

1 **Collapse displacements of masonry arch with geometrical** 2 **uncertainties on spreading supports**

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4 This work is aimed at evaluating the collapse displacement of masonry arch
5 subjected to spreading supports. This is achieved through a general application of
6 the virtual works principle. The problem is described in a finite displacements
7 formulation and investigated with a probabilistic approach, also considering the
8 effects of the geometrical uncertainties. This aspect is related to the imperfections
9 of the voussoirs, which affect the structural shape. The comparison between the
10 numerical and experimental results, derived both by the literature and laboratory
11 tests, confirms that the geometrical irregularities can significantly affect the
12 results obtained on the nominal structural geometry. Moreover, the disagreement
13 observed in the experimental tests is explained.

14 Keywords: masonry arch; spreading support; irregular geometry; uncertainties;
15 limit equilibrium

16

17 **1. Introduction**

18 The masonry arch is one of the most commonly used structural components in the
19 historical constructions. In the last centuries, the understanding of its behaviour has
20 received a growing interest of architects, engineers and researchers, especially for the
21 development of the scientific method. As for the more general cases of vaulted systems,
22 the main function of a masonry arch is to bring the upper loads through specific ways of
23 the structure to the ground, covering small or large spaces. The definition of the bearing
24 capacity is a crucial task for the right dimensioning of an arch. In the case of restoration
25 and/or retrofitting of existing buildings, bearing capacity is also fundamental for its
26 check and validation. In the last decades, the scientific literature on this topic has
27 considerably grown and the level of knowledge has significantly increased. In the

28 second half of the XX century, a fundamental contribution was provided by Heyman
29 [1,2], who used limit analysis for the study of masonry structures with an efficient
30 approach for the rapid evaluation of the structural limit conditions. In this work,
31 conceivable simplified hypotheses were assumed: no-tensile material, infinite
32 compressive strength and no-sliding condition at failure between the voussoirs. The
33 method is based on the well-known safe theorem, which states that “if a set of internal
34 forces in a masonry structure can be found that equilibrate the external loads, and which
35 lie everywhere within the masonry, then the structure is safe – safe in the sense that it
36 cannot collapse under those loads” [3]. After Heyman’s model, the upper bound and the
37 lower bound methods or, alternatively, the limit equilibrium state analysis have been
38 largely used. These methods were applied with several purposes, as the definition of the
39 minimum thickness and/or the bearing capacity under vertical and lateral loads for
40 different shapes of arches [4-11], the study of arches and vaults behaviour by using the
41 thrust network analysis [12-15] or advanced numerical methods [16-19], the analysis of
42 the strengthening effects through innovative materials [20-25] and many others.

43 During its life, a masonry arch has to withstand several threats that could
44 significantly reduce its bearing capacity. This problem can be mainly related to two
45 aspects: (i) structural damages of the arch (e.g. openings or sliding between the
46 voussoirs due to load actions) and/or material degradation (reduction of the arch
47 thickness or the strength of materials); (ii) springing settlements.

48 As far as it concerns the evaluation of structural and material degradation effects,
49 in the last years several works have been focused on the assessment of the strength or
50 stability reduction of a masonry arch due to its irregular geometry. The problem was
51 investigated by modelling masonry arches taking into account the actual stones

52 dimensions [26,27]. Elsewhere, parametric studies were applied to investigate the
53 influence of localized damages [28,29] or probabilistic approaches were used for the
54 estimation of uncertainties effects on the bearing structural capacity considering
55 horizontal loads, both in static [30] and dynamic conditions [31]. These works
56 emphasized that in the most cases the reduction of the collapse conditions, with respect
57 to the results obtained on the structures having nominal geometries, cannot be
58 neglected.

59 Regarding the study of the springing settlements effects, it can be stated that some
60 aspects concerning the structural response of masonry arches – and more in general of
61 masonry vaults – still present open problems. Differential settlements can be considered
62 one of the main causes of collapse of vaulted structures [4], occurring for slow long-
63 term deformations, for example due to static loads, or very quickly dynamic behaviour
64 of the building, as in the case of earthquake actions. In a study concerning settled
65 pushing structures, in particular arches and domes, Como [32] demonstrated “that, if the
66 geometry changes are negligible, the structure will attain the minimum thrust state,
67 saving its safety margin as in the perfect state”. Ochsendorf [33] analysed the collapse
68 conditions of the masonry arch on spreading supports in horizontal direction as a
69 function of the geometrical parameters, namely the curvature radius, the thickness and
70 the angle of embrace. Experimental results pointed out that the hinges may move with
71 the increase of the settlements before reaching the collapse. Galassi *et al.* [34] studied
72 the response of masonry structures to settlements considering rigid blocks connected by
73 unilateral contact and frictional links, through a non-linear numerical procedure
74 experimentally validated. Starting from the work of Ochsendorf, Coccia *et al.* [35]
75 developed an incremental procedure, based on the kinematic theorem applied to the
76 deformed configuration. They aimed at attaining the collapse conditions of the masonry

77 arch with horizontal spreading supports by varying the geometrical parameters and the
78 number of voussoirs. Zang *et al.* [36] analysed the masonry arch on spreading supports
79 through a mesoscale modelling strategy, considering solid elements for bricks
80 connected by interface elements for mortar joints. Constitutive models allow to consider
81 the effects related to the possible presence of damages. Recently, Zampieri *et al.*
82 analysed the effects of local pier scour in a multi-span masonry bridges [37] and the
83 influence of no-horizontal springing supports of the masonry arch on the collapse
84 mechanisms, with a numerical approach supported by experimental observations
85 [38,39].

86 As pointed out by literature works, numerical simulations carried out on nominal
87 geometry models seem to overestimate experimental results [4,33,35]. Starting from this
88 point, this paper is aimed at investigating the role of geometrical irregularities,
89 evaluated through a probabilistic approach, on the collapse conditions of a masonry arch
90 subjected to spreading supports, which could be also related to abutments or piers
91 deformations. In particular, the collapse conditions are studied through an incremental
92 numerical procedure using the virtual works principle applied at the deformed
93 configuration. For each deformed configuration, the limit equilibrium approach is used
94 to assure the structural equilibrium and the strength condition defining the right hinges
95 configurations. This condition occurs when the thrust line is contained inside the arch
96 and passes through the hinge points.

97 Considering two experimental tests, in this work it is demonstrated for the first
98 time that the reduction of the ultimate displacement observed at collapse, can be related
99 to geometrical uncertainties, if compared with numerical simulations. This aspect leads
100 to the opportunity of introducing safety factors in order to take into account such effects
101 also in engineering practice [30].

102 2. Problem statement and numerical procedure

103 2.1 Basic hypotheses

104 Let us consider a circular masonry arch of radius r , thickness t and angle of embrace α
105 made by n voussoirs under only its own weight in an equilibrium state (Figure 1). The
106 generic i th voussoir is subjected to the vertical force

$$107 \quad g_i = \gamma A_i d \quad (1)$$

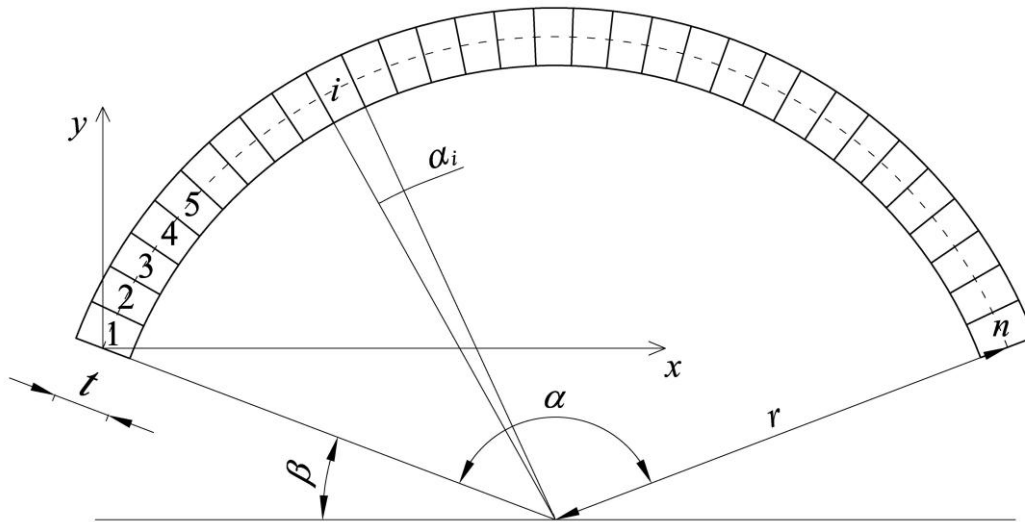
108 where γ is the specific weight, A_i the area of the i th voussoir and d the constant out-of-
109 plane depth. The arch is supposed be fixed on a spreading support, in particular the left
110 support without loss of generality (point $P_0(x_0, y_0)$ in Figure 2), and the direction of the
111 settlement δ_0 identified by the angle θ with respect to the horizontal. Given the
112 geometrical parameters, the Cartesian coordinates of a generic point belonging to the
113 arch can be indicated as a function of the radius r , thickness t and angle of embrace α .
114 As an example, with reference to the Oxy system indicated in Figure 1, the coordinates
115 of the centre of mass of the i th voussoir are

$$116 \quad x_i = r \cos \beta - r \cos \left(\beta + \frac{\alpha_i}{2} + (i-1)\alpha_i \right) \quad (2)$$

$$117 \quad y_i = -r \sin \beta + r \sin \left(\beta + \frac{\alpha_i}{2} + (i-1)\alpha_i \right) \quad (3)$$

118 being $\beta = (\pi - \alpha)/2$ and $\alpha_i = \alpha/n$.

119 The passage from the initial unsettled configuration Ω^0 to an equilibrated settled
120 configuration Ω^k induced by the spreading support is described by a kinematic
121 mechanism consisting of a three-hinged rigid body chain.

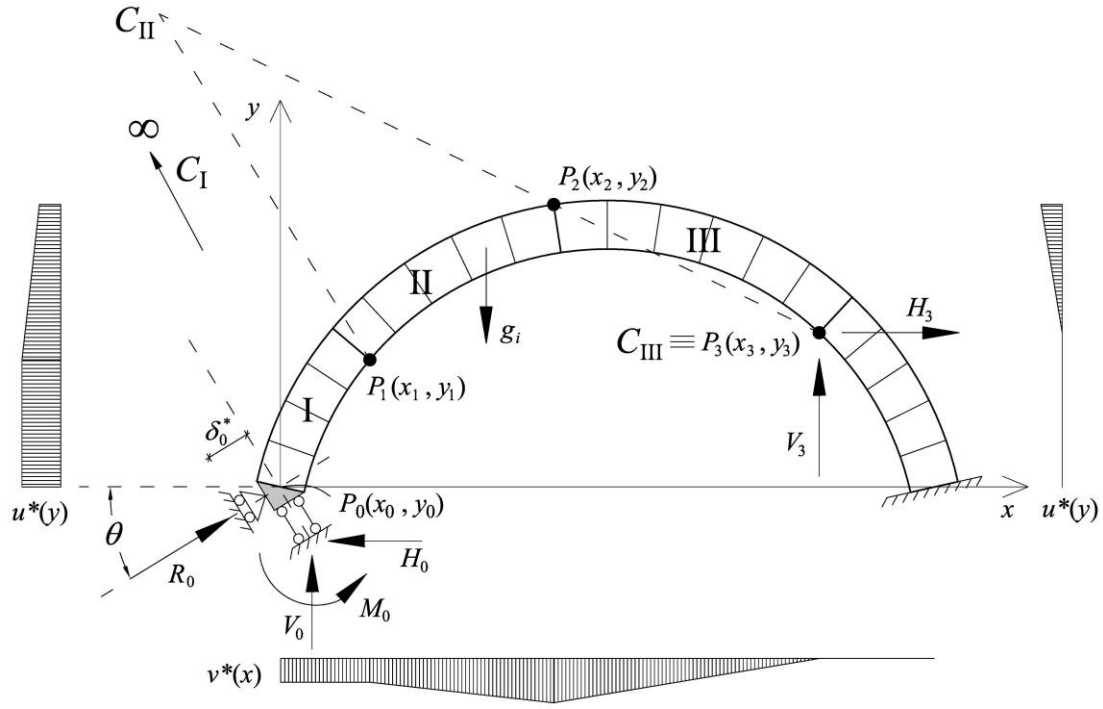


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123 Figure 1. Illustration of a masonry arch, divided in n voussoirs, with its geometrical
 124 parameters: radius r , thickness t and angle of embrace α .

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126 The mechanism can be analysed with the well-known hypotheses proposed by
 127 Heyman [1]: (i) mechanism condition, (ii) resistance criterion and (iii) equilibrium
 128 condition. The first condition (i) requires that the mechanism is only of rotational type,
 129 so that no sliding can occur at each joint; the second (ii) considers a material with
 130 infinite compressive strength and no-tensile strength; the third (iii) corresponds to the
 131 individuation of a thrust line – equilibrated with the external loads – everywhere
 132 contained inside the arch parts profile and passing through the hinges. The ultimate state
 133 of equilibrium is reached by progressively increasing the value of the displacement δ_0
 134 up to the loss of stability of the arch. This condition leads to the structural collapse with
 135 a mechanism which may involve either all the voussoirs, with a five-hinges symmetric
 136 mechanism, or a part of them, with the occurrence of an asymmetric configuration. In
 137 this case the collapse may develop starting from a four-hinges mechanism, or due to the
 138 alignment of the three hinges already present (three-hinges mechanism).



139

140 Figure 2. Virtual displacement diagrams applied to the masonry arch for the
 141 determination of the reaction force R_0 .
 142

143 Let us consider the settlement δ_0 assigned in P_0 along θ direction and the
 144 resulting kinematically admissible displacement field $\delta(u, v)$ of the structure, with u
 145 and v as displacement components in x and y directions respectively. The equation of
 146 the virtual works – employed in the small displacement field – provided by the
 147 equilibrated settled configuration Ω^k and a virtual displacement field δ^{k*} having the
 148 same properties previously described (i.e. δ_0^{k*} defined in θ direction and δ^{k*}
 149 kinematically admissible) is

$$150 \quad \langle g, \delta^{k*} \rangle + R_0^k \cdot \delta_0^{k*} = \langle \sigma^k, \varepsilon^{k*} \rangle \quad (4)$$

151 where σ^k and ε^{k*} are the stress and strain fields respectively, and R_0^k is the reaction
 152 force acting on P_0 along θ direction. In the Equation (4), the notation $\langle \bullet, \circ \rangle$ indicates the
 153 work calculus given by the system “•” of forces or stresses, and the system “◦” of

154 displacements or strains [40]. Assuming that the elastic deformations can be considered
 155 everywhere negligible, the right side of Equation (4) vanishes

$$156 \quad \langle \sigma, \varepsilon^{k*} \rangle = 0, \quad (5)$$

157 so that it is possible to calculate the reaction force along the settlement direction

$$158 \quad R_0^k = -\frac{\langle g, \delta^{k*} \rangle}{\delta_0^{k*}} \quad (6)$$

159 Following the notation of Figure 2, the solution of the equilibrium problem is easily
 160 given by a system of equilibrium equations (three global equilibrium conditions and a
 161 balance equation around the point P_2) expressed in the matrix form [37]

$$162 \quad \mathbf{Q} \cdot \mathbf{r} = \mathbf{q} \quad (7)$$

163 where \mathbf{Q} is the coefficient matrix

$$164 \quad \mathbf{Q} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -\tan \theta & 0 & 0 & 1 \\ 0 & -1 & (x_0 - x_3) & (y_3 - y_0) \\ 0 & 0 & (x_2 - x_3) & (y_3 - y_2) \end{bmatrix} \quad (8)$$

165 \mathbf{r} is the vector of the unknown reaction forces

$$166 \quad \mathbf{r} = \begin{Bmatrix} V_0 \\ M_0 \\ V_3 \\ H_3 \end{Bmatrix} \quad (9)$$

167 and \mathbf{q} is the vector of the known terms

$$168 \quad \mathbf{q} = \begin{Bmatrix} F_{03} - R_{0V}^k \\ -R_{0H}^k \\ -F_{03} b_{03} \\ -F_{23} b_{23} \end{Bmatrix} \quad (10)$$

169 in which F_{03} and F_{23} are the resultant vertical forces of the structural parts comprised
 170 between the points $P_0 - P_3$ and $P_2 - P_3$ respectively, $R_{0V}^k = R_0^k \sin \theta$ and $R_{0H}^k = R_0^k \cos \theta$

171 are the vertical and horizontal components of the reaction R_0^k , while b_{03} is the
172 perpendicular distance from P_0 to the line of action of F_{03} , and b_{23} is the perpendicular
173 distance from P_3 to the line of action of F_{23} .

174 The problem solution is achieved through the following equation

$$175 \quad \mathbf{r} = \mathbf{Q}^{-1} \cdot \mathbf{q} \quad (11)$$

176 from which the horizontal component H_0 is derived through the relation $H_0 = V_0 \tan \theta$.

177 If Heyman hypothesis about the criterion resistance is satisfied, namely the thrust line is
178 everywhere inside the arch profile in each block, the solution is admissible, otherwise
179 the hinges must be moved and the solution is achieved by means of few iterations.

180 As discussed above, the collapse condition can be reached for the arising of
181 several types of mechanisms. Several authors asserted that the type of collapse
182 mechanism depends on several features, in particular the arch geometry (e.g. the rise,
183 the span and the thickness) and the settlement direction [4,33,35]. It is well-known from
184 the literature that a three hinges mechanism suddenly develop with an even small
185 settlement and then, with the increasing of the displacement, evolves up to the arch
186 collapse. During this process, it is also possible to observe a change of mechanism,
187 characterized by a movement of the hinges before the collapse.

188 In this perspective, the description of the mechanism evolution requires a finite
189 displacements formulation of the problem, in order to define the geometrical
190 configuration of the kinematic structural mechanism, also considering the possible
191 change of the hinges position, until reaching collapse.

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195 *2.2 Numerical procedure in finite displacement field*

196 The structural problem introduced in the previous section, regarding the research of the
 197 ultimate condition of an arch subjected to a spreading support, is solved through an
 198 incremental numerical procedure based on increasing values of the assigned settlement.
 199 The search algorithm of the ultimate condition, developed in the finite displacements
 200 field, consists of three main steps.

201 The first step is dedicated to the identification of the kinematic mechanism
 202 corresponding to the initial unsettled configuration Ω^0 (Figure 2). Through an iterative
 203 procedure, it is possible to identify a virtual displacement field δ^{0*} associated to a
 204 kinematically admissible mechanism. The procedure, based on the three Heyman
 205 hypotheses previously recalled, allows to obtain an equilibrated system in which the
 206 thrust line is tangent to the arch profile at the three hinges ($P_1^0(x_1^0, y_1^0)$, $P_2^0(x_2^0, y_2^0)$ and
 207 $P_3^0(x_3^0, y_3^0)$) associated to the mechanism. The Equation (4) of virtual works, taking into
 208 account the assumption (5), becomes

$$209 \quad \langle g, \delta^{0*} \rangle + R_0^0 \cdot \delta_0^{0*} = 0 \quad (12)$$

210 so that the reaction force is obtained by

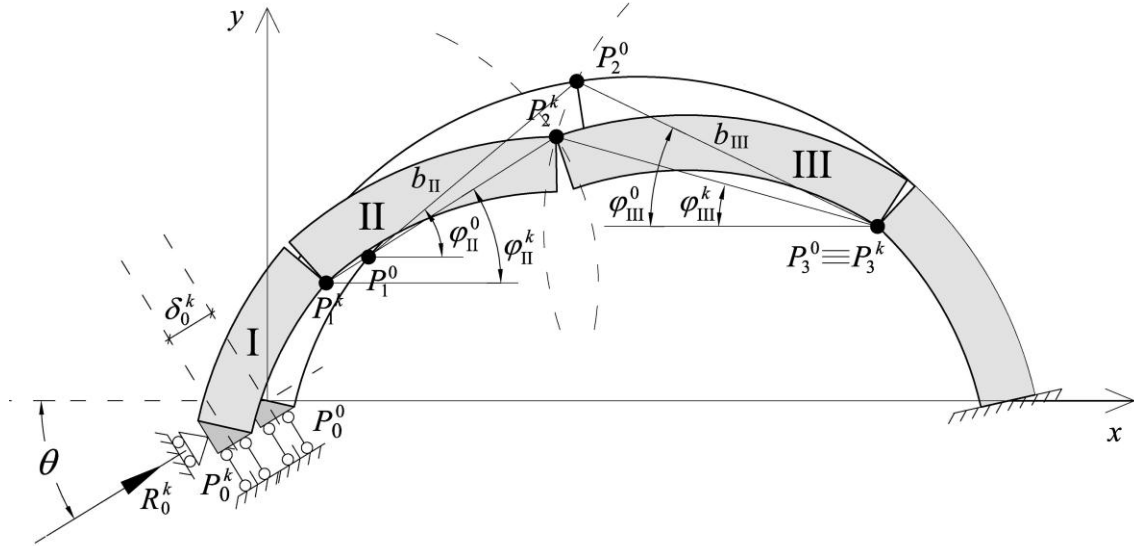
$$211 \quad R_0^0 = - \frac{\langle g, \delta^{0*} \rangle}{\delta_0^{0*}} \quad (13)$$

212 Given the hinges position in the initial configuration Ω^0 , it is possible to study
 213 the settled configuration Ω^k (second step) characterized by a settlement $\delta_0^k(u_0^k, v_0^k)$,
 214 applied at $P_0(x_0, y_0)$, which components are

$$215 \quad u_0^k = \delta_0^k \cos \theta \quad (14)$$

$$216 \quad v_0^k = \delta_0^k \sin \theta \quad (15)$$

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218

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Figure 3. Graphical representation of the kinematic mechanism in the generic

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settled configuration Ω^k .

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Also in this step, the right mechanism in the Ω^k configuration, resulting by the

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application of the settlement $\delta_0^k(u_0^k, v_0^k)$, is reached through an iterative procedure,

224

assuring the validity of Heyman's conditions. In fact, the position of the hinges

225

$P_1^k(x_1^k, y_1^k)$, $P_2^k(x_2^k, y_2^k)$ and $P_3^k(x_3^k, y_3^k)$ may not coincide with P_1^0 , P_2^0 and P_3^0 of Ω^0 ,

226

due to the possible occurrence of the change mechanism phenomenon previously

227

described.

228

The mechanism, illustrated in Figure 3, is described by the motion of three hinged

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bodies, namely I, II and III, in the finite displacements field resulting by the assigned

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settlement $\delta_0^k(u_0^k, v_0^k)$. The rotational parameters $(\varphi_{II}^0, \varphi_{III}^0)$ and $(\varphi_{II}^k, \varphi_{III}^k)$ – which define

231

the placement of the body II and III in the unsettled Ω^0 and settled Ω^k configuration

232

respectively – are related to the settlement components through the following relations

233

$$u_0^k = u_1^k = x_1^k - x_1^0 = b_{II}^0 \cos \varphi_{II}^0 + b_{III}^0 \cos \varphi_{III}^0 - b_{II}^k \cos \varphi_{II}^k - b_{III}^k \cos \varphi_{III}^k \quad (16)$$

234

$$v_0^k = v_1^k = y_1^k - y_1^0 = -b_{II}^k \sin \varphi_{II}^k + b_{III}^k \sin \varphi_{III}^k \quad (17)$$

235 where b_{II} and b_{III} are the distances $\overline{P_1P_2}$ and $\overline{P_2P_3}$ respectively, evaluated both in the
 236 unsettled or settled configuration.

237 The Equations (15) and (14) lead to the expressions of the rotational parameters
 238 $(\varphi_{II}^k, \varphi_{III}^k)$ evaluated in the deformed configuration

$$239 \quad \varphi_{II}^k = \arcsin \left[\frac{1}{b_{II}^k} (b_{III}^k \sin \varphi_{III}^k - \delta^k \sin \theta) \right] \quad (18)$$

$$240 \quad \varphi_{III}^k = \arccos \left[\frac{1}{b_{III}^k} (b_{II}^0 \cos \varphi_{II}^0 + b_{III}^0 \cos \varphi_{III}^0 - b_{II}^k \cos \varphi_{II}^k - \delta_0^k \cos \theta) \right] \quad (19)$$

241 The incremental values of the rotational parameters $(\Delta\varphi_{II}^k, \Delta\varphi_{III}^k)$ associated to the
 242 passage from the unsettled Ω^0 and settled Ω^k configurations are directly obtained by
 243 the relations

$$244 \quad \Delta\varphi_{II}^k = \varphi_{II}^0 - \varphi_{II}^k \quad (20)$$

$$245 \quad \Delta\varphi_{III}^k = \varphi_{III}^0 - \varphi_{III}^k \quad (21)$$

246 Given the above Equations (14)-(21), the displacement components of each point of the
 247 arch in the settled configuration can be obtained. With reference to the body I, the
 248 horizontal and vertical components, u_i^k and v_i^k respectively, of the displacement vector
 249 at a generic point $Q^k(x_i^k, y_i^k)$ are

$$250 \quad u_i^k = u_0^k = \delta_0^k \cos \theta \quad (22)$$

$$251 \quad v_i^k = v_0^k = \delta_0^k \sin \theta \quad (23)$$

252 Concerning with the body II, the displacement components in the case of $x_i^k > x_1^0$ are

$$253 \quad u_i^k = u_0^k - \sqrt{(x_1^0 - x_i^k)^2 + (y_1^0 - y_i^k)^2} \cos \Delta\varphi_{II}^k \quad (24)$$

$$254 \quad v_i^k = v_0^k - \sqrt{(x_1^0 - x_i^k)^2 + (y_1^0 - y_i^k)^2} \sin \Delta\varphi_{II}^k \quad (25)$$

255 while in the case of $x_i^k < x_1^0$ are

$$256 \quad u_i^k = u_0^k + \sqrt{(x_1^0 - x_i^k)^2 + (y_1^0 - y_i^k)^2} \cos \Delta\varphi_{\Pi}^k \quad (26)$$

$$257 \quad v_i^k = v_0^k + \sqrt{(x_1^0 - x_i^k)^2 + (y_1^0 - y_i^k)^2} \sin \Delta\varphi_{\Pi}^k \quad (27)$$

258 As far as it concerns the body III, the following relations are given for the case of

259 $x_i^k < x_3^0$

$$260 \quad u_i^k = \sqrt{(x_3^0 - x_i^k)^2 + (y_3^0 - y_i^k)^2} \cos \Delta\varphi_{\Pi}^k \quad (28)$$

$$261 \quad v_i^k = \sqrt{(x_3^0 - x_i^k)^2 + (y_3^0 - y_i^k)^2} \sin \Delta\varphi_{\Pi}^k \quad (29)$$

262 and for the case of $x_i^k > x_3^0$

$$263 \quad u_i^k = -\sqrt{(x_3^0 - x_i^k)^2 + (y_3^0 - y_i^k)^2} \cos \Delta\varphi_{\Pi}^k \quad (30)$$

$$264 \quad v_i^k = -\sqrt{(x_3^0 - x_i^k)^2 + (y_3^0 - y_i^k)^2} \sin \Delta\varphi_{\Pi}^k \quad (31)$$

265

266 After the kinematic definition of the configuration Ω^k , the third step of the

267 procedure leads to the value of the reaction force R_0^k at the spreading support along θ

268 direction through the Equation (6) and to the solution of the equilibrium problem (11).

269 If the equilibrated system is statically admissible and the thrust line is everywhere inside

270 the arch profile of each block passing through the hinges, is possible to increase the

271 settlement to study the new configuration Ω^{k+1} . Otherwise is necessary to move the

272 hinges, achieving the solution through few iterations in the configuration Ω^k before

273 passing to the new configuration Ω^{k+1} .

274 Finally, the collapse condition, and then the ultimate admissible settlement, is

275 reached with the occurrence of a kinematic chain which activates a mechanism.

276

277 **3. Modelling of geometrical uncertainties with a probabilistic approach**

278 *3.1 Description of the random geometry*

279 The analysis of masonry structures are generally affected by uncertainties due to both
280 the geometrical irregularities and the variability of the materials mechanical properties.
281 In this work, having the material infinite compressive strength and no-tensile property,
282 only the effects of the geometrical irregularities are taken into account, following the
283 probabilistic approach introduced by Cavalagli *et al.* [30]. Dealing with masonry arches,
284 the geometrical uncertainties may be due to several causes: irregularities in the
285 fabrication of the blocks (bricks and/or stones); imperfections due to the construction of
286 both the arch and of the provisional structures placed for the supporting of the arch
287 itself; irregularities related to the degradation of the materials over time. The
288 geometrical irregularities of the arch has been modelled by means of a statistical
289 approach, with the introduction of uncertain geometrical parameters. Such uncertainties
290 are introduced with the aim to represent the geometrical irregularities that are generally
291 unknown, in order to describe the different structural behaviour of arches having the
292 same nominal geometry.

293 The following hypotheses are considered: random values of the angle of
294 embrace α_i , the thickness t_i and the radius r_i of each voussoir, and deterministic value
295 of the angle of embrace α (Figure 4). It should be noted that in this work the joint
296 direction is not considered as random parameter, so that each joint of the random arch
297 has a radial direction. The random parameters are defined by independent uniform
298 probability density functions in a range of variability limited by a dimensional tolerance
299 value ε as follows:

$$\begin{aligned}
\alpha_i &= E[\alpha_i] + \varepsilon \alpha / n \cdot p_{\alpha_i} = \alpha / n + \varepsilon \alpha / n \cdot p_{\alpha_i} = \alpha / n (1 + \varepsilon p_{\alpha_i}) \\
300 \quad \tilde{t}_i &= E[\tilde{t}_i] + \varepsilon t \cdot p_{t_i} = t + \varepsilon t \cdot p_{t_i} = t (1 + \varepsilon t p_{t_i}) \\
\tilde{r}_i &= E[\tilde{r}_i] + \chi r \cdot p_{r_i} = r + \chi r \cdot p_{r_i} = r (1 + \chi p_{r_i})
\end{aligned} \tag{32}$$

301 where n is the number of voussoirs, χ is a curvature tolerance related to ε as
302 $\chi = \varepsilon(t/r)$ and p_{α_i} , p_{t_i} , p_{r_i} are independent samples taken from a uniform probability
303 density function in the range $[-1,1]$ (Figure 5). The Equations (32) show that the mean
304 values $E[\tilde{\bullet}]$ of the random geometrical parameters are assumed equal to the nominal
305 values. Concerning the variable parts, the number of extracted samples are n for the
306 random parameters \tilde{t}_i and \tilde{r}_i , and $n-1$ for α_i in order to assure the deterministic value
307 of the angle of embrace of the arch; the n th value of the sample results from the
308 difference

$$309 \quad \alpha_n = \alpha - \sum_{i=1}^{n-1} \alpha_i \tag{33}$$

310 The random nature of the geometrical parameters affects the description of the
311 Cartesian coordinates of a generic point belonging to the arch. As an example, the
312 Equations (2) and (3) indicating the centre of mass of the generic i th voussoir become

$$313 \quad \tilde{x}_i = r \cos \beta - \tilde{r}_i \cos \left(\beta + \frac{\tilde{\alpha}_i}{2} \right) \tag{34}$$

$$314 \quad \tilde{y}_i = -r \sin \beta + \tilde{r}_i \sin \left(\beta + \frac{\tilde{\alpha}_i}{2} \right) \tag{35}$$

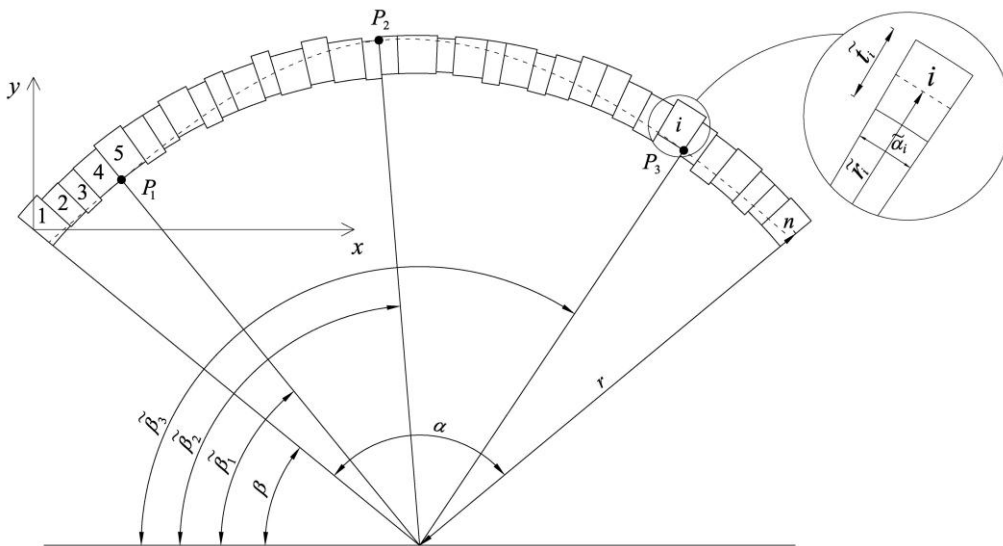
315 for $i=1$, and

$$316 \quad \tilde{x}_i = r \cos \beta - \tilde{r}_i \cos \left(\beta + \frac{\tilde{\alpha}_i}{2} + \sum_{m=1}^{i-1} \tilde{\alpha}_m \right) \tag{36}$$

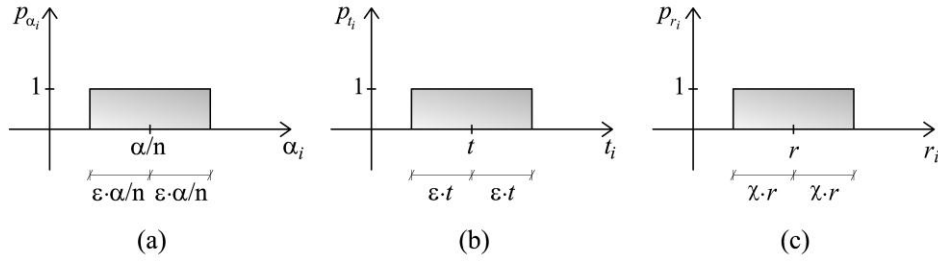
$$317 \quad \tilde{y}_i = -r \sin \beta + \tilde{r}_i \sin \left(\beta + \frac{\tilde{\alpha}_i}{2} + \sum_{m=1}^{i-1} \tilde{\alpha}_m \right) \quad (37)$$

318 for $2 \leq i \leq n$.

319 In the analysis of the results, the probabilistic approach considers the mean values
 320 of loads and/or displacements, obtained at collapse, evaluated over a total number h of
 321 randomly generated cases equal to 1000 of each sample $(\alpha_i)_n^h$, $(\tilde{t}_i)_n^h$ and $(\tilde{r}_i)_n^h$, for a
 322 fixed number of voussoirs n . In [30] it has been already demonstrated that the
 323 introduction of geometrical uncertainties in the model affects the bearing capacity of an
 324 arch, obtaining lower values of the mean collapse loads with respect to the nominal
 325 values provided by a deterministic geometry. This aspect is related to the variability of
 326 the effective contact length between the voussoirs, which directly affects the strength
 327 criterion by limiting the position of the thrust line. In this paper this effect is taken into
 328 account in the kinematic description of the problem, developed in the finite
 329 displacement field, which considers the possible occurrence of the hinges at the extreme
 330 points of each effective contact length.



331
 332 Figure 4. Masonry arch with geometrical irregularities described by the random values
 333 of the angle of embrace $\tilde{\alpha}_i$, the thickness \tilde{t}_i and the radius \tilde{r}_i of the i th voussoir.



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Figure 5. Probability density functions for the angle of embrace $\tilde{\alpha}_i$ (a), the thickness \tilde{t}_i (b) and the radius \tilde{r}_i (c).

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3.2 A parametric investigation about the random effect

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This Section reports the results of a parametric analysis of an arch subjected to a spreading support δ on a direction having an inclination $\theta = 45^\circ$ on the horizontal (see Figure 2). The arch has the following nominal geometrical parameters: angle of embrace $\alpha = 102.78^\circ$, radius $r = 1.40$ m and thickness $t = 0.12$ m, from which derived a dimensionless ratio $t/r = 0.08545$. The specific nominal geometry refers to a real masonry arch, which has been tested in the laboratory and described more in detail in the Section 4.2. The parametric analysis exposed in the following aims at investigating the uncertainties effects on the ultimate displacement to be expected in the experimental test. Following the probabilistic approach described in the previous Section, the geometrical irregularities of the voussoirs are considered assuming parameter $\epsilon = 0.03$, due to the intrinsic values of tolerance affecting the bricks of the actual specimen [30]. Moreover, the effect of stereotomy is also taken into account by assuming several values of the number n of voussoirs in the range of 10 to 50. Following the approach previously described, for each value of n , 1000 samples of arches affected by geometrical uncertainties have been obtained.

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The results, expressed in terms of the random variable $\left(\tilde{\delta}_u\right)_n^h$, has been

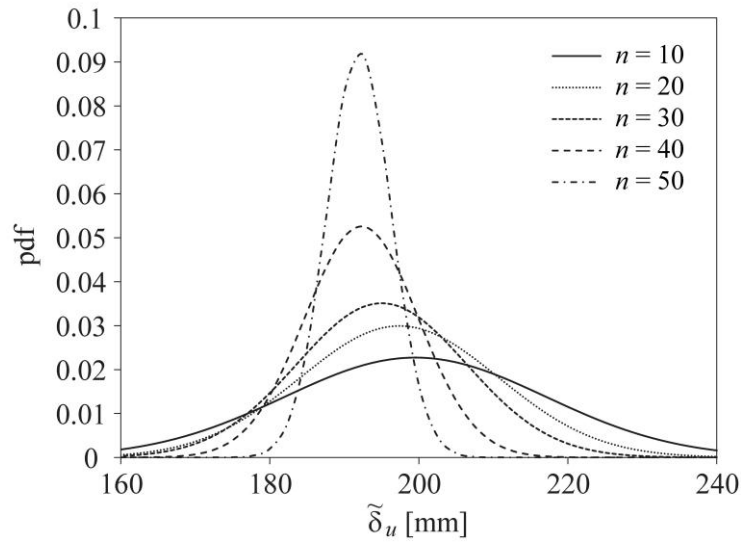
354

interpolated through the normal probability density function

$$p_{\tilde{\delta}_u} = f(\tilde{\delta}_u | \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\tilde{\delta}_u - \mu)^2}{2\sigma^2}} \quad (38)$$

356 where $\mu = E[\tilde{\delta}_u]$ and $\sigma^2 = \sigma^2[\tilde{\delta}_u] = E[\tilde{\delta}_u - \mu]^2$ are the mean value and the variance of
 357 the sample of the random ultimate displacement (Figure 6). It is worth noting as the
 358 spread and the mean values of the ultimate displacement decrease with the increase of
 359 the number of the voussoirs. Furthermore, for each fixed value of n , the mean final
 360 displacement is always lower than the displacement value obtained by using the
 361 nominal geometry. Table 1 summarizes the results. It should be also noted that, for the
 362 arch geometry in in exam, as the displacement δ_k of the settled springing increases, the
 363 position $\tilde{\beta}_j$ of the cracking hinges changes. This change occurs at between 80% and
 364 90% of the final displacement, as shown by the graph in Figure 7, which represents, for
 365 the case of $n=50$, the value of the position of the hinges in the model with nominal
 366 geometry and the mean value of the random position of the hinges ($\tilde{\beta}_j - \beta$) obtained in
 367 the model with irregular geometry, both normalized with respect to the angle of
 368 embrace α . The graph highlights that, for the case of $n=50$, the mean position of the
 369 hinges obtained by probabilistic analysis on the irregular geometry is quite close to the
 370 position obtained from deterministic analysis. Figure 8 shows the position of the hinges,
 371 as a function of n , in the initial configuration (Figure 8a) and in the final configuration
 372 (Figure 8b), in which can be observed a higher uncertainty in the definition of the
 373 cracking hinges for a low number n of voussoirs. However, in terms of mean values, the
 374 position of hinges 1, 2 and 3 differs slightly from the value obtained from deterministic
 375 analysis.

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378 Figure 6. Probability density function of the random ultimate displacement for arches at
 379 different values of the number n of voussoirs (n equal to 10, 20, 30, 40 and 50).

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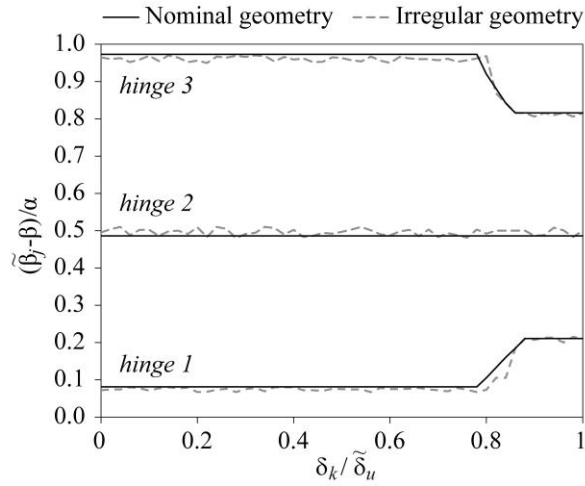
382

383 Table 1. Ultimate displacements obtained by numerical analysis at different values of
 384 the number n of voussoirs (n equal to 10, 20, 30, 40 and 50) using nominal and irregular
 385 geometry.

Nominal geometry		Irregular geometry	
δ_u [mm]	n	μ [mm]	σ [mm]
234.10	10	199.54	17.56
230.01	20	197.41	13.33
226.04	30	195.02	11.37
222.00	40	192.31	7.59
217.90	50	191.99	4.32

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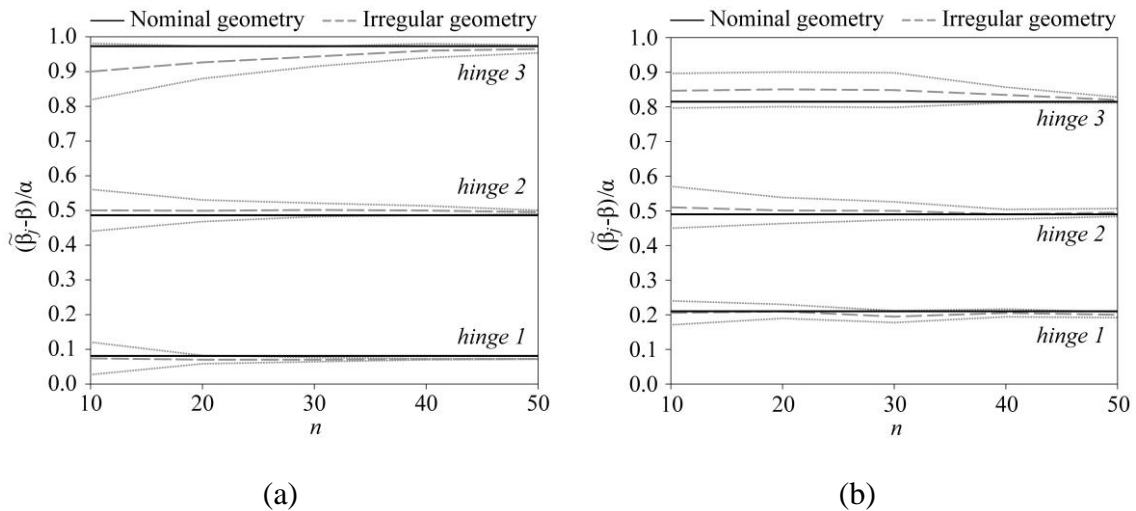
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388

389 Figure 7. Evolution of the normalized position $\left[\left(\tilde{\beta}_j - \beta \right) / \alpha \right]$ of hinges 1, 2 and 3 in
 390 function of the normalized settlement $\left(\delta_k / \tilde{\delta}_u \right)$ at the springing, for the case of nominal
 391 (continuous lines) and irregular (dashed lines) geometry, evaluated in average.

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395 Figure 8. Normalized position $\left[\left(\tilde{\beta}_j - \beta \right) / \alpha \right]$ of hinges 1, 2 and 3 in function of the
 396 number n of voussoirs, for the case of nominal (continuous lines) and irregular (dashed
 397 lines) geometry, in the initial (a) and ultimate (b) configuration. The dotted lines below
 398 and above the continuous curve are related to the values $\mu - \sigma$ and $\mu + \sigma$ respectively.

399

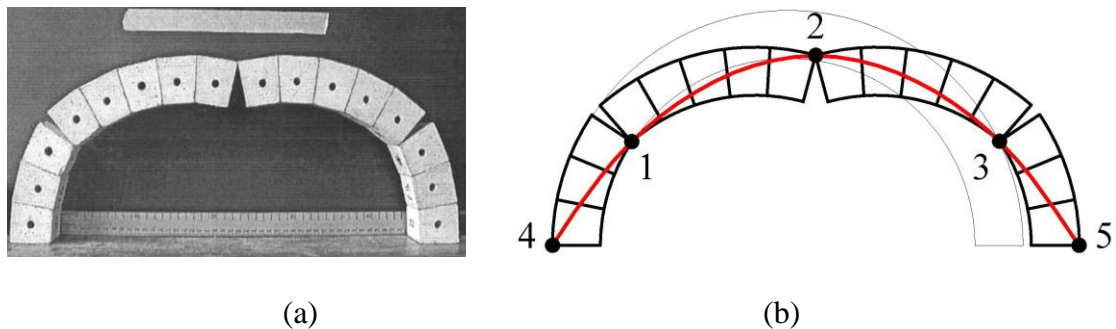
400 **4. Experimental tests**

401 In this section, the comparison between the results obtained by proposed numerical
402 procedure and two related experimental tests is reported. The first refers to a round arch
403 subjected to horizontal settlement developed by Ochsendorf [33]; in the second, a
404 springing of a segmental arch has been subjected to incremental settlements along a
405 direction inclined of 45° angle from the horizontal (Figure 2).

406 *4.1 Horizontal spreading support*

407 The small-scale experimental test carried out by Ochsendorf [33] considered as a case
408 of study refers to an arch comprising 16 concrete blocks (Figure 9a), with a 50 mm
409 radial thickness, mean radius r of 220 mm and thickness-radius ratio $t/r = 0.23$.

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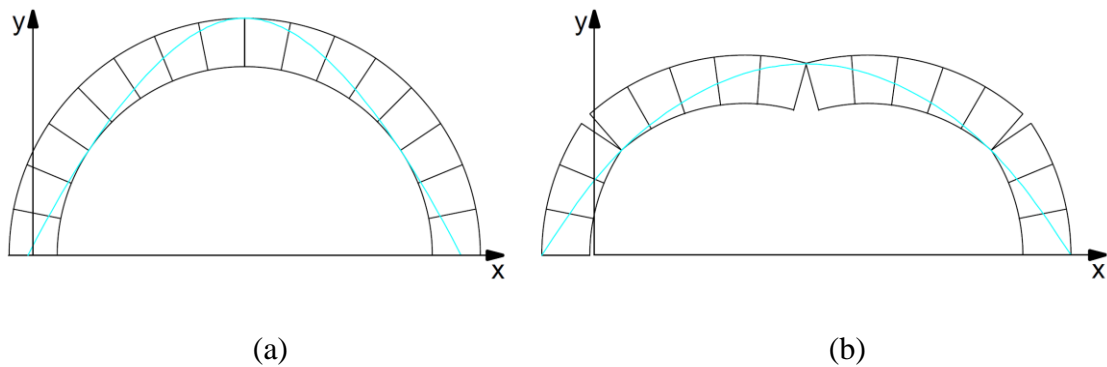
413 Figure 9. (a) Experimental test carried out by Ochsendorf [33]: equilibrium state of a
414 settled configuration before collapse. (b) Theoretical symmetric five-hinges collapse
415 mechanism.

416

417 On this arch, a springing was subjected to incremental horizontal settlement until
418 reaching the final condition. Experimental testing showed that the cracking hinges (1, 2
419 and 3) do not change in position from the initial to the final condition. Furthermore, the
420 maximum measured displacement (30 mm) was reached with an increment of 15.4% of
421 the span with respect to the internal radius (Figure 9b). In Figure 9a the instant

422 immediately before the collapse is shown. From this instant on, the arch has lost its
 423 stability and the collapse occurred with a pure rotational mechanism. An important
 424 observation made by Ochsendorf was that the expected theoretical five-hinges
 425 mechanism did not occur owing to model imperfections, which reduced the ultimate
 426 displacement δ_u at springing from the predicted value of about 33 mm to 30 mm. The
 427 theoretical collapse condition was obtained through the study of the limit equilibrium,
 428 which is commonly used considering the nominal arch geometry. In general, the results
 429 quite accurately represents both the arch configurations and the position at which the
 430 hinges occur (Figure 10), nonetheless, an error of the final displacement estimation,
 431 which in some cases may not be neglected, could be done. The same results of
 432 Ochsendorf's analyses are obtained by Coccia *et al.* [35], in which the problem of the
 433 right ultimate displacement estimation is highlighted, making the focus on the
 434 geometric imperfections that are found in the real arch.

435

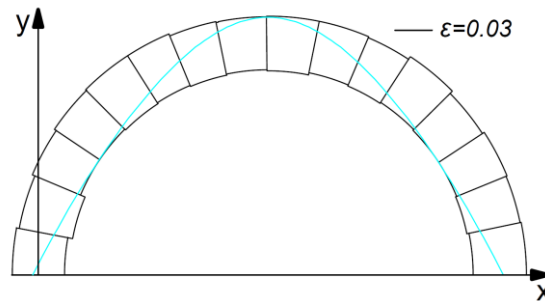


438 Figure 10. Masonry arch with nominal geometry: thrust line in the initial (a) and
 439 ultimate (b) configuration.

440

441 In this work, an interpretation of the experimental observation has been provided
 442 by introducing geometrical uncertainties in the model through the probabilistic
 443 approach described in Section 3. Fixing the number of the voussoirs ($n=16$), 1000

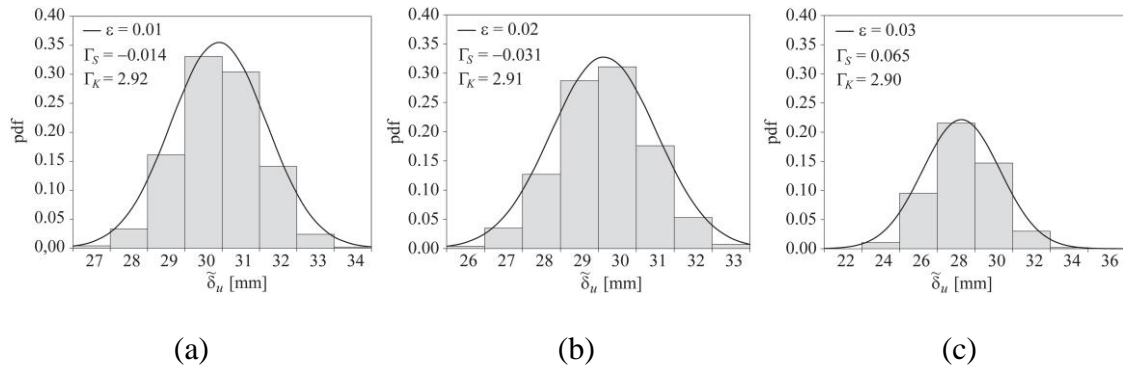
444 samples of arches affected by geometrical uncertainties have been generated. The
 445 random parameters have been generated using the Equations (32) and setting three
 446 levels of tolerance ε (0.01, 0.02 and 0.03). Figure 11 shows the case of a random arch,
 447 with $\varepsilon = 0.03$, in the initial state characterized by the three-hinges chain. The geometrical
 448 irregularities determine the natural loss of symmetry in the mechanism, so that the
 449 ultimate condition is reached, by increasing the displacement of a springing, with the
 450 occurrence of a fourth hinge at the extrados of the left or right springing alternatively.
 451



452
 453 Figure 11. Masonry arch with irregular geometry ($\varepsilon = 0.03$): thrust line in the initial
 454 configuration.

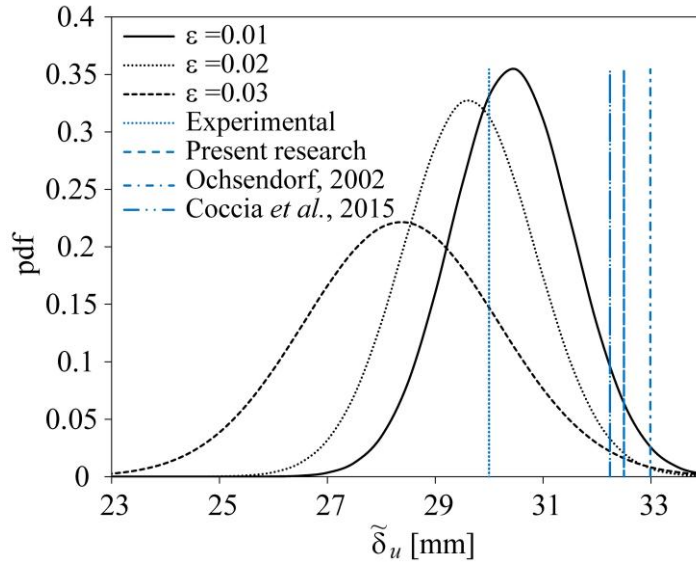
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 456 In Figure 12 the histogram of the probability density of the random ultimate
 457 displacement $\tilde{\delta}_u$ has been represented for ε equal to 0.01, 0.02 and 0.03. Given the
 458 Skewness (Γ_S) and Kurtosis (Γ_K) values related to each population of $(\tilde{\delta}_u)^\varepsilon$, the
 459 normal probability density function $p_{\tilde{\delta}_u}$ expressed by the Equation (38) has been used
 460 to interpolate in first approximation the numerical results.

461



462
463 (a) (b) (c)
464 Figure 12. Histogram of the probability density of the ultimate displacement $(\tilde{\delta}_u)^\varepsilon$, with
465 its interpolant normal probability density function, for the case of ε equal to 0.01 (a),
466 0.02 (b) and 0.03 (c).

467
468 The normal distributions of Figure 12 have been superimposed with the
469 experimental and numerical results performed using nominal geometry (Figure 13). The
470 graph highlights that the greater the level of tolerance ε , the greater the spread of the
471 interpolant normal distribution and the lower the mean value of the random ultimate
472 displacement. It is interesting to note that the probabilistic approach provides a more
473 consistent prediction of the experimental displacement observed by Ochsendorf,
474 considering a value of ε between 0.01 and 0.02. Table 2 summarizes the comparison
475 between the experimental observations and the numerical results obtained using
476 nominal geometry and the probabilistic approach, with reference to the mean and
477 standard deviation values. Finally, it is conceivable to consider that the deviation
478 between the experimental results and those by from numerical simulation with nominal
479 geometry can be related to uncertainties in the actual geometry of the arch.



480

481 Figure 13. Comparison between the ultimate displacement values obtained by the
 482 experimental test, by numerical analysis using nominal geometry and the normal
 483 distributions of the random ultimate displacement values given by the probabilistic
 484 approach (ε equal to 0.01, 0.02 and 0.03).

485

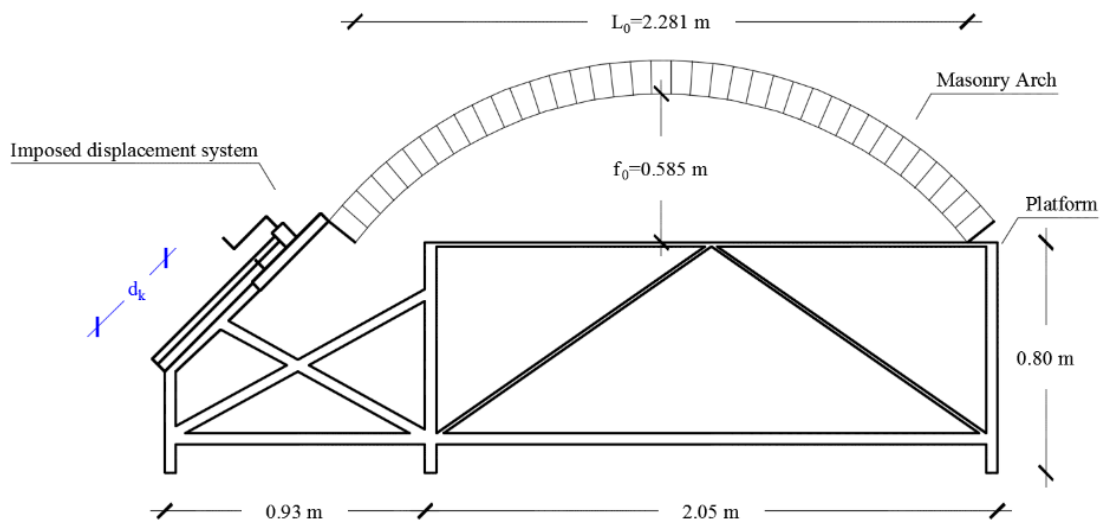
486

487 Table 1. Ultimate displacements obtained by experimental test and numerical analysis
 488 using nominal and irregular geometry.

Experimental test		Numerical analysis			
Real geometry	Nominal geometry	Irregular geometry			
δ_u [mm]	Reference	δ_u [mm]	ε	μ [mm]	σ [mm]
30.0	Ochsendorf, 2002	32.9	0.01	30.42	1.1
	Coccia <i>et al.</i> , 2015	32.2	0.02	29.63	1.2
	Present research	32.5	0.03	28.37	1.8

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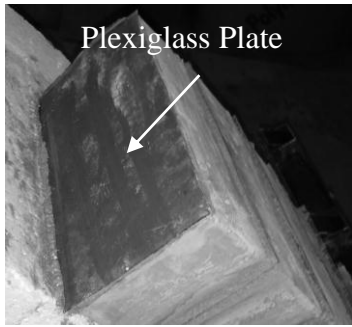
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492 Figure 14. Specimen geometry and test layout.

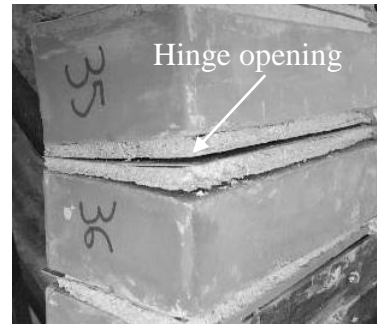
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494 4.2 No-horizontal spreading support

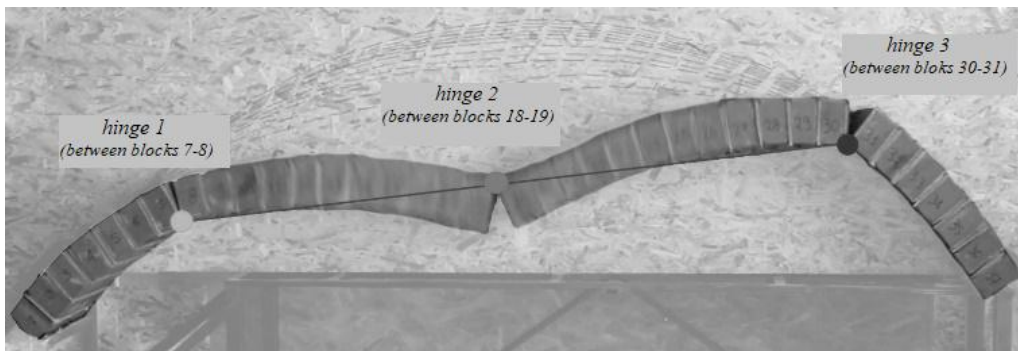
495 The masonry arch described in Section 3.2 refers to a real structure constructed and
 496 tested in the laboratory. The arch has a nominal span of 2.281 m, a nominal net rise of
 497 0.585 m and is constituted by 37 bricks assembled with mortar joints (Figure 14). The
 498 arch complies with Heyman's condition of zero resistance between the interfaces of the
 499 blocks, as a Plexiglas plate was inserted (Figure 15a) in the middle of each mortar joint.
 500 For this reason, mortar hinges formed at the interface between the Plexiglas plate and
 501 the mortar (Figure 15b). The structure is placed on a steel frame system featuring a
 502 moveable springing (left springing) along a direction inclined of 45° with respect to the
 503 horizontal. The test system provides an instant-by-instant displacement measurement,
 504 until the structural collapse. The support movement has been imposed with a manual
 505 system, and the displacement measured with an LVDT activated simultaneously with a
 506 video recording of the test. The collapse condition occurred with a three-hinges
 507 mechanism in correspondence to an ultimate displacement of about 195 mm (Figure
 508 15c).



(a)



(b)



(c)

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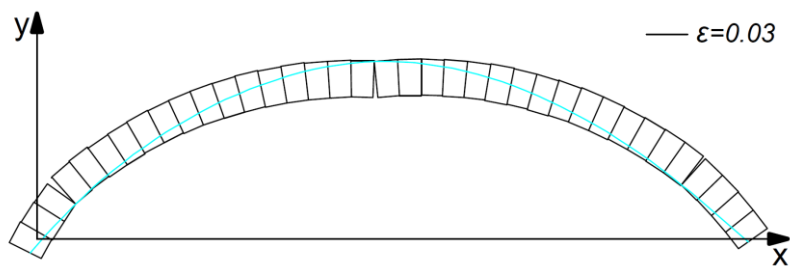
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513 Figure 15. (a) Plexiglas plate inserted in the middle of the mortar joint. (b) Opening of
 514 the hinge corresponding to the Plexiglas plates (initial hinge configuration). (c) Arch
 515 collapse condition.

516

517



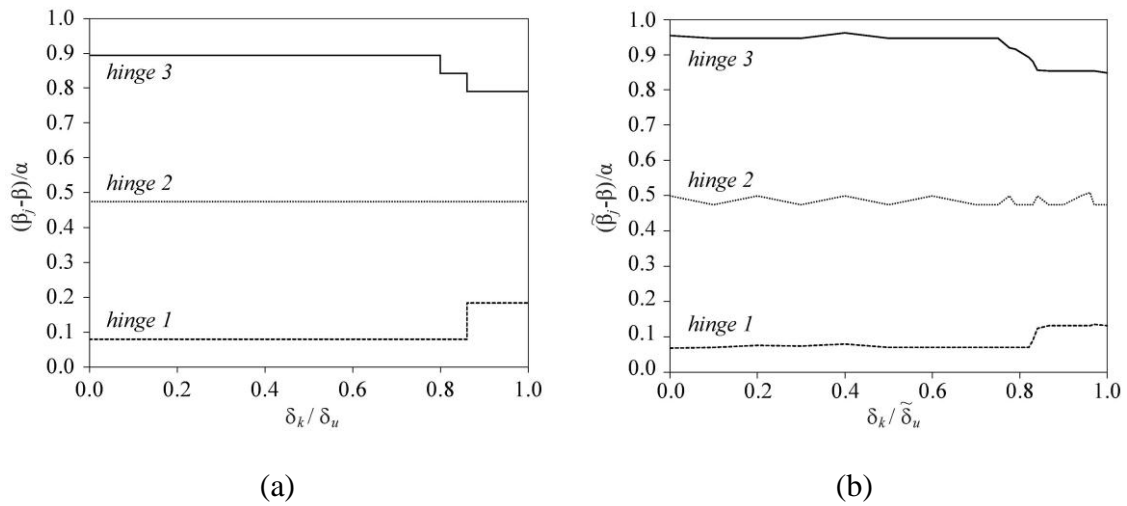
518

519 Figure 16. A case extracted from random arch samples with a tolerance value $\varepsilon = 0.03$
 520 in an equilibrated settled configuration Ω^k ($\tilde{\delta}_k = 69.6$ mm).

521

522 The numerical procedure proposed in this paper has been used to simulate the
523 experimental test, both with nominal and irregular geometry ($\varepsilon=0.03$) using the
524 probabilistic approach. In Figure 16 a case extracted from random arch samples is
525 illustrated in an equilibrated settled configuration Ω^k .

526 Fixing the number of voussoirs ($n=37$), 1000 samples have been generated
527 considering $(\alpha_i)^h$, $(\tilde{t}_i)^h$ and $(\tilde{r}_i)^h$ as random geometrical parameters and analysed
528 through the proposed procedure. As expected from the parametric analysis carried out in
529 Section 3.2, the configuration of hinges 1, 2 and 3 changes with the increase of the
530 displacement δ_k imposed to the left springing. The proposed algorithm, by updating the
531 position of the hinges via the thrust line, is able to accurately represent this phenomenon
532 of change in hinge position throughout the development of the mechanism up to the
533 collapse, as can be seen from the graph in Figure 17. The figure shows the evolution of
534 the normalized hinge position $\left[(\tilde{\beta}_j - \beta) / \alpha \right]$ in function of the normalized settlement at
535 the springing $(\delta_k / \tilde{\delta}_u)$ obtained during the experimental test (Figure 17a) and by
536 numerical simulations (Figure 17b). The localizations of hinges 1, 2 given by numerical
537 analysis are quite similar to those observed in the experimental test, while the position
538 of hinge 3 are quite different. However, it must be considered that the results reported in
539 Figure 17b are evaluated as a mean over 1000 samples, thus the presence of consistent
540 solutions cannot be a priori excluded. In Figure 18 the results in terms of ultimate
541 displacement are shown.



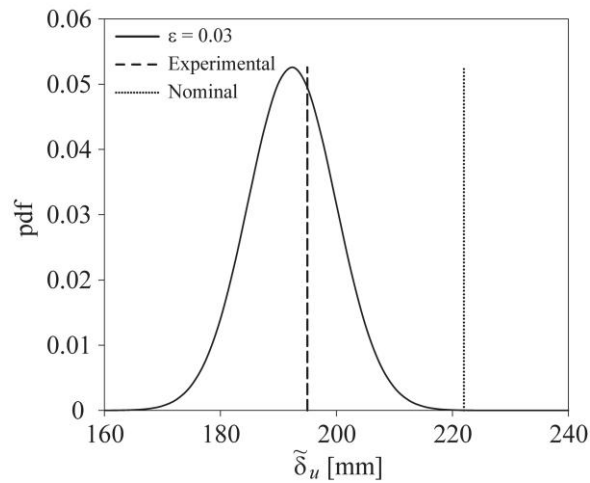
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543

544 Figure 17. Evolution of the normalized position $\left[(\tilde{\beta}_j - \beta) / \alpha \right]$ of hinges 1, 2 and 3 in
 545 function of the normalized settlement $(\delta_k / \tilde{\delta}_u)$ at the springing. (a) Results of
 546 experimental test. (b) Mean values provided by the numerical simulations based on the
 547 probabilistic approach.

548

549



550

551 Figure 18. Comparison between probabilistic results (continuous curve), numerical
 552 simulations with nominal arch geometry (dotted line) and experimental results (dashed
 553 line).

554

555 The normal distribution which interpolates the obtained random ultimate displacement
556 are plotted in Figure 18 with the deterministic values obtained by the experimental test
557 and the numerical simulation carried out considering the nominal geometry. The graph
558 highlights that, in average, the results given by irregular geometry (mean value of about
559 192.5 mm) provide a more consistent prediction of the experimental ultimate
560 displacement (195 mm) than the case of nominal geometry (222 mm), reducing the
561 error from 13.8% to 1.3%.

562 Finally, it has been demonstrated that the overestimations of the collapse
563 condition, in terms of both displacements and reaction forces, generally observed by the
564 direct application of the limit equilibrium approach on structures having nominal
565 geometry, with respect to the experimental observations, can be corrected by
566 introducing uncertainties in the model.

567 **5. Conclusions**

568 In this paper, the behaviour of the masonry arch on no-horizontal spreading supports
569 has been analysed, taking into account the geometrical irregularities effects. A
570 numerical procedure based on the limit equilibrium approach has been developed in
571 large displacements field, in order to follow the evolution of the mechanism until the
572 collapse with the incremental increase of the imposed settlement. The algorithm makes
573 use of the Principle of Virtual Work to solve static problem, and it is able to reach the
574 collapse conditions characterized by all the mechanisms described above. The
575 geometrical irregularities have been considered as intrinsic uncertainties of the
576 structures and spread on the arch model by means of three random variables. These
577 random parameters, namely the radius of curvature, the thickness and the angle of
578 embrace of each voussoir, have been described through independent uniform probability

579 density functions. It should be noted that each type of random structural analysis, is a
580 result of a significant number of samples analysed in a probabilistic sense.

581 The procedure has been applied to two experimental tests. The former is the
582 well-known test carried out by Ochsendorf concerning a semi-circular arch made by 16
583 blocks subjected to horizontal settlements at both the supports. The numerical
584 simulations of the test, provided by Ochsendorf himself and recently by Coccia and co-
585 authors, show a little overestimation of the ultimate admissible displacement with
586 respect to the experimental observations. In this paper has been demonstrated that the
587 overestimation (about 6.8%) of the ultimate condition obtained by a structure with
588 nominal geometry could be corrected by including uncertainties in the model. In
589 particular, it has been shown that considering an error between 1% and 2% of the
590 dimensions of the blocks, the experimental results could be better reproduced. The latter
591 test refers to a segmental arch, made of 37 bricks, on a springing support with an
592 inclination of 45° on the horizontal. Also in this case the numerical simulations carried
593 out on the nominal geometry has provided an overestimation of the ultimate
594 displacement (about 13.8%) with respect to the experimental results, while including the
595 geometrical uncertainties in the model with an error of 3% of the brick dimensions a
596 more consistent estimation of the actual structural capacity can be achieved.

597 Finally, the obtained results highlight that the uncertainties effects cannot be
598 neglected in the performance evaluations of experimental tests and, even more, this
599 aspect should be considered more in general in structural analysis. The role of
600 uncertainties will be as significant as the level of structural and/or material degradation
601 will be. The choice of the tolerance level, which describes the irregularities in the
602 statistical model, determines the quality of the results and must be defined in function of

603 the case in exam. In this context, geometrical safety factors could be introduced, in
604 order to take into account the uncertainties effects on the analysis of actual structures.

605

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611

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