DESIGN OF EQUIRIPPLE MINIMUM PHASE FIR-FILTERS

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ABSTRACT

The numerical robustness and applicability of a recent approach to the design of minimum phase FIR filters by cascading a passband and a stopband filter [Go 80] is improved by the following measures: i) Spectral factorization (root finding) is avoided by exploiting the Hilbert relations between attenuation and phase ii) Unbounded gain in don't care regions is restricted by prescription of appropriate desired values. As a result, highly selective minimum phase FIR filters requiring a considerably reduced number of multiplications are obtained.

1 INTRODUCTION

Unconditional stability, the linear-phase property, and the ease of design and implementation are major reasons for the widespread use of finite impulse response (FIR) filters in digital signal processing [Mi 93]. However, there exist a couple of important applications, where the high group delay resulting from linear phase is crucial, such as in two-way speech communication systems calling for a low round-trip delay. Furthermore, various types of filter banks for perfect reconstruction are commonly derived by spectral factorisation of a prototype into a minimum and maximum phase filter [Va 93].

The objective of this contribution is to present a novel approach to the design of minimum phase FIR-filters that is highly robust thus allowing the design of high-performance or / and high-order filters. This goal is achieved with a filter order that is at or close to the theoretical minimum, whilst the number of multiplications may even be considerably smaller than the filter length.

2 STATE-OF-THE-ART

The classical approach [Sch 70] starts with a linear-phase FIR-filter with double zeros on the unit circle of the z-plane, from which the zeros inside and every other zero on the unit circle is retained for the minimum phase filter by spectral factorisation. The linear phase prototype is designed with the MPR-program [Pa 73] with subsequent lifting of the zero phase frequency response to produce the unit circle double zeros. The major restrictions of this approach are as follows:

- Numerical inaccuracies in finding all double zeros of high-order polynomials. Note that the order of the lifted linear phase FIR filter is twice the order of the minimum phase filter to be designed.
- Allowance of only one constant stopband requirement.

As a result, high-order and/or high-attenuation designs as well as particular requirements are not possible with this approach.

An algorithm avoiding the root finding process is suggested by Boite and Leich [Bo 81]. By exploiting the relation between phase and attenuation of a minimum phase filter given by the Hilbert transform, the coefficients are directly derived via the IFFT of the minimum phase frequency response. Although avoiding the spectral factorisation, this approach still requires the design of a linear phase FIR-filter with a degree of twice the order of the desired minimum phase filter.

In [Go 80] an algorithm is proposed that decomposes the stated design problem into the design of two cascaded filters of lower order

$$H_{min}(z) = H_s(z) H_p(z)$$

In this iterative approach the stopband filter $H_s(z)$ and the passband filter $H_p(z)$ are designed in an alternating manner. Thus, all zeros of $H_s(z)$ are always forced to lie on the z-plane unit circle, while $H_p(e^{j\Omega})$ equalizes the monotonic passband response of $|H_s(e^{j\Omega})|$ in order to obtain the overall desired frequency response of the minimum phase filter to be designed. As a result, $H_s(z)$ is both linear and minimum phase and can be designed by means of the MPR-program [Pa 73]. Thus, $H_s(z)$ is directly designed avoiding the root finding process, whereas spectral factorisation is only required for the low-order zero phase prototype passband filter $P(z) = H_p(z)H_p(z^{-1})$ encompassing only single zeros off the z-plane unit circle.

The design of $H_p(z)$ is only specified for the desired passband that attracts all zeros. As a consequence, the response of the passband filter can attain an extremely high magnitude in its don't care region giving rise to severe numerical errors in the MPR-program. The same is observed with $H_s(z)$ that has to compensate for the high gain of $H_p(e^{j\Omega})$ with extremly high attenuation. Subsequently it is shown how these problems can efficiently be overcome.

3 THE NOVEL APPROACH

The third approach to the design of equiripple minimum phase FIR filters [Go 80] is revisited. Its shortcomings are analysed and overcome with the following measures. Without any loss of generality, only lowpass filter design is considered.

3.1 Replacement of Spectral Factorisation

Root finding for spectral factorisation of the zero phase prototype passband filter transfer function $P(z)$ is avoided. Similar to [Bo 81] the Hilbert relation between the components of the cepstrum, attenuation and phase, is exploited by using a cepstral series expansion. Since all zeros of $P(z)$ are off the unit circle, this method is applicable without any numerical problem in contrast to [Bo 81], where in the vicinity of the zeros on the unit circle the true cepstrum has to be replaced by an approximation.

3.2 Additional Constraint for Passband Filter

In the MPR-design the out of passband gain of $|H_p(e^{j\Omega})|^2 = |P(e^{j\Omega})|$ must be restricted to avoid unbounded numerically intractable increase. One approach to this end is to prescribe a suitable desired function in parts of or the whole don't care region for the design of $P(z)$. It is most appropriate to specify a constant desired value corresponding to the actual gain of $P(e^{j\Omega})$ at the passband edge. Weighting in this region has to compromise between increase of filter order and the restriction that $P(e^{j\Omega})$ must be strictly positve. Although this approach attracts more zeros to $H_p(e^{j\Omega})$ (figure 1), the overall degree of the minimum phase filter is, nevertheless, close to the minimum filter degree according to [Sch 70]. This is due to the need of lower requirements for the stopband attenuation of $H_s(z)$. As a consequence the stopband filter is of lower degree.

3.3 Modified IFIR-Method

An alternative approach to reduce the impact of unrestricted gain in the don't care region of the passband filter $H_p(z)$ is based on a modification of the IFIR-method [Va 93]. Instead of $P(z)$ a model passband filter $P(z')$ is designed for the K-fold stretched frequency axis.Widening the passband width by K and, thus, reducing the don't care region, a slightly higher order as compared to that of $P(z)$ is required. By substituting z' by z^K the number of multiplications remains unchanged, whereas the gain of $H_p(e^{j\Omega K})$ (figure 2) in the don't care region is constrained to the magnitude of the images of the frequency response being periodic with F/K (F: sample rate). Although the number of coefficients unequal to zero is unchanged, the group delay for the filter $H_p(z)$ is increased by K.

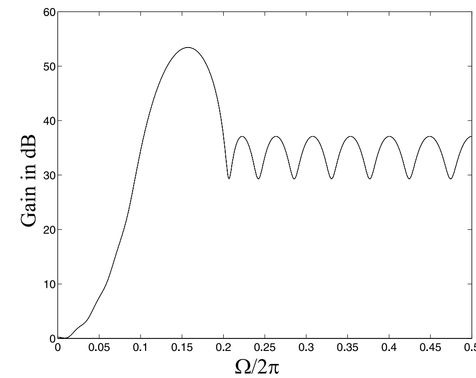

Figure 1: Modified algorithm applied to the design of the passband filter $H_p(z)$ ($\Omega = 2\pi f/F$; F: sample rate)

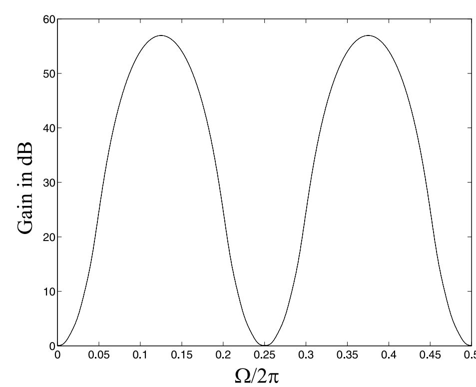

Figure 2: Modified IFIR-method applied to the design of the passband filter $H_p(e^{j\Omega}) = \sqrt{P(e^{jK\Omega})}$. Example with $K = 4$

Thus again, the stopband requirements for $H_s(z)$ (figure 3) are greatly alleviated, resulting in considerably shorter filter lengths for the stopband filter as compared to [Go 80]. Due to the delay replacement the overall filter order is, however, slightly higher than that required by the direct design according to [Sch 70].

One particular area of application of this approach is the

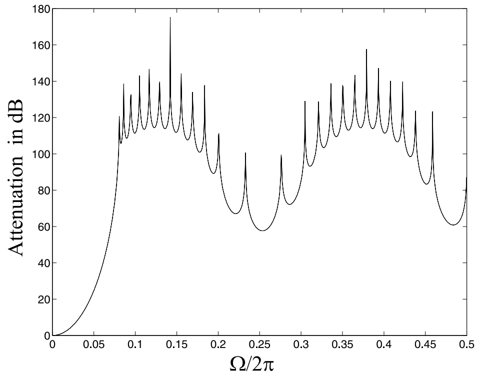

Figure 3: Attenuation of stopband filter $H_s(z)$ suitable for the passband filter obtained with the modified IFIR-method. Example with $K = 4$ of figure 2

design of lowpass filters for sampling rate alteration. A passband filter designed with the modified IFIR-method has only K-fold delays, where K represents the sample rate alteration factor. For a decimator, for instance, a downsampler can be shifted to the input of the passband equalizer by using the noble identity $H_p(z^K) = H_p(z')$ [Va 93]. As a consequence, all operations of $H_p(z')$ can be carried out at the K-fold reduced sample rate (figure 4).

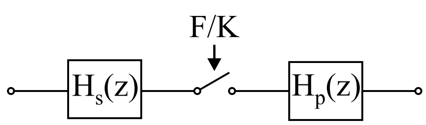

Figure 4: A decimator based on IFIR design

4 EXAMPLES

The proposed algorithms are illustrated by the following two examples. The first one demonstrates the high stopband attenuation, whereas the second one shows an application to sampling rate alteration.

4.1 Filter with high stopband attentuation

Example 1 is based on the prescription of a constant desired value in most part of the don't care region of the passband prototype filter $P(z)$. Following this approach, a narrowband lowpass filter of length $N = 124$ is designed for a passband edge frequency $\Omega_p = 0.1\pi$ and a stopband edge frequency $\Omega_s = 0.2\pi$.

The passband filter with all its zeros off the z-plane unit circle of length $N_p = 61$ is designed for the above Ω_p, a don't care region in the interval $[\Omega_p, \Omega_s] = [0.1\pi, 0.2\pi]$

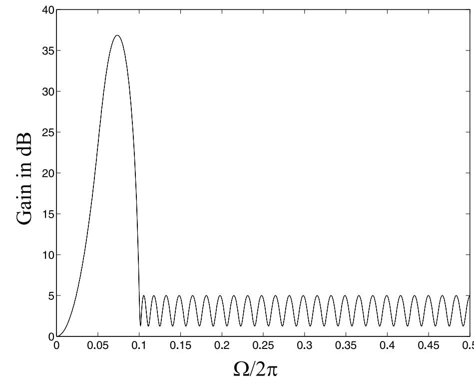

Figure 5: Gain of passband filter $H_p(z)$ of example 1; $N_p = 61$

and a constant desired value of 1.5 in $[\Omega_s, \pi]$. Its magnitude is depicted in figure 5.

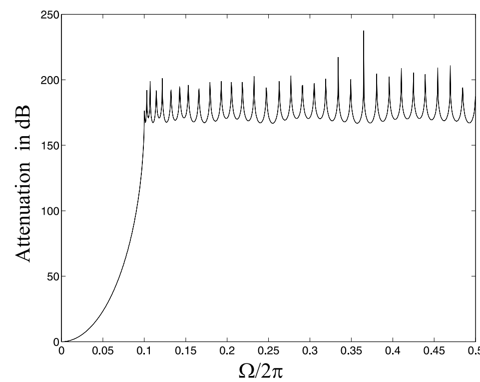

Figure 6: Attenuation of stopband filter $H_s(z)$ of example 1; $N_s = 64$

The attenuation of the corresponding minimum phase stopband filter of length $N_s = 64$ with symmetric impulse response is shown in figure 6. Cascading both filters results in the attenuation response of the desired lowpass filter $H_{min}(z) = H_p(z)H_s(z)$ with a minimum overall rejection of 164 dB as depicted in figure 7. From figure 7 it is evident that the classical design of this filter according to [Sch 70] would have failed due to the aforementioned various numerical errors.

4.2 Mth band filter for sampling rate alteration

Example 2 presents the design of a minimum phase Mth band filter [Mi 82] based on the modified IFIR-method of section 3.3 for sample rate alteration by a factor of M. To this end we set $K = M$. Following this approach,

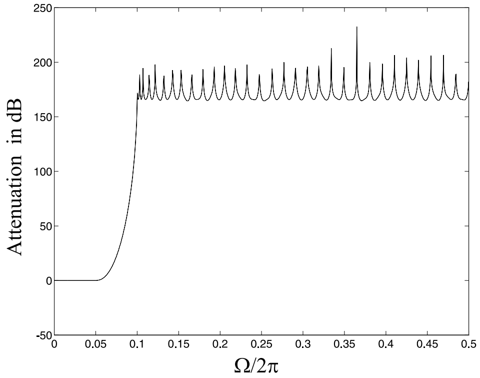

Figure 7: Attenuation of a narrowband lowpass with $N = 124$, $\Omega_p = 0.1\pi$ and $\Omega_s = 0.2\pi$, minimum stopband attenuation: 164 dB

a minimum phase 8th band filter of length $N = 104$ is designed with a passband edge frequency of $\Omega_p = 0.0625\pi$ and disjoint stopbands symmetric about $i \cdot F/8$ for i = 1,2,3,4 and of width $2\Omega_p$ [Mi 82]. The passband filter is assigned a filter length of $N_p = 41$, while the stopband filter requires a length of $N_s = 64$. Note, however, that the implementation of $H_p(z)$ requires just 6 multiplications per output sample, since the former filter is derived from a prototype filter of length 6 by 8-fold delay multiplication. The design result is depicted in figure 8, exhibiting again a rejection of at least 160 dB in the prescribed stopbands. Due to the impact of the lifting of the zero-phase frequency response by the stopband deviation, minimum phase Mth band filters of this type cannot be designed by the classical design method according to [Sch 70].

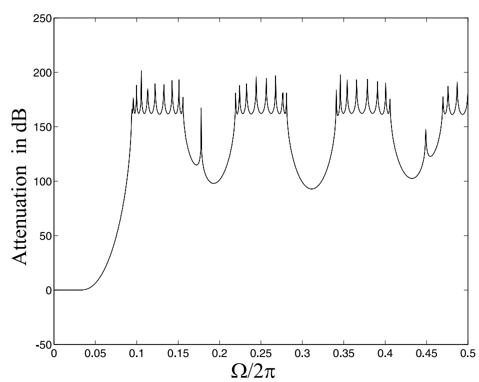

Figure 8: Minimum phase 8th band filter designed with the modified IFIR-method ($N = 104$, $\Omega_p = 0.0625\pi$, $K = 8$)

5 CONCLUSIONS

A recently proposed algorithm for the design of equiripple minimum phase FIR filters has been revisited and extended such that it is numerically robust enough to solve essentially any practical design problem up to extremely tight specifications such as very high attenuation, high order, narrow transition bands or combinations thereof.

The lowest number of multiplications per output sample is obtained if the minimum phase filters, based on the design method as reported in this paper, are realised as a cascade of $H_p(z)$ and $H_s(z)$. Since the latter is also linear phase, its coefficient symmetry can be exploited. As a result the number of multiplications per sample is always below that required for closed-form realisations being based on designs according to [Sch 70, Bo 81]. The savings amount up to more than 50 %. Using this hardware approach, an example 2 implementation of a decimator according to figure 4 in conjunction with an efficient multirate realisation of $H_s(z)$ according [Va 93] requires only 38 multiplications per output sample rather than 104. Furthermore, filters with stopband attenuations of up to 165 dB can be designed.

6 REFERENCES

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