Cross-asset holdings and the interbank lending market*

Olessia Caillé † Louis Raffestin ‡

September 27, 2019

Abstract

Recent empirical papers found that, contrary to popular belief, wholesale funding markets did not cease to function during the Subprime crisis. This paper shows that this resilience may be explained by the fact that strong interconnections between banks give them an incentive to subsidize each other. We specify a model in which financial institutions that share similar portfolios provide each other with favorable lending conditions in order to protect themselves against fire sales. The resulting subsidy drives rates down and leads to a more clustered interbank network, particularly during crises. Overall the subsidy is desirable from a systemic risk perspective.

Keywords: Wholesale funding, Short-term unsecured lending, Fire sales, Systemic risk.

JEL classification: E43, G01, G21.

*We thank Jean-Edouard Colliard, Christophe Hurlin, Christophe Pérignon, Georges Hübner, Marcel Voia, Hendrik Kruse, Sullivan Hué, José Garcia, Matthieu Picault, Daria Onori, as well as seminar/conference participants at LEO, International Rome Conference on Money 2018, ANR Multirisk Workshop, AFSE 2019 for helpful comments and suggestions. O. Caillé gratefully acknowledges the ANR MultiRisk (ANR-16-CE26-0015-01) for research funding.

†University of Orléans, CNRS, LEO, FRE 2014, F45067, Orléans, France. email: olessia.caille@univ-orleans.fr.

‡University of Orléans, CNRS, LEO, FRE 2014, F45067, Orléans, France. email: louis.raffestin@univ-orleans.fr.

1 Introduction

At the height of the financial crisis, the fed put in place the dollar swap line, a lending scheme designed to accommodate the need for dollars of the other central banks. The mechanism initially took the form of a market format auctioning which led the ECB to borrow at a maximum rate of 12% on October 8, 2008. However on October 13, the Fed changed the lending scheme to full-allotment format which allowed central banks to borrow limitless amounts at a predetermined, much lower rate.

When explaining why the Fed decided to forego interest payments, the chairman of the Fed of New York, T. Geithner, said: *"Not that we have an obligation in this sense, but we have an interest in helping these guys mitigate the problems they face"* (FOMC Transcripts, 2008: Geithner, Oct. 28-29, p. 21). In other words the decision by the Fed to subsidize the ECB was motivated by the strength of the interconnections between both central banks. Because private financial institutions also share strong links in terms of direct exposures, common asset holdings, or reputational risk; a similar mechanism may be at play in wholesale funding markets.

This paper develops a model in which common asset holdings give banks an incentive to provide each other with favourable funding conditions, in order to protect themselves from fire sales. The intuition is as follows. Consider a financial system in which banks engage in fire selling when in financial distress. Two banks L (for lender) and B (for borrower) have overlapping portfolios, and a liquidity shock leads bank B to be in need of short-term funds. Because asset sales by B would depress asset prices and consequently negatively impact the value of portfolio of L, L has an interest in limiting the risk of B by lending at an artificially low rate. This mechanism is referred to as the *common interest subsidy.* It is found to be particularly strong when liquidity risk is high because the incentive to protect borrowers is higher in that context.

Formally the model is based on including potential asset sales in the profits functions of the borrowing and the lending bank. Because a lower rate decreases the odds attached to fire sales, the profit-maximizing rate for the lender r^* falls with ω, the amount of common asset holdings between both banks. This relationship is studied analytically and computationally.

The framework easily generalizes to N banks for a given network of asset holdings, leading to interesting results at the interbank market level. First, the common interest subsidy implies that the average rate on the interbank market *should be below what borrower risk suggests, particularly during crises.* Such a mechanism is compatible with the recent literature on wholesale funding markets during the Subprime crisis who showed that – contrary to popular belief – rates rose moderately and with limited dispersion in 2008-2009 (see for instance Angelini et al., 2011, Afonso et al., 2011, or Gabrieli and Georg, 2014). [1]

Second, the model implies that a loan is more likely to take place between banks that are closely related in terms of asset holdings, since related banks provide each other with favourable conditions. Therefore *the interbank network should be clustered, particularly during crises.* This prediction is also in line with the dedicated literature (e.g., Craig et al., 2015, Bräuning and Fecht, 2016, Kobayashi and Takaguchi, 2018).

[1] The same broad conclusion applies to the market for certificates of deposits (Pérignon et al., 2018) or to the repo markets (Mancini et al., 2015, Boissel et al., 2017, Krishnamurthy et al., 2014, Copeland et al., 2014).

This paper contributes to a small theoretical literature linking asset holdings to the interbank market. Rogers and Veraart (2013) consider the incentive for a given set of banks to bail out a bank B, depending on the losses that B incurs to the system through asset prices. Acharya et al. (2012) provide a model in which a cash rich bank may adopt predatory behavior with respect to a cash poor bank when assets are bank-specific. Caballero and Simsek (2013) show that fire sales are positively correlated to the uncertainty around interbank lending exposures. Finally the closest paper to this study comes from Leitner (2005), who specifies a model in which the threat of contagion gives safe banks an incentive to bail out risky ones.

The key difference between this literature and our study is that papers such as Leitner (2005) take a normative approach, focusing on the impact of the subsidy on the choices for banks at the micro level. In contrast, the main contribution of our model is to have clear-cut implications at the interbank market level – namely a lower average rate and a clustering of the lending network – that are consistent with the seemingly puzzling features observed during the 2008 crisis.

The dual impact of the common interest subsidy raises the question of its total impact on financial stability. Indeed while privileged relationship help limit borrower risk through lower rates (Temizsoy et al., 2015), an undiversified lending network usually lowers the ability of the system to absorb shocks (Gai and Kapadia, 2010). The second contribution of this paper is to estimate both effects in order to draw a conclusion on the desirability of the common interest subsidy from a systemic risk perspective. We do so by running Monte-Carlo simulations of the N-bank model in which bankruptcies spread across bank balance sheets through sales until the situation stabilizes, as in Greenwood et al. (2015). We then draw the probability that all banks go bankrupt, which indicates systemic risk; as well as contagion statistics such as the likelihood that one bank i survives if another bank j failed.

We find that the subsidy does foster contagion, particularly between banks with a high level of asset commonality. Nevertheless this effect is dominated by the stabilizing impact of lower rates on borrower risk, and overall *the common interest subsidy enhances systemic stability.* In this way the paper suggests that popular models of contagion (Eisenberg and Noe, 2001, Cifuentes et al., 2005) may slightly overestimate the level systemic risk by failing to account for the common interest of all participants to see it survive.

The rest of the paper is organized as follows. Section 2 presents a 2-banks framework, and describes the solving procedure to obtain the equilibrium interest rate for a given bank couple. Section 3 presents the equilibrium interest rate, and studies how it changes with the level of commonality between the 2 banks and the size of the loan. Section 4 generalizes the model to many banks and studies the features of the interbank market through simulations. Section 5 discusses the impact of the common interests subsidy on systemic stability. Section 6 concludes.

2 The model

2.1 Framework

2.1.1 Balance sheets in $t = 0$

Consider an economy in which credit is intermediated by two banks: B (for borrower) and L (for lender). There are three periods: $t = 0, 1, 2$. On the liability side, both banks finance themselves through capital K and deposits D. On the asset side, three assets coexist:

- asset b, which is held exclusively by bank B at a quantity of $q_{b,0}$

- asset l, which is held exclusively by bank L at a quantity of $q_{l,0}$

- asset c, which is held by both banks, at quantities of $q^B_{c,0}$ and $q^L_{c,0}$ for banks B and L respectively.

Denoting by $p_{i,t}$ the price of an asset i in time t, the balance sheet in $t = 0$ of a given bank I is given by:

Assets	Liabilities
Asset held by I only $\quad p_{i,0}q_{i,0}$	Deposits D^I_0
Asset held by both banks $\quad p_{c,0}q^I_{c,0}$	Capital K^I_0

This balance sheet will evolve following random shocks to two variables: deposits D and asset prices p. To capture the fact that the banks perform maturity transformation, shocks to deposits occur in $t = 1-\epsilon$ while shocks to asset prices occur in $t = 2-\epsilon$, meaning that a maturity mismatch is embedded in the balance sheet.

2.1.2 Liquidity shock and the lending market in $t = 1$

The $t = 1 - \epsilon$ shock to deposits (or liquidity shock) for a given bank I is defined as:

$$\varepsilon^D_I = D^I_{1-\epsilon} - D^I_0$$

where ε^D_I follows any symmetric distribution with moments denoted by $\{\mu_d,\, \sigma^2_d\}$.

To ensure that a market for liquidity can always exist, we set that ε^D_I is symmetric: an inflow of liquidity ε^D in bank L is matched by an outflow ε^D in the bank B. Mathematically this means that $\varepsilon^D_B = -\varepsilon^D$ and $\varepsilon^D_L = \varepsilon^D$.

Bank B, who received the negative liquidity shock, must find a sum of ε^D between $t = 1 - \epsilon$ and $t = 1$ to honor its commitment to its depositors. This can be done in two ways. First, bank B may borrow ε^D from bank L at a rate r. Second, it may quickly sell a quantity $\triangle q_{i,1}$ of assets on the market to price elastic investors who clear the market, for a total amount of ε^D. The timing is as follows: in $t = 1 - \epsilon$ bank B demands a loan. Bank L then responds by offering a given interest rate r. Bank B either accepts the loan in $t = 1$, or refuses and immediately sells assets.

Outside investors − who take the other side of bank sales − are set to be long-termistic agents who only care about the long term fundamental value of assets FV, and have a standard mean-variance utility function with a risk aversion of $\frac{1}{\tau}$. This modeling yields the well-known expression for desired holdings of a given asset i at a given time t :

$$q_{i,t}^d = \tau \frac{FV_i - p_{i,t}}{\sigma_{FV}^2}.$$

First-differencing this expression gives the change in demand from outside investors between $t-1$ and t, which must be equal to the the quantity of asset i sold by banks over the same period $\triangle q_{i,t}$. Market equilibrium then yields :

$$\triangle p_{i,t} = -\lambda \triangle q_{i,t} \tag{1}$$

where $\lambda = \frac{\sigma_{FV}^2}{\tau}$. Equation (1) states that selling assets leads to a drop in asset prices.

The balance sheets for both banks in $t = 1$ in the loan and no-loan case can be found in appendix A.

2.1.3 Asset shocks and bankruptcies in $t = 2$

For any asset i, the $t = 2 - \epsilon$ price shock is defined as:

$$\varepsilon_i^p = p_{i,2-\epsilon} - p_{i,1}$$

where $\varepsilon_{i,t}^p$ follows any symmetric distribution with moments $\{\mu_p, \sigma_p^2\}$.

A bank immediately defaults in $t = 2 - \epsilon$ if it loses more than its capital, i.e. if $\Pi_I = K_{2-\epsilon}^I - K_0^I < -K_0^I$. Both banks may be declared bankrupt at this point, but bank L may also be declared bankrupt later, in $t = 2$.[2] Indeed a failure of bank B involves two types of costs for bank L. First, any existing loan is defaulted on, leading a net loss of ε_i^p for bank L. Second, when a bank fails, a share δ of its assets is immediately sold between $t = 2 - \epsilon$ and $t = 2$. Formally this means that a bankrupt bank B sells a total quantity $\delta q_{c,0}^B$ of the common asset, leading to a price decline of $\lambda \delta q_{c,0}^B$, that lowers the value of the portfolio of bank L by an amount $q_{c,0}^L \lambda \delta q_{c,0}^B$.

The last modeling feature is to specify that bankruptcy is hurtful for banks by assigning a lump sum cost of $-F$ to it. Denoting by d_I, the event of a default in bank I in $t = 2$, and by $\bar{d}_I$ its survival, the utility function of a bank I is expressed as follows:

$$\begin{cases} U_I|\bar{d}_I = \Pi_I & if \ \Pi_I > -K_0^I \\ U_I|d_I = -F & if \ \Pi_I < -K_0^I \end{cases} \tag{2}$$

where the likelihood of a bankruptcy is:

$$P(\Pi_I < -K_0^I) \tag{3}$$

Figure 1 summarizes the possible utilities for both banks.

[2] Note that we only consider knock-on effects from the liquidity poor bank B to the liquidity rich bank L. Including the possibility of contagion from bank L to bank B would change the expression for the expected utility of bank B in the case of a loan. This would make the model much more complex, with little benefit because the quantitative impact of contagion on the expected utility of bank B is very small, as the probability that contagion occurs is near zero.

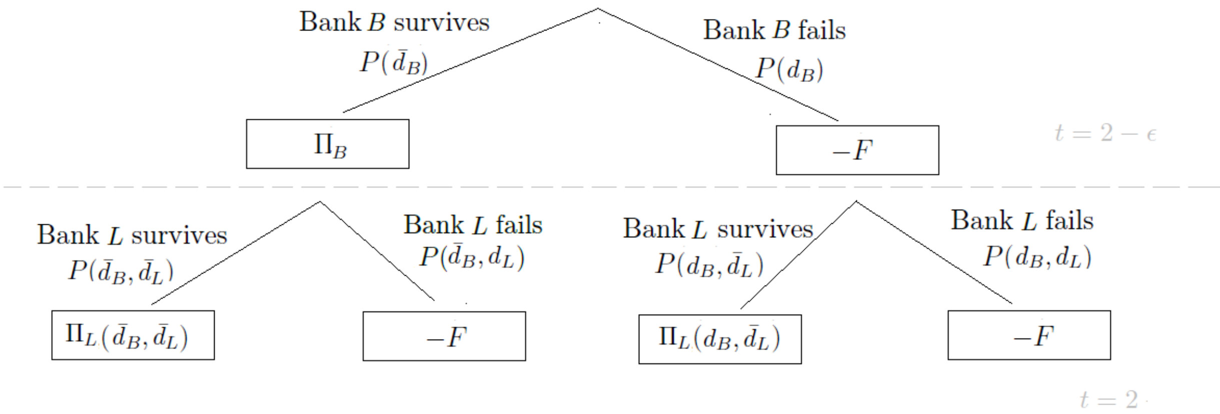


Figure 1: Profit tree diagram
This figure summarizes the profit for each bank in each possible case and the probability associated with each case, in $t = 2 - \epsilon$ and $t = 2$.

Having laid down the framework, we may turn to the solving procedure. To end this section, figure 2 provides a summary of the sequence of events in the model. The figure shows that two rounds of asset sales may occur: the first $\triangle q_{i,1}$ at the end of the first period to obtain liquidity in the absence of a loan, the second $\triangle q_{i,2}$ at the end of the second period in the event of a bankruptcy.

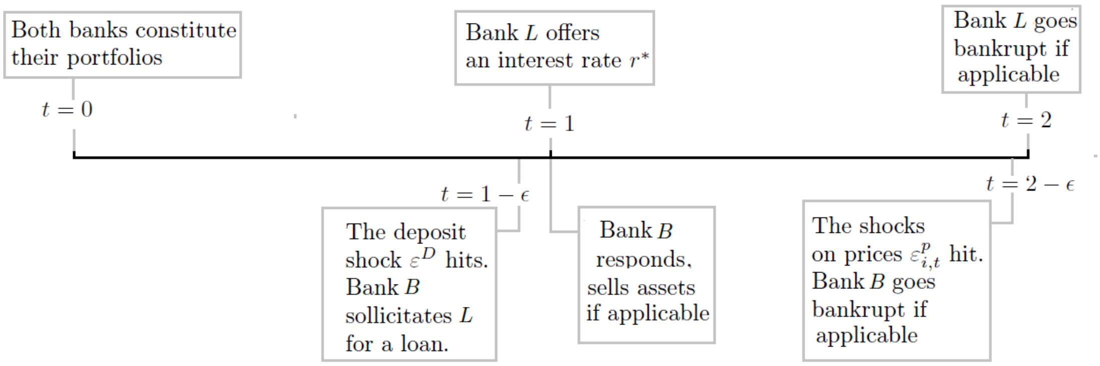


Figure 2: Model timeline
This figure presents the chronology of occurrence for the events considered in this paper.

2.2 Bank profits and solving procedure

2.2.1 Bank profits

This section presents the profit functions for both banks, which will be used to obtain the equilibrium on the lending market. Two cases must be studied in parallel: one in which a loan occurred in $t = 1$, and on in which no loan occurred. Detailed derivations using balance sheets can be found in appendix B.

The loan case

When a loan has been granted in $t = 1$, the interests must be repaid in $t = 2$. Profit for the borrower between $t = 0$ and $t = 2$ is then given by:

$$\Pi_B = -r\varepsilon^D + q_{b,0}\varepsilon_b^p + q_{c,0}^B\varepsilon_c^p \tag{4}$$

where $r\varepsilon^D$ represents interest payments, while $q_{b,0}^B\varepsilon_b^p$ and $q_{c,0}^B\varepsilon_c^p$ represents the profits/losses from the realizations of asset shocks.

For bank L, the profit function depends on whether bank B survived in $t = 2 - \epsilon$. In case of survival, B simply pays back the loan which yields:

$$\Pi_L|\bar{d_B} = r\varepsilon^D + q_{l,0}\varepsilon_l^p + q_{c,0}^L\varepsilon_c^p. \tag{5}$$

However if bank B failed, it defaults on its loan and sells assets between $t = 2 - \epsilon$ and $t = 2$, leading to the following profit for L:

$$\Pi_L|d_B = -\varepsilon^D + q_{l,0}\varepsilon_l^p + q_{c,0}^L\varepsilon_c^p - q_{c,0}^L\lambda\delta q_{c,0}^B \tag{6}$$

where $-\varepsilon^D$ represents the net loss from loan default, while $q_{c,0}^L\lambda\delta q_{c,0}^B$ captures the impact of fire sales from bank B on the portfolio of bank L.

The no-loan case

Without a loan, profit for bank B is impacted by asset sales in the first period $\triangle q_1$, as well as the realizations of asset shocks in $t = 2$ on the remaining assets. This yields:

$$\Pi_B^{nl} = -q_{b,0}\lambda\triangle q_{b,1} - q_c\lambda\triangle q_{c,1}^B + (q_{b,0} - \triangle q_{b,1})\varepsilon_b^p + (q_{c,0}^B - \triangle q_{c,1}^B)\varepsilon_c^p \tag{7}$$

where the first two terms refer to the losses from early asset sales, and the last two represent the profits stemming from the performance of the assets that were not sold in $t = 1$.

For bank L if B survives, the sales of the common asset by bank B in $t = 1$ negatively impacts profits. Formally:

$$\Pi_L^{nl}|\bar{d_B} = -q_{c,0}^L\lambda\triangle q_{c,1}^B + q_{l,0}\varepsilon_l^p + q_{c,0}^L\varepsilon_c^p. \tag{8}$$

Finally when bank B fails, it sells some of its remaining assets until it reaches the proportion δ associated with failed banks. The expression for the profit of L is thus :

$$\Pi_L^{nl}|d_B = -q_{c,0}^L\lambda\delta q_{c,0}^B + q_{l,0}\varepsilon_l^p + q_{c,0}^L\varepsilon_c^p. \tag{9}$$

2.2.2 Solving procedure

Let us now introduce the method for finding the lending market equilibrium between B and L. This section provides a general outline suited to the use on numerical methods, on which the rest of the paper is based. Nevertheless a tractable version of the model exists when the variance of the common asset c is set to zero. This case is solved in appendix C and provides a step by step derivation of the equilibrium rate which the reader may find helpful.

Step 1: finding the optimal rate for bank L

The first step towards obtaining the equilibrium is to find the optimal rate for the lending bank r^*, which is done by maximizing the $t = 2$ utility of bank L as expected in $t = 1$. Formally this expected utility is given by:

$$E(U_L) = P(d_B, d_L) * (-F) + P(\bar{d}_B, d_L) * (-F) + P(d_B, \bar{d}_L) * \Pi_L | d_B + P(\bar{d}_B, \bar{d}_L) * \Pi_L | \bar{d}_B. \qquad (10)$$

All the probabilities of failure/survival that feature in (10) can be drawn from the profit functions in the loan case, given in the previous section. For instance, the probability that both banks survive $P(\bar{d}_B, \bar{d}_L)$ can be decomposed as:

$$P(\bar{d}_B, \bar{d}_L) = P(\bar{d}_B) * P(\bar{d}_L | \bar{d}_B) = P(\Pi_B > -K_0^B) * P(\Pi_L > -K_0^L | \Pi_B > -K_0^B)$$

Therefore plugging expressions (4) to (6) into (10) gives $E(U_L)$ its final expression. The expected profit maximizing rate r^* then appears as the interest rate r that satisfies the equality $\partial E(U_L)/\partial r = 0$, which is uniquely defined.

Step 2: finding the reservation rate for bank B

The second step to obtain the equilibrium rate is to get the maximum rate $\bar{r}$ that the borrower is willing to accept. Asset sales are costly for the seller since they lower the value of the assets that remain in his portfolio. In $t = 1$, the borrower must compare this cost to that of paying an amount $r\varepsilon_D$ in interests to the lender. The maximum interest rate that bank B is willing to pay will then be defined by the value of r for which expected utilities in the loan and no-loan cases are equivalent. Formally:

$$E(U_B) = E(U_B^{nl}).$$

Both $E(U_B)$ and $E(U_B^{nl})$ are obtained in the same way as $E(U_L)$, using the profit functions of section 2.2.1. The reservation rate $\bar{r}$ then appears, and is also unique.

Step 3: drawing the equilibrium

Once r^* and $\bar{r}$ are obtained, two possibilities emerge. First, the rate that bank L wishes to implement is lower than the threshold rate, i.e. $r^* < \bar{r}$. In this case bank B accepts $r = r^*$ since the borrower benefits unambiguously from a lower rate. In the second case, the rate that the lender would to like to charge is higher than the maximum rate that the borrower is willing to accept, i.e. $r^* > \bar{r}$. Here bank L is constrained, and the best possible rate it can get is $r = \bar{r}$.

The utility maximizing rate for bank L, under the constraint that bank B participates, is thus given by:

$$r = min(r^*, \bar{r})$$

which represents the equilibrium interest rate on the interbank market.[3]

[3]Note that r is the interest rate should a loan take place, yet it is not clear that a loan will take place at this point. Indeed in theory bank L has no obligation to lend, and may prefer to let bank B sell assets if the interest rate r is too low. However this case never occurs in the parameter sets used in this paper, and thus we do not consider it any further.

The algorithm follows the three-steps procedure just described to obtain the market equilibrium.[4]

3 The equilibrium interest rate between two banks

This section calibrates the model and studies the equilibrium interest rate between bank B and bank L.

3.1 Calibration and implementation

We first set that both banks have symmetric asset holdings, i.e. $q_{l,0} = q_{b,0} = q_{i,0}$ and $q^L_{c,0} = q^B_{c,0} = q^I_{c,0}$; and normalize all asset prices in $t = 0$ to 1.

This allows us to define:

$$q^I_{c,0} = \omega(D^I_0 + K^I_0) \tag{11}$$

and

$$q_{i,0} = (1-\omega)(D^I_0 + K^I_0) \tag{12}$$

where ω is the share of the common asset over total assets for both banks. This parameter represents the amount of commonality between them, which will be allowed to vary.

The quantity of deposits in $t = 0$ is set to 9 (i.e. $D^B_0 = D^L_0 = 9$) while the amount of capital is set to 1 (i.e. $K^B_0 = K^L_0 = 1$). This yields a capital ratio of 10%, which appears consistent with casual observation.[5] Assets shocks are normally distributed with moments $\mu = 0.1$ and $\sigma = 0.2$, which implies a Sharpe ratio of 0.5 that is in line with the S&P500 since its creation.[6] The impact of a sale on prices λ is set to 0.1, following a study by Coval and Stafford (2007) who found an average abnormal returns of -10.1% following a period of fire sales on a specific asset.

Two parameters are more difficult to calibrate because they are unobservable and/or constantly changing: the utility cost of failure F, and the share δ of the bank's assets that are immediately sold following failure. For both parameters we follow the same method: we choose a given value, and consider alternatives in appendix D.[7]

λ is set to 0.1 which means that sales will have a small but significant effect on prices. F is set to 5, which represents half of the total exposure at period $t = 0$. Finally we specify $\delta = 0.3$, meaning that a bank that goes bankrupt immediately sells 30 percent of its remaining assets.

[4] A slight complexity arises from the fact that both banks are not independent since they share the common asset. A failure/survival of bank B has implications for the distribution of the common asset c, and consequently on the distribution of the profits of bank L. To account for this, we use Bayesian inference to compute the probability distribution of ε^p_c, knowing that bank B failed or survived in $t = 2 - \epsilon$. We then use this new distribution of ε^p_c to get the probability that bank L fails in $t = 2$.

[5] According to the IMF, the world's banks capital to asset ratio ranges between 9.78% and 10.75% from 2010 to 2017 and the tier 1 leverage ratio of U.S. banks was 9.76% in the first quarter of 2019.

[6] Assuming reinvested dividends. Data taken from Robert Shiller's page http://www.econ.yale.edu/~shiller/data.htm.

[7] We also consider alternative values for λ.

3.2 The equilibrium interest rate as a function of cross-asset holdings

This subsection presents the equilibrium interest rate r as a function of ω, the level of commonality in the banks' portfolios. The following applies to a specific liquidity shock $\varepsilon^D = 1$, but other values for ε^D are considered in appendix D.

Figure 3 plots the maximum rate that the lender is willing to accept $\bar{r}$ (solid line), the rate that the lender would like to charge r^* (dashed line), and the equilibrium interest rate $r = min(\bar{r}, r^*)$ (line with circle markers); for levels of commonality ω between 0.1 and 0.9.

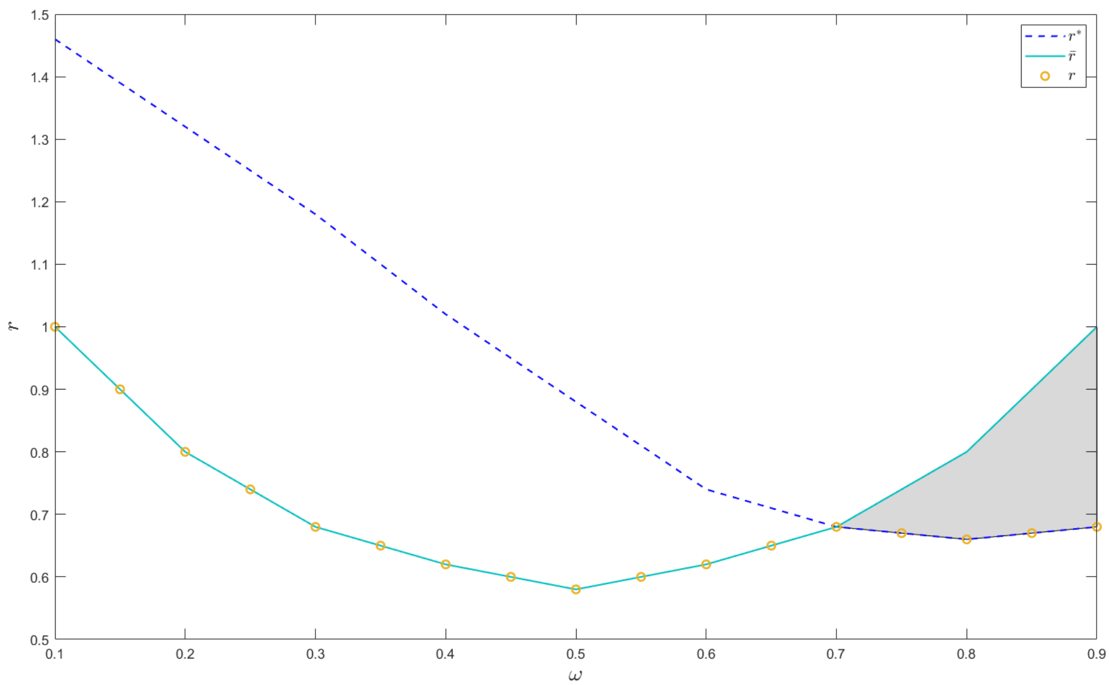


Figure 3: Equilibrium interest rate with $\varepsilon^D = 1$
This figure represents the interest rate that bank L would like to charge r^* (dashed line), the maximum rate that bank B is willing to accept $\bar{r}$ (solid line) and the equilibrium interest rate r (with circle markers) for different shares of common asset ω. The equilibrium interest rate r is defined as $min(r^*, \bar{r})$.

The first observation is that r^*, the profit-maximizing rate for the lender, is globally decreasing with the level of commonality ω. This captures the key mechanism: when the portfolios of the two banks become similar, bank L becomes more exposed to a default of bank B, through the possibility of fire sales in the common asset c. This gives bank L an incentive to charge low interest rates to keep the default risk of bank B at a low level, and thus indirectly protect itself.

Nevertheless, it is interesting to note that above a given threshold of similarity between both portfolios, the incentive to subsidize the borrower starts to decrease. The reason is that past a certain level of similarity in the asset structure, the fates of the banks become so closely tied together that bank L is bounded to fail if bank B fails, regardless of the loan. Bank L then becomes better off betting on survival, and r^* increases slightly.

Turning to $\bar{r}$, we note that the maximum rate that bank B may accept is U-shaped. This reflects the evolution of the probability of default for bank B, which finds its minimum when the bank holds both assets in equal measure. The intuition is as follows: when default is less likely, the borrowing bank is more willing to take the risk of selling assets, which translates into a lower threshold interest rate.

Moving on to the equilibrium rate $r = min(r^*, \bar{r})$, we observe that $r = \bar{r}$ when ω is low. However as r^* falls with ω relative to $\bar{r}$, the situation reverses and we have $r = r^*$. This latter case implies that the lender offers a rate that is lower than the one it could obtain $\bar{r}$, in order to keep the risk of its borrower at bay. The distance between the offered rate and the rate that bank B was prepared to pay, $\bar{r} - r^*$, can be seen as an indicator of the amount of interest revenue that the lender is willing to forego. It is represented by the shaded zone in the figure, and rises monotonically with the level of similarity between both banks, *implying that the incentive to subsidize increases with the level of commonality between banks.*

Appendix D checks the robustness of the relationship between r and ω. All tests appear to have the expected impact.

3.3 Interest rate and liquidity risk

Having established that a common interest subsidy may exist between two given banks, we seek to study how its magnitude may change with market conditions. In particular, an interesting question in our context lies with whether the incentive to subsidize borrowers grows stronger when liquidity risk rises, since the presence of a safety net for borrowers may be particularly desirable in that context.

To investigate, figure 4 plots the evolution of equilibrium rate as a function of the size of the liquidity shock (between $\varepsilon^D = 0.1$ and $\varepsilon^D = 3$), for two banks with a fixed level of asset commonality $\omega = 0.5$. To highlight the impact of the common asset subsidy, we compare the results of our framework (full line) to a framework in which bank L does not account for the possibility of fire sales by bank B in its maximization problem (dashed line).

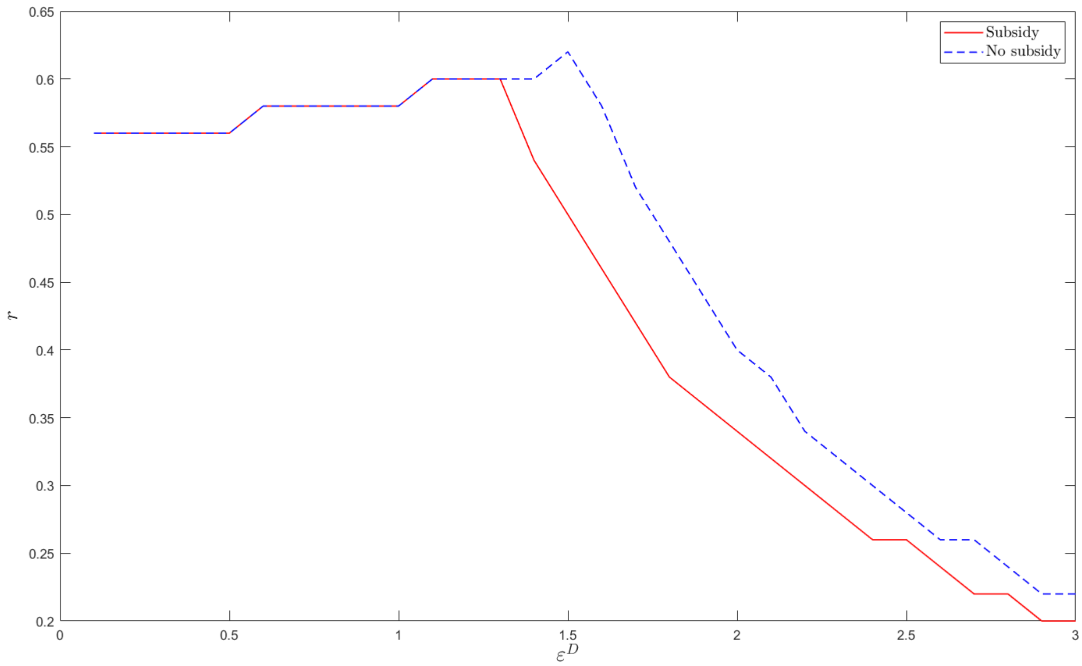


Figure 4: Equilibrium interest rate across values of ε^D

This figure plots the equilibrium interest rate r as a function of the size of the liquidity shock ε^D, when $\omega^D = 0.5$. The full line gives r when banks include potential fire sales in their utility function (the subsidy case), the dashed line represents the case in which banks do not account for asset commonalities (no subsidy).

The key information conveyed by figure 4 is that the common interest subsidy has a negative impact on rates for realizations of ε^D above $\varepsilon^D = 1.3$. The subsidy kicks in when the liquidity needs of borrowers are large because borrowers carry more risk and the impact of a default is larger in this context. This gives lenders a stronger incentive to forego interest payments in exchange for lower borrower risk. Nevertheless it is interesting to see that the impact of the subsidy decreases between $\varepsilon^D = 1.3$ and $\varepsilon^D = 3$. This reflects the "betting on survival" effect highlighted in the previous section.

Despite this small counter-balancing effect, figure 4 clearly implies *that the size of the common interest subsidy increases as the variance of liquidity shocks rises.* This notably means that, in times of crisis, the common interest subsidy should exert a calming influence on rates.

This result may help to shed light on the seemingly puzzling behavior of wholesale markets during the Subprime crisis, which have been found to have been remarkably resilient by recent studies (Angelini et al., 2011, Afonso et al., 2011, Pérignon et al., 2018). If the common interest subsidy draws a wedge between the rate a given borrower secures and the rate suggested by his risk profile, it is not surprising that rates reacted mildly to rising counterparty risk in 2008.

It is worth noting that empirical studies on relationship lending seem supportive of the idea that common interests have a calming influence the interbank market during crises. Indeed several studies (e.g. Bräuning and Fecht, 2016; Affinito, 2012; Cocco et al., 2009) have showed that "relationship lending has a negative effect on the bilateral interest rate when market conditionssuffer from credit risk uncertainty", in the words of Bräuning and Fecht (2016). This cross sectional pattern can be explained, among other factors, by the common interest subsidy.

4 Common interests subsidy and interbank lending

This section extends the framework of section III to a system of N-banks, and studies the interbank lending produced by this structure.

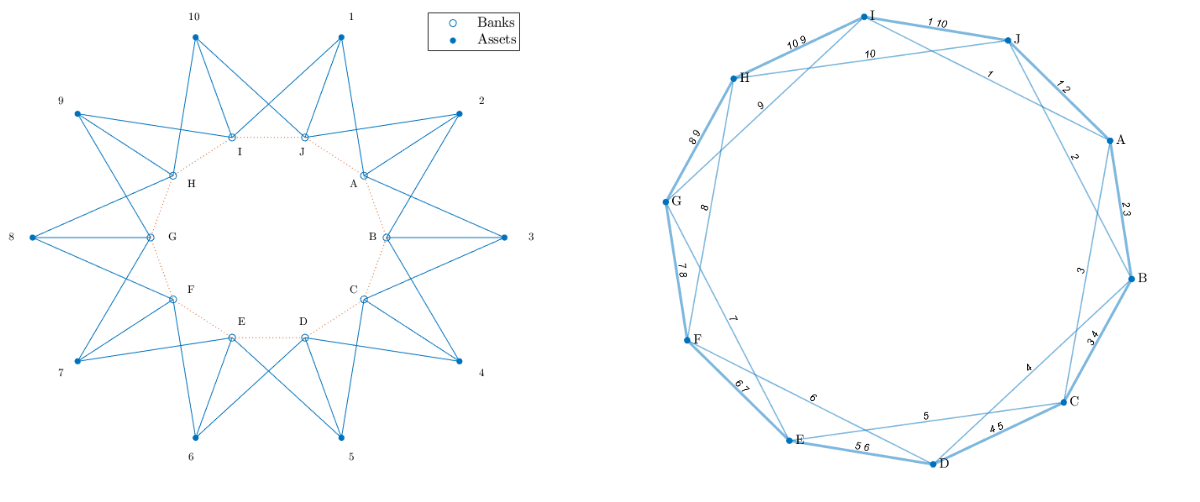
4.1 Generalizing the 2 banks model

4.1.1 modeling

The baseline structure of the N-banks model is designed to allow any equilibrium interest rate between two given banks in the system to be defined by the model calibrated in section 3. This translates into two conditions. First, the asset holding network is such that, for any bank couple with assets in common, the quantity of the common assets held by both banks is the same. Second, the vector of bank liquidity shocks must ensure that negative shock experienced by borrowers is equal to the positive shock received by lenders.

To respect the latter condition, the liquidity shock ε^D is set to be unique and symmetric, meaning that it will be positive for half of the banks and negative for the other half. This structure is designed to yield an open market for liquidity in $t = 1$ in which all banks appear. An alternative specification will be considered in section 5.

To respect the former condition, we specify the following network. In an economy where banks collectively own $N = 10$ assets, bank A owns assets 1, 2, 3; while bank B owns assets 2, 3, 4, etc. This structure is called circulant and is fairly classic in the network literature (see for instance Raffestin, 2014). Figure 5-a provides a graphical expression of the asset holdings, and figure 5-b shows how these holdings translate into links between banks.



(a) Asset holdings **(b) Links between banks**

Figure 5: Circulant network
This figure represents the assets held by every bank. 5-a connects each bank to the assets it owns, where banks are indicated with letters and assets are numbered. For instance bank A owns assets 1,2, and 3. Figure 5-b connects banks according to the assets they have in common, where a thick line indicates 2 assets in common, and a thin line means 1 asset in common.

This asset holding network implies that each bank has two "close neighbors" with whom it shares 2 assets, and 2 "distant neighbors" with whom it shares only one asset. Close and distant relationships are represented in figure 5-b by the thick and thin lines, respectively. Finally each bank co-exists with 5 "outsiders" with whom it shares no assets.

As with the liquidity shock, an alternative structure for asset holdings will be considered in section 5. However the structure of 10 banks each holding 3 assets is kept throughout the paper. The reason is that both the number of banks and the number of assets have no qualitative impact on the result. The only real requirement is to have different levels of asset commonalities for different bank couples.

4.1.2 Matching procedure and Monte-Carlo simulations

The framework presented defines a single equilibrium interest rate for a given bank couple and liquidity shock ε^D. However to characterize the interbank network one must specify how banks are matched.

We set the matching procedure to work sequentially. One randomly selected cash poor bank is served first, and its lender is removed from the pool of eligible lenders; and so on. This method is the simplest way of modeling the network creation process, and has already been used in the network literature (see for instance Anand et al., 2012). We store, for each lending relationship, the rate offered and the banks involved.

For some realizations of ε^D the close neighbor, the distant neighbor, and the outsider all offer the same rate $r = \bar{r}$. In this case the lender is chosen randomly by the borrower. For other realizations of ε^D, the rates offered to bank B will change with the distance in the network. In this case, the borrower chooses the lowest rate, as he benefits unambiguously from cheaper loans.

Contrary to the previous section who studied a single realization of the liquidity shock ε^D, this section presents the results from a Monte-Carlo simulation of $100,000$ realizations of ε^D, which gives $100,000$ interbank lending networks.

The liquidity shock ε^D is drawn from a Gaussian. Three different values for the variance of ε^D are considered, in order to study the interbank market for varying levels of liquidity risk: $\sigma_d^2 = 0.5$, $\sigma_d^2 = 1$ and $\sigma_d^2 = 1.5$.

4.2 Results

4.2.1 Equilibrium interest rates at the system level

We start by considering interest rates at the system level. As a first step, figure 6 plots the bilateral equilibrium interest rates as a function of the liquidity shock, depending on the nature of the of the two banks considered (close, distant, outsider). As expected, the figure resembles figure 4: banks who share connections through asset holdings use lower rates on average.

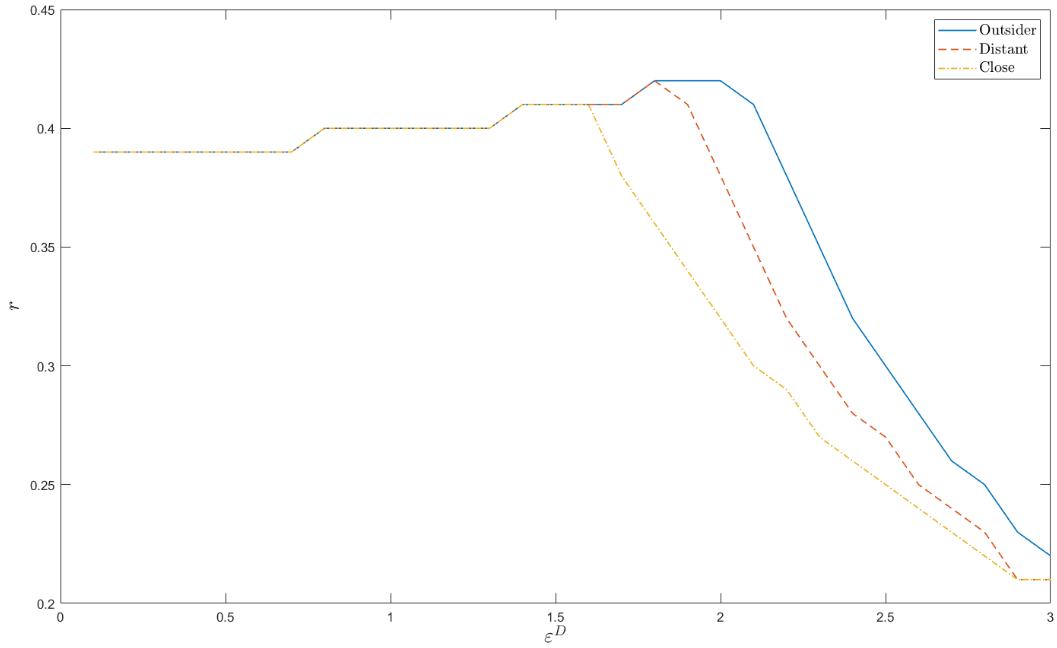


Figure 6: Equilibrium interest rate with $\omega = \{0; 0.33; 0.66\}$
This figure plots the bilateral equilibrium interest rates as a function of the liquidity shock, depending on the nature of the of the two banks considered (close, distant, outsider). The full line gives r when the two banks share no assets, the dashed line represents the case in which banks share 1 asset and the dotted-dashed line the case in which banks share 2 assets.

We now describe how these bilateral rates scatter in the interbank market. Figure 7 plots the distribution of lending rates from the Monte-Carlo simulation, grouping all loans, for the three levels of liquidity risk. The y-axis uses a logarithmic scale.

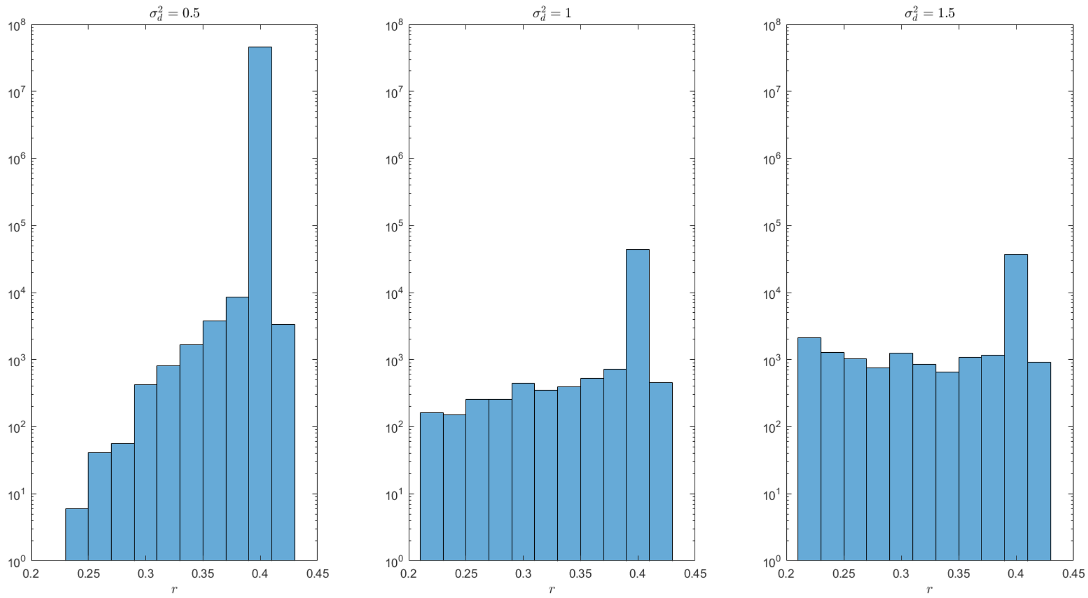


Figure 7: Distribution of lending rates when liquidity risk rises
This figure represents the the distribution of lending rates from the Monte-Carlo simulation, for different levels of liquidity risk σ_d^2. The y-axis uses a logarithmic scale.

Figure 7 shows that lower liquidity risk is associated with a lower variance and a higher expected value for rates. This comes from the fact that small realizations of ε^D do not incentivize lenders to subsidize, because borrowers carry less risk and the impact of a default is more manageable. Nearly all lending is done at the same rate of 0.40 which corresponds to the maximum rate that the borrower can accept. However, as liquidity shocks become larger, so does the incentive to help borrowers. A growing proportion of lending is done below 0.40, and the average rate on the market falls under the influence of the common interests subsidy.

The force behind the change in the distribution of equilibrium interest rates is the increase in the share of lending that is being done between close (or to a lesser extent distant) lenders. The following section investigates this point by studying the cross-sectional patterns of interbank lending.

4.2.2 The interbank lending network

Figure 8 plots the average interbank lending network produced by the Monte Carlo simulation. Again we consider three values for the variance $\sigma_d^2 = 0.5$; $\sigma_d^2 = 1$, and $\sigma_d^2 = 1.5$.

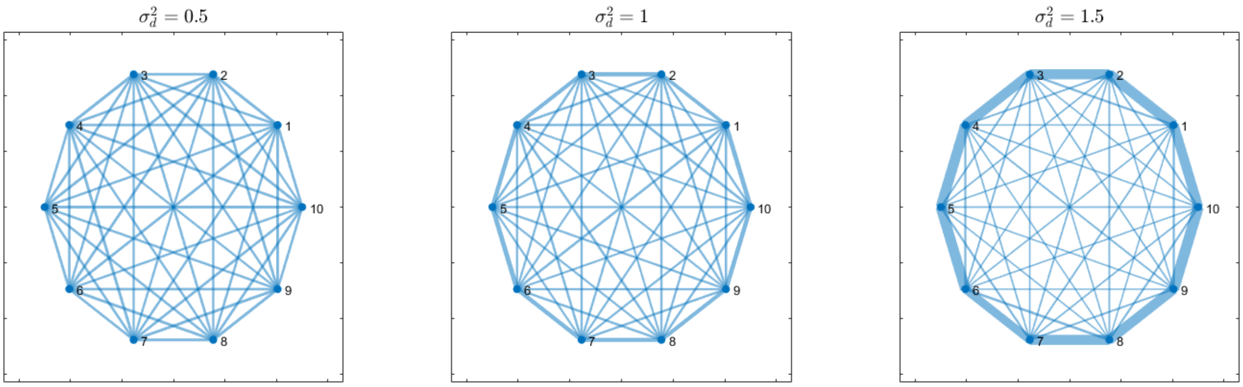


Figure 8: The interbank lending network when liquidity risk rises
This figure represents the lending patterns in the $N = 10$ interbank network for different levels of liquidity risk σ_d^2. The width of the lines represents the average volume of loans between two banks in the the Monte-Carlo simulation.

The figure shows that *the interbank lending network becomes increasingly clustered as liquidity risk rises*. When the variance of the liquidity shock σ_d^2 is equal 0.5 and for a given borrower, a close neighbor is as likely to enter a loan agreement as a distant neighbor or an outsider. This case resembles the network that would prevail without a common interest subsidy, in which all banks would have an equal chance of entering a lending relationship. However when the variance equals 1, a close neighbor becomes 27% more likely to lend than a distant neighbor, who is himself 2% more likely to lend than an outsider. When σ_d^2 reaches 1.5, these two values rise to 63% and 11% respectively. Note how the interbank lending network converges towards the asset holding network represented in figure 5-b when σ_d^2 rises.

The tendency to cluster is natural: because neighbors sometimes provide each other with favourable interest rates, they are more likely to engage in a lending relationship. This finding is in line with several empirical studies. In particular, Affinito (2012) and Cocco et al. (2009) document that close banks lent to each other at higher volumes in addition to lower rates during the crisis.

5 Common interests subsidy and financial stability

The previous section highlighted two opposing impacts of the common interest subsidy on financial stability. On one hand, it allows borrowers to secure lower rates which enhances their robustness (Temizsoy et al., 2015). On the other hand, it leads to a clustered banking network, which should foster contagion (Gai and Kapadia, 2010). This section seeks to compare both effects and draw a conclusion of the desirability of the common interest subsidy from a systemic perspective.

5.1 In the baseline framework

Systemic risk is studied by running a Monte Carlo simulation of the following procedure:

(i) in $t = 1$ a liquidity shock enters the system[8] and spurs an interbank lending network following the modeling presented in section 4,

(ii) in $t = 2 - \epsilon$ shocks to asset returns are realized, leading some banks to go bankrupt

(iii) in $t = 2$ bankruptcies impact the capital of the remaining banks through interbank loans and asset price contagion, possibly leading to new failures. This last step is ran recursively until no bank fails.

This procedure yields a vector of failed banks through which systemic risk may be studied. In particular the following four statistics may be computed:

- "odds of complete collapse" gives the likelihood that all banks in the system are bankrupt at the end of the contagion process. This metric is viewed as the main indicator of systemic risk.

- "odds of immediate failure" represents the likelihood for a given bank to be bankrupt because of asset shocks, before the contagion process begins.

- "odds of contagion to the close neighbors" represents the likelihood that a bank is bankrupt at the end of the contagion process, conditional on one of its close neighbors being bankrupt before it starts.

- "odds of contagion to the distant neighbors" represents the likelihood that a bank is bankrupt at the end of the contagion process, conditional on one of its distant neighbors being bankrupt before it starts.

Table 1 gives some of the values obtained after 100 000 iterations. Again our model is compared to one in which fire sales are not accounted for by lenders, meaning that the common interest subsidy vanishes:

	subsidy	no subsidy	relative value
Odds of complete collapse	2.11	2.68	-21.27%
Odds of immediate failure	4.96	5.13	-3.31%
Odds of contagion to close neighbor	57.91	55.64	$+4.08\%$
Odds of contagion to distant neighbor	28.61	28.47	$+0.49\%$

Table 1: Systemic implication of common interest subsidy

This table reports the values of four systemic risk statistics with and without common interest subsidy.

[8] For conciseness we only consider one value for the distribution of ε^D here: $\sigma_d^2 = 1$

The first line shows that the likelihood of a generalized failure is 2.11% in the subsidy set-up, versus 2.68% in the no subsidy set-up. This means that the common interest subsidy lowers the risk of a generalized failure by 21.27%.

This stabilizing effect is captured by the "odds of immediate failure" statistic. Indeed, individual failure before contagion is 3,31% less likely in the subsidy set-up compared to the no subsidy one, which is logical since borrowers are less risky when obtain more favourable lending conditions.

The other two statistics show that contagion is indeed higher in the subsidy network, as expected from clustering. In particular, the likelihood that a bank fails if its close neighbour failed is 4.08% higher compared to the no subsidy network. However the odds of contagion from the distant neighbour are nearly similar in both frameworks. This comes from the fact that generalized failure is more likely in the no subsidy set-up, which increases measured contagion.

Overall, the key result from this simulation is that *the contagion effect is dominated by the individual risk reduction effect, leading the common interest subsidy to be desirable from a systemic risk perspective.*

5.2 Alternative frameworks

This section runs the systemic analysis in two alternative frameworks in terms of asset holdings and liquidity shocks.

5.2.1 A systemically important bank in the asset holding network

The baseline case used a symmetric asset holding network: all assets were held by three banks, and all banks had similar structures. In this extension we remove symmetry, allowing one asset to be held by all banks ("asset 4"), and one bank to hold only this common asset ("Bank C"). The resulting network is represented in figure 9.

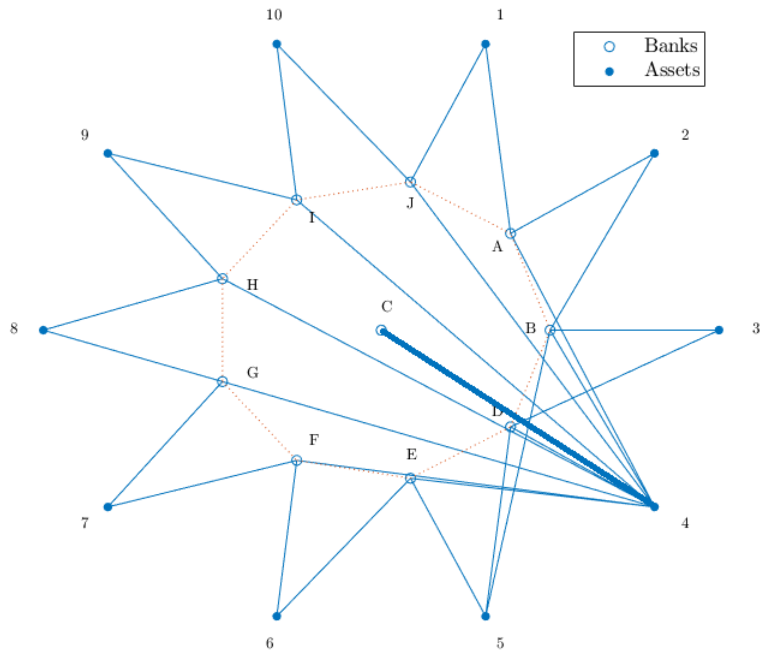


Figure 9: Asymmetric network

This figure represents the assets held for every bank. Banks are indicated with letters and assets are numbered. For instance bank A owns assets 1, 2 and 4. Each bank holds asset 4 and two other assets, bank C only holds asset 4.

The interesting feature of this set-up is that bank C will have particular importance in the network, because its failure would trigger massive sales of an asset that has more potential for contagion. In other words, bank C is systemically important. The goal is to study whether the common interest subsidy is more desirable in this context.

Following the same method as in section 5.1, we run a Monte-Carlo simulation from which several systemic risk statistics are drawn. Table 2 presents the results.

	subsidy	no subsidy	relative value
Odds of complete collapse	2.32	3.06	−24.18%
Odds of immediate failure	0.53	0.77	−31.17%
Odds of contagion to close neighbor	84.44	84.60	−0.19%
Odds of contagion to distant neighbor	68.98	70.16	−1.68%

Table 2: Systemic implication of common interest subsidy for an asymmetric network

This table reports the values of four systemic risk statistics with and without common interest subsidy. The statistics are computed for the asymmetric network, in this framework one of the banks is more systemic and the network is more clustered.

The absolute values of this table are not directly comparable to those of table 1, because banks are less diversified on average and thus the system is riskier. The most interesting statistic is the comparison of the subsidy and no subsidy cases, in the last column.

It appears that the common interest subsidy is more helpful in reducing systemic risk when a systemically important bank appears. The likelihood that the entire system collapses is 24.18% lower allowing for a common interest subsidy, to be compared with 21.27% in the previous section.

The reason for this added impact that a failure of the SIFI leaves the banking network with little chance of survival. Therefore a key objective from a systemic risk perspective is to keep bank C healthy. The common interest subsidy helps in this endeavour because it allows bank C to obtain an interest rate that is much lower than it would if banks did not account for its particular status. For instance, when $\varepsilon^D = 1$, the SIFI gets an interest rate of 1.24% in the subsidy network, versus 0.72% in the no subsidy one. This effect shows in the "odds of immediate failure" statistics, which indicates that the risk of non-contagion-triggered failure falls by 31.7% when the subsidy appears.

To end this section we note that in this SIFI network both contagion statistics are lower in the subsidy set-up. This is surprising at first sight since the interbank lending is more clustered in that case. However this simply reflects the fact that generalized failure is more likely in the no subsidy framework, which again increases measured contagion.

5.2.2 Different liquidity shock

So far the liquidity shock has also been considered symmetric: a negative shock for half of the banks was matched by a positive shock of equal magnitude for the other half. This subsection changes this modeling by allowing liquidity shocks to be positive on aggregate.

More precisely, the liquidity shock is still the same in absolute value for all banks, but we allow for the possibility that more than half of the banks be potential lenders. This means that 5 to 10 banks may have experienced a positive liquidity shock. The probabilities attached to each number between 5 and 10 is drawn from a truncated gaussian with a mean of 5 and a standard deviation of 2.5.

We then run the Monte Carlo simulations using the same steps as in section 5.1. Table 3 summarizes the results:

	subsidy	no subsidy	relative value
Odds of complete collapse	1.28	1.46	−12.33%
Odds of immediate failure	4.70	4.75	−1.05%
Odds of contagion to close neighbor	57.45	55.55	+3.42%
Odds of contagion to distant neighbor	26.46	26.41	+0.19%

Table 3: Systemic implication of common interest subsidy for an asymmetric liquidity shock

This table reports the values of four systemic risk statistics with and without common interest subsidy. The liquidity shock is here asymmetric, allowing the possibility that more than half of the banks be potential lenders, and the system as a whole to have a liquidity surplus in period $t = 2$.

The impact of the subsidy remains qualitatively similar, but decreases in magnitude. The probability of a generalized failure falls by 12.33% when accounting for the subsidy, versus 21.27% in the baseline scenario. This finding is not surprising, since lending needs and borrower risk are both lower with positive liquidity shocks $t = 1$, which implies that the common interest subsidy should have a lesser role.

Nevertheless it is interesting to note that for the banks who do experience a negative liquidity shocks, the common interest subsidy kicks in more often in this set-up. Indeed, because more banks have positive liquidity shock, the likelihood that a close neighbour has cash to lend increases. This enhances the impact of the subsidy on cross-sectional lending. Put simply, the subsidy is at play for a larger proportion of a lesser number of loans.

6 Conclusion

In a banking model where fire sales may spread shocks throughout the banking system, similar banks are tempted to lend to each other at favorable conditions compared to what borrower risk suggests. This explains why rates and volumes on the interbank lending market have showed strong resilience during the 2008 crisis, and why the lending network became more clustered at that time. This common interest subsidy appears desirable for the perspective of systemic risk.

The simple findings derived from our model have interesting implications. For instance, if banks of similar profiles exhibit solidarity during crises, international risk diversification becomes less desirable from the perspective of a social planner as it lowers the amount of asset commonality between banks of similar countries. Such unintended effects do not call into questions the fact that asset diversification has a positive impact on financial stability, but the regulator should keep them in mind when assessing the scope of the next banking crisis.

References

Acharya, V. V., Gromb, D., and Yorulmazer, T. (2012). Imperfect competition in the interbank market for liquidity as a rationale for central banking. *American Economic Journal: Macroeconomics*, 4(2):184–217.

Affinito, M. (2012). Do interbank customer relationships exist? and how did they function in the crisis? learning from italy. *Journal of Banking & Finance*, 36(12):3163–3184.

Afonso, G., Kovner, A., and Schoar, A. (2011). Stressed, not frozen: The federal funds market in the financial crisis. *The Journal of Finance*, 66(4):1109–1139.

Anand, K., Gai, P., and Marsili, M. (2012). Rollover risk, network structure and systemic financial crises. *Journal of Economic Dynamics and Control*, 36(8):1088–1100.

Angelini, P., Nobili, A., and Picillo, C. (2011). The interbank market after august 2007: what has changed, and why? *Journal of Money, Credit and Banking*, 43(5):923–958.

Boissel, C., Derrien, F., Ors, E., and Thesmar, D. (2017). Systemic risk in clearing houses: Evidence from the european repo market. *Journal of Financial Economics*, 125(3):511–536.

Bräuning, F. and Fecht, F. (2016). Relationship lending in the interbank market and the price of liquidity. *Review of Finance*, 21(1):33–75.

Caballero, R. J. and Simsek, A. (2013). Fire sales in a model of complexity. *The Journal of Finance*, 68(6):2549–2587.

Cifuentes, R., Ferrucci, G., and Shin, H. S. (2005). Liquidity risk and contagion. *Journal of the European Economic Association*, 3(2-3):556–566.

Cocco, J. F., Gomes, F. J., and Martins, N. C. (2009). Lending relationships in the interbank market. *Journal of Financial Intermediation*, 18(1):24–48.

Copeland, A., Martin, A., and Walker, M. (2014). Repo runs: Evidence from the tri-party repo market. *The Journal of Finance*, 69(6):2343–2380.

Coval, J. and Stafford, E. (2007). Asset fire sales (and purchases) in equity markets. *Journal of Financial Economics*, 86(2):479–512.

Craig, B. R., Fecht, F., and Tümer-Alkan, G. (2015). The role of interbank relationships and liquidity needs. *Journal of Banking & Finance*, 53:99–111.

Eisenberg, L. and Noe, T. H. (2001). Systemic risk in financial systems. *Management Science*, 47(2):236–249.

Gabrieli, S. and Georg, C.-P. (2014). A network view on interbank market freezes. *Bundesbank Discussion Paper*.

Gai, P. and Kapadia, S. (2010). Contagion in financial networks. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 466(2120):2401–2423.

Greenwood, R., Landier, A., and Thesmar, D. (2015). Vulnerable banks. *Journal of Financial Economics*, 115(3):471–485.

Kobayashi, T. and Takaguchi, T. (2018). Social dynamics of financial networks. *EPJ Data Science*, 7(1):15.

Krishnamurthy, A., Nagel, S., and Orlov, D. (2014). Sizing up repo. *The Journal of Finance*, 69(6):2381–2417.

Leitner, Y. (2005). Financial networks: Contagion, commitment, and private sector bailouts. *The Journal of Finance*, 60(6):2925–2953.

Mancini, L., Ranaldo, A., and Wrampelmeyer, J. (2015). The euro interbank repo market. *The Review of Financial Studies*, 29(7):1747–1779.

Pérignon, C., Thesmar, D., and Vuillemey, G. (2018). Wholesale funding dry-ups. *The Journal of Finance*, 73(2):575–617.

Raffestin, L. (2014). Diversification and systemic risk. *Journal of Banking & Finance*, 46:85–106.

Rogers, L. C. and Veraart, L. A. (2013). Failure and rescue in an interbank network. *Management Science*, 59(4):882–898.

Temizsoy, A., Iori, G., and Montes-Rojas, G. (2015). The role of bank relationships in the interbank market. *Journal of Economic Dynamics and Control*, 59:118–141.

Appendix A

This appendix studies the balance sheets of both banks in $t = 1$, following the occurrence of the liquidity shock ε^D. Two cases need to be considered separately:

(i) bank B accepts the interest rate offered by bank L, and a loan occurs.

(ii) bank B refuses the rate offered by bank L, and sells assets.

The "loan" case

In the event of a loan, a debt (claim) of $B_B = \varepsilon^D$ appears in the books of bank B (bank L). The balance sheet for bank B in $t = 1$ is then given by :

bank B:

assets	liabilities
$p_{b,0}q_{b,0}$	$D_1^B = D_0^B - \varepsilon^D$
$p_{c,0}q_{c,0}^B$	$K_1^B = K_0^B$
	$B_B = \varepsilon^D$

While the balance sheet for bank L writes:

bank L:

assets	liabilities
$p_{l,0}q_{l,0}$	$D_1^L = D_0^L + \varepsilon^D$
$p_{c,0}q_{c,0}^L$	$K_1^L = K_0^L$
$B_B = \varepsilon^D$	

The "no-loan" case

If there has been no agreement on a loan in $t = 1$, bank B sells a quantity $\triangle q_{b,1}$ of asset b at a price $p_{b,1} = p_{b,0} - \lambda\triangle q_{b,1}$, and a quantity $\triangle q_{c,1}$ of asset c at a price $p_{c,1} = p_{c,0} - \lambda\triangle q_{c,1}$, in order to obtain a sum of ε^D. Mathematically this translates into the following condition:

$$\triangle q_{b,1}(p_{b,0} - \lambda\triangle q_{b,1}) + \triangle q_{c,1}(p_{c,0} - \lambda\triangle q_{c,1}) = \varepsilon^D. \tag{13}$$

The total value of assets in $t = 1$ is then given by:

$(q_{b,0} - \triangle q_{b,1})(p_{b,0} - \lambda\triangle q_{b,1}) + (q_{c,0}^B - \triangle q_{c,1})(p_{c,1} - \lambda\triangle q_{c,1}) = K_1^B + D_1^B$

Using (13), this can be re-expressed as:

$K_1^B + D_1^B = q_{b,0}(p_{b,0} - \lambda\triangle q_{b,1}) + q_{c,0}^B(p_{c,0} - \lambda\triangle q_{c,1}) - \varepsilon^D$

$K_1^B + D_1^B = p_{b,0}q_{b,0} + p_{c,0}q_{c,0}^B - \lambda q_{b,0}\triangle q_{b,1} - \lambda q_{c,0}^B\triangle q_{c,1} - \varepsilon^D$

Using the $t = 0$ balance sheet equilibrium.

$K_1^B + D_1^B = K_0 + D_0 - \varepsilon^D$-$\lambda q_{b,0}\triangle q_{b,1} - \lambda q_{c,0}^B\triangle q_{c,1}$

Denoting by $\Pi_{B,1} = K_1^B - K_0^I$ the profit of bank B between $t = 0$ and $t = 1$, this expression can be rearranged as:

$\Pi_{B,1} = -(\lambda q_{b,0}\triangle q_{b,1} + \lambda q_{c,0}^B\triangle q_{c,1})$

which allows us to draw the balance sheet for bank B in $t = 1$ when no loan occurred:

assets	liabilities
$(q_{b,0}-\triangle q_{b,1})(p_{b,0}-\lambda\triangle q_{b,1})$	$D_1^B = D_0^B - \varepsilon^D$
$+(q_{c,0}^B-\triangle q_{c,1})(p_{c,0}-\lambda\triangle q_{c,1})$	$K_1^B = K_0^B$
	$-\lambda q_{b,0}\triangle q_{b,1} - \lambda q_{c,0}^B\triangle q_{c,1}$

To obtain the asset sales for each asset $\triangle q_{b,1}$ and $\triangle q_{c,1}$, we assume that bank B keeps the same proportion of assets b and c as in $t=0$, since the characteristics of both assets are unchanged. Formally:

$$\frac{q_{c,0}^B}{q_{c,0}^B + q_{b,0}} = \frac{q_{c,1}^B}{q_{c,1}^B + q_{b,1}} = w \tag{14}$$

which, along with equation (13) allows us to uniquely define $\triangle q_{b,1}$ and $\triangle q_{c,1}$.

Turning to bank L, two new elements appear in the balance sheet in $t=1$. The first element is a surplus of liquidity of ε^D, since the bank received a positive liquidity shock. We simply set that this excess liquidity goes into a liquid short-term market at one period horizon that yields no interests $L_1 = \varepsilon^D$. The second element is a fall in asset values due to the fact that the price of asset c has fallen as a result of the sales from bank B. The balance sheet in $t=1$, for bank L in the no loan case, is then given by :

bank L:

assets	liabilities
$p_{l,0}q_{l,0}$	$D_1^L = D_0^L + \varepsilon^D$
$q_{c,0}^L(p_{c,0}-\lambda\triangle q_{c,1}^B)$	$K_2^L = K_0^L - \lambda q_{c,0}^L\triangle q_{c,1}^B$
$L_1 = \varepsilon^D$	

Appendix B

The loan case

Bank B

In $t=2$, the asset shocks ε_b^p and ε_c^p materialize, and the loan must be repaid with interests. Because there has not been any sales in $t=1$ we have $p_{i,0} = p_{i,1}$. Capital adjusts to maintain the equality between assets and liabilities, which yields:

bank B:

assets	liabilities
$q_{b,0}(p_{b,0}+\varepsilon_b^p)$	$D_2^B = D_0^B - \varepsilon^D$
$q_{c,0}^B(p_{c,0}+\varepsilon_c^p)$	$K_2^B = K_0^B - r\varepsilon^D + q_{b,0}\varepsilon_b^p + q_{c,0}^B\varepsilon_c^p$
	$B_{B,2} = \varepsilon^D(1+r)$

Bank L, given survival of bank B

The same logic applies, except that the loan now appears on the asset side since bank L is the lender:

bank L:

assets	liabilities
$q_{l,0}(p_{l,0}+\varepsilon_l^p)$	$D_2^L = D_0^L + \varepsilon^D$
$q_{c,0}^L(p_{c,0}+\varepsilon_c^p)$	$K_2^L = K_0^L + r\varepsilon^D + q_{c,0}^L\varepsilon_c^p + q_{l,0}\varepsilon_l^p$
$B_{B,2} = \varepsilon^D(1+r)$	

Bank L, given failure of bank B

If bank B has gone bankrupt, the loan is defaulted on, which implies a net loss of ε^D for the lender. A share δ of assets is also liquidated by bank B to manage its bankruptcy, which lowers the market value of asset c. The balance sheet of bank L writes:

	assets	liabilities
bank L:	$q^L_{c,0}(p_{c,0} - \lambda\delta q^B_{c,0} + \varepsilon^p_c)$	$D^L_1 = D^L_0 + \varepsilon^D$
	$q_{l,0}(p_{l,0} + \varepsilon^p_l)$	$K^L_2 = K^L_0 + q_{l,0}\varepsilon^p_l + q^L_{c,0}\varepsilon^p_c$
		$-\varepsilon^D + q^L_{c,0}(-\lambda\delta q^B_{c,0})$

The no-loan case

When there has not been a loan in $t = 1$, the only change in $t = 2$ comes from the asset shocks. One simply needs to add these shocks to the balance sheets provided in appendix A.

Bank B

	assets	liabilities
bank B:	$(q_{b,0} - \triangle q_{b,1})(p_{b,0} - \lambda\triangle q_{b,1} + \varepsilon^p_b)$	$D^B_1 = D^B_0 - \varepsilon^D$
	$(q^B_{c,0} - \triangle q^B_{c,1})(p_{c,0} - \lambda\triangle q^B_{c,1} + \varepsilon^p_c)$	$K^B_2 = K^B_0+$
		$-\lambda q_{b,0}\triangle q_{b,1} - \lambda q^B_{c,0}\triangle q^B_{c,1}$
		$+(q_{b,0} - \triangle q_{b,1})\varepsilon^p_b+(q^B_{c,0} - \triangle q^B_{c,1})\varepsilon^p_c$

For bank L, when bank B survives

The same logic applies: only asset shocks need to be added to the $t = 1$ situation.

	assets	liabilities
bank L:	$q_{l,0}(p_{l,0} + \varepsilon^p_l)$	$D^B_1 = D^B_0 + \varepsilon^D$
	$q^L_{c,0}(p_{c,0} - \lambda\triangle q^B_c + \varepsilon^p_c)$	$K^B_2 = K^B_0 - \lambda q^L_{c,0}\triangle q^B_{c,1}$
	$L_1 = \varepsilon^D$	$+q_{l,0}\varepsilon^p_l + q^L_{c,0}\varepsilon^p_c$

For bank L, when bank B fails

In this case bank B sells some of its remaining asset of the common asset until it reaches the maximum proportion of asset that can be sold, δ. Mathematically: $\triangle q^B_{c,2} + \triangle q^B_{c,1} = \delta q_{c,0}$. Because the impact of sales is linear both rounds of selling (in $t = 1$ and $t = 2$) can be considered together.

	assets	liabilities
bank L:	$q_{l,0}(p_{l,0} + \varepsilon^p_l)$	$D^B_1 = D^B_0 + \varepsilon^D$
	$q^L_{c,0}(p_{c,0} - \lambda\delta q^B_{c,0} + \varepsilon^p_c)$	$K^B_2 = K^B_0 - \lambda\delta q^L_{c,0}q^B_{c,0}$
	$L_1 = \varepsilon^D$	$+q_{l,0}\varepsilon^p_l+q^L_{c,0}\varepsilon^p_c$

Appendix C

This appendix describes the steps involved in solving the analytically tractable version of the model in section 3.2.

Conditions for a tractable solution

A closed-form solution requires making specific assumptions regarding the distributions for the asset shocks. In particular, this appendix sets that:

(i) the shocks on the privately held assets ε_b^p and ε_l^p are both uniformly distributed over $[\mu - V,\ \mu + V]$, where V is small enough that $P(d_L|\bar{d}_B) = 0$. This means that the liquidity rich bank may not default if the liquidity poor bank has survived and paid back its loan. Formally this translates into the following condition: $V_{pL} < \frac{r\varepsilon^D + K_0^L + q_l\mu_{pL}}{2q_{l,0}}$.

(ii) The liquidity shock ε^p is also uniformly distributed over the interval $[0, C]$, where C may not exceed a threshold value C' defined by $C' = \frac{q_i(mu+V)+2F}{3}$. This upper bound states that the best possible return on assets is sufficient to offset the worst possible withdrawal of funds in $t = 1$.

(iii) the shock on the commonly held asset ε_c^p is set to zero. This rules out shocks on the common asset c allows both banks to be independent in terms of asset returns, which simplifies the analysis while keeping the intuition intact.

Expected profits and probabilities of failure

In order to find r^* one must draw the probabilities of failure/survival and expected from the profit functions given in section 2.3.2.

Let us for instance consider equation (4) which gives the expression for Π_B, the profit for bank B in the loan case. Because there is no shock on the common asset in the context of this appendix, Π_B simplifies to:

$$\Pi_B = -r\varepsilon^D + q_i\varepsilon_b^p$$

Since ε_b^p is uniformly distributed over the interval $[\mu - V,\ \mu + V]$, the likelihood that bank B survives $P(\bar{d}_B)$ is thus :

$$P(\bar{d}_B) = P(\Pi_B > -K) = \frac{q_i\mu + q_iV - r\varepsilon^D + K}{2q_iV} \tag{15}$$

Let us now consider the profit of bank L when bank B defaulted on its loan, which writes:

$$\Pi_{L,F} = q_i\varepsilon_l^p - \varepsilon^D - \lambda\delta(q_c)^2$$

This yields the following expression for the probability that bank L survives while B defaults:

$$P(\bar{d}_L, d_B) = \frac{q_i\mu + q_iV - \varepsilon^D - \lambda\delta(q_c)^2 + K}{2q_iV} \tag{16}$$

while the expression for the expected utility of bank L knowing that it has survived while bank B failed, denoted by $E(U_L|\bar{d}_L, d_B)$, is:

$$E(U_L|\bar{d}_L, d_B) = \frac{1}{2}[q_i(\mu + V) - \varepsilon^D - \lambda\delta(q_c)^2 - K] \tag{17}$$

Finally let us consider the profit for bank L when bank B survives :

$$\Pi_{L,S} = r\varepsilon^D + q_{l,0}\varepsilon_l^p$$

In this case the probablity of survival for bank L is, by definition:

$$P(\bar{d}_L, \bar{d}_B) = 1 \tag{18}$$

which implies that the expected utility of bank L in this case is:

$$E(U_L|\bar{d}_L, \bar{d}_B) = q_i\mu + r\varepsilon^D \tag{19}$$

Solving

Solving first involves plugging expressions (15) to (19) into (10), and maximizing the resulting expression. (10) may be rewritten as:

$$E(U_L) = \underbrace{[1 - P(\bar{d}_L|d_B)] * [1 - P(\bar{d}_B)] * (-F)}_{A} + \underbrace{P(\bar{d}_L|d_B) * [1 - P(\bar{d}_B)] * E(U_L|\bar{d}_L, d_B)}_{B} + \underbrace{P(\bar{d}_B) * E(U_L|\bar{d}_B, \bar{d}_L)}_{C} \tag{20}$$

Plugging expressions (15) and (16) in A yields:

$$A = (-F)(\frac{1}{2q_iV})^2 * r\varepsilon^D * (q_iV - q_i\mu - K + b(q_c)^2 + \varepsilon^D) + cst$$

where cst gathers all the terms that are unrelated to r. Taking the derivative of A with respect to r, and rearranging gives :

$$\frac{\partial A}{\partial r} = -F(\frac{1}{2q_iV})^2 * \varepsilon^D * 2q_iV + F(\frac{1}{2q_iV})^2 * \varepsilon^D * (q_iV + q_i\mu + K - b(q_c)^2 - \varepsilon^D)]$$

Plugging expressions (15), (16) and (17) in B yields:

$$B = \frac{1}{2}(\frac{1}{2q_iV})^2 * (q_i\mu + q_iV - \lambda\delta(q_c)^2 - \varepsilon^D - K) * (q_i\mu + q_iV - \lambda\delta(q_c)^2 - \varepsilon^D + K) * r\varepsilon^D + cst$$

Taking the derivative and re-arranging :

$$\frac{\partial B}{\partial r} = \frac{1}{2}(\frac{1}{2q_iV})^2 * (q_i\mu + q_iV - \lambda\delta(q_c)^2 - \varepsilon^D - K) * (q_i\mu + q_iV - \lambda\delta(q_c)^2 - \varepsilon^D + K) * \varepsilon^D$$

Plugging expressions (15) and (19) in C yields:

$$C = (\frac{1}{2q_1V}) * [(q_i\mu + q_iV + K - q_i\mu) * r\varepsilon^D - (r\varepsilon^D)^2] + cst$$

Taking the derivative and re-arranging :

$$\frac{\partial C}{\partial r} = (\frac{1}{2q_iV})\varepsilon^D * [q_iV + K - 2r\varepsilon^D]$$

Using the fact that $\frac{\partial E(U_L)}{\partial r} = \frac{\partial A}{\partial r} + \frac{\partial B}{\partial r} + \frac{\partial C}{\partial r}$, and re-arranging, we obtain the final result:

$$r^* = \frac{1}{2 * \varepsilon^D} \left(q_iV + K - F + (\frac{1}{2q_iV}) * [q_i\mu + q_iV + K - \lambda\delta(q_c)^2 - \varepsilon^D] * [F + \frac{1}{2}(q_i\mu + q_iV - \lambda\delta(q_c)^2 - \varepsilon^D - K)] \right) \tag{21}$$

This form shows that the rate that lenders are willing to offer depends in non monotonic fashion on all variables, particularly the amount of common exposure q_c.

Expression (21) can be re-expressed as a polynomial of the second degree of q_c, whose behavior can easily be studied. This study, available on request, shows that over an interval $[0, T]$ where $T > 0$, the rate that a lending bank is willing to offer to a borrowing banks falls with the degree of commonality between the two banks, which is evidence that the common asset subsidy is present in this tractable version of the model.

Appendix D

We study how the equilibrium interest rate changes with the characteristics of the financial markets. In particular, we examine the impact of a change in the value of the following four parameters:

- the share of assets liquidated following bankruptcy δ. - the impact of sales on prices λ. - the disutility of bankruptcy F. - the size of the liquidity shock ε^D.

For each parameter we consider two values, on the low and high side of the default value. Figure 10 plots the results:

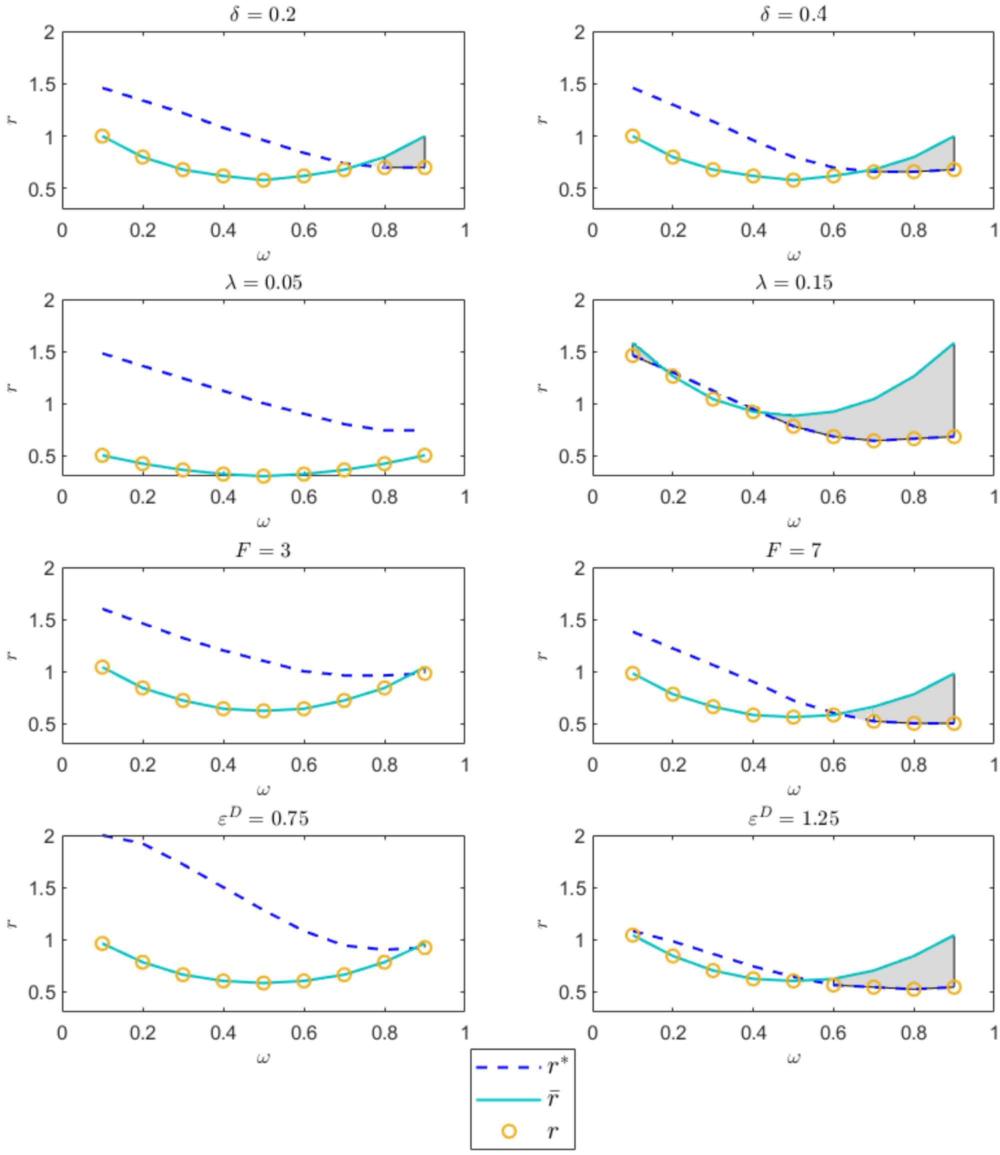


Figure 10: Equilibrium interest rate for different values of ε^D, λ, F and δ.
This figure represents the behaviour of the three interest rates r^*, $\bar{r}$ and r for low and high values of the share of assets liquidated following bankruptcy δ, the impact of sales on prices λ, the disutility of bankruptcy F and the size of the liquidity shock ε^D. This figure plots the interest rate that bank L would like to charge r^* (dashed line), the maximum rate that bank B is willing to accept $\bar{r}$ (full line) and the equilibrium interest rate r (line with circle markers) for different level of commonality ω.

The first comment is that the behavior of both r^* and $\bar{r}$ are very consistent across subplots. In particular, the interest rate that maximizes the utility of the lender initially falls with the level of asset commonality, before stabilizing. Different parameters values may lead the initial drop to be more or less pronounced, but do not change the overall pattern. What changes is the point of appearance of the common interest subsidy, i.e. the level of commonality ω such that

$r^* = \bar{r}$.

The parameter that appears to have the strongest effect is λ, the price impact of fire sales. When λ drops to 0.05, the equilibrium interest rate is always the maximum rate that the borrower is willing to pay, $r = \bar{r}$. In other words, it is never in the interest of the lender to subsidize the borrower when $\varepsilon^D = 1$. However, if $\lambda = 0.15$, r^* and $r = \bar{r}$ are very close even for low levels of commonality, and the subsidy permanently sets in as soon as $\omega = 0.45$. This strong sensibility is not surprising: when the lender has more to fear from fire sales, his incentive to avoid them grows stronger.

F and ε^D appear to have a qualitatively similar yet more moderate impact. When the cost of failure increases, the lender is more willing to accept the opportunity cost of lower rates in exchange for lower bankruptcy risk. When the liquidity needs of the borrower rise, the amount of sales required to face those needs also increases, which makes fire sales more costly, and gives the lender more reasons to subsidize. Note that the impact of ε^D on fire sales is non-linear: because the sales lower the price of the asset, the amount of sales required to obtain the first unit of liquidity is lower than that required to obtain the second unit.

The last parameter is δ, which represents the proportion of asset that gets liquidated following bankruptcy. Qualitatively, its effect is in line with expectations: when a failure involves more fire sales, the cost it imposes on banks with similar asset structure rises, which increases the incentive to subsidize. However it is interesting to note that this impact is very modest. The reason for this is that an immediate sale of 20% already has a strong negative impact on the expected profit of the lender, implying that there is little room for worsening.