ERRATUM ON "ENTROPY-ENERGY INEQUALITIES AND IMPROVED CONVERGENCE RATES FOR NONLINEAR PARABOLIC EQUATIONS"

JOSÉ A. CARRILLO, JEAN DOLBEAULT, IVAN GENTIL, AND ANSGAR JÜNGEL

There is a gap in the proof of Proposition 1-ii) and Theorem 4-ii) claiming that

$$\lim_{t\to\infty} \Sigma_k[u(\cdot,t)] = 0\ .$$

In the following, we explain how this gap can be closed.

Indeed, by a Sobolev-Poincaré inequality and the dissipation of entropy estimate of Lemma 2 or Theorem 3, for some constant $c > 0$,

$$\int_0^\infty \left\| u^{(k+m)/2}(\cdot,s) - \int_{S^1} u^{(m+k)/2}(x,s)\,dx \right\|^2_{L^\infty(S^1)} ds \leq c \int_0^\infty \int_{S^1} \left|(u^{(k+m)/2})_x(x,s)\right|^2 dx\,ds < +\infty.$$

Thus, there exists an increasing diverging sequence $(t_n)_{n\in\mathbb{N}} \to +\infty$ such that

$$\left\| u^{(k+m)/2}(\cdot,t_n) - \int_{S^1} u^{(m+k)/2}(x,t_n)\,dx \right\|_{L^\infty(S^1)} \to 0 \qquad (1)$$

as $n \to \infty$. On the other hand, for the same subsequence, we can assume without loss of generality that $(u^{(k+m)/2})_x(\cdot,t_n) \to 0$ in $L^2(S^1)$. From here, due to the compact embedding of $H^1(S^1)$ into $L^2(S^1)$, there exists a constant B such that

$$u^{(k+m)/2}(x,t_n) \to B \text{ a.e. in } S^1 \quad \text{and} \quad \int_{S^1} u^{(k+m)/2}(x,t_n)\,dx \to B. \qquad (2)$$

Consequently from (1), we deduce that the sequence $(u^{(k+m)/2}(\cdot,t_n))$ is bounded in $L^\infty(S^1)$ and thus, also the sequence $(u(\cdot,t_n))$.

Now, taking into account the uniform bound of $u(\cdot,t_n)$ and that from (2), we infer

$$u(x,t_n) \to B^{2/(k+m)} \quad \text{a.e. in } S^1,$$

and we deduce by Lebesgue's theorem that

$$\bar{u} = \int_{S^1} u(x,t_n)\,dx \to B^{2/(k+m)}.$$

and thus, $B = \bar{u}^{(k+m)/2}$. Consequently, $u(\cdot,t_n) - \bar{u} \to 0$ a.e. in S^1 with the sequence $u(\cdot,t_n)$ uniformly bounded in $L^\infty(S^1)$. From now on, the proof follows as in the published paper. This argument was also used in (A. Jüngel, and I. Violet, First-order entropies for the Derrida-Lebowitz-Speer-Spohn equation, *Discrete Contin. Dyn. Syst.* B 8 (2007), 861-877) and we thank I. Violet for pointing out to us this gap.