CONTRIBUTIONS TO MODELING THE BEHAVIOR OF CHAOTIC SYSTEMS WITH APPLICABILITY IN ECONOMIC SYSTEMS

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Abstract: *The surrounding reality can be viewed as the result of the interaction of dynamic systems nonlinear complexes. It has been shown, however, that some very simple systems can have it complicated and seemingly random behaviors. The chaos theory aims to explain and to predict in a short time the seemingly random and unpredictable behavior of the systems Nonlinear. Although the ideas preceding the emergence of chaos theory had been around for a long time, they were crystallized for the first by* Lorenz *(1963) in the work Deterministic Nonperiodic Flow. Lorenz created a mathematical model of the circulation of atmospheric currents of convection and observed that when there is a slight difference between the initial conditions, completely different results are obtained thus rediscovering the phenomenon of sensitivity to the variation of the initial conditions. The phenomenon observed has become a very popular paradigm of chaos theory called the „butterfly effect" and states that if the flapping of the wings of a butterfly changes the weather conditions in the jungle in a minor way Amazonian, this fact can have the effect, at the end of a complex causal chain, of the appearance of a tornadoes in Texas.*

Keywords*: Chaos theory, nonlinear dynamics, nonlinear time series analysis, chaos identification, Lyapunov exponent, neural networks prediction of chaotic time series, multilayer, neural networks of support vectors, ARIMA model*
JEL Classification: *C23, C26, C38, C55, C81, C87*

Introduction

The „butterfly effect" paradigm captures the essence of the chaos phenomena: in firstly the sensitivity to the initial conditions and secondly the deterministic character of them by highlighting structures in the phase space, structures called chaotic attractors. A system whose behavior was considered until recently random becomes predictable short term. The quality of the prediction decreases as the dynamics of a chaotic system evolve time due to the divergence of the initial trajectories. To predict the future behavior of one system, the current state of the system must be known with infinite precision – otherwise impossible in reality.

Objectives

The main objective of the article is to develop models that simulate as accurately as possible the behavior of the systems that generated the analyzed time series, namely that of the variation in the spot price of the electricity consumed in Romania. In the studied time series the presence of some indications of existence was highlighted chaos. The identification of chaos was made possible by the determination of some values whose values suggests the existence of such small-scale chaotic actors. In order to reach this objective, we started from the analysis of the current stage in modeling chaotic systems, the necessary steps have been taken to identify the chaos in the time series through the application of nonlinear analysis methods and finally the actual modeling was made using of two techniques: hybrid modeling ARIMA-time neural network and multilayer perceptron modeling with Echo State Network.

Until the foundation of the chaos theory, the complicated and inexplicable and unpredictable evolution of a system was considered random behavior. With the advent of chaos theory and a its instruments, a new world became visible, and things could be explained from a another perspective. The main advantage of chaos theory is that it allows prediction short-term evolution of chaotic systems and provides explanations of the type of behavior respectively dynamic.

Research

The research is structured in six stages, starting with stage 1 in which it is presented at sea the main concepts related to chaos theory in nonlinear systems.

Some patterns are presented which is repeated in the case of chaotic systems due to the chaos production mechanism itself - the phenomenon of doubling the period. Feigenbaum's number allows predictions to be made regarding the moment when there will be a qualitative change in the dynamic evolution of of the system as a result of a fork. Also here is briefly one of the most important notions of chaos theory, a notion that derives directly from the property of systems chaotic to be sensitive to the variation of the initial values, namely the exponent Lyapunov.

In stage 2, titled The Current Stage in Chaotic Systems Modeling, they are presented the main tools for modeling the evolution of these systems, and in the end a a comparative analysis of them. There are various examples from the literature that describe modeling the logistic function using fuzzy logic, modeling using networks neurons of a test data set, use of Markov chains in statistical modeling of Henon attractor.

Stage 3 describes the classical techniques for modeling and predicting time series. Are presented, with illustrative examples, simple regression, sliding mean method, method Holt, Winter method and finally ARIMA modeling applicable to linear systems. Stage 4 attempts to synthesize the entire process of nonlinear modeling of the series time, according to a procedure presented schematically at the beginning of the chapter and elaborated on the basis consulting the specialized literature. The main methods of identification are presented at large visual manifestation of chaos in time series, methods and sizes that allow identification numerical of the chaotic dynamics, as well as the main theorem on which the systems study is based chaotic. The reconstruction of the phase space is based on Takens' theorem and allows the reconstruction the dynamics of the initial system without knowing anything about the initial system, available only a univariate time series. The purpose of the phase space reconstruction is to characterize the behavior of a dynamic system that manifests itself through an unknown process and highlighting the existence of an attractor with a fractal dimension. The results obtained are presented following nonlinear analysis in case of three time series chosen by the author. Stage 5 presents various methods for predicting time series from the theory perspective chaos. The prediction is made using the support vector neural networks Machines and multilayer perceptron. A time series prediction model is also presented on defining a size on the space of the shapes that represent input data for a neural network time multilayer perceptron. In the last stage, time series modeling is carried out, in which case it could be set in highlights the existence of a chaotic dynamic. Modeling is done by a hybrid method

ARIMA-neural networks and, as a variant that allows a comparative analysis, with the help of a type of neural networks, relatively recent, called networks with echo state (Echo State Network).

Presentation of the main research concepts

Nonlinear analysis of time series

Nonlinear behavior allows a better understanding of complex natural phenomena. Nonlinear dynamics introduced a set of new concepts and tools that allow analysis and analysis investigating the dynamics generated by nonlinear processes. It can be said that at the time of in front, there is a conceptual unification of the notions (attractors, doubling of the period, bifurcations, Lyapunov exponent, sensitivity to initial conditions). Techniques that study the concepts introduced of nonlinear dynamics are grouped under the generic name of nonlinear signal processing or nonlinear analysis.

The behavior of a nonlinear dynamic system is shown in the state space or phase space - a conceptual space in which the dimensions correspond to the variables in the system. Changes over time of the system described by differential equations are reflected by the movements of a point in space condition, movements called trajectories. An image of changing the state of a multi-system time intervals is called the phase portrait. The portraits of the phase reveal the existence of attractors that are regions or points of the phase space to which all the close trajectories converge. A strange attractor will occupy a region of the phase space in which all trajectories will be captured which, apparently randomly they will cover the entire surface of it without repeating. A state space can be created starting from a series of time by graphically representing the observations offset with a certain time interval, a process called state space reconstruction.

Reconstruction of the phase space. Takens' theorem

During the last 20 years various techniques have been developed for signal analysis and processing framed in the category of non-linear chaotic dynamics. These techniques are called generic „Nonlinear signal processing techniques" and is based in particular on the Takens theorem (Takens, 1981). The theorem allows the reconstruction of a one-dimensional or multi-dimensional trajectory equivalent to the initial trajectory extracted from the space of the time series from which it departed. The idea a however, it was used long before its publication. In

1927 Yule (1927) used a graph in x (t), x (t + 1) -x (t-1) coordinates for the time series analysis of the sunspots. Inclusion in the phase space is a process by which a series of time is transformed into one series of coordinates in the reconstructed phase space. The set of reconstructed coordinates defines a trajectory in the reconstructed phase space. If the space of the reconstructed phase is m in size, then each reconstructed spatial coordinate Xi is a vector of size m obtained from the series of initial time, taking into account only some components of the original time series separately through a time delay called (Gao, Cao, Tung, and Hu, 2007). In practice, there are two important problems regarding the delay method as a method of phase space reconstruction. The first problem, common to all methods of reconstruction is that of establishing the minimum size of inclusion. This minimum size is unknown and must be determined.

Three methods are used to determine the minimum inclusion size: the method of the false nearest neighbors and that of the saturation of the attractor invariants and the method decomposition into singular values. The method of the false nearest neighbors is based on the general property of the attractor referred to above, namely that the distant points in the original phase space are close in the reconstructed space (false close neighbors), if the inclusion size is incorrectly chosen. The minimum inclusion dimension can also be determined by the saturation method to the attractor invariants (Li, 2006). This technique is based on the fact that once the attractor is deployed completely, its fractal dimension is independent of the inclusion dimension. Through therefore, when the attractor is represented in an incorrect inclusion dimension, its size is dependent on the size of inclusion. To determine the delay, the mutual information method and the function method are used of self-correlation.

ARIMA hybrid modeling - neural networks

The process of determining the ARIMA model consists of two stages:

- Identification - the model that best corresponds to the behavior of the series is determined of time. If the time series is affected by the seasonal component, the model to be determined is called SARIMA (Seasonal ARIMA) and is represented as ARIMA (p, d, q) (P; D, Q) S, where d and D ordinally represent seasonal differences and non-seasonal, p and P represent the orders of the seasonal and non-seasonal autoregressive terms, and q and Q ordinally represent the moving average terms. S stands for seasonal difference. The model parameters are determined by studying the graphs according to autocorrelation (ACF) and partial correlation (PACF).

- Verification - consists of evaluating the performance of the model in the description time series behavior. If the performance is not satisfactory it will come back to the previous stage, modifying the structure of the model until the moment when it it becomes suitable for capturing the dynamics of the time series. It is considered to consider the initial time series as being composed of a linear component autocorrelated and a nonlinear component, so that the initial time series can be decomposed so:

Following the modeling of ARIMA, the time series will be formed of the linear component modeled and the residues obtained:

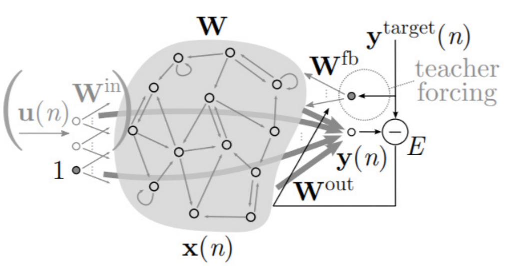
The residues obtained will in fact contain nonlinear relationships that could not be captured by the ARIMA model. The nonlinear component of the time series will be modeled using a network multi-layered perceptron neurons (Gonzales, 2000).

Neural networks Echo state networks

A general problem that arises when training recurrent networks is that, because the weights of the connections between neurons have variable values, the space of solutions is vast and the possibility of reaching a local minimum instead of the global minimum is significant. Networks ESN tends to solve this problem by training the weights of neuronal connections between reservoir and neurons in the output layer. As a result, tanks that contain fewer neurons tend to generalize better in the case of new data to multi-neuron reservoirs, indicating that, as in the case of FNN networks, the phenomenon of over-specialization may occur or memorizing the results (Jaeger, 2001). In the prediction problems it has been shown that recurrent (backward) neural networks have superior performance to neural networks feedforward. Higher predictive performance is is due to the ability of the RNN to form more complex temporal patterns by using units delay between exits and the hidden layer. Recurring connections and delays must used in problems where information is temporally correlated because they play the role of a short-term memories, memory that, given the fact that the above information influences the present state of the network which, in turn, determines the future values from the outputs network, contributes to the improvement of the generalization and thus, to obtain very good results in prediction problems (Donner, 2011). The same connections, however, cause the complexity of the interaction to increase dynamic between neurons, which will require very complex network training algorithms that partially converge.

ESN and another alternative to the training and use of recurring networks, Liquid State Machine (LSM) laid the foundation for data processing techniques included in the field called Reservoir Computing. ESN represents a new paradigm for the use of recurrent neural networks trained with a much simpler training algorithm than the classical ones. The idea that led to the creation of this type of network is based on the fact that, under certain conditions, the state of the network becomes asymptotically independent of the input values, their influence decreases over time. The basic idea from which he started his research in the field of ESN networks involves generating a recurrent neural network whose input is presented a signal that determines the creation of a wide spectrum of dynamics in the reservoir of neurons that form signals nonlinear output. The linear combination of these and the input signal determines the obtaining a reading function, that is, a prediction of the desired output signal. The main reason for the increasingly frequent use of this type of neural network is is the very simple learning algorithm. The only weights that are updated are those between the dynamic reservoir, figure 1 (hidden layer) and exits, thus avoiding the situation where the trained network reaches a local minimum, thus altering the quality of learning, its results and especially of the generalization being suboptimal (Lukosevicius, 2012). Selective update of weights eliminate the situation where the simultaneous updating of all weights determines the occurrence the chaotic behavior of the network, in which case a slight change in the value of the inputs produces a completely different result at the output of the network. The occurrence of such a phenomenon prevents touching network convergence.

Figure 1. Training with teacher forcing



The results of the research are:

1. Conduct a synthesis in the very broad field of time series analysis and prediction. I have especially focused on nonlinear analysis of time series for identification purposes manifestation of chaos according to a methodology that involves going through some stages succession. For this purpose, we analyzed the time series corresponding to the leueuro exchange rate. Although studies on this type of problem abound and most put it evidence of the existence of a chaotic dynamics, in the case of this time series there are no indications to confirm the hypothesis from which I left.

2. I applied nonlinear analysis methods and techniques over the spot price time series a electricity and time series of daily flows of rivers and I emphasized the existence of a chaotic dynamics. Chaotic dynamics implies the existence of a relationship deterministic causes and guarantees the possibility of short-term prediction of values future.

3. In the research, we made a comparison between two types of neural networks that are falls into the category of global methods for predicting chaotic time series, namely MLP network (Multilayer Perceptron) and SVM network (Support Vector Machines) support vector machines. I focused on these prediction tools because the network MLP is a universal approximator, being able to learn nonlinear relations between inputs to could predict future values. We tested the performance of these two types of networks neurons over a set of three time series: the Mackey-Glass time series, the time series Lorenz and the one corresponding to the daily evolution of the spot price of electricity. Into the following the results analysis it was established that MLP time networks perform better, and performance depends on the number of neural network inputs.

4. I have demonstrated empirically that, in the case of chaotic time series, the number of entries of The MLP neural network, for which optimal predictions are obtained, must be approximate equal to the product between the inclusion size and the delay parameter used for reconstruction of the state space.

5. Developing a neural model that allows the prediction of economic growth or decline given the individual evolution of a set of indicators. Graphic representation of the evolution of activity in an x-y axis system can be obtained as follows: each form represents a point in the plane where x is the time period corresponding to the form, and y is the class to which the form belongs. The accuracy of the representation depends on the number of classes. This representation first implies the ordering of classes and their

corresponding renumbering in the sense from negative evolution to positive evolution and then defining a size that characterizes each class. In the study, I took considering a set of indicators describing the economic performance of Romania in 2000-2016.

6. I determined models of the two time series by applying two modeling methods: hybrid modeling that involves developing a valid ARIMA model and then using a a multilayer perceptron and the second method involving the use of a network type neural network called the Echo State Network. This second time series is in especially difficult to model because it has special features (return to average, atypical values, seasonality, negative prices).

7. I designed a program, written in Matlab, to simulate ESN time networks.

Conclusion

In my opinion, future research that extends those in this research may focus in the following directions:

- Modeling the time series considered in this research using data assimilation with the help of Kalman Unscented filters and Wavelet neural networks;
- Identification of chaos and other time series corresponding to the manifestations of some real-world processes and establishing possible correlations between the onset of chaos and qualitative changes in the evolution of the system;
- Creating a prediction algorithm that uses the combined capabilities of ESN and wavelet neural networks;
- Identification of the optimal order of the neural model used for modeling and prediction of chaotic time series.

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