{cases}$$

$$\iota_{A^{\aleph}}(x) = \begin{cases} \dfrac{(a^m - x)}{(a^m - a^l)} i, & a^l \le x < a^m, \\ i, & x = a^m, \\ \dfrac{(x - a^u)}{(a^u - a^m)} i, & a^m \le x < a^u, \\ 1, & otherwise. \end{cases}$$

$$\nu_{A^{\aleph}}(x)=\begin{cases}\dfrac{(a^{m}-x)}{(a^{m}-a^{l})}\omega, & a^{l}\le x<a^{m},\\ \omega, & x=a^{m},\\ \dfrac{(x-a^{u})}{(a^{u}-a^{m})}\omega, & a^{m}\le x<a^{u},\\ 1, & otherwise.\end{cases}$$

Where, $0\le \tau_{A^{\aleph}}(x)+\iota_{A^{\aleph}}(x)+\nu_{A^{\aleph}}(x)\le 3, x\in A^{\aleph}.$ Additionally, when $a^{l}\ge 0,$ $A^{\aleph}$ is called a nonnegative TNN. Similarly, when $a^{l}<0,$ $A^{\aleph}$ becomes a negative TNN.

Definition 4 [50]. Suppose $A_1^{\aleph}=<(a_1,b_1,c_1),(\mu_1,\nu_1,\omega_1)>$ and $A_2^{\aleph}=<(a_2,b_2,c_2),(\mu_2,\nu_2,\omega_2)>$ be two TNNs. Then the arithmetic relations are defined as:

$(i)A_1^{\aleph}\oplus A_2^{\aleph}=<(a_1+a_2,b_1+b_2,c_1+c_2),(\mu_1\wedge\mu_2,\nu_1\vee\nu_2,\omega_1\vee\omega_2)>$

$(ii)A_1^{\aleph}-A_2^{\aleph}=<(a_1-c_2,b_1-b_2,c_1-a_2),(\mu_1\wedge\mu_2,\nu_1\vee\nu_2,\omega_1\vee\omega_2)>$

$(iii)A_1^{\aleph}\otimes A_2^{\aleph}=$
$<(a_1a_2,b_1b_2,c_1c_2),(\mu_1\wedge\mu_2,\nu_1\vee\nu_2,\omega_1\vee\omega_2)>,\ \ if\ a_1>0,b_1>0,$

$$(iv)\lambda A_1^{\aleph}=\begin{cases}<(\lambda a_1,\lambda b_1,\lambda c_1),(\mu_1,\nu_1,\omega_1)>, & if\ \ \lambda>0\\ <(\lambda c_1,\lambda b_1,\lambda a_1),(\mu_1,\nu_1,\omega_1)>, & if\ \ \lambda<0\end{cases}$$

3. Proposed method

Consider the following trapezoidal neutrosophic linear programming (TNLP) with *m* constraints and *n* variables;

$$\begin{aligned}&\text{Max (Min) } (c^{t}\tilde{x})\\ &\text{subject to}\\ &\qquad A\tilde{x}\le\tilde{b},\end{aligned}\qquad (1)$$

$\tilde{x}$ is a non-negative TNN.

Where $A=\left[a_{ij}\right]_{m\times n}$ is the coefficient matrix, $\tilde{b}=\left[\tilde{b}_1,\tilde{b}_2,\tilde{b}_3,\ldots,\tilde{b}_m\right]^{t}$ is the triangular neutrosophic available resource vector, $c=\left[c_1,c_2,c_3,\ldots,c_n\right]^{t}$ is the objective coefficient vector and $\tilde{x}=\left[\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\ldots,\tilde{x}_n\right]^{t}$ is the triangular neutrosophic decision variable vector.

The steps of the proposed method are as follows:

Step 1: Assuming $\tilde{b}=<b^{l},b^{m},b^{r};T_{\tilde{b}},I_{\tilde{b}},F_{\tilde{b}}>,$ $\tilde{x}=<x^{l},x^{m},x^{r};T_{\tilde{x}},I_{\tilde{x}},F_{\tilde{x}}>,$ and using Definition 4, the LP problem (1) can be transformed into the problem (2).

$$\begin{aligned}
&\text{Max (Min)} \sum_{j=1}^{n} c_j < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} >, \\
&\text{subject to} \\
&\quad \sum_{j=1}^{n} a_{ij} < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > \ \leq \ < b_i^l, b_i^m, b_i^r; T_{\tilde{b}_i}, I_{\tilde{b}}, F_{\tilde{b}} >, \forall i \\
&\quad\quad < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > \ \geq 0, \quad \forall j.
\end{aligned} \tag{2}$$

Step 2: Using definition 2 -4, and with the following assumptions:

$$\sum_{j=1}^{n} c_j < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} >=< u^l, u^m, u^r; T, I, F >,$$

$$\sum_{j=1}^{n} a_{ij} < x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} >=< v^l, v^m, v^r; T, I, F >,$$

the LP problem (2) can be transformed into the problem (3).

$$\begin{aligned}
&\text{Max (Min)} < u^l, u^m, u^r; T, I, F >, \\
&\text{subject to} \\
&< v^l, v^m, v^r; T, I, F > \ \leq \ < b_i^l, b_i^m, b_i^r; T_{\tilde{b}_i}, I_{\tilde{b}}, F_{\tilde{b}} >, \forall i \\
&T \leq \min(T_{\tilde{b}_i}), I \geq \max(I_{\tilde{b}}), F \geq \max(F_{\tilde{b}}), \\
&T = \min(T_{\tilde{x}_j}), I = \max(I_{\tilde{x}_j}), F = \max(F_{\tilde{x}_j}), \\
&< x_j^l, x_j^m, x_j^r; T_{\tilde{x}_j}, I_{\tilde{x}_j}, F_{\tilde{x}_j} > \ \geq 0.
\end{aligned} \tag{3}$$

Step 3: By the neutrosophic nature, the LP problem (3) can be transformed into a multi-objective problem (4).

$$\begin{aligned}
&\text{Max (Min) } u^l, \\
&\text{Max (Min) } u^m, \\
&\text{Max (Min) } u^r,
\end{aligned} \tag{4}$$

$$\begin{aligned}
&\text{Max (Min)} \sum_{j=1}^{n} T_{\tilde{x}_j}, \\
&\text{Min (Max)} \sum_{j=1}^{n} I_{\tilde{x}_j}, \\
&\text{Min (Max)} \sum_{j=1}^{n} F_{\tilde{x}_j},
\end{aligned}$$

$$
\begin{aligned}
&\text{subject to}\\
&v^{l} \le b_i^{l}, \forall i\\
&v^{m} \le b_i^{m}, \forall i\\
&v^{r} \le b_i^{r}, \forall i\\
&\sum_{j=1}^{n} T_{\tilde{x}_j} \le n \min(T_{\tilde{b}_i}),\\
&\sum_{j=1}^{n} I_{\tilde{x}_j} \ge n \max(I_{\tilde{b}}),\\
&\sum_{j=1}^{n} F_{\tilde{x}_j} \ge n \max(F_{\tilde{b}})\\
&x_j^{l} \ge 0, x_j^{m} - x_j^{l} \ge 0, x_j^{r} - x_j^{m} \ge 0,\\
&0 \le T_{\tilde{x}_j} \le 1, 0 \le I_{\tilde{x}_j} \le 1, 0 \le F_{\tilde{x}_j}\ 0 \le 1,\\
&T_{\tilde{x}_j} + I_{\tilde{x}_j} + F_{\tilde{x}_j} \le 3,\\
&T_{\tilde{x}_j} \ge F_{\tilde{x}_j}, T_{\tilde{x}_j} \ge I_{\tilde{x}_j}.
\end{aligned}
$$

Step 4: Using summation of all object functions, the Model (4), obtained in Step 3, can be converted into the crisp linear programming problem as follows:

$$
\text{Max (Min) } u^{l} + u^{m} + u^{r} + \sum_{j=1}^{n} T_{\tilde{x}_j} - \sum_{j=1}^{n} I_{\tilde{x}_j} - \sum_{j=1}^{n} F_{\tilde{x}_j}, \tag{5}
$$

Subject to: all constraints of Model (4).

Step 4: Find the optimal solution $\tilde{x}$ by solving the crisp linear programming problems obtained in problem (5) and find the neutrosophic optimal value by putting in the objective function.

4. Numerical example

In this section, a numerical example problem has been solved using the proposed method to illustrate the applicability and efficiency of it.

Example 1 .

$$
Max\ (\tilde{z}) = 5\tilde{x}_1 + 4\tilde{x}_2
$$

subject to

$$\begin{aligned}&6\tilde{x}_1+4\tilde{x}_2\le\ <3,5,6;0.6,0.5,0.6>,\\&\tilde{x}_1+2\tilde{x}_2\le\ <5,8,10;0.3,0.6,0.6>,\\&-\tilde{x}_1+\tilde{x}_2\le\ <12,15,19;0.6,0.4,0.5>,\\&\qquad\tilde{x}_2\le\ <14,17,21;0.8,0.2,0.6>,\\&\tilde{x}_1,\tilde{x}_2\ge 0.\end{aligned}\tag{6}$$

Now. To solve the problem with the proposed method, we have the following steps:

Step 1: Assuming $\tilde{x}=<x^l,x^m,x^r;T_{\tilde{x}},I_{\tilde{x}},F_{\tilde{x}}>,$ and using Definition 4, the LP problem (4) can be transformed into the problem (7).

$$\begin{aligned}&\text{Max}\,\tilde{z}=5<x_1^l,x_1^m,x_1^r;T_{\tilde{x}_1},I_{\tilde{x}_1},F_{\tilde{x}_1}>\oplus 4<x_2^l,x_2^m,x_2^r;T_{\tilde{x}_2},I_{\tilde{x}_2},F_{\tilde{x}_2}>\\&\text{subject to}\\&6<x_1^l,x_1^m,x_1^r;T_{\tilde{x}_1},I_{\tilde{x}_1},F_{\tilde{x}_1}>\oplus 4<x_2^l,x_2^m,x_2^r;T_{\tilde{x}_2},I_{\tilde{x}_2},F_{\tilde{x}_2}>\ \le\ <3,5,6;0.9,0.1,0.2>,\\&<x_1^l,x_1^m,x_1^r;T_{\tilde{x}_1},I_{\tilde{x}_1},F_{\tilde{x}_1}>\oplus 2<x_2^l,x_2^m,x_2^r;T_{\tilde{x}_2},I_{\tilde{x}_2},F_{\tilde{x}_2}>\ \le\ <5,8,10;0.7,0.2,0.1>,\\&-<x_1^l,x_1^m,x_1^r;T_{\tilde{x}_1},I_{\tilde{x}_1},F_{\tilde{x}_1}>\oplus<x_2^l,x_2^m,x_2^r;T_{\tilde{x}_2},I_{\tilde{x}_2},F_{\tilde{x}_2}>\ \le\ <12,15,19;0.8,0.3,0.1>,\\&<x_2^l,x_2^m,x_2^r;T_{\tilde{x}_2},I_{\tilde{x}_2},F_{\tilde{x}_2}>\ \le\ <14,17,21;0.8,0.2,0.1>,\\&<x_j^l,x_j^m,x_j^r;T_{\tilde{x}_j},I_{\tilde{x}_j},F_{\tilde{x}_j}>\ \ge 0,\ \ \forall j.\end{aligned}\tag{7}$$

Step 2: Using definition 2 -4, the LP problem (7) can be transformed into the problem (8).

$$\begin{aligned}&\text{Max}\,\tilde{z}=<5x_1^l+4x_2^l,5x_1^m+4x_2^m,5x_1^r+4x_2^r;T,I,F>\\&\text{subject to}\\&<6x_1^l+4x_2^l,6x_1^m+4x_2^m,6x_1^r+4x_2^r;T,I,F>\ \le\ <3,5,6;0.9,0.1,0.2>,\\&<x_1^l+2x_2^l,x_1^m+2x_2^m,x_1^r+2x_2^r;T,I,F>\ \le\ <5,8,10;0.7,0.2,0.1>,\\&<-x_1^r+x_2^l,-x_1^m+x_2^m,-x_1^l+x_2^r;T,I,F>\ \le\ <12,15,19;0.8,0.3,0.1>,\\&<x_2^l,x_2^m,x_2^r;T_{\tilde{x}_2},I_{\tilde{x}_2},F_{\tilde{x}_2}>\ \le\ <14,17,21;0.8,0.2,0.1>,\\&<x_j^l,x_j^m,x_j^r;T_{\tilde{x}_j},I_{\tilde{x}_j},F_{\tilde{x}_j}>\ \ge 0,\ \ \forall j.\end{aligned}\tag{8}$$

Step 3: Using the Step 4 of our proposed method, the Model (8), can be converted into the crisp linear programming problem as follows:

$$\begin{aligned}&\text{Max}\,\tilde{z}=5x_1^l+4x_2^l+5x_1^m+4x_2^m+5x_1^r+4x_2^r+T_{\tilde{x}_1}+T_{\tilde{x}_2}-I_{\tilde{x}_1}-I_{\tilde{x}_2}-F_{\tilde{x}_1}-F_{\tilde{x}_2}\\&\text{subject to}\\&6x_1^l+4x_2^l\le 3,x_1^l+2x_2^l\le 5,-x_1^r+x_2^l\le 12,x_2^l\le 14,\end{aligned}\tag{9}$$

$$6x_1^m + 4x_2^m \le 5, x_1^m + 2x_2^m \le 8, -x_1^m + x_2^m \le 15, x_2^m \le 17,$$
$$6x_1^r + 4x_2^r \le 6, x_1^r + 2x_2^r \le 10, -x_1^l + x_2^r \le 19, x_2^r \le 21,$$
$$T_{\tilde{x}_1} + T_{\tilde{x}_2} \le 1.4,$$
$$I_{\tilde{x}_1} + I_{\tilde{x}_2} \ge 0.6,$$
$$F_{\tilde{x}_1} + F_{\tilde{x}_2} \ge 0.4,$$
$$x_j^l \ge 0, x_j^m - x_j^l \ge 0, x_j^r - x_j^m \ge 0,$$
$$0 \le T_{\tilde{x}_j} \le 1, 0 \le I_{\tilde{x}_j} \le 1, 0 \le F_{\tilde{x}_j} 0 \le 1,$$
$$T_{\tilde{x}_j} + I_{\tilde{x}_j} + F_{\tilde{x}_j} \le 3,$$
$$T_{\tilde{x}_j} \ge F_{\tilde{x}_j}, T_{\tilde{x}_j} \ge I_{\tilde{x}_j}.$$

Step 4: Using Matlab or any software, we can solve the optimal solution as follows:

$$\tilde{x}_1 = < 0, 0, 0; 0.6, 0.6, 0.4 >,$$
$$\tilde{x}_2 = < 0.75, 1.25, 1.5; 0.8, 0, 0 >,$$
$$\tilde{z} = < 3, 5, 6; 0.6, 0.6, 0.4 >.$$

5. Conclusion

In this paper, we proposed a new direct algorithm for solving the linear programming problems, including neutrosophic variables and the right-hand side. In the proposed model, we maximize the degrees of acceptance and minimize indeterminacy and rejection of objectives. Meantime, a numerical example was provided to show the efficiency of the proposed method and illustrate the solution process. The new model not only richens uncertain linear programming methods but also provides a new effective way for handling indeterminate optimization problems

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