ESTIMATING UNCERTAINTY IN LOSS MEASUREMENT OF POWER TRANSFORMERS

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Abstract: The Eco-design directive issued by the European Commission has led to requirements on efficiency of power transformers. In the case of large power transformers used in grid applications, serious problems are encountered in establishing how reliable the loss measurements are. An effort is currently on-going within IEC to produce a documentary standard on “Rules for the determination of uncertainties in the measurement of the losses of power transformers”. An IEC standard should be clear and easy to understand by all users in the industry. Background theory and material, whilst necessary for understanding, is not required for the day-to-day application of the standard. This paper presents a more detailed background and theory on the measurement of transformer losses and how to quantify precision. The authors are all members of the IEC maintenance team working with the standard.

The development of an uncertainty budget for a measurement starts from a model function of the measurement. The basic model function for power loss measurement will be developed for the case where instrument transformers are used to adapt the high voltages and currents to the inputs of a watt meter. Input data will be errors and uncertainties of errors for the instrument transformers and of the watt meter. Ratio and magnitude errors are of course important, but in measurements at low power factor, phase displacement will be the dominant factors.

In adherence to the Guide to the expression of uncertainty in measurement (GUM), using the partial derivatives of the model function with respect to its parameters, an expression for the sensitivity to uncertainties of each parameter can be formally expressed and quantified.

For power transformers, there are two important loss measurements: no-load loss measurements, where voltage and current may be distorted, and load-loss measurements, where the power factor can be very small. For each of these two cases, slightly different modifications of the basic model function are needed.

For no-load loss measurements a change in voltage level will have a nonlinear (approximately quadratic) influence on the measured loss. Although tests shall be performed as close as possible to nominal voltage, there will be an unavoidable uncertainty in the measurement of this voltage, and a corresponding term has to be added to the model function. A second additional term relates to the fact that the no-load loss test signals are distorted, requiring a correction to the measured losses, as required by IEC standards, albeit with some uncertainty.

For load loss measurements, the test current shall be in the range 50 to 100 % of nominal, and a recalculation is to be performed to extrapolate the result from test current to nominal current, considering that resistive losses vary with the square of current. A further correction is to be made to refer the result to nominal reference temperature, which can be appreciably higher than the temperature during test. Both corrections require modifications of the basic model function.

The paper derives the expressions for uncertainty propagation from the modified model functions and sets out the resulting uncertainty budgets.

1 INTRODUCTION

The losses of power transformers (no-load and load losses) are object of guarantee and penalty in many contracts and play an important role in the evaluation of the total (service) costs and therefore in the investments involved. Furthermore, regional regulations, such as the European Union directive for EcoDesign [1], may also pose requirements on establishment of reliable values for losses.

According to ISO/IEC 17025 and ISO/IEC Guide 98-3 (GUM) [2], [3], the result of any measurement should be qualified with the evaluation of its uncertainty. A further requirement is that known corrections should have been applied before evaluation of uncertainty.

Corrections and uncertainties are also considered in IEC 60076-2 [4] where some general indications are given for their determination.

Earlier work has been published by both CENELEC and IEC [5, 6], which have been used as springboard for the work presented here.

Measurement of the losses, can from a measuring point of view, be seen to consist of the estimate of a measurand and the evaluation of the uncertainty that affects the measurand itself. The procedures can also be applied to loss measurements on power transformers as evaluation of the achievable performance of a test facility in the course of prequalification processes, as estimations of achievable uncertainty in the enquiry stage of an order or prior to beginning final testing at manufacturer´s premises and for evaluations of market surveillance measurements.

The uncertainty will depend on the quality of the test installation and measuring system, on the skill of the staff and on the intrinsic measurement difficulties presented by the tested objects.

In cases where the losses are required to conform to stated tolerance limits, it is recommended that the estimated uncertainty should be less than the tolerance limit. This situation will obtain for example in market surveillance activities. Achieving a desired uncertainty of 3 % in measurement of load-loss of a low-loss transformer is in the experience of the authors a challenging task.

In an aside it can be noted that reference for loss at high voltage refers back to loss factor of compressed gas capacitors, and that their performance is adequate for loss of transformers but can be just about sufficient for large reactors [7].

2 MODEL FUNCTION

2.1 Remarks on error and uncertainty

In most cases, a measurand Y is not measured directly, but is determined from N other quantities $X_1, X_2, \ldots, X_N$ through a functional relationship $Y = f(X_1, X_2, \ldots, X_N)$, called the model function.

From ISO/IEC Guide 98-3 we have the estimated standard deviation associated with the output estimate or measurement result y, termed combined standard uncertainty and denoted by $u_c(y)$, is determined from the estimated standard deviation associated with each input estimate x_i, termed standard uncertainty and denoted by $u(x_i)$

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \quad (1)$$

where the partial derivatives are the *sensitivity coefficients* for the propagation of uncertainty of the measured quantities x_i into the measurand. The standard uncertainty is understood to have a coverage probability of 67 %.

Example of input quantity: Ratio error of voltage and current transducer. The known error is the measured ratio error at calibration of the transducer. The unknown error is the uncertainty of its calibration and possibly ageing and environmental effects.

2.2 Basic formulation

Considering only sinusoidal signals, the power can be expressed as

$$P = I_{rms} \cdot U_{rms} \cdot cos(\phi) \quad (2)$$

Alternatively, one can use the general formulation

$$P = \frac{1}{T} \int_0^T I(t) \cdot U(t) dt \quad (3)$$

where in both cases the current and voltage are those appearing in the test object. Eq. (3) is valid for any cyclic function, but in practice formulation (2) is sufficient for evaluation of uncertainties in transformer loss measurements.

Let us assign subscript $_{to}$ to the parameters valid in the tested transformer and subscript $_{tr}$ to parameters observed after transducers. Transducer scale factors are designated *k*, ratio errors by ε and subscript $_V$ for voltage and $_C$ for current. We have the true power loss P_{to}:

$$P_{to} = U_{to} \cdot I_{to} \cdot cos(\phi_{to}) \quad (4)$$

The power observed with the measuring system is however influenced by the precision of the transducers:

$$U_{tr} = \frac{U_{to} \cdot (1+\varepsilon_V)}{k_{VN}} \text{ and } I_{tr} = \frac{I_{to} \cdot (1+\varepsilon_C)}{k_{CN}}$$

$$\text{Phase correction: } \Delta_{\phi V} - \Delta_{\phi C} \tag{5}$$

$$P_{tr} = U_{tr} \cdot I_{tr} \cdot cos(\phi_{tr}) \text{ where } \phi_{tr} = \phi_{to} + (\Delta_{\phi V} - \Delta_{\phi C})$$

Strictly speaking, this correction for the phase displacement is true for inductive loads, which holds true for measurement of transformer loss.

We can now define a correction factor C:

$$P_{to} = C \cdot P_{tr} \text{ or } C = \frac{P_{to}}{P_{tr}} \tag{6}$$

Recognising that the wattmeter measures P_{tr}, disregarding the possible errors of the wattmeter and denoting the readings as P_m, U_m and I_m

$$P_{to} = C \cdot P_{tr} \approx \mathrm{C} \cdot \mathrm{P}_m$$

$$C = \frac{k_{VN}}{1+\varepsilon_V} \cdot \frac{k_{CN}}{1+\varepsilon_C} \cdot \frac{cos(\phi_{to})}{cos(\phi_{to} + (\Delta_{\phi V} - \Delta_{\phi C}))} \tag{7}$$

$$U_{to} \approx U_m \cdot \frac{k_{VN}}{1+\varepsilon_V} \qquad I_{to} \approx I_m \cdot \frac{k_{VN}}{1+\varepsilon_C}$$

The true phase angle ϕ between voltage and current under sinusoidal conditions is obtained by correcting for the known phase displacement difference between CT and VT, which for inductive loads is given by $(\Delta_{\phi V} - \Delta_{\phi C})$:

$$\phi = \phi_M - (\Delta_{\phi V} - \Delta_{\phi C}) \approx arccos\left(\frac{P_W}{I_M U_{rms}}\right) - (\Delta_{\phi V} - \Delta_{\phi C}) \tag{8}$$

which is not strictly applicable for cases where distortion is present, but still suitable for the purpose of uncertainty evaluation.

The term $\frac{P_W}{I_M U_{rms}}$ is the power factor as measured by the power meter.

Consideration is made only of single-phase circuits, three-phase results are to be obtained from the single-phase case.

2.3 No-load loss

No-load loss is to be referred to a specific test voltage, normally nominal voltage U_N. The standard does not give a formula to recalculate loss to the specified voltage, thus correction is not possible. It is however required that the test shall be performed as close as possible to the specified test voltage. Uncertainty can however be estimated.

The relevant quantity for test voltage in no-load loss measurement is rectified average, U_{avg}, as this reflects the flux in the core.

The standard states that temperature of the core has no influence on the loss, and the same stands for winding resistance loss. The reason for the latter is that the winding resistance loss is small, thus changes in winding resistance are negligible.

The impedance seen by the supply during no-load loss test is not linear, thus the test voltage waveshape may be influenced, and this will influence the loss. The standard has given a formula to correct loss for voltage waveshape. The current waveshape is of course not sinusoidal for the same reason, even if test voltage is a pure sine wave. No correction for this is recognised by the standard. The model function is extended to cover this by adding parameters U_{avg} and U_{rms}.

$$P_{NLL} = P_{to} \cdot \left[\frac{U_N}{k_{VN} \frac{1}{1+\varepsilon_V} \cdot U_{avg}}\right]^n \cdot \left(1 + \frac{U_{avg} - U_{rms}}{U_{avg}}\right) \tag{9}$$

For the purpose of this analysis, we assume that the losses follow a power law where the exponent n is on the order of 2. If needed, this exponent can be estimated from several measurements performed at different voltages.

The ratio uncertainty of the voltage transformer now appears repetitively, and must be taken properly into account to avoid double counting:

$$P_{NLL} = \frac{k_{CN}}{1+\varepsilon_C} \cdot \frac{1}{\left(\frac{k_{VN}}{1+\varepsilon_V}\right)^{n-1}} \cdot$$

$$\cdot \frac{cos\,\phi}{cos(\phi + (\Delta_{\phi V} - \Delta_{\phi C}))} \cdot P_m \cdot \left[\frac{U_N}{U_{avg}}\right]^n \cdot \left(1 + \frac{U_{avg} - U_{rms}}{U_{avg}}\right) \tag{10}$$

2.4 Load loss

2.4.1 General

Load loss is to be stated for a given current, normally nominal current I_N , and also at a given temperature, normally the reference temperature θ_{ref} , which can be e.g. 75, 85 or 120 ºC.

The standard has defined a formula for loss versus current, loss versus temperature and winding DC resistance versus temperature. The temperature mentioned here is the winding mean temperature. Before being able to recalculate load loss to a different temperature, loss needs to be split in the loss of the winding DC resistance $I^2 \cdot R$ and loss due to stray flux, called other losses P_O in the standard. This latter calculation is formulated in a separate model function for clarity reason.

The winding resistance R presents an additional complexity as this resistance can be measured at a different temperature θ.

The model function is extended for load loss by adding parameters: P_2, I_m, θ_1 θ_2, R_2, R_1 and I_N.

2.4.2 Model function for load loss at rated current

The single-phase model function for the measured power P_2 measured at temperature θ_2 and referred to the rated current I_N is:

$$P_2 = P_{to}\left[\frac{I_N}{k_{CN}\dfrac{1}{1+\varepsilon_C}I_m}\right]^2 \qquad (11)$$

Recognising that the ratio error now appears repetitively we can rearrange as

$$P_2 = k_{CN}\left(1+\varepsilon_C\right)\cdot k_{VN}\frac{1}{1+\varepsilon_V}\cdot \cdot\frac{cos\,\phi}{cos\left(\phi+(\Delta_{\phi V}-\Delta_{\phi C})\right)}\cdot P_m\cdot\left[\frac{I_N}{k_{CN}I_m}\right]^2 \qquad (12)$$

2.4.3 Load loss at rated current and reference temperature

The measured loss P_2 is assumed to be composed of I^2R loss and additional loss P_{a2}. The latter varies as the inverse of resistance, according to the standard. The relation between these at the reference current I_N is described by the model function:

$$P_2 = I_N^2R_2 + P_{a2} \text{ where } I_N^2R_2 = I_{HV}^2R_{HV} + I_{LV}^2R_{LV}$$

$$P_{a2} = P_2 - I_N^2R_2 \qquad (13)$$

The total load loss P_{LL} for the I^2R_r loss and additional loss P_{ar} at reference temperature is defined in IEC 60076-1:2011, Annex E [8] as:

$$\begin{aligned} P_{LL} &= I_N^2R_r + P_{ar} = \\ &I_N^2R_2\frac{t+\theta_r}{t+\theta_2} + P_{a2}\frac{t+\theta_2}{t+\theta_r} = \\ &I_N^2R_2\frac{t+\theta_r}{t+\theta_2} + (P_2 - I_N^2R_2)\frac{t+\theta_2}{t+\theta_r} \end{aligned} \qquad (14)$$

where the resistance R_2 of the windings as attained during the load test performed at temperature θ_2 .Parameter t is a constant set to 235 for copper and to 225 for aluminium windings and relates to the temperature coefficient of the resistivity.

3 UNCERTAINTY

3.1 General

In accordance with the Guide to the Expression of Uncertainty in Measurement, ISO/IEC Guide 98-3, you derivate the model function for the measurand (the quantity to be determined) with respect to each input quantity in order to obtain the sensitivity factor for the uncertainty fo this quantity.

Each input quantity is characterised by its uncertainty, usually given as the standard uncertainty, i.e. “uncertainty of the result of a measurement expressed as a standard deviation” (GUM). For practical reasons, uncertainties derived by statistical analysis of series of observations are designated Type A, whereas contributions obtained by other means are designated Type B. In the context of this Paper, only Type B contributions will be discussed. In the actual measurement situation, Type A will need to be considered as well.

The combined standard uncertainty is the positive square root of the combined variance of all input quantities, while taking into account the sensitivity factors. In the general case, the combined uncertainty is given in the same unit as the measurand.

In the case that the model function consists entirely of multiplications (or divisions) a simplified method can be employed where relative contributions are considered.

Formal derivations will not be shown explicitly here, except for calculation of load loss to reference temperature.

3.2 No load loss uncertainties

Final equation (10) for P_{NLL} derivated with respect to the input quantities and analysed with the simplified method for multiplicative contributions are summarised in **Table 1**.

Table 1: No-load loss uncertainties

Quantity	Component	Std. uncert.	Sens. Coeff.
CT ratio error	ε_C	u_C	1
VT ratio error	ε_V	u_V	n-1
Measured power	P_W	u_{PW}	1
Phase	$\frac{cos(\phi)}{cos\left(\phi+(\Delta_{\phi V}-\Delta_{\phi C})\right)}$	$u_{FD}\approx 0$	1
Voltage	U_N	u_{UM}	n
Correction to sinusoidal waveform	$1+\frac{U_{avg}-U_{rms}}{U_{avg}}$	u_{WF}	1

As no-load loss does not exhibit very small power factor, the contribution from phase displacement has been neglected.

It can be noted that contribution u_{WF} when using two independent instruments for U_{rms} and U_{avg} will require an advanced analysis to determine the standard uncertainty. However, in the common case that the same sampling instrument is used for both measurements, it is reasonable to see them as fully correlated, and furthermore small enough that the uncertainty contribution can be neglected.

The combined standard uncertainty is given by;

$$u_{NLL}=\sqrt{u_C^2+(n-1)^2\cdot u_V^2+u_{PW}^2+n^2\cdot u_{UM}^2+u_{WF}^2} \quad (15)$$

The expanded relative uncertainty is $U_{NLL}=2u_{NLL}$, which corresponds to a coverage probability of approximately 95 %.

3.3 Load loss uncertainties

3.3.1 Uncertainties for load loss at rated current

Final equation (12) for power at rated current P_2 derivated with respect to the input quantities and analysed with the simplified method for multiplicative contributions are summarised in **Table 2**.

Table 2: Measured load loss uncertainties

Quantity	Component	Std. uncert.	Sens. Coeff.
CT ratio error	ε_C	u_C	1
VT ratio error	ε_V	u_V	1
Power meter	P_W	u_{PW}	1
Phase	$\frac{cos(\phi)}{cos\left(\phi+(\Delta_{\phi V}-\Delta_{\phi C})\right)}$	u_{FD}	1
Ampere meter	I_M	u_{IM}	2

Where $u_{FD}\approx u_{\Delta\varphi}\tan\varphi\approx u_{\Delta\varphi}/\cos\varphi$

and $u_{\Delta\varphi}$ shall be given in radians. It is here evident that at low power factor, the phase displacement uncertainty is dominant.

Combined standard relative uncertainty calculated as:

$$u_{P2}=\sqrt{u_C^2+u_V^2+u_{PW}^2+u_{FD}^2+4u_{IM}^2} \quad (16)$$

3.3.2 Uncertainties for recalculation to reference temperature

The results of the load loss test shall be reported to the reference temperature in accordance with IEC 60076-1 as shown by equation (14).

The loss power and the associated uncertainty contributions are to be expressed as absolute uncertainties (e.g in watts) in order to obtain correct calculation of the total uncertainty at reference temperature (model function is not multiplicative only).

Quantity	Compo-nent	Absolute standard uncertainty	Sensitivity coefficient
$I_N^2R_r$ loss	R_r	$R_2\cdot u_{R2}$	$I_N^2\left(\frac{t+\theta_r}{t+\theta_2}-\frac{t+\theta_2}{t+\theta_r}\right)$
Measured loss	P_2	$u_{P2}\cdot P_2$	$\frac{t+\theta_2}{t+\theta_r}$
Mean winding temperature	θ_2	$\dot{u}_{\theta 2}$	$I_N^2R_2\frac{t+\theta_r}{(t+\theta_2)^2}+$ $+\left(P_2-I_N^2R_2\right)\frac{1}{t+\theta_r}$

A case is made here of the process of finding the Sensitivity coefficient for the propagation of uncertainty due to the resistance R_2 of winding at the temperature θ_2 valid during load-loss measurement and the impact on the result as recalculated to reference temperature θ_r . The partial derivative of equation (14) with respect to R_2

$$\frac{\partial P_{LL}}{\partial R_2} = I_N^2 \left(\frac{t+\theta_r}{t+\theta_2} - \frac{t+\theta_2}{t+\theta_r} \right) \qquad (17)$$

Given that the absolute uncertainty for R_2 is $R_2 \cdot u_{R2}$, where u_{R2} is the relative standard uncertainty for R_2 , we can now state the contribution from R_2 to absolute standard uncertainty of load-loss as:

$$\dot{u}_{P_{LL}} = \left(\frac{t+\theta_r}{t+\theta_2} - \frac{t+\theta_2}{t+\theta_r} \right) \cdot I_N^2 R_2 \cdot u_{R2} \qquad (18)$$

Since the uncertainty is absolute, it is expressed in Watt or multiples thereof.

3.4 Note on corrections vs uncertainty

The general rule is that measurements shall be corrected for known errors. The question then boils down to what does constitute a "known error".

Errors can only be corrected for if they are stable over time, as for example magnetic voltage and current transformers. In general, electronic devices cannot be regarded as stable over time, thus their error(s) established at calibration cannot be used as a known error. It is of course fervently assumed that this error will not drift outside the given accuracy of the device.

A corollary is then that an advanced measuring system with electronically enhanced devices cannot be corrected for, and that uncertainty must be based on specifications that have been verified by calibrations.

4 CONCLUSIONS

Measurement of losses of transformers is complicated and a compromise has to be found between scientific precision and practical work by transformer testing laboratories. In this treatise we have made an effort to reduce the formal mathematics to what is necessary to achieve a meaningful assessment of uncertainty in loss measurement. The authors are however cognizant of the difficulties in applying the theory to practical situations, and look forward to a future dialogue, especially with experts from industry.

This work can serve as a backdrop for the discussion of preference of calibration of the diverse components coupled with a complex analysis of propagation of uncertainty, versus system wide calibration of the entire loss measuring system. There is no clear answer to this, where complexity the component evaluation is set against time expenditure necessary to cover the parameter space when using the system wide calibration.

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