

An efficient numerical approach to the prediction of laminate tolerance to Barely Visible Impact Damage

Abrar H. Baluch^{1a}, Olben Falcó^a, José Luis Jiménez^a, Bas H. A. H. Tijs^b,
Cláudio S. Lopes^{1a}

^a*IMDEA Materials Institute, c/ Eric Kandel 2, 28906 Getafe, Madrid, Spain*

^b*GKN Aerospace: Fokker, Papendrecht, The Netherlands*

Abstract

An efficient numerical approach for the prediction of the Compression After Impact (CAI) strength of aerospace-grade CFRP laminates when exposed to Barely Visible Impact Damage (BVID) is proposed. The approach is based on mapping relevant BVID features, i.e. delaminations, onto an efficient CAI finite element model based on continuum shell discretization, and can be used on Low-Velocity Impact (LVI) results obtained experimentally or by means of high-fidelity virtual tests. It is proposed that delaminations may be represented by simplified shapes, and only the ones at critical through-thickness locations need to be mapped, allowing the clustering of several plies in a single shell layer. General guidelines, that are potentially valid for a wide range of unidirectional CFRP laminates, are proposed to identify relevant and critical BVID features to be mapped onto the efficient CAI modelling. The approach was validated for five laminates of AS4/8552 material, covering a range of different thicknesses, overall achieving CAI strength predictions within 5% of the experimental results. In comparison with the alternative high-fidelity CAI virtual testing approach, this method leads to computational efficiency gains of an order of magnitude. Moreover, the full simulation of the sequence LVI plus CAI steps can be accelerated by a factor of four.

Keywords: Virtual Testing, Low-Velocity Impact (LVI), Compression After Impact (CAI), Barely Visible Impact Damage (BVID)

¹Corresponding authors: abrar.baluch@imdea.org, claudiosaul.lopes@imdea.org

1. Introduction

The usage of laminated composite materials is increasing exponentially in all engineering sectors, specially aerospace and automotive, wherein they allow many advanced structures with desirable properties which were otherwise not possible. However, such structures also have unique ways towards failure [1–3]. Their failure mechanisms vary according to the loading scenario and include fiber breakage and kinking, matrix cracks and delamination. The contribution of these mechanisms towards damage also varies according to the loading type and direction. One sensitive aspect for aerospace composite structures is the event of Low-Velocity Impact (LVI) that leads to Barely Visible Impact Damage (BVID). The circumstances leading to BVID vary widely, from dropped tools on composite panels to in-service impacts, among other reasons. This type of damage plays a critical role in the structural integrity of composite structures, mainly because of its difficult identification. BVID consists mostly in local delamination, hidden from the naked eye underneath a possibly unsuspected impact point, that can have a critical detrimental effect on the strength of composite structures when in-service.

The mechanisms of initiation and progression of delamination in LVI events have been described in the literature [4–7]. Hull et al. [8] observed that delaminations tend to develop along major axes parallel to the fiber orientation of the neighbouring ply furthest away from the impact point, and that their area is highly dependent on the specimen out-of-plane displacement during impact. Such damage then has the major contribution towards laminate failure in the event of Compression After Impact (CAI) loads. This is fundamentally because of the reduction of structural stability caused by the delaminated composite. The CAI event can reduce the component strength up to 60% in comparison to pristine samples [9–13]. Therefore, the CAI strength is considered a critical design parameter for a large number of aircraft structures, and it has been standardized to assess the tolerance to BVID in composite laminates.

The CAI strength of composite laminates affected by BVID depends on delamination and its characteristics, including geometry, size and orientation, as well as on its location through the thickness of the composite [14–16]. In support of these assertions, experimental studies have been carried out by means of artificially generated delaminations, with different shapes, and at different locations through the thickness of composites [17, 18]. Multiple delamination configurations have been explored in the literature. Some re-

38 searchers used circular delaminations [19] with regular intervals, while some
39 others used elliptical shapes, and yet a few others used peanut-shaped,
40 which is claimed to be a more realistic profile [16]. Wang et al. [20] reported
41 that the distribution of delaminations at different locations through thickness
42 of laminates severely affects their CAI strength. Aslan et al. [21] reported
43 that delamination at interfaces near the laminate surface resulted in the high-
44 est reductions of the CAI strength. In fact, it has been proposed that a single
45 circular delamination near the surface dominates the CAI response [22]. Mi-
46 nak et al. [23] discussed in detail the effects of intra-ply matrix cracking,
47 and its interaction with interface delamination propagation, on the CAI re-
48 sponse of thick and thin laminates. At present, most of the researchers agree
49 that the influence of matrix cracking on the tolerance to BVID is negligible
50 compared to the effect of delamination.

51 A substantial number of researchers have studied multiple BVID configura-
52 tions and developed analytical and semi-analytical CAI strength reduction
53 models [1, 14, 15, 18, 22, 24–27] based on one or more through-thickness
54 delaminations of idealized shapes. Judging by one of the recent works [22],
55 besides their readiness and efficiency, such models can be accurate to within
56 30-40% error bounds.

57 To obtain the delamination profile on impacted laminated specimens,
58 the most straight-forward way is to measure it experimentally by conduct-
59 ing ultrasonic inspection (C-scan). Alternatively, the capabilities to predict
60 the delaminated surfaces with substantial accuracy have been developed in
61 recent years and go by the mark of virtual testing, i.e. high-fidelity finite el-
62 ement (FE) simulations [28–31]. Notwithstanding their disputable accuracy,
63 the virtual testing approach to LVI has the advantage that it can straight-
64 forwardly include the simulation of the CAI event as a follow-up simulation
65 step, as proposed in [29, 30], and be used for BVID resistance and tolerance
66 predictions. However, the high-fidelity simulation of both loading steps can
67 be quite expensive computationally, as will be shown in this paper.

68 In this work, an efficient numerical methodology to accurately predict tol-
69 erance to BVID is proposed and validated. The approach can be seen as an
70 alternative to the expensive CAI simulations, and can be applied on results
71 of either experimental or high-fidelity simulations of LVI. It consists on map-
72 ping relevant BVID features, i.e. delaminations, onto an efficient CAI finite
73 element model based on continuum shell discretization. To aide the devel-
74 opment and validation of the methodology, LVI and CAI experiments were
75 carried on different laminate configurations based on the AS4/8552 carbon

76 fibre-reinforced material system. Nonetheless, the guidelines proposed in this
 77 paper to perform delamination mapping into efficient CAI modelling are po-
 78 tentially applicable to a wide range of composite laminates. The applicability
 79 of the proposed methodology is, however, limited to impact configurations
 80 that are dominated by delamination.

81 2. Experimental evaluation of laminate tolerance to BVID

82 This section describes the LVI experiments that were performed to acquire
 83 the BVID data on different specimens, as well as the CAI tests to validate
 84 the proposed damage tolerance assessment methodology.

85 2.1. Test specimens

86 Unidirectional AS4/8552 pre-preg material was used to manufacture five
 87 different kinds of laminates. These configurations were selected on the basis
 88 of their use in aerospace applications at industrial level. The configurations
 89 constitute limits of the design space in terms of stiffness properties. This
 90 is reflected in the following approximate percentage ratios of plies in the 0,
 91 ± 45 , and 90 directions which are used in the aerospace industry for config-
 92 uration identification: 30/60/10 (361); 30/50/20 (351); 25/50/25 (252); and
 93 10/80/10 (181 and 181T). The laminates were cured in autoclave by adopting
 94 standard procedures at GKN Aerospace: Fokker. Their stacking sequences
 95 and average ply properties are presented in Tables 1 and 2 respectively.

Table 1: Laminate configurations analyzed.

| Lam- inate | # of plies | thick [mm] | Stacking Sequence | 0° plies % | 45° plies % | 90° plies % |
|---------------|---------------|---------------|--|------------------|-------------------|-------------------|
| 181 | 16 | 2.944 | $[\pm 45/0/45_2/ - 45_2/90]_S$ | 12.5 | 75.0 | 12.5 |
| 181T | 20 | 3.68 | $[\pm 45/0/ \pm 45/90/ \pm 45_2]_S$ | 10.0 | 80.0 | 10.0 |
| 252 | 24 | 4.416 | $[(45/0/-45/90)_3]_S$ | 25.0 | 50.0 | 25.0 |
| 351 | 24 | 4.416 | $[45/0/-45/0/45/0/-45/90/45/90/-45/0]_S$ | 33.0 | 50.0 | 17.0 |
| 361 | 34 | 6.256 | $[45/0/-45/0/\pm 45/0/ - 45/90/45_2/90/ - 45/0_2/ \pm 45]_S$ | 30.0 | 60.0 | 10.0 |

Table 2: Average AS4/8552 ply material properties (taken from [32]).

| Ply elastic properties | | | | | | |
|---------------------------------|-------------------|--|---|-------------------------|-------------------|----------|
| E_{1t} (GPa) | E_{1c} (GPa) | E_{2t} (GPa) | E_{2c} (GPa) | $G_{12} = G_{13}$ (GPa) | $v_{12} = v_{13}$ | v_{23} |
| 137.1 | 114.3 | 8.8 | 10.1 | 4.9 | 0.314 | 0.487 |
| Ply strengths properties | | | | | | |
| X^T (MPa) | X^C (MPa) | Y^T (MPa) | Y^C (MPa) | S^L (MPa) | | |
| 2106.4 | 1675.9 | 74.2 | 322.0 | 110.4 | | |
| Interface properties | | | | | | |
| τ_n^0 (MPa) | τ_{sh} (MPa) | $G_{Ic,\theta=0}$ (kJ/m ²) | $G_{IIc,\theta=0}$ (kJ/m ²) | η_{bk} | | |
| 74.2 | 110.4 | 0.30 ± 0.01 | 0.87 ± 0.06 | 1.45 | | |

96 *2.2. Barely Visible Impact Damage (BVID)*

97 The pristine specimens were subjected to drop-weight impact experiments
 98 according to the Airbus standard test method AITM 1-0010 and ASTM
 99 D7136 [33, 34] by means of a CEAST 9350 Drop Tower Impact System (In-
 100 stron). Different impact energies, in the range of 10J to 70J, were experi-
 101 mented in order to determine the threshold of BVID. At least two repeti-
 102 tions of each impact test were performed. The permanent indentation on
 103 each specimen was measured after 30 min of the impact event by using a
 104 depth gauge. The compiled results are shown in curves of increasing inden-
 105 tation depth for increasing impact energy (Figure 1). For each configuration,
 106 interpolation was performed to estimate the energy corresponding to the
 107 threshold of BVID, defined by the test standard as relating to a permanent
 108 indentation of 1 mm [35–37]. This criterion establishes that an impact event
 109 is identifiable when the resulting permanent indentation is deeper than 1 mm.
 110 Three impact test per configurations were then performed at approximately
 111 those energies and the corresponding results added to the database (Figure
 112 1).

113 From this point onward, the presented research focus on the impact con-
 114 figurations corresponding to the determined energy threshold for BVID. The
 115 proposed analysis methodology can only be considered valid for impact dam-
 116 age dominated by delamination, such as BVID.

117 Non-Destructive Testing (NDT) was conducted on the samples impacted
 118 at the thresholds of BVID to analyze the extent of damage. To determine the
 119 impact damage footprint, i.e. the superposition of all damage features in the
 120 specimens, the technique of ultrasonic C-scan (Tecnitest Triton 1500 USPC-

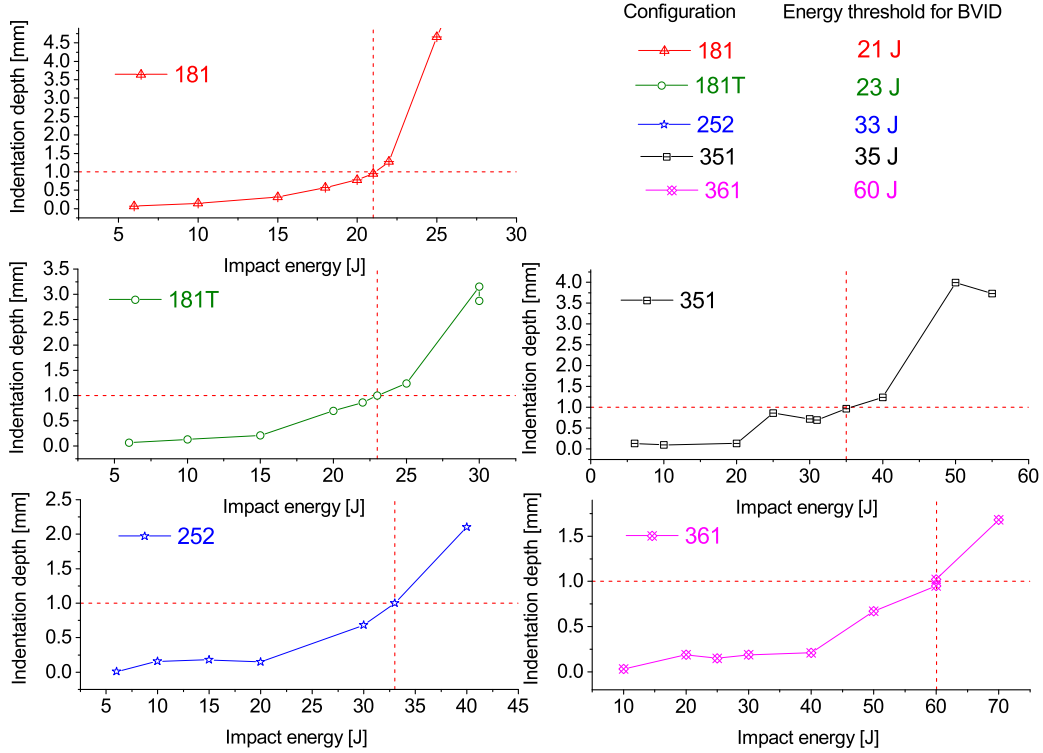


Figure 1: Experimentally-obtained relations between impact energy and permanent indentation for all configurations analyzed, and identification of BVID threshold.

121 3100 ultrasonic) was employed. An attenuation of 12 db was established as
 122 the threshold of damage. The location of delaminations through the thick-
 123 ness of the laminate was also determined by means of the same ultrasound
 124 equipment in depth scan mode. Such depth scans were performed on both
 125 faces of the samples, impact and back (opposite-to-impact) sides, with results
 126 shown in Figure 2. For all cases, a delamination can be observed a the ply
 127 furthest away from the impact point. Moreover, these delaminations have
 128 approximately oval shape with a large aspect ratio and major axis aligned
 129 with the fibre orientation of the delaminated ply. This phenomenon has been
 130 extensively reported in the literature and is due to back-face splitting (see
 131 Figure 8). From the impact face, the first major delamination was found
 132 always at a depth over 1 mm. At shallower depths, major delaminations
 133 were suppressed by high out-of-plane compressive stresses [38]. For the inner

134 specimen interfaces, delaminations seem to be bounded by circumferences of
135 approximately constant diameter.

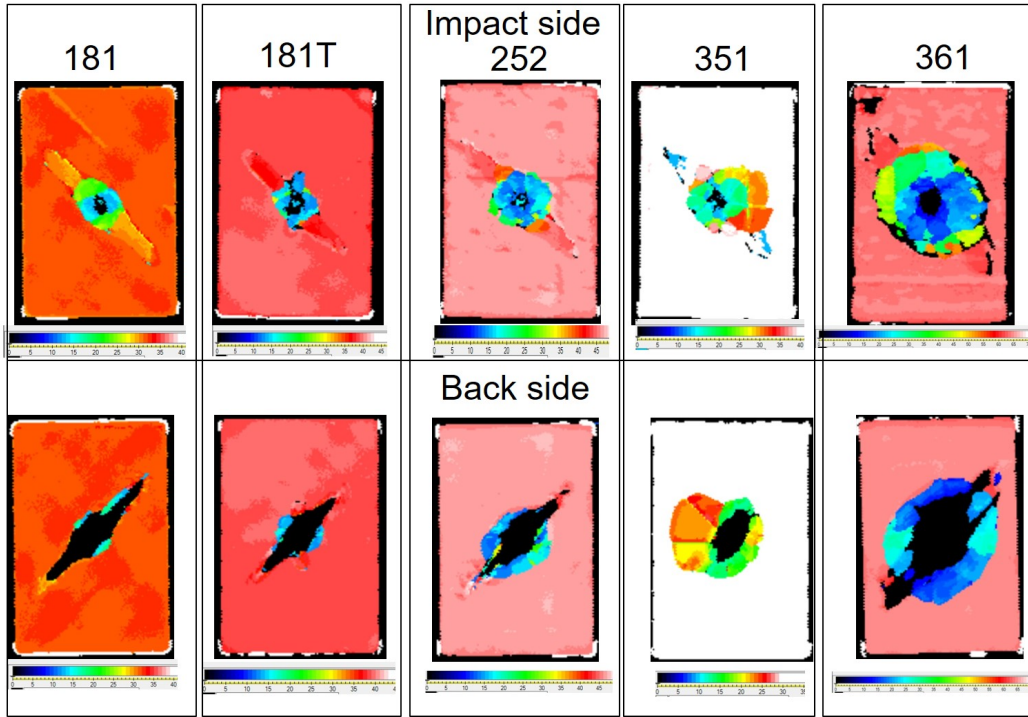


Figure 2: Depth scan for multiple laminates after BVID. The colour scheme at bottom of each figure represents the depth of delamination (scale: x0.1 mm).

136 2.3. Damage tolerance

137 In order to determine the tolerance to BVID, standard Compression After
138 Impact (CAI) tests were performed on three specimens of each configuration
139 following the procedures defined by ASTM D7137 and AITM 1-0010 [34, 39],
140 using an electromechanical testing machine (Instron 3384) equipped with a
141 150 kN load-cell in combination with an anti-buckling support fixture. To
142 monitor out-of-plane displacements and correct eventual global buckling be-
143 haviour, a Digital Image Correlation (DIC) system was used (VIC-3DTM,
144 Correlated Solutions). The failed specimens were then again analyzed by
145 means of ultrasonic C-Scan to determined the delamination patterns resul-
146 tant of the CAI tests.

147 The results of the CAI tests in terms of specimen deformation and super-
148 imposed delamination patterns can be appreciated in Figure 8. A significant
149 amount of damage to the backface plies is visible, mainly in the form of
150 ply splitting. The average experimentally-obtained CAI strength values are
151 represented in Figure 9.

152 **3. State-of-the-art virtual testing of BVID and CAI**

153 As pointed out in the introduction, high-fidelity simulations have been
154 proposed by different authors [28–31] to predict the LVI and CAI behaviours
155 of composite laminates. Two-step analyses were proposed to describe both
156 loading cases in sequence [29, 30]. The first step tackles the simulation of
157 LVI. Once it finishes, after enough physical time to allow for the rebound of
158 the impactor plus the dissipation of major vibrations in the virtual specimen
159 (typically requiring 5 to 10% of the total contact time), a second step ensures
160 addressing the simulation of the CAI. This type of approach was also pursued
161 in this work. The purpose was twofold. On the one hand, the high-fidelity
162 simulation of LVI was as a way to predict the threshold of BVID in all an-
163 alyzed specimens with high realism without having to conduct experiments.
164 On the other hand, the high-fidelity simulation of compression after BVID
165 was used in order to establish a reference case for comparison against a more
166 efficient approach proposed in this research. Hence, the simulation of CAI
167 using this method was conducted only for one of the laminate configurations.

168 *3.1. Modelling approach*

169 FE modelling of the composite coupons were carried out in Abaqus/Explicit
170 [40], following the methodology proposed in [28] and briefly described as fol-
171 lows. The virtual standard coupons were split into parts employing differ-
172 ent discretization levels, and whose kinematic compatibility was ensured by
173 means of tie constraints, as proposed earlier by the authors [32]. A global
174 discretization was employed to the coupon end sections by using continuum
175 shell elements describing the linear-elastic laminate behaviour. The central
176 damage zone, where the possibility of damage is taken into account, used
177 three-dimensional ply-by-ply discretization (with reduced-integration C3D8R
178 elements in Abaqus [40]).

179 Delamination and intraply damage mechanisms were tackled by two dis-
180 tinct approaches. The ply damage model was based on the work of Maimí
181 et al. [41, 42], ensuring the correct dissipation of energy associated with

182 different ply failure modes, and was implemented by means of a VUMAT
183 user subroutine. Ramberg-Osgood laws [43] were used to describe the ma-
184 terial nonlinear shear responses. The onset of ply damage was determined
185 by means of the three-dimensional failure criteria proposed by Catalanotti
186 et al. [44], and it was followed by exponential damage laws. Matrix cracking
187 was assumed to occur under mixed-mode behaviour. To enable delamination
188 among different plies, the surface-based cohesive zone modelling approach
189 implemented in Abaqus [40] was employed. This allowed the description
190 of inter-ply damage and ply decohesion by means of a generalized traction-
191 separation behavior [45] in combination with a general contact algorithm
192 applied to ply surfaces. Frictional contact among delaminated plies was sim-
193 ulated by coupling a Coulomb-type model ($\mu = 0.5$) to the cohesive zone
194 formulation. The interface penalty stiffness was set to 2×10^5 N/mm³ in all
195 traction directions, after checking that it provided correct results while al-
196 lowing efficient computations. Delamination initiation was predicted by a
197 quadratic interactive stress criterion while the propagation of this damage
198 mode was tacked with the energy-based Benzeggagh-Kenane (BK) criterion
199 [46].

200 The mesh generation was governed by mesh alignment with orthotropic
201 material directions and directional biasing (mesh ratio of 3 to 1), as proposed
202 in [32]. Moreover, the mesh regularization approach proposed by Bažant and
203 Oh [47] was employed to ensure the proper computation of the crack energy
204 release rate for each damage mode. The application of this regularization
205 scheme imposes severe limits to the maximum element dimensions, and this
206 leads to element counts in excess of 1.5 million for a typical LVI-CAI speci-
207 men.

208 The described two-step modelling approach is schematically illustrated
209 in Figure 3. Of crucial importance is the application of the appropriate
210 boundary conditions in each step. These were applied by explicitly modelling
211 the contacts between the specimens and supporting fixtures. In the LVI step,
212 four clamps and a rigid support with a central window interacted with the
213 specimen by means of contact interactions with friction, allowing also to
214 absorb oscillations after rebound of the rigid impactor. For the simulation of
215 the CAI step, two new fixture parts, loading plates and side fixture plates,
216 were put in contact with the composite specimen, as shown in Figure 3.
217 While the side fixture plates and one of the loading plates were prevented
218 of movement during the CAI step, a gradual compressive displacement was
219 imposed to the remaining loading plate.

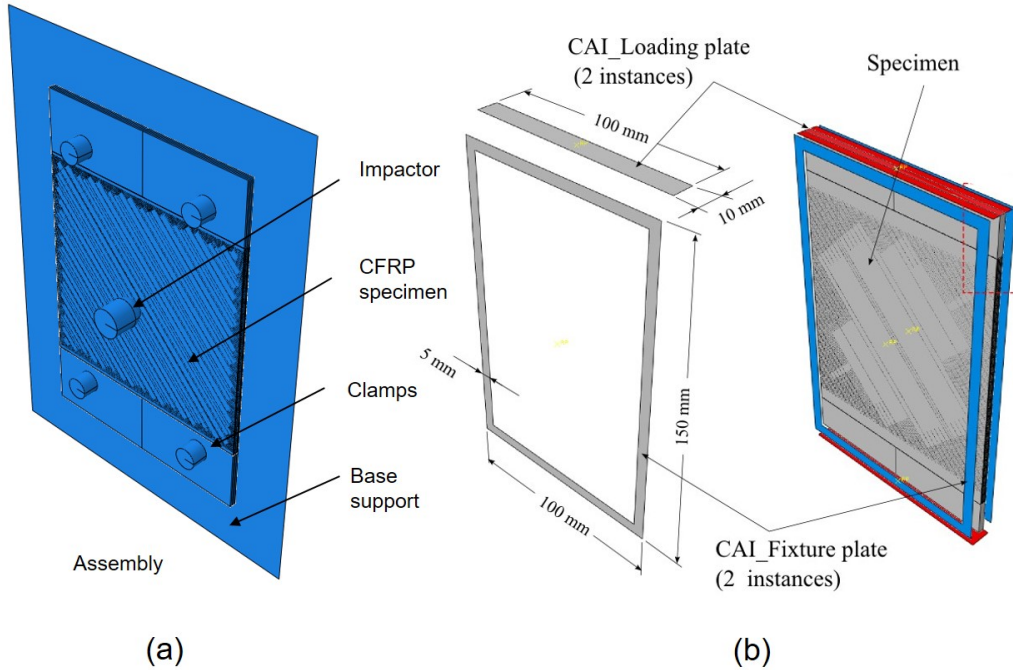


Figure 3: Details of high-fidelity simulation of BVID tolerance: (a) LVI virtual test set-up. (b) CAI virtual test set-up.

220 A velocity speed-up factor was applied to the CAI loading in order to
 221 improve the efficiency of the computational analyses while keeping quasi-
 222 static loading conditions, i.e. that the kinetic energy ($E_k = 1/2MV^2$) is
 223 relatively small (typically $< 5\%$) compared to the overall specimen internal
 224 energy.

225 3.2. Results and limitations of the approach

226 As a reference case, the two-step LVI+CAI simulation process for 181
 227 configuration impacted at the threshold of BVID (21J) is schematically de-
 228 picted in Figure 4. It can be observed that the delamination profile is fully
 229 developed at about half of the LVI step, corresponding to the peak impact
 230 load. Such behaviour has been observed in earlier works [30, 31], and is due
 231 to the quasi-static nature of the LVI event. Moreover, the LVI delamina-
 232 tion profile does not grow further until the peak compression load point in
 233 the CAI step, at which stage it rapidly propagates towards the edges of the
 234 sample causing its unstable crushing associated with sublaminar buckling.

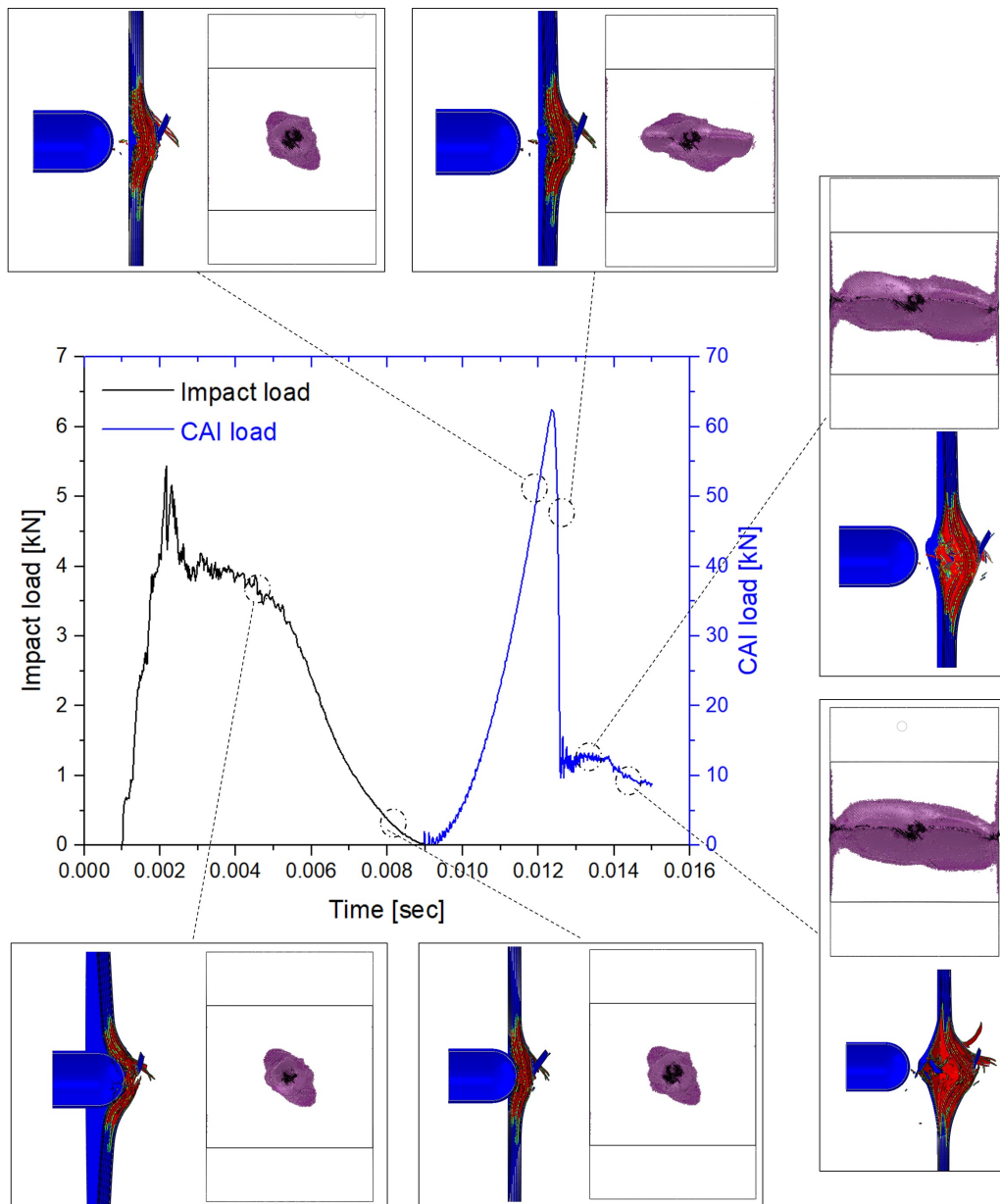


Figure 4: Schematics of the LVI+CAI virtual testing on the 181 configuration impacted at the threshold of BVID (21J).

235 Although this modelling approach provides high-fidelity results as well
236 as very good insights on the development of BVID, its propagation under
237 compression loads and eventually good predictions of damage tolerance, it
238 is very expensive computationally. The two-step simulation represented in
239 Figure 4 took in excess of 100 hours to process in 48 parallel computing
240 cores (6x Intel Xeon IvyBridge 2.5GHz CPU). The CAI step alone took
241 around 50 hours. For samples with a higher number of plies, as in all the
242 remaining configurations analyzed in this work, the computation time would
243 increase proportionally due to higher number of elements, and would result
244 unpractical.

245 **4. Proposed methodology to predict tolerance to BVID based on** 246 **damage mapping**

247 It has been demonstrated that, for the range of BVID damage events,
248 the induced intraply damage is limited and/or has negligible detrimental
249 effect on compressive strength, i.e. the CAI behaviour after BVID depends
250 almost exclusively on the delamination damage produced during LVI [48].
251 This is because the CAI response is dominated by the loss of stability due
252 to the splitting of the specimen into sublaminates rather than by the loss
253 of ply compressive strength produced by intralaminar damage which, in this
254 range of impact energies, is mostly constituted by matrix cracks which can
255 close under compression. In addition, it is assumed in this work that the
256 permanent indentation in the range of BVID (< 1 mm) has negligible effect
257 on such loss of stability as compared to the effect of the size of the produced
258 delaminations.

259 The above-mentioned observation and assumption motivate possible sub-
260 stantial simplifications in the CAI modelling. On the one hand, no intraply
261 damage model is needed, with the addition that the mesh requirements cease
262 to be bounded by the limits imposed by the mesh regularization scheme. On
263 the other hand, it may become possible to cluster several plies into a sublami-
264 nate modelled by a single through-thickness element, and account for delami-
265 nations only at certain critical interfaces. Hence, the efficient CAI modelling
266 strategy proposed in this work consists on mapping relevant BVID features,
267 i.e. delaminations, onto an efficient CAI finite element model based on con-
268 tinuum shell discretization. The approach can be seen as an alternative to
269 the expensive CAI simulations that follow equally expensive high-fidelity LVI
270 simulations, but can also be applied on experimental LVI results. Moreover,

271 when applied on LVI virtual test results, it leads not only to a simplification
272 of the CAI step but also of the required LVI process simulation since it does
273 not need to include the rebound of the impactor, i.e. the LVI simulation
274 only needs to be conducted up to maximum impact load point when the
275 delamination profile gets to be fully developed.

276 *4.1. Modelling approach*

277 In contrast to the high-fidelity simulation approach described above, the
278 efficient methodology proposed in this work to model CAI consists in the
279 use of continuum shell elements (SC8R elements in Abaqus [40]) to model
280 delaminated sublaminates. From the modelling point of view, these ele-
281 ments resemble three-dimensional elements, allowing for contact relations to
282 be specified at different surfaces, however their constitutive behaviour is that
283 of a conventional shell. This has the added advantage that the bending be-
284 haviour of plies and sublaminates is addressed with better accuracy than
285 with three-dimensional elements.

286 Similarly to the high-fidelity simulation approach, the virtual coupons
287 were split into parts employing different discretization levels, which were
288 constrained kinematically by means of tie constraints, as shown in Figure 5.
289 In the central zone, multiple sublaminates were created with surface contact
290 relations between them. Delaminations resulting from BVID were modelled
291 as circular or oval regions wherein only contact and friction conditions were
292 specified between sublaminate surfaces. These were then allowed to propa-
293 gate under CAI loads. This was achieved by using cohesive-frictional contact
294 relations outside those pre-delaminated areas, in a similar way as defined in
295 high-fidelity models. The meshing in this region was governed by the require-
296 ments of the cohesive modelling approach to achieve a reasonable description
297 of the fracture process zone [49]. Within the regions away from the central
298 sections, a coarser mesh was used in combination with a single continuum
299 shell element through-the-thickness of the entire laminate. The total num-
300 ber of SC8R elements was in the order of 500.000, varying according to the
301 laminate configuration analyses.

302 Standard CAI boundary conditions and loads were applied in simplified
303 form, i.e. by applying nodal constraints and velocities instead of explicitly
304 modelling the testing fixtures as required by the high-fidelity simulation ap-
305 proach. Both loading velocity speed-up and mass scaling (1000x) were used
306 to improve the efficiency of the computational analyses while keeping quasi-
307 static loading conditions similar to the high-fidelity CAI simulation approach

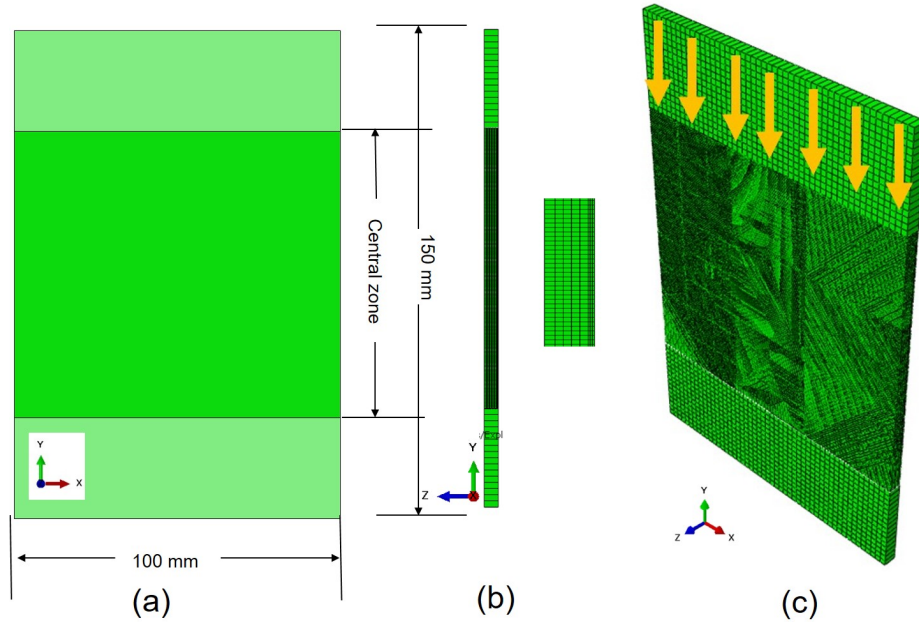


Figure 5: Specimen geometry as per ASTM D7137 and Airbus standard AITM 1-0010. (a) Front view. (b) Side view along with clustered plies. (c) CAI loading applied.

308 described above ($E_k < 5\%$ of specimen overall internal energy). As an ex-
 309 ample, the computation time necessary for the simulation of the CAI of the
 310 181 configuration, using the same parallel computing configuration used for
 311 the high-fidelity analyses (48-core), was 8 hours on average, representing a
 312 speed-up factor of about 6.5 w.r.t. the high-fidelity virtual testing.

313 One further modelling simplification was also employed and validated,
 314 consisting in coarsening the mesh in the central region of the specimens
 315 behind the limits imposed by the nominal interface properties to guarantee
 316 accurate description of delamination initiation and propagation. According
 317 to recommendations given in the literature [49], a few cohesive elements are
 318 required to model appropriately the cohesive fracture process zone, whose
 319 length (l_{cz}) can be estimated with

$$l_{cz} = ME \frac{G_c}{(\tau^0)^2} \quad (1)$$

320 wherein E is the Young modulus of the material, G_c is the interface fracture
 321 energy release rate, τ^0 is the maximum interfacial strength (see Table 2), and

322 M is a model parameter which was proposed by Rice [50] to be 0.88. In this
323 work, cohesive surfaces are used instead of cohesive elements, but a similar
324 number of degrees of freedom within the fracture process zone is adopted to
325 capture the fracture mechanism in a smooth progressive way. Considering
326 the above guidelines and mesh convergence studies, an average element size
327 of 0.4 mm was adopted.

328 According to the engineering solution for using coarse meshes in the sim-
329 ulation of delamination with cohesive zone models proposed by Turon et al.
330 [49], the propagation of delamination can still be predicted with reasonable
331 accuracy if l_{cz} is enlarged by lowering τ^0 while keeping G_c constant, as well
332 as the other parameters in Equation 1. This approach is, however, likely to
333 compromise the correct capturing of delamination initiation. Though, most
334 problems involving delamination such as CAI are governed by its propagation
335 instead by its initiation.

336 The application of the engineering solution for coarse meshes proposed by
337 Turon et al. [49] to the current case allowed mesh coarsening from element
338 lengths of 0.4 mm to 1.0 mm. The result, in terms of efficiency gain, was a
339 reduction of the computation time by another factor of 8.1 w.r.t the baseline
340 BVID mapping strategy, and an overall speed-up factor of 53 (the virtual
341 CAI test of the 181 configuration took 45-50 minutes).

342 *4.2. Simplified BVID mapping strategy*

343 The damage footprints for all configurations impacted at the threshold of
344 BVID, as determined by means of ultrasonic C-Scan inspection, are shown
345 in Figure 6. The damage footprints roughly consist of the superimposition
346 delaminations at each interface (see also Figure 2)

347 It is proposed that the performance of the virtual testing approach can
348 be further increased, while maintaining effectiveness, by mapping only de-
349 laminations at a few critical interfaces. Supported by extensive research
350 on typical delamination profile characteristics, related literature and exper-
351 imental observations, multiple configurations were explored to provide the
352 artificial maps of delamination in the samples impacted at the threshold of
353 BVID.

354 It has been demonstrated that delamination size, depth location, pre-
355 dominant orientation, number of delaminated interfaces, as well as specimen
356 dimensions, boundary conditions and stacking sequence affect CAI behaviour
357 which is eventually dominated by the mechanisms of local sublaminar buck-
358 ling and global laminate buckling [22, 25, 26, 51]. It was also found that the


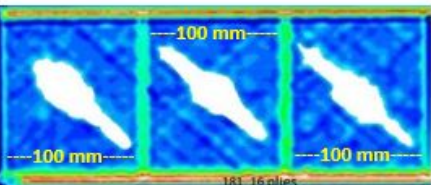

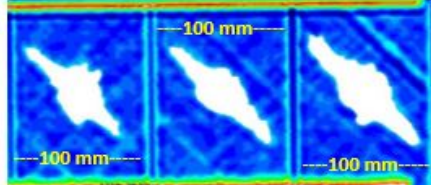

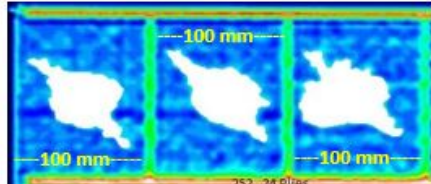

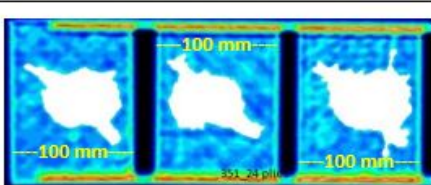

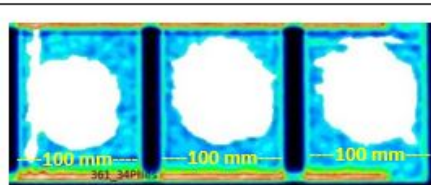
| Delamination provided in Continuum shell model | | Experimentally measured area by C-SCAN | |
|--|---|--|--|
| 181 |  |  | Delaminated area = 1610 mm ² (Approx), =10.7% of total area 181_16 Plies |
| 181T |  |  | Delaminated area = 1610 mm ² (Approx), =10.7% of total area 181T_20 Plies |
| 252 |  |  | Delaminated area = 2399 mm ² (Approx), =15.9% of total area 252_24 Plies |
| 351 |  |  | Delaminated area = 2763 mm ² (Approx), =18.4% of total area 351_24 Plies |
| 361 |  |  | Delaminated area = 5607 mm ² (Approx), =37.3% of total area 361_34 Plies |

Figure 6: BVID profiles obtained by means of ultrasonic C-scan (three repetitions for each impact configuration). Simplified delamination mapping profiles are shown for comparison.

359 ratio of delamination width to laminate width has a significant effect on these
360 buckling modes [52]. Hence, in the present research, sensitivity analyses were
361 performed for different delamination sizes and geometries including circular,

362 oval and peanut-shaped along with their location through the thickness of
363 the laminate.

364 In previous research, near-surface delaminations were found to be critical
365 for in CAI strength reduction [15, 22]. Single delaminations near to surface
366 appeared to be more severe in comparison to multiple delaminations at the
367 interior of laminate. Moreover, the strength reduction is more accentuated
368 when the previous circumstance is combined with the existence of one or
369 more of the external-most plies oriented in loading direction. In contrast,
370 other studies pointed out that only delaminations located deeper than a
371 critical depth of 10 to 20 % of the laminate thickness trigger the buckling
372 phenomenon [53].

373 Based on the present and previous studies, a set of guidelines was es-
374 tablished that is applicable to typical aerospace laminates, such as the ones
375 analyzed in this work, and possibly many other configurations irrespective
376 of the material system. The following guidelines can either be used with
377 experimentally- or numerically-obtained data.

- 378 **1. The mapped delaminations should be represented either by**
379 **circular or elliptical shapes, in the later case, aligned with the**
380 **orientation of the neighbouring ply furthest away from the**
381 **impact face.**

382 The majority should be circular with diameter equal to the one of the
383 smallest circumference that is able to encompass the BVID footprint
384 (leaving out the effects of back-face splitting). Oval shapes should
385 be used for mapped delaminations at ply interfaces close to the lam-
386 inate surfaces, and have major axes parallel to the orientation of the
387 innermost neighboring ply. The observation that the major axes of LVI-
388 resulting delaminations match with the orientation of the neighbouring
389 ply furthest away from the impact face was made by several authors,
390 as for example Obdržálek [24] Lopes et al. [5]. The dimensions of the
391 ellipses should be made relative to the diameter of BVID footprint, as
392 detailed below.

- 393 2. **Clustered plies should be modelled as unique thick plies, and**
394 **delamination between those and inner neighbouring plies should**
395 **always be mapped.**

396 Delamination should not be allowed between clustered plies, which can
397 be considered thick plies. However, under LVI loads, thick plies induce
398 high interlaminar shear stresses at their interfaces with neighboring
399 plies, specially at interfaces away from the impact face which are, then,
400 highly prone to delaminate. Not only such delaminations have impor-
401 tant size, but they also become critical under CAI loads, both because
402 of the relevant strain energy accumulated in the clustered plies and
403 because of the relatively high interlaminar stresses that continue to be
404 induced under CAI loads and promote further delamination propaga-
405 tion. Hence, BVID delamination between thick and inner neighbouring
406 plies should always be mapped.

- 407 3. **The impact face sublaminates should consist of all plies down**
408 **to the depth of permanent indentation (interfaced by a de-**
409 **lamination mapped by an oval shape).**

410 In general, delaminated interfaces at shallower depths than the value
411 of permanent indentation are either implausible or non-critical for CAI
412 strength, and may be disregarded. In support of the former assertion,
413 Wisnom et al. [38] pointed out that right underneath the impact point
414 delamination is suppressed due to the effect of out-of-plane compressive
415 stresses. For the threshold of BVID, this corresponds to 1 mm through-
416 thickness distance from the impact point. Hence, the first delamination
417 to be mapped should be located after such distance from the impact face
418 resulting in a sublaminates of thickness 1 mm or higher. An exception to
419 this rule is made if at inferior but close distance, one or more laminae
420 exist in loading direction, i.e. 0° plies. Due to the relatively high
421 accumulated strain energy, the delamination of such a sublaminates from
422 the main one may result in significant loss of stability. To account
423 for this circumstance, the delamination of the 0° ply from the main
424 sublaminates should be represented. If multiple 0° plies exist within the
425 indentation depth, then only the delamination of the innermost one
426 should be mapped, and under no circumstance a delamination in the
427 two interfaces closer to the impact face need be modelled. The pre-
428 delamination should be represented by an oval shape, with an aspect

429 ratio of 2x1, with major axis of length equal to the diameter of the
430 measured impact footprint.

- 431 **4. The backface sublaminates should consist of a single ply (in-**
432 **terfaced by a delamination mapped by an oval shape).**

433 The delamination of the backface ply is governed by the phenomenon
434 of fibre splitting resulting in an elongated shape that overgrows the
435 innermost LVI delaminations and is found to have a relevant effect
436 on CAI behaviour. Hence, the backface sublaminates should consist of
437 a single ply with delamination between the neighbouring sublaminates
438 mapped as an oval shape with major axis doubling the diameter of the
439 impact footprint, and an aspect ratio of 5x1.

- 440 **5. The second sublaminates from the backface should be mod-**
441 **elled as consisting of two plies (interfaced by a delamination**
442 **mapped by an oval shape).**

443 To account for the fading influence of ply splitting towards the center of
444 the impacted sample, the second sublaminates from the backface should
445 be modelled as consisting of only two plies. The delamination towards
446 the inside should be made oval with length of the major axis equalling
447 1.5 times the diameter of the impact footprint. Hwang and Huang [54]
448 mentioned that a short delamination under a much larger delamination
449 nearer the laminate surface, as defined by the previous guideline, has
450 no effect on the CAI behaviour. The present research corroborates such
451 observation for the case of relatively small delaminations underneath
452 the impact point but leads to the opposite conclusion for the case of
453 much larger delaminations close to the backface of the laminate.

- 454 **6. Once the above rules are satisfied, the remaining inner sub-**
455 **laminates should be divided into thinner sublaminates whose**
456 **thicknesses depends on the thickness of the original laminate.**

457 Hwang and Huang [54] observed that the local buckling load for multi-
458 ple delaminated plies or sublaminates can be predicted by using a single
459 sublaminates delaminated at a critical depth of the laminated coupon.
460 This observation was supported by sensitivity studies conducted in the

461 framework of the present research to investigate the number of plies
462 that could reasonably be modelled by a single delaminated sublami-
463 nate. Based on the gathered information it was devised that, once
464 the above rules are satisfied, the remaining inner sublaminates should
465 be divided into thinner sublaminates whose thicknesses depend on the
466 category of the original laminate:

- 467 (a) For thin laminates (laminates with less than 25 plies), the in-
468 ner sublaminates should have 8-15% of the total original laminate
469 thickness.
- 470 (b) For thicker laminates (laminates with 25 plies or more), the inner
471 sublaminates should be further categorized into
 - 472 i. Sublaminates near the centre plane with 8-15 % of the original
473 laminate thickness.
 - 474 ii. Sublaminates away from the centre plane with 15-20 % of the
475 original laminate thickness.

476 In all these cases, the delamination profile should be circular with di-
477 ameter equal to that of the impact footprint.

478 This rule was devised based on sensitivity analyses, and it does not nec-
479 essarily result in the mapping of specific important or critical delam-
480 inations. Rather, it leads to the minimum amount, size and through-
481 thickness distribution of delamination damage that needs to be repre-
482 sented in order to achieve accurate prediction of BVID tolerance.

483 The simplified BVID mapping resulting of the application of the above
484 guidelines to the laminates analysed in this work is schematically represented
485 in Figure 6 for comparison with the delamination profiles obtained experi-
486 mentally by means of ultrasonic C-Scan. The detailed sublaminates-based
487 configurations are given in Table 3.

488 *4.3. Results of CAI simulation based on BVID mapping*

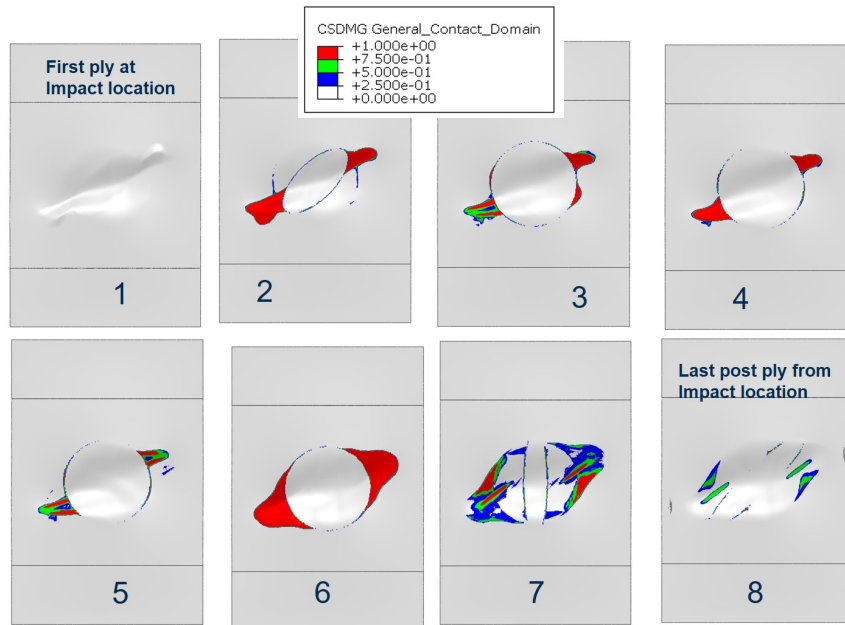
489 Examples of simulations of deformation modes and delamination prop-
490 agation resulting from using the methodology just described are shown in
491 Figures 7 and 8. As a general trend, it was numerically predicted that de-
492 lamination starts to propagate at the higher stages of compressive loading
493 and mainly along the width-wise direction of the coupons, as experimentally
494 observed by Chai et al. [55] among other authors. Such damage propagation
495 accompanied the local buckling and bulging of the sublaminates and occurred
496 within a relatively narrow loading window before the CAI specimen finally

Table 3: Sublaminates-based configurations for simulation of BVID tolerance. The locations of mapped delaminations are identified with '///'.

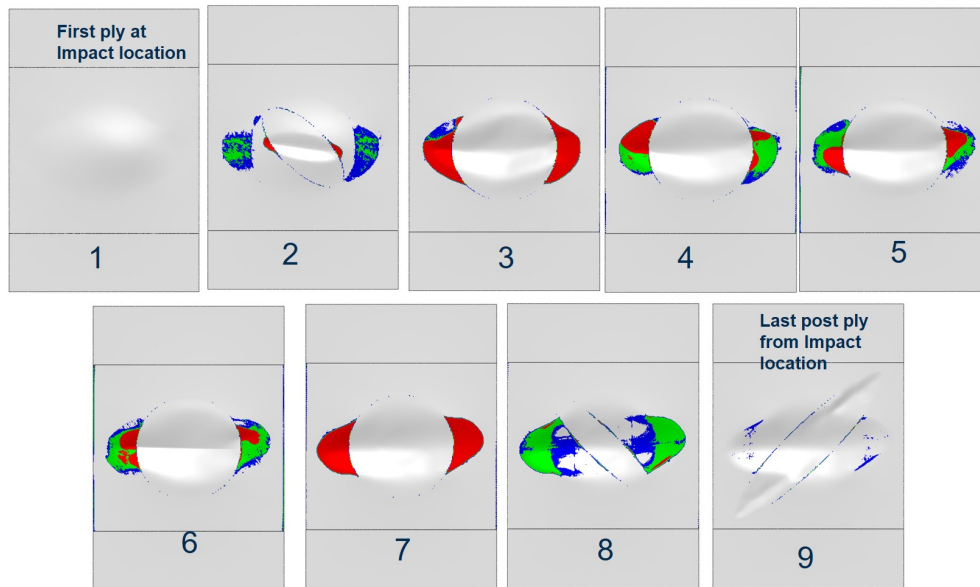
| Laminate | Sublaminates-based configuration |
|----------|---|
| 181 | $[\pm 45/0//45_2// - 45_2//90_2// - 45_2//45_2//0/ - 45//45]$ |
| 181T | $[\pm 45/0/ \pm 45//90/ \pm 45//45/ - 45_2// \pm 45/45//$ $90/\mp 45//0/ - 45//45]$ |
| 252 | $[45/0/-45/90/45/0// - 45/90//45/0// - 45/90_2// - 45/0/45//$ $90/-45/0//45,90// - 45,0//45]$ |
| 351 | $[45/0/-45/0/45/0// - 45/90//45/90// - 45/0_2// - 45/90/45//$ $90/-45/0//45/0// - 45/0//45]$ |
| 361 | $[45/(0/-45)_2//45/0/ - 45/90/45_2//90/ - 45/0_2// - 45/45_2//$ $-45/0_2// - 45/90/45_2//90/ - 45/0/ \pm 45/0// - 45/0//45]$ |

497 collapsed as the result of global or combined local-global buckling modes. By
 498 configuration, the thinner specimens of types 181 and 181T initially faced
 499 local buckling and collapsed by a global buckling mode whilst the thicker
 500 laminates 252, 351 and 361, initially developed local sublaminates buckling
 501 and failed by a mixture of both local and global buckling modes. Such buck-
 502 ling phenomena and their dependence on the thickness of the compressed
 503 specimen has been experimentally observed by other researchers [56]. At
 504 collapse, the delaminations grew rapidly towards the edges of the specimen.
 505 These predictions correlate well with the experimental observations gath-
 506 ered during the CAI tests and represented in Figure 8 for all configurations
 507 analyzed.

508 The progressive failure mechanisms explained above occur suddenly and
 509 close to specimen collapse, as corroborated by the high fidelity CAI simula-
 510 tions (see Figure 4). As a result, the load-displacement behaviors obtained
 511 experimentally and by means of simulations based on BVID mapping are
 512 fairly linear up to failure. The correlation between CAI strength results
 513 obtained by both methods (with and without applying Turons engineering
 514 solution for mesh coarsening [49]) is presented in Figure 9, which also in-
 515 dicates the obtained ranges of experimental results. The baseline virtual
 516 testing approach based on simplified delamination mapping is remarkably
 517 accurate, resulting in predictions within a 5% range of the average experi-
 518 mental values. The coarse mesh variant provides, in general, less accurate
 519 results but still within 90% accuracy.



(a)



(b)

Figure 7: Simulated sublaminate deformations and delamination propagation at peak compressive load point: (a) configuration 181 (16 plies in 8 sublaminates); (b) configuration 252 (24 plies in 9 sublaminates).

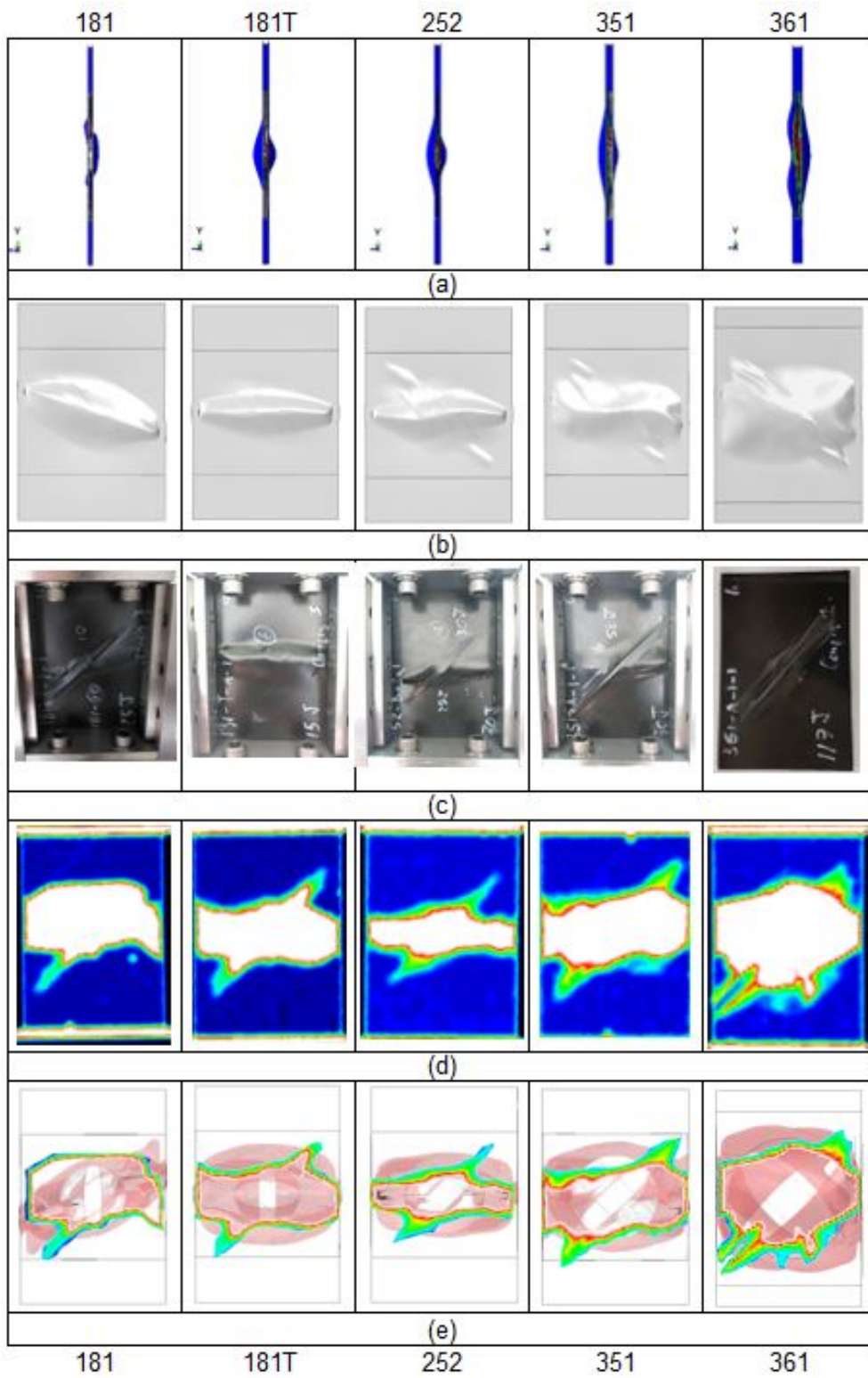


Figure 8: Correlation between experimental and numerical results obtained after collapse for different configurations subject to CAI: (a) Simulated deformation behaviour consisting of local, global or combined (mixed) buckling phenomena; (b) Simulated backface deformation; (c) Backface deformation and damage obtained experimentally; (d) Delamination profiles obtained by means of ultrasonic C-scan; (e) Simulated delamination profiles along with mapped C-scan.

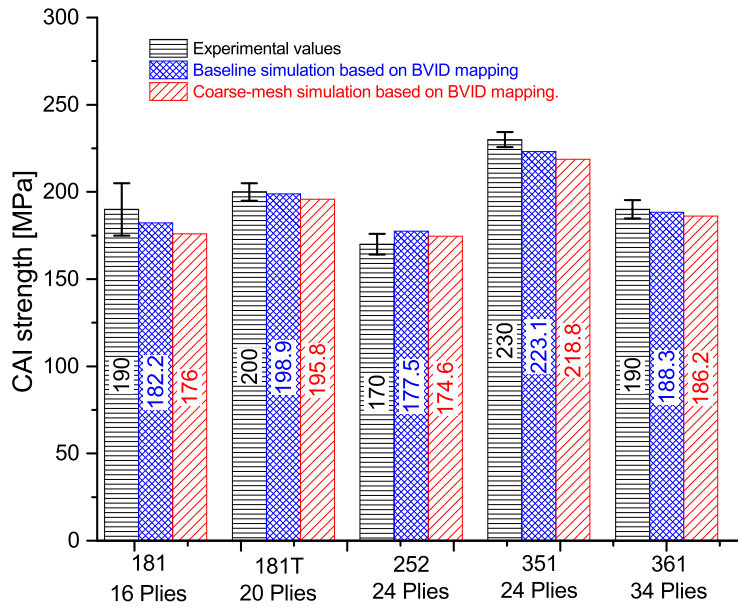


Figure 9: Comparison of CAI strength results obtained experimentally (ranges of experimental results indicated) and by means of simulation based on BVID mapping, with and without applying Turon's engineering solution for mesh coarsening [49].

520 **5. Conclusions**

521 This work proposes and validates an efficient numerical methodology to
522 accurately predict the structural tolerance of composite laminates to BVID.
523 It consists on mapping relevant BVID features, i.e. delaminations, onto an ef-
524 ficient CAI finite element model based on continuum shell element discretiza-
525 tion. It is proposed that delaminations may be represented by simplified
526 shapes, and that only the ones at critical through-thickness locations need
527 to be mapped, allowing the clustering of several plies in a single shell layer.
528 To aid the development and validation of the methodology, LVI and CAI
529 experiments were carried on different laminate configurations based on the
530 AS4/8552 carbon fibre-reinforced material system. Nonetheless, the guide-
531 lines proposed in this paper to perform delamination mapping into efficient
532 CAI modelling are potentially applicable to a wide range of composite lami-
533 nates.

534 The proposed approach can be seen as an alternative to the expensive
535 CAI simulations that follow equally expensive high-fidelity LVI simulations,
536 but can also be applied on experimental LVI results. Moreover, when applied
537 on LVI virtual test results, the methodology leads not only to a simplifica-
538 tion of the CAI step but also of the required LVI process simulation since
539 this does not need to include the rebound of the impactor. This allows an
540 acceleration of the full simulation of LVI plus CAI sequence by a factor of
541 four. Considering the CAI step alone, two variants of the approach were
542 presented: one allowing a speed-up factor above 6, in comparison with the
543 alternative high-fidelity CAI virtual testing, and an accuracy level within
544 the 5 % error range; and another allowing to speed-up analyses by over 50
545 times while obtaining values that differ as much as 10 % of the experimental
546 results. In either case, the confidence and robustness of the predictions, al-
547 though not competitive in computational cost, are substantially higher than
548 for alternative analytic or semi-analytic models in the literature. Hence, this
549 last type of analyses are very promising as a fast tool to asses BVID tolerance
550 and predict material allowables in the aerospace industry environment and
551 other alike.

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562 **Data availability**

563 The raw/processed data required to reproduce these findings cannot be
564 shared at this time as the data also forms part of an ongoing study.

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