HIGH ACHIEVING STUDENTS IN LEAVING CERTIFICATE MATHEMATICS: WHY HAS THE GENDER GAP WIDENED?

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Research has identified a gender gap in the mathematical attainment of post-primary students around the world, favouring male students. In Ireland, following a review of the outcomes of a high-stakes examination taken by students at the end of post-primary schooling over an 18-year period, a similar such gap has been identified here and is widening. Data are presented to show that this gender gap widened with the introduction of a revised post-primary mathematics curriculum, colloquially known as Project Maths. This paper explores potential reasons behind the widening gap. Problem solving appears to be the pivotal issue and spatial ability may be a contributory factor. Addressing students' spatial ability is explored as way to address the gender gap and enable students to reach their full mathematical potential.

INTRODUCTION: THE GENDER GAP IN POST-PRIMARY MATHEMATICS

This paper represents an initial phase of research establishing the existence of a widening gender gap at the highest attainment level of post-primary mathematics in Ireland. Investigation is needed to understand the underlying issues: Why is there a discrepancy? What has caused the relative situation for female students to worsen? What solutions and actions might help address this imbalance?

High-achieving students and the gender gap at Leaving Certificate

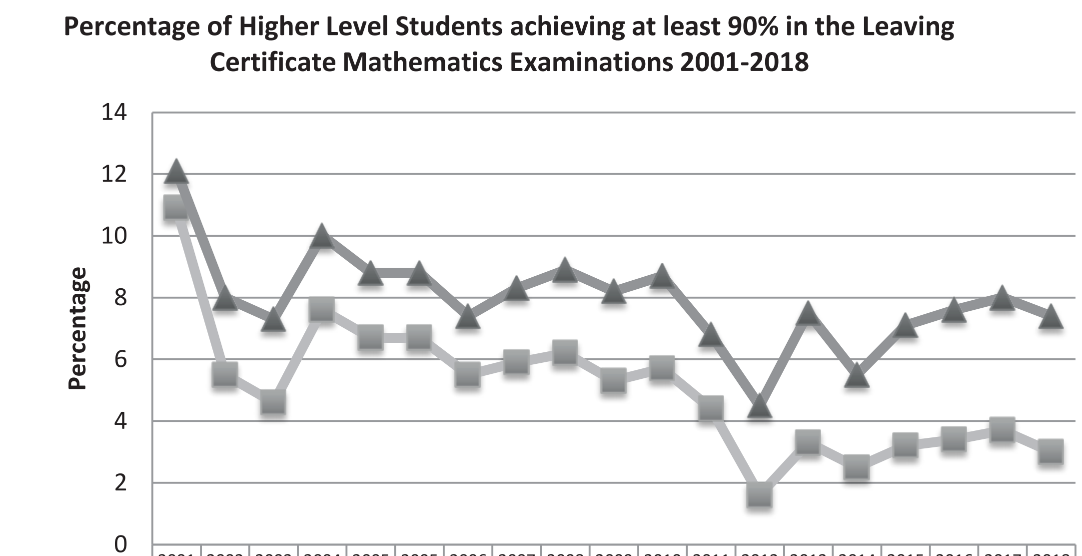
The Leaving Certificate (LC) examination in Ireland constitutes the end of post-primary education assessment, and also acts as university matriculation examination. In the LC mathematics examinations there are three different levels that students may study over a two-year, senior cycle course and sit the examination: Foundation, Ordinary or Higher Level (HL). The curriculum and final examination for these courses vary in breath, depth and difficulty. The State Examinations Commission has published annual statistical reports since 2001 categorising the attainment of 55,000 or so students who sit the LC examinations each year (SEC, 2019).

Inspecting SECs statistics for 2018, we see that approximately one in every three students sat the HL mathematics papers, with one in twenty of these students achieving the highest grade (90%-100%). Within this there are three notable gender differences: More male than female students sat the LC HL mathematics examination; more males achieved the highest grade; and, of those who sat the HL examination, a greater proportion of males than females achieved the highest grade (Table 1).

Table 1: Gender participation in the Leaving Certificate Mathematics Examination 2018

	Male	Female
Mathematics students at all levels	26,429	26,953
Higher Level students	8,741	8,096
% Gender taking Higher Level	33%	30%
Achieving at least 90%	646	245
% Total student cohort achieving at least 90%	2.4%	0.9%
% HL students achieving at least 90%	7.4%	3.0%

These three findings have been constant in the years 2001-2018 (n ≈ 1 million students). That more male students always sit the HL LC examination might be a little surprising given that every year from 2003 to 2016 a greater number of female students completed the preparatory HL Junior Cycle examination (Shiel & Kelleher, 2017, p. 75). Allowing for the weighted difference in participation when one compares the percentage of males and females who sit the LC HL examination who achieve the highest grade (SEC, 2019) we can see that a prevailing gender gap still exists and that this gap has widened (Figure 1).



	2001	2002	2003	2004	2005	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
% Female	10.9	5.5	4.6	7.6	6.7	6.7	5.5	5.9	6.2	5.3	5.7	4.4	1.6	3.3	2.5	3.2	3.4	3.7	3.0
% Male	12.1	8	7.3	10	8.8	8.8	7.4	8.3	8.9	8.2	8.7	6.8	4.5	7.5	5.5	7.1	7.6	8.0	7.4

Figure 1: Comparing the percentage of Higher Level Students achieving at least 90% in the Leaving Certificate Mathematics Examinations 2001-2018 by gender.

The gap appears to change at the time of the phased introduction of the new post-primary mathematics curriculum 2012-2015, colloquially known as *Project Maths*. Prior to this (2001-2012), of those who took the HL paper, on average 44% more males than females achieved the highest grade. Since the new LC HL examination was fully implemented (2015-2018), that ratio has considerably worsened from a gender equality viewpoint (k = 2.9) with 127% more males than females who took the examination achieving this grade.

International context of a gender gap in mathematics

The existence of a gender gap in students' achievement in mathematics is not unique to Ireland. Research in the US found that while there are no mean differences between boys and girls upon entry to primary school, girls lose one-quarter of a standard deviation relative to boys over the first six years of school (Fryer & Levitt, 2010). Robinson and Lubienski (2011) also established that the gender gap in mathematics widens at post-primary level.

There is international evidence that it is among the highest achieving students that the distinction between the sexes is most pronounced. In the TIMSS 2015 (Trends in International Mathematics and Science Study) comparative assessment of Grade 8 student achievement in Advanced Mathematics, for each of the three assigned cognitive domains of knowing, applying and reasoning, males achieved a significantly higher mean score than females (Mullis, Martin, Foy & Hooper, 2016). Similarly, in PISA (Programme for International Student Assessment), a triennial international survey testing the knowledge and skills of 15-year-old students, more male students than female students on average across Organisation for Economic Co-operation and Development (OECD) countries performed at the highest levels in 2015 (Perkins & Shiel, 2016).

Irish students have performed well in these international post-primary mathematics tests. In TIMSS 2015 Irish students ranked 9th out of 39 countries in Grade 8 Mathematics, and in

PISA 2012 Irish students achieved a mean score significantly above the OECD average score and ranked 18th out of 70 participating countries/economies (Perkins & Shiel, 2016). However, at the more advanced levels of mathematics Irish students do less well. In TIMSS 2015 only 7% of Irish students reached the Advanced Benchmark Ireland compared to 43-54% in some of the highest-ranking regions of Singapore, Chinese Taipei and Korea (Mullis, Foy & Hooper, 2016; Clerkin, Perkins & Cunningham, 2016). This is similar to the results of PISA 2012 when just 9.8% of students in Ireland performed at the highest proficiency levels (Levels 5-6) – below the OECD average of 10.7%. A number of countries have significantly higher percentages of students performing at Levels 5-6, including Japan, Korea and Singapore (20.3% -34.8%). This relatively poorer performance of our highest achieving students along with a "topic of concern: space and shape" were identified by Perkins and Shiel (2016, p.51) as having particular implications for teaching and learning.

Focusing on gender differences, in PISA 2015 Irish male students achieved a mean score of 511.6 while females achieved a mean score of 495.4. This difference of 16.1 points is larger than the corresponding OECD average difference of 7.9 (Perkins & Shiel, 2016). In addition, almost twice as many male students in Ireland perform at Proficiency Levels 5-6, compared with female students (12.4% and 6.5% respectively). In this regard, the mean score of male students was similar to the OECD average, but it is of particular concern that our female students were 0.27 SD below the international mean for females. This was similar to TIMSS 2015 where 5% of Irish female students compared to 8% of male students achieved the advanced benchmark (Clerkin, Perkins & Cunningham, 2016). This suggests that gender-gap between the highest achieving males and females is more pronounced in Ireland than in other countries.

POTENTIAL REASONS FOR THE GENDER GAP AT LEAVING CERTIFICATE FOR HIGH-ACHIEVING STUDENTS

The widening of the gender gap in LC HL attainment might be due to several reasons including: affective factors, changes to the nature of the examination, synchronous changes within the educational system, and perhaps gender differences in spatial reasoning.

What has changed?

Beginning in 2012 students passing HL LC mathematics received 25 bonus Third Level entry points. This is recognised as being a major factor in doubling the number of students sitting the LC HL examination from 15.8% in 2011 to 31.5% in 2018 (SEC, 2019). It is likely the attraction of a bonus point reward has increased the proportion of students with lower mathematical ability in the HL population. It is also probable that, on average, HL mathematics class sizes have increased. It could be argued that the effect of these changes on high-achieving students might be gender neutral.

With the phased introduction of the new mathematics curriculum (NCCA, 2013) teachers across the country engaged in substantial professional development focused on methodologies, use of dynamic software, and teaching through problem solving (Shiel & Kelleher, 2017). In parallel, hundreds of out-of-field teachers (Ní Ríordáin & Hannigan, 2011) were up skilled through universities. However, in TIMSS 2015 one fifth of students were still taught by teachers whose main area of study was something other than mathematics, which was considerably larger than corresponding proportions in the highest achieving countries (Clerkin, Perkins & Chubb, 2018).

Perhaps the most influential change was in the HL LC mathematics examination itself. The new LC HL examination phased in between 2012 and 2015 became far less predictable than the 'old' papers as more questions required solving problems in unfamiliar contexts. Also, topics that would have previously been contained in stand-alone questions are now interconnected in expansive, layered questions. Commenting on the overall percentage decrease in the A-rate, the Chief Examiner's Report (SEC, 2015) noted a substantial increase in the number of candidates taking HL and a deliberate attempt to increase the emphasis on problem solving and higher order thinking skills, "Skills that students find difficult to master and teachers may find difficult to instil" (SEC, 2015, p. 9). We might reasonably conjecture that a student's problem solving ability is now more important than ever for examination success, but why are female students in Irish classrooms finding mastery more elusive than their male counterparts?

Affective measures relating to gender performance mathematics

Research has demonstrated that affective measures impact students' performance in mathematics. Many studies have considered the influence of mathematical-gender stereotypes (Good, Aronson, and Harder, 2008; Song, Zuo & Wen, 2017), the gender attitudes of parents and teachers (Gunderson, Ramirez & Levine, 2012; Hyde et al. 2008), and how these negatively impact on female students' performance in mathematics. Other affective factors such as: confidence and grit (Flanagan & Einarson, 2017); self-efficacy, attitude and self-concept (Erturan & Jansen, 2015; Franceschini et al., 2014; Nosek & Smyth, 2011); the role of competition (Niederle & Vesterlund, 2010); and the role of culture (Nollenberger, Rodriguez-Planas & Sevilla, 2016) affect female students' performance in mathematics across many countries. Fryer and Levitt (2010) observed a gender gap that was evident across every strata of society and could not be explained by either less investment by girls in mathematics or low parental expectations. While these considerations may be worthy of further exploration in the Irish context, it is not apparent how any of these effects would have changed considerably between 2012 and 2015, leading to a widening of the gender gap for high-achieving students in LC mathematics.

Gender differences in spatial ability

Researchers have frequently found gender differences in spatial ability in favour of males (Reilly & Newman, 2013). Flaherty's (2005) study, which included Irish participants, found that while gender gaps in spatial ability in favour of males were global phenomena, they are not stable because culture and experience influence these gender differences. Early stage research in Ireland suggests that this spatial ability gap widens as students move through post-primary school (Harding, 2018). This is not unique to Ireland. Mix and Cheng (2012, p. 219) found that for children with visuospatial defects the gap in spatial ability widens over time.

Gender differences in spatial ability might be a contributory factor to the mathematical gender gap being discussed in this paper. Spatial ability is an aspect of intelligence that "depends on understanding the meaning of space and using the properties of space as a vehicle for structuring problems, for finding answers and for expressing solutions" (National Research Council, 2006, p. 3). Mix and Cheng (2012, p. 5) argue: "The relation between spatial ability and mathematical performance is so well established that it no longer makes sense to ask if they are related." Spatial reasoning is a strong predictor of success in mathematics (Casey, Nuttall, Pezaris & Benbow, 1995; Cheng & Mix, 2014; Lowrie, Logan & Ramful, 2016; Moè, 2015; Newcombe, 2013; Wai, Lubienski & Benbow, 2009).

Spatial ability and problem solving ability are related. Spatial reasoning can reduce working memory load and increasing success in solving mathematical 'word problems' (Duffy, 2017). Hill, Laird and Robinson (2014) identified gender differences in working memory in favour of males. When dealing with novel tasks and problems set in unfamiliar contexts, such as the new HL LC examination, students are rewarded less by rote-learning and algorithmic practice and more by spatial ability and application of working memory (Ma, Husain, & Bays, 2014). Students with high levels of spatial ability tend to be much more adept at mentally representing word problems in mathematics which leads to significantly higher success rates in problem solving (Boonen, van Wesel, Jolles & van der Schoot, 2014; Kozhevnikov, Motes & Hegarty, 2007).

Teaching and learning characteristics of the Irish mathematics classroom

Clerkin, Perkins adn Chubb (2018) identify that working on problems for which there was no immediately obvious solution was more common internationally than in Ireland.
Also teachers' ratings of confidence were slightly lower than the TIMSS average in relation to showing students a variety of problem-solving strategies (*Ibid*). This contrasts with the cultural script in Japanese classrooms (Stigler & Hiebert, 1999).

Considering the amount of time students experience mathematics in school, the average annual instructional hours devoted to mathematics, reported by teachers, was 109 in Ireland compared with a TIMSS average of 138. In particular, time spent teaching geometry in Ireland was significantly lower than TIMSS average (Clerkin, Perkins & Chubb, 2018) and this may explain why Irish students underperform in this topic in international tests. In PISA 2012 students in Ireland performed significantly less well on the Space and Shape (i.e. geometry) subscale (Perkins & Shiel, 2013). This weakness in Shape and Space is line with Ireland's relative underperformance on the geometry subscale in TIMSS (Clerkin, Perkins & Cunningham, 2016). More research is required to determine if Irish mathematics students are missing out on activities that would develop their spatial thinking skills and the consequences of this shortcoming.

Participation rates in subjects requiring high levels of mathematical and spatial skills

Spatial skills instruction is found to improve spatial cognition (Sorby, Veurink & Streiner 2018) and improve spatial performance (Uttal, Meadow, Tipton, & Hand, 2013), thereby improving students' problem solving skills. Furthermore, participating in spatial skills instruction has been found to advance outcomes for gifted STEM students (Miler & Halpern, 2012) and eliminate the spatial ability gender gap (Tzuriel & Egozi 2010). Research on spatial skills has demonstrated that exposure to subjects that stimulate spatial thinking result in indirect but long-lasting skills in mental rotation (Moè, 2015), a commonly-assessed aspect of spatial ability. Gender differences in mental rotation skills can disappear if females have the opportunity engage with spatial tasks and are frequently exposed to spatial thinking.

In Ireland, there is a persistent gender imbalance in subjects that may have consequences relating to mathematical achievement including applied mathematics, physics, engineering, technical graphics, woodwork and metalwork (McGrath, 2016). Even though the number of students completing the HL examination in subjects such as applied mathematics, physics

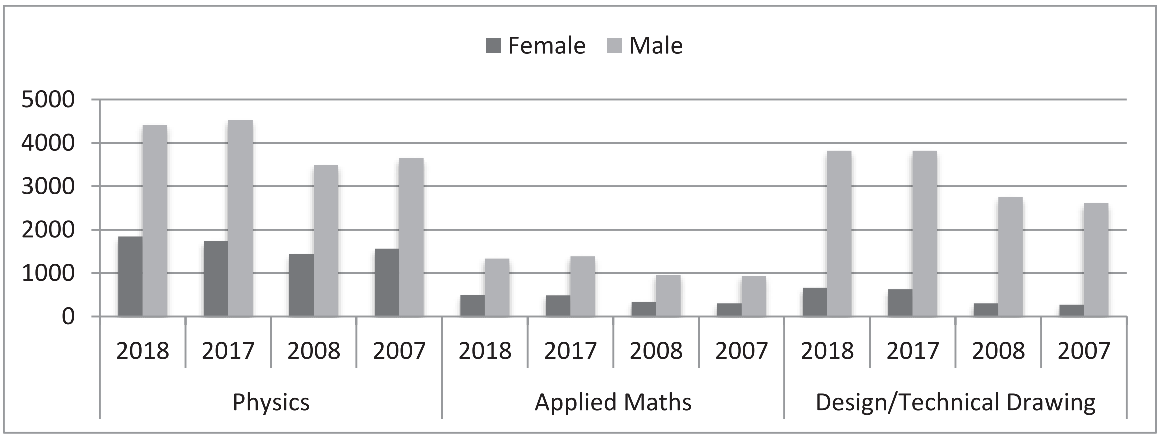


Figure 2: Leaving Certificate Higher Level candidates in 'spatial' subjects by gender for particular years.

and technical drawing has increased in recent years (SEC, 2019), the proportion of female students has not notably changed (SEC, 2019; Figure 2).

Further research on the opportunities for Irish female students to engage in spatial skills development and the impact of such experiences on female students' outcomes in mathematics may be worthwhile.

CONCLUSION

A gender gap at the highest level of attainment in LC HL mathematics seems to have always existed but this paper highlights that it has widened with the phased introduction of the new mathematics examinations in 2012-2015. This widening of the gender gap seems to be connected to changes in the exam, which now requires greater problem solving proficiency.

It is notable that, at the advanced level of mathematics in PISA 2012 and TIMSS 2015, the gender gap disadvantaging Irish female students is larger than average. Yet attainment is higher and the gender gap in mathematics is less pronounced in countries that facilitate more geometry and seem to give greater support for problem solving in their classrooms. An underlying issue of the gender gap for high-achieving students in LC HL mathematics may be cognitive differences in spatial ability between male and female students.

Spatial ability can be developed through spatial skills training and exposure to spatially rich learning experiences in subjects including mathematics (Reilly, Neumann & Andrews, 2017). This has implications for classroom practice (Ontario, 2014) and for teachers whose own comfort level with spatial reasoning is related to students' growth in this area (Gunderson, Ramirez, Beilock & Levine, 2013). Further research is required to support the claim that spatial ability is contributing to a widening gender gap in attainment at LC HL mathematics and to consider appropriate interventions to address this imbalance, thereby supporting equality in gender representation at all levels of mathematics.

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