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Interval neutrosophic numbers Choquet integral operator for multi-criteria decision making

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Abstract. In this paper, the Choquet integral and the interval neutrosophic set theory are combined to make multi-criteria decision for problems under neutrosophic fuzzy environment. Firstly, a ranking index is proposed according to its geometrical structure, and an approach for comparing two interval neutrosophic numbers is given. Then, a $\leq_L$ implied operation-invariant total order which satisfies order-preserving condition is proposed. Secondly, an interval neutrosophic number Choquet integral (INNCI) operator is established and a detailed discussion on its aggregation properties is presented. In addition, the procedure of multi-criteria decision making based on INNCI operator is given. Finally, a practical example for selecting the third party logistics providers is provided to illustrate the feasibility of the developed approach.

Keywords: Neutrosophic set (NS), order relation, fuzzy measure, Choquet integral, multi-criteria decision making (MCDM)

1. Introduction

The concept of neutrosophic set (NS) is introduced by Smarandache [22], which generalizes the classic set, fuzzy set (FS), interval valued fuzzy set (IVFS), intuitionistic fuzzy set (IFS), as well as interval valued intuitionistic fuzzy set (IVIFS). A NS is characterized independently by a truth-membership, an indeterminacy-membership and a falsity-membership. It is a powerful tool to deal with incomplete, indeterminate and inconsistent information. Comparing with NS, IFSs and IVIFSs can only handle incomplete information but not the indeterminate information and inconsistent information which exist commonly in real situations. For example, in a decision making process, a manager decides whether he should select the third party logistics provider A or not. When we ask about the opinion of an expert about a certain statement, he may say that the possibility that he select A is between 0.5 and 0.6, that he does not select A is between 0.1 and 0.2, and the degree that he is not sure is between 0.2 and 0.3. For a neutrosophic notation, it can be expressed as $x([0.5, 0.6], [0.1, 0.2], [0.2, 0.3])$. The sum of the degree of truth, indeterminacy, and falsity may be greater or less than 1. For example, if $x = (0.6, 0.2, 0.3)$, x is a NS but is not an IFS. If $x = (0.5, 0.1, 0.2)$, x is a NS but is not an IFS. If $x = (0.5, 0.2, 0.3)$, x is not only a NS, but also an IFS. Another example, assuming there are 10 voters during a voting process, in time t_1, four vote 'yes', three vote 'no' and three are undecided. For neutrosophic notation, it can be expressed as $x(0.4, 0.3, 0.3)$; in time t_2, two vote 'yes', three vote 'no', two give up, and three are

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undecided, then it can be expressed as $x(0.2, 0.3, 0.3)$. That is beyond the scope of the IFS. So the notion of neutrosophic set is more general [22, 33].

In a NS, the degree of truth, indeterminacy, and falsity belong to $]0^-, 1^+[$, where $]0^-, 1^+[$ is the non-standard unit interval [21]. Obviously, it is difficult to apply in real applications. Therefore, Wang et al. [27] proposed the concept of a single valued neutrosophic set (SVNS), which is the subclass of a NS. Sometimes the degree of truth, falsity, and indeterminacy of a certain statement cannot be defined exactly in the real situations but denoted by several possible interval values. In order to research this problem, Wang et al. [26] proposed the concept of interval neutrosophic set (INS) and gave the set-theoretic operators of INS. Recently, many researchers have shown great interest in multi-criteria decision making (MCDM) problems with neutrosophic information. Ye [30, 31] proposed correlation coefficients between SVNSs and applied them to MCDM problems with single valued neutrosophic information. Ye [32] proposed single valued neutrosophic cross entropy and applied it to MCDM. Ye [29] introduced the concept of simplified neutrosophic sets (SNSs), which can be described by singleton subintervals/subsets in the real unit interval [0, 1], and proposed a MCDM method using aggregation operators for SNSs. Chi and Liu [2] extended a TOPSIS method to interval neutrosophic multiple attribute decision-making problems. Zhang et al. [34] defined the operations of INSs and gave the aggregation operators of interval neutrosophic number weighted averaging(INNWA) and interval neutrosophic number weighted geometric(INNWG), then a MADM method is established based on the proposed operators. Broumi and Smarandache [1] introduced the concept of correlation coefficients of interval valued neutrosophic set.

Aggregation function plays an important role in MCDM problems. All above aggregation operators only consider situations where criteria (attribute) and preferences of decision makers are independent of one another, which means that their effects are viewed as additive. However, in many real decision making problems, it is common to find that there is interaction among preference of decision makers. As an aggregation function, the Choquet integral [3] with respect to fuzzy measures [23] is able to flexibly describe the relative importance of decision criteria as well as their interactions [11, 19]. Therefore, it is interesting to combine the Choquet integral and the INS theory for MCDM under neutrosophic fuzzy environment. On one hand, we can deal with the imprecise and uncertain decision information; on the other hand, we can efficiently take into account of the various interactions among the decision criteria. Based on the above discussion, there are three aims in the paper. First, it will propose a new ranking method for interval neutrosophic numbers (INNs). Second, it will propose an interval neutrosophic number Choquet integral (INNCI) operator and discuss its properties. Third, it will establish a decision making method based on the proposed ranking method and the INNCI operator to handle decision making problems with interval neutrosophic information.

The paper is organized as follows. In Section 2, the concepts of NS, SNS, INS and operations of INS are reviewed. In Section 3, a ranking index is proposed according to geometrical quantities which reflect the inferiors and superiors of INNs, and an approach to compare two INNs is proposed. Furthermore, a $\leq_L$ implied operation-invariant total order which satisfies order-preserving condition is proposed. In Section 4, INNCI operator is proposed, and some of its properties are researched. In Section 5, the MCDM procedure based on INNCI operator is presented under neutrosophic environment. In Section 6, an example is given to illustrate the concrete application of the method and to demonstrate its feasibility and applicability. Conclusions are made in Section 7.

2. Interval neutrosophic Set

Definition 2.1. [22] *Let X be a space of points (objects), with a generic element in X denoted by x. A NS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, i.e., $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$, and $F_A(x) : X \to]0^-, 1^+[$, where $]0^-, 1^+[$ is the non-standard unit interval. The sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.*

Since it is difficult to apply NSs to practical problems, Wang et al. [27] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2. *Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by truth-membership function $T_A(x)$,*

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indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. A SVNS A can be written as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$.

Similar to interval-valued intuitionistic fuzzy set, Wang et al. [26] proposed the concept of INS.

Definition 2.3. [26] *Let X be a space of points (objects) with generic elements in X denoted by x. An INS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x) = [\inf T_A(x), \sup T_A(x)] \subseteq [0, 1]$, $I_A(x) = [\inf I_A(x), \sup I_A(x)] \subseteq [0, 1]$, $F_A(x) = [\inf F_A(x), \sup F_A(x)] \subseteq [0, 1]$.*

Remark 2.1. *From Definition 2, an INS A can be expressed as*

$$\begin{aligned} A &= \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\} \\ &= \{\langle x, [\inf T_A(x), \sup T_A(x)], [\inf I_A(x), \sup I_A(x)], \\ &\qquad [\inf F_A(x), \sup F_A(x)]\rangle \mid x \in X\} \end{aligned}$$

Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

Definition 2.4. [26] *An INS A is contained in the other INS B, $A \subseteq B$, if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$, and $\sup F_A(x) \geq \sup F_B(x)$ for any x in X.*

For convenience, let $\tilde{a} = \langle [T_{\tilde{a}}^L, T_{\tilde{a}}^U], [I_{\tilde{a}}^L, I_{\tilde{a}}^U], [F_{\tilde{a}}^L, F_{\tilde{a}}^U]\rangle$ denotes an INN and L be the set of all INNs in X.

Definition 2.5. [2] *Let $\tilde{a}$ and $\tilde{b}$ be two INNs, and λ be a real number. Then, the operational rules are defined as follows:*

(1) $\tilde{a} \oplus \tilde{b} = \langle [T_{\tilde{a}}^L + T_{\tilde{b}}^L - T_{\tilde{a}}^L \cdot T_{\tilde{b}}^L, T_{\tilde{a}}^U + T_{\tilde{b}}^U - T_{\tilde{a}}^U \cdot T_{\tilde{b}}^U], [I_{\tilde{a}}^L \cdot I_{\tilde{b}}^L, I_{\tilde{a}}^U \cdot I_{\tilde{b}}^U], [F_{\tilde{a}}^L \cdot F_{\tilde{b}}^L, F_{\tilde{a}}^U \cdot F_{\tilde{b}}^U]\rangle$.

(2) $\tilde{a} \otimes \tilde{b} = \langle [T_{\tilde{a}}^L \cdot T_{\tilde{b}}^L, T_{\tilde{a}}^U \cdot T_{\tilde{b}}^U], [I_{\tilde{a}}^L + I_{\tilde{b}}^L - I_{\tilde{a}}^L \cdot I_{\tilde{b}}^L, I_{\tilde{a}}^U + I_{\tilde{b}}^U - I_{\tilde{a}}^U \cdot I_{\tilde{b}}^U], [F_{\tilde{a}}^L + F_{\tilde{b}}^L - F_{\tilde{a}}^L \cdot F_{\tilde{b}}^L, F_{\tilde{a}}^U + F_{\tilde{b}}^U - F_{\tilde{a}}^U \cdot F_{\tilde{b}}^U]\rangle$.

(3) $\lambda\tilde{a} = \langle [1 - (1 - T_{\tilde{a}}^L)^\lambda, 1 - (1 - T_{\tilde{a}}^U)^\lambda], [(I_{\tilde{a}}^L)^\lambda, (I_{\tilde{a}}^U)^\lambda], [(F_{\tilde{a}}^L)^\lambda, (F_{\tilde{a}}^U)^\lambda]\rangle$, $\lambda > 0$.

(4) $\tilde{a}^\lambda = \langle [(T_{\tilde{a}}^L)^\lambda, (T_{\tilde{a}}^U)^\lambda], [1 - (1 - I_{\tilde{a}}^L)^\lambda, 1 - (1 - I_{\tilde{a}}^U)^\lambda], [1 - (1 - F_{\tilde{a}}^L)^\lambda, 1 - (1 - F_{\tilde{a}}^U)^\lambda]\rangle$, $\lambda > 0$.

The operations of two INNs has been defined by Zhang et al. [34] by using of a strict Archimedean t-norm $T(x, y) = k^{-}(k(x) + k(y))$ and its dual t-conorm $S(x, y) = l^{-}(l(x) + l(y))$ with $l(x) = k(1 - x)$. When $k(x) = -\log(x)$, the operational rules defined in [2] and [34] are identifiable.

Theorem 2.1. *Let $\tilde{a}$ and $\tilde{b}$ be two INNs, then $\tilde{a} \oplus \tilde{b}$ and $\lambda \otimes \tilde{a}$ are also INNs.*

The results of Theorem 2.1 are obvious. Furthermore, the operations of two INNs have the following properties [34].

Proposition 2.1. *Let $\tilde{a}$,$\tilde{b}$ and $\tilde{c}$ be INNs and $\alpha, \beta \geq 0$, then*

(1) $\tilde{a} \oplus \tilde{b} = b \tilde{\oplus} \tilde{a}$, $\tilde{a} \otimes \tilde{b} = \tilde{b} \otimes \tilde{a}$.

(2) $(\tilde{a} \oplus \tilde{b}) \oplus \tilde{c} = \tilde{a} \oplus (\tilde{b} \oplus \tilde{c})$, $(\tilde{a} \otimes \tilde{b}) \otimes \tilde{c} = \tilde{a} \otimes (\tilde{b} \otimes \tilde{c})$.

(3) $\alpha(\tilde{a} \oplus \tilde{b}) = \alpha\tilde{a} \oplus \alpha\tilde{b}$, $(\tilde{a} \oplus \tilde{b})^\alpha = \tilde{a}^\alpha \oplus \tilde{b}^\alpha$.

(4) $\alpha\tilde{a} \oplus \beta\tilde{a} = (\alpha + \beta)\tilde{a}$, $\tilde{a}^\alpha \otimes \tilde{a}^\beta = \tilde{a}^{\alpha+\beta}$.

3. Comparison method of INNs

In this section, we will propose two approaches to compare two INNs. Firstly, a ranking index is developed according to some geometrical quantities which reflect the inferiors and superiors of INNs, and then order relation $\leq_H$ is proposed. Furthermore, due to limitations of $\leq_H$, a $\leq_L$ implied operation-invariant order is proposed.

3.1. Ranking method for INNs based on geometrical structure

Let us consider a three-dimensional coordinate of the technical neutrosophic cube that shown in Fig. 1, where x is the truth axis with value range in [0, 1], y is the false axis with value range in [0, 1], and similarly z

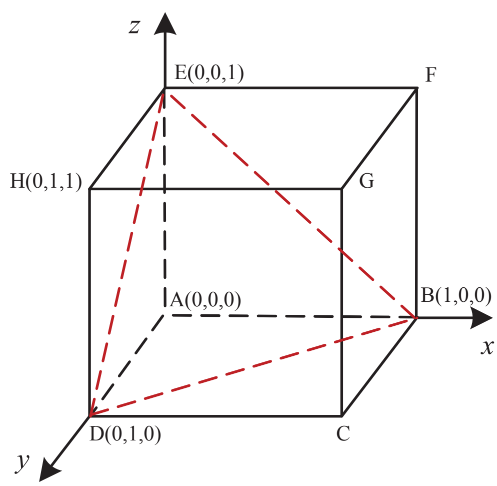

Fig. 1. The three-dimensional coordinates of the INNs.

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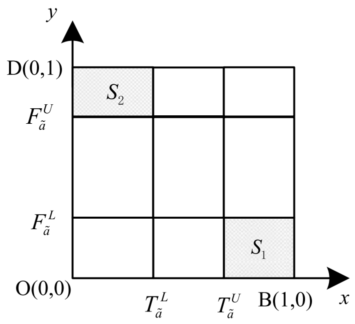

Fig. 2. The two-dimensional projection of INNs.

is the indeterminate axis with value range in [0, 1]. The neutrosophic cube can be divided into three disjoint regions [5]:

(1) Triangle BDE , whose sides are equal to $\sqrt{2}$, represents the geometrical locus of the points whose sum of the coordinates is 1.

(2) The pyramid EADB is the locus of the points whose sum of coordinates is less than 1.

(3) the solid EHDCGFB (excluding $\triangle$ BDE) is the locus of points whose sum of their coordinates is greater than 1.

Considering a point a in the technical neutrosophic cube. The superiors point is $B(1, 0, 0)$ and the inferiors point is $H(0, 1, 1)$, therefore, the shorter distance between a and $B(1, 0, 0)$ is, and the longer distance between a and $H(0, 1, 1)$ is, the bigger a is.

Given a INN $\tilde a$, the two-dimensional projection of $\tilde a$ in plan (x, y) is shown in Fig. 2. The smaller area of S_1 is, and the bigger area of S_2 is, the bigger $\tilde a$ is. The shorter distance between $\left(T^L_{\tilde a} + T^U_{\tilde a}/2,\ I^L_{\tilde a} + I^U_{\tilde a}/2,\ F^L_{\tilde a} + F^U_{\tilde a}/2\right)$ and $B(1, 0, 0)$ is, and the longer distance between $\left(T^L_{\tilde a} + T^U_{\tilde a}/2,\ I^L_{\tilde a} + I^U_{\tilde a}/2,\ F^L_{\tilde a} + F^U_{\tilde a}/2\right)$ and $H(0, 1, 1)$ is, the bigger $\tilde a$ is. According to the geometrical description of INNs, the ranking index of INNs is proposed.

Definition 3.1. *Let $\tilde a$ be an INN. The ranking index of $\tilde a$ is defined as follow*

$$H(\tilde a) = \frac{S_2 + D^-}{S_1 + S_2 + D^* + D^-} \tag{1}$$

where $S_1 = (1 - T^U_{\tilde a}) \cdot F^L_{\tilde a}$, $S_2 = (1 - F^U_{\tilde a}) \cdot T^L_{\tilde a}$, $D^* = \sqrt{(1 - \bar T_{\tilde a})^2 + \bar I^2_{\tilde a} + \bar F^2_{\tilde a}}$, $D^- = \sqrt{\bar T^2_{\tilde a} + (1 - \bar I_{\tilde a})^2 + (1 - \bar F_{\tilde a})^2}$, $\bar T_{\tilde a} = \dfrac{T^L_{\tilde a} + T^U_{\tilde a}}{2}$, $\bar I_{\tilde a} = \dfrac{I^L_{\tilde a} + I^U_{\tilde a}}{2}$ *and* $\bar F_{\tilde a} = \dfrac{F^L_{\tilde a} + F^U_{\tilde a}}{2}$.

From the Definition 3.1, it is easy to get the following proposition.

Proposition 3.1. *For any INNs $\tilde a = \left\langle \left[T^L_a, T^U_a\right], \left[I^L_a, I^U_a\right], \left[F^L_a, F^U_a\right]\right\rangle$, then $H(\tilde a) \in [0, 1]$. Furthermore, if $\tilde a = \langle[1, 1], [0, 0], [0, 0]\rangle$, then $H(\tilde a) = 1$, and if $\tilde a$=$\langle[0, 0], [1, 1], [1, 1]\rangle$, then $H(\tilde a) = 0$.*

Theorem 3.1. *Let $\tilde a = \left\langle[T^L_{\tilde a}, T^U_{\tilde a}], [I^L_{\tilde a}, I^U_{\tilde a}], [F^L_{\tilde a}, F^U_{\tilde a}]\right\rangle$ and $\tilde b = \left\langle[T^L_{\tilde b}, T^U_{\tilde b}], [I^L_{\tilde b}, I^U_{\tilde b}], [F^L_{\tilde b}, F^U_{\tilde b}]\right\rangle$ be two INNs. If $T^L_{\tilde a} \le T^L_{\tilde b}$, $T^U_{\tilde a} \le T^U_{\tilde b}$, $I^L_{\tilde a} \ge I^L_{\tilde b}$, $I^U_{\tilde a} \ge I^U_{\tilde b}$, $F^L_{\tilde a} \ge F^L_{\tilde b}$ and $F^U_{\tilde a} \ge F^U_{\tilde b}$, then $H(\tilde a) \le H(\tilde b)$.*

Proof. Let $G(\tilde a) = \dfrac{1}{H(\tilde a)} = \dfrac{S_1 + D^*}{S_2 + D^-} + 1$, then

$$\frac{\partial G(\tilde a)}{\partial T^L_{\tilde a}} = \frac{-\frac{1}{2}(D^*)^{-1}\left(1 - \frac{T^L_{\tilde a} + T^U_{\tilde a}}{2}\right)}{S_2 + D^-} - \frac{S_1 + D^*}{(S_2 + D^-)^2} \times \left[1 - F^U_{\tilde a} + \tfrac{1}{2}\left(D^-\right)^{-1}\left(\frac{T^L_{\tilde a} + T^U_{\tilde a}}{2}\right)\right],$$

$$\frac{\partial G(\tilde a)}{\partial T^U_{\tilde a}} = \frac{-F^L_{\tilde a} - \frac{1}{2}(D^*)^{-1}\left(1 - \frac{T^L_{\tilde a} + T^U_{\tilde a}}{2}\right)}{S_2 + D^-} - \frac{S_1 + D^*}{(S_2 + D^-)^2} \times \frac{1}{2}\left(D^-\right)^{-1}\left(\frac{T^L_{\tilde a} + T^U_{\tilde a}}{2}\right).$$

Since $T_a \subseteq [0, 1]$, $F_a \subseteq [0, 1]$, then $\dfrac{\partial G(\tilde a)}{\partial T^L_{\tilde a}} \le 0$, $\dfrac{\partial G(\tilde a)}{\partial T^U_{\tilde a}} \le 0$. Similarly, $\dfrac{\partial G(\tilde a)}{\partial I^L_{\tilde a}} \ge 0$, $\dfrac{\partial G(\tilde a)}{\partial I^U_{\tilde a}} \ge 0$, $\dfrac{\partial G(\tilde a)}{\partial F^L_{\tilde a}} \ge 0$, $\dfrac{\partial G(\tilde a)}{\partial F^U_{\tilde a}} \ge 0$.

Thus, $G(\tilde a)$ is a decreasing function of $T^L_{\tilde a}$ and $T^U_{\tilde a}$, an increasing function of $I^L_{\tilde a}$, $I^U_{\tilde a}$, $F^L_{\tilde a}$ and $F^U_{\tilde a}$. Therefore, $H(\tilde a)$ is an increasing function of $T^L_{\tilde a}$ and $T^U_{\tilde a}$, a decreasing function of $I^L_{\tilde a}$, $I^U_{\tilde a}$, $F^L_{\tilde a}$ and $F^U_{\tilde a}$. The proof of Theorem is completed.

Based on the ranking index of INNs, an approach to compare two INNs is proposed as follow.

Definition 3.2. *Let $\tilde a$ and $\tilde b$ be two INNs, $H(\tilde a)$ and $H(\tilde b)$ be ranking index of $\tilde a$ and $\tilde b$, respectively, then*

(1) If $H(\tilde a) < H(\tilde b)$, then $\tilde a$ is smaller than $\tilde b$, denoted by $\tilde a <_H \tilde b$.

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(2) *If $H(\tilde{a}) = H(\tilde{b})$, then $\tilde{a}$ and $\tilde{b}$ represent the same information, denoted by $\tilde{a} =_H \tilde{b}$.*

The order relation $\leq_H$ is reflexive, antisymmetric, transitive and total, and hence defines a total order on INNs.

3.2. $\leq_L$ implied operation-invariant order

Definition 3.3. *Let $\tilde{a} = \langle [T_{\tilde{a}}^L, T_{\tilde{a}}^U], [I_{\tilde{a}}^L, I_{\tilde{a}}^U], [F_{\tilde{a}}^L, F_{\tilde{a}}^U] \rangle$ and $\tilde{b} = \langle [T_{\tilde{b}}^L, T_{\tilde{b}}^U], [I_{\tilde{b}}^L, I_{\tilde{b}}^U], [F_{\tilde{b}}^L, F_{\tilde{b}}^U] \rangle$ be two INNs. An order relation $\leq_L$ on L is defined by*

$$(1)\ \tilde{a} \leq_L \tilde{b} \text{ iff } T_{\tilde{a}}^L \leq T_{\tilde{b}}^L, T_{\tilde{a}}^U \leq T_{\tilde{b}}^U, I_{\tilde{a}}^L \geq I_{\tilde{b}}^L, I_{\tilde{a}}^U \geq I_{\tilde{b}}^U, F_{\tilde{a}}^L \geq F_{\tilde{b}}^L \text{ and } F_{\tilde{a}}^U \geq F_{\tilde{b}}^U;$$

$$(2)\ \tilde{a} <_L \tilde{b} \text{ iff } \tilde{a} \leq \tilde{b} \text{ and } \tilde{a} \neq \tilde{b}$$

The order relation $\leq_L$ is reflexive, antisymmetric and transitive, and so a partial order. As compared with the order relation $\leq_H$, the order relation $\leq_L$ has some important properties although it is not a total order. According to Theorem 3.1 and Definition 3.3, it is easy to obtain the following proposition.

Proposition 3.2. *Let $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ be INNs and $\alpha \geq 0$.*

(1) *If $\tilde{a} \leq_L \tilde{b}$, then $\tilde{a} \leq_H \tilde{b}$.*

(2) *If $\tilde{a} \leq_L \tilde{b}$, then $\tilde{a} \oplus \tilde{c} \leq_L \tilde{b} \oplus \tilde{c}$ and $\tilde{a} \otimes \tilde{c} \leq_L \tilde{b} \otimes \tilde{c}$.*

(3) *If $\tilde{a} \leq_L \tilde{b}$, then $\alpha\tilde{a} \leq_L \alpha\tilde{b}$ and $\tilde{a}^{\alpha} \leq_L \tilde{b}^{\alpha}$.*

Monotonicity is one of the most important properties of an aggregation function. Therefore it is necessary to define an operation-invariant total order. Inspired by the concept of the operation-invariant total order on IFVs proposed by Liu [15–17] and an order implied operation-invariant total order proposed by Wu et al. [25], the following definition is given.

Definition 3.4. *Let $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ be INNs and $\alpha \geq 0$. A $\leq_L$ implied operation-invariant total order, denoted by $\dot{\leq}$, is a total order on INNs if it satisfies the following conditions*

(1) *If $\tilde{a} \leq_L \tilde{b}$, then $\tilde{a} \dot{\leq} \tilde{b}$.*

(2) *If $\tilde{a} \dot{\leq} \tilde{b}$, then $\tilde{a} \oplus \tilde{c} \dot{\leq} \tilde{b} \oplus \tilde{c}$ and $\tilde{a} \otimes \tilde{c} \dot{\leq} \tilde{b} \otimes \tilde{c}$.*

(3) *If $\tilde{a} \dot{\leq} \tilde{b}$, then $\alpha\tilde{a} \dot{\leq} \alpha\tilde{b}$ and $\tilde{a}^{\alpha} \dot{\leq} \tilde{b}^{\alpha}$.*

In the following, we will propose a $\leq_L$ implied operation-invariant total order.

Definition 3.5. *Let $\tilde{a} = \langle [T_{\tilde{a}}^L, T_{\tilde{a}}^U], [I_{\tilde{a}}^L, I_{\tilde{a}}^U], [F_{\tilde{a}}^L, F_{\tilde{a}}^U] \rangle$ and $\tilde{b} = \langle [T_{\tilde{b}}^L, T_{\tilde{b}}^U], [I_{\tilde{b}}^L, I_{\tilde{b}}^U], [F_{\tilde{b}}^L, F_{\tilde{b}}^U] \rangle$ be two INNs. A $\leq_L$ implied operation-invariant total order, denoted by $\leq_S$, can be defined as follows:*

- *If $T_{\tilde{a}}^U < T_{\tilde{b}}^U$, then $\tilde{a} <_S \tilde{b}$;*
- *If $T_{\tilde{a}}^U = T_{\tilde{b}}^U$, then*
  - ♦ *If $T_{\tilde{a}}^L < T_{\tilde{b}}^L$, then $\tilde{a} <_S \tilde{b}$;*
  - ♦ *If $T_{\tilde{a}}^L = T_{\tilde{b}}^L$, then*
    - ⊛ *If $F_{\tilde{a}}^U < F_{\tilde{b}}^U$, then $\tilde{a} <_S \tilde{b}$;*
    - ⊛ *If $F_{\tilde{a}}^U = F_{\tilde{b}}^U$, then*
      - ⊚ *If $F_{\tilde{a}}^L < F_{\tilde{b}}^L$, then $\tilde{a} <_S \tilde{b}$;*
      - ⊚ *If $F_{\tilde{a}}^L = F_{\tilde{b}}^L$, then*
        - ∗ *If $I_{\tilde{a}}^U < I_{\tilde{b}}^U$, then $\tilde{a} <_S \tilde{b}$;*
        - ∗ *If $I_{\tilde{a}}^U = I_{\tilde{b}}^U$, then*
          - ⋆ *If $I_{\tilde{a}}^L < I_{\tilde{b}}^L$, then $\tilde{a} <_S \tilde{b}$;*
          - ⋆ *If $F_{\tilde{a}}^L = F_{\tilde{b}}^L$, then $\tilde{a} =_S \tilde{b}$.*

It is easy to verify that the order relation $\leq_S$ satisfies the implication conditions and the order-preserving conditions given in the Definition 3.4.

3.3. Comparative analysis with score, accuracy and certainty functions for INNs

Zhang et al. [34] defined the score function, accuracy function and certainty function for an INN $\tilde{a} = \langle [T_{\tilde{a}}^L, T_{\tilde{a}}^U], [I_{\tilde{a}}^L, I_{\tilde{a}}^U], [F_{\tilde{a}}^L, F_{\tilde{a}}^U] \rangle$ as follows:

$$s(\tilde{a}) = \left[T_{\tilde{a}}^L + 1 - I_{\tilde{a}}^U + 1 - F_{\tilde{a}}^U, T_{\tilde{a}}^U + 1 - I_{\tilde{a}}^L + 1 - F_{\tilde{a}}^L \right], \tag{2}$$

$$a(\tilde{a}) = \left[\min\{T_{\tilde{a}}^L - F_{\tilde{a}}^L, T_{\tilde{a}}^U - F_{\tilde{a}}^U\}, \min\{T_{\tilde{a}}^L - F_{\tilde{a}}^L, T_{\tilde{a}}^U - F_{\tilde{a}}^U\} \right], \tag{3}$$

$$c(\tilde{a}) = [T_{\tilde{a}}^L, T_{\tilde{a}}^U]. \tag{4}$$

where $s(\tilde{a})$, $a(\tilde{a})$ and $c(\tilde{a})$ represent the score function, accuracy function, and certainty function of the INN $\tilde{a}$, respectively.

Zhang et al. [34] gave the ranking method as follows:

Definition 3.6. *Let $\tilde{a}$ and $\tilde{b}$ be two INNs, then*

(1) If $p(s(\tilde{a}) \geq s(\tilde{b})) > 0.5$, then $\tilde{a}$ is greater than $\tilde{b}$, denoted by $\tilde{a} >_P \tilde{b}$;

(2) If $p(s(\tilde{a}) \geq s(\tilde{b})) = 0.5$ and $p(a(\tilde{a}) \geq a(\tilde{b})) > 0.5$, then $\tilde{a}$ is greater than $\tilde{b}$, denoted by $\tilde{a} >_P \tilde{b}$;

(3) If $p(s(\tilde{a}) \geq s(\tilde{b})) = 0.5$, $p(a(\tilde{a}) \geq a(\tilde{b})) = 0.5$ and $p(c(\tilde{a}) \geq c(\tilde{b})) > 0.5$, then $\tilde{a}$ is greater than $\tilde{b}$, denoted by $\tilde{a} >_P \tilde{b}$;

(4) If $p(s(\tilde{a}) \geq s(\tilde{b})) = 0.5$, $p(a(\tilde{a}) \geq a(\tilde{b})) = 0.5$ and $p(c(\tilde{a}) \geq c(\tilde{b})) = 0.5$, then $\tilde{a}$ is equal to $\tilde{b}$, denoted by $\tilde{a} =_P \tilde{b}$.

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where $P(a \geq b)$ denotes the degree of possibility of $a \geq b$ and is formulated for $a = [a^L, a^U]$ and $b = [b^L, b^U]$ by

$$P(a \geq b) = \max\left\{1 - \max\left\{\frac{b^U - a^L}{a^U - a^L + b^U - b^L}, 0\right\}, 0\right\}.$$

Remark 3.1. *The order relations $\leq_H$ and $\leq_P$ can reflect the characteristics of INNs and have their own advantages. The order relation $\leq_H$ puts emphasis on geometrical structure of INNs, and $\leq_P$ can reflect the possibility degree of $\tilde{a} \geq \tilde{b}$. Comparing with $\leq_H$ and $\leq_P$, the order relation $\leq_S$ puts a heavy emphasis on the truth-membership degree of an INN and more or less neglects the indeterminacy-membership degree and falsity-membership degree. It does not adequately reflect the characteristics of INNs.*

The order relation $\leq_H$ and $\leq_P$ have some limitations which will be shown in examples below.

Example 3.1. *Let* $\tilde{a} = \langle[0.65, 0.75], [0.10, 0.30], [0.25, 0.55]\rangle$, $\tilde{b} = \langle[0.55, 0.85], [0.10, 0.30], [0.35, 0.45]\rangle$.

(1) *The degrees of possibility of $s(\tilde{a})$ and $s(\tilde{b})$), $a(\tilde{a})$ and $a(\tilde{b})$), $c(\tilde{a})$ and $c(\tilde{b})$) are computed as follows:*

$$p(s(\tilde{a}) \geq s(\tilde{b})) = p(a(\tilde{a}) \geq a(\tilde{b})) = p(c(\tilde{a}) \geq c(\tilde{b})) = 0.5.$$

then from Definition 3.6, $\tilde{a} =_P \tilde{b}$. But it is obvious that $\tilde{a}$ and $\tilde{b}$ are not equal.

(2) *By Eq. (1), $H(\tilde{a}) = 0.7157$, $H(\tilde{b}) = 0.7205$, then $\tilde{a} <_H \tilde{b}$.*

Example 3.2. *Let* $\tilde{a} = \langle[0.65, 0.75], [0.20, 0.45], [0.32, 0.38]\rangle$, $\tilde{b} = \langle[0.62, 0.68], [0.25, 0.40], [0.25, 0.35]\rangle$.

(1) *The degrees of possibility of $s(\tilde{a})$ and $s(\tilde{b})$), $a(\tilde{a})$ and $a(\tilde{b})$), $c(\tilde{a})$ and $c(\tilde{b})$) are computed as follows:*

$$p(s(\tilde{a}) \geq s(\tilde{b})) = p(a(\tilde{a}) \geq a(\tilde{b})) = 0.5,$$

$$p(c(\tilde{a}) \geq c(\tilde{b})) = 0.8125.$$

then from Definition 3.6, $\tilde{a} >_P \tilde{b}$.

(2) *By Eq.* (1), $H(\tilde{a}) = H(\tilde{b}) = 0.7095$, *then $\tilde{a} =_H \tilde{b}$. But it is obvious that $\tilde{a}$ and $\tilde{b}$ are not equal.*

The order relation $\leq_H$ and $\leq_P$ are not order-preserving for operations $\tilde{a} \oplus \tilde{b}$, $\tilde{a} \otimes \tilde{b}$, $\lambda\tilde{a}$ and $\tilde{a}^\lambda$. In the following, counterexamples are given.

Example 3.3. *Let* $\tilde{a} = \langle[0.66, 0.71], [0.35, 0.26], [0.26, 0.30]\rangle$, $\tilde{b} = \langle[0.66, 0.70], [0.32, 0.42], [0.20, 0.23]\rangle$. *From Definition 4.3, $H(\tilde{a}) = 0.7351$, $H(\tilde{b}) = 0.7435$, then $\tilde{a} <_H \tilde{b}$. From Definition 3.6, $p(s(\tilde{a}) \geq s(\tilde{b})) = 0.4848$, then $\tilde{a} <_P \tilde{b}$.*

(1) *Let* $\tilde{c} = \langle[0.64, 0.90], [0.51, 0.52], [0.10, 0.70]\rangle$, *then $\tilde{a} \oplus \tilde{c} >_H \tilde{b} \oplus \tilde{c}$, $\tilde{a} \oplus \tilde{c} >_P \tilde{b} \oplus \tilde{c}$.*

(2) *Let* $\tilde{c} = \langle[0.64, 0.90], [0.51, 0.52], [0.68, 0.70]\rangle$, *then $\tilde{a} \otimes \tilde{c} >_H \tilde{b} \otimes \tilde{c}$, $\tilde{a} \otimes \tilde{c} >_P \tilde{b} \otimes \tilde{c}$.*

(3) *Let $\lambda = 2$, then $\lambda\tilde{a} >_H \lambda\tilde{b}$, $\lambda\tilde{a} >_P \lambda\tilde{b}$.*

(4) *Let $\lambda = 0.2$, then $\tilde{a}^\lambda >_H \tilde{b}^\lambda$, then $\tilde{a}^\lambda >_P \tilde{b}^\lambda$.*

Remark 3.2. *From Example 3.1 and Example 3.2, in some situations, if we use ranking method as given in Definition 3.2 and Definition 3.6 to compare two INNs, the results might be irrational. We can use the order relation $\leq_H$ to compare two INNs if the comparing results is obviously irrational by using the order relation $\leq_P$ and vice versa. Therefore, the order relation $\leq_H$ and $\leq_P$ can be viewed as mutually complementary. From Example 3.3, it is easy to obtain that the order relation $\leq_S$ is order-preserving for operations ranking method. Meantime, it can avoid the irrational results as mentioned in Example 3.1 and Example 3.2, therefore, comparing with $\leq_H$ and $\leq_P$, the order relation $\leq_S$ is more superior.*

4. Interval neutrosophic Choquet integral operator and its properties

4.1. Fuzzy measure and Choquet integral

Definition 4.1. [9, 12, 13, 20, 23] *A fuzzy measure on X is a set function $\mu : P(X) \to [0, 1]$ satisfies the following conditions.*

(1) $\mu(\emptyset) = 0$, $\mu(X) = 1$ (boundary condition)

(2) *If $A, B \in P(X)$ and $A \subseteq B$ then $\mu(A) \leq \mu(B)$* (monotonicity)

One can see that a fuzzy measure is a normal monotone set function which vanishes at the empty set. Furthermore, A fuzzy measure on X is said to be

- additive if $\mu(A \cup B) = \mu(A) + \mu(B)$ for all disjoint subsets $A, B \subseteq X$.
- cardinality-based if, for any $A \subseteq X$, $\mu(A)$ depends only on the cardinality of A.

In the framework of the MCDM, X can be interpreted as a finite decision criteria set. $\mu(A)$ can be viewed as the grade of subjective importance of subset $A \subseteq X$. The monotonicity of the fuzzy measure means that the importance of a subset of criteria cannot decrease when new criteria are added to it [12].

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Definition 4.2. *Let f be a real-valued function on X, the Choquet integral of f with respect to a fuzzy measure μ on X is defined as*

$$(C)\int f d\mu = \sum_{i=1}^{n}(\mu(X_{(i)}) - \mu(X_{(i+1)}))f(x_{(i)})$$

where $(\cdot)$ indicates a permutation on X such that $f(x_{(1)}) \leq f(x_{(2)}) \cdots \leq f(x_{(n)})$ and $X_{(\cdot)} = \{x_{(i)}, \cdots, x_{(n)}\}$, $X_{(n+1)} = \emptyset$.

When using a fuzzy measure to model the importance of the subsets of criteria, the Choquet integral can be viewed as a aggregation function [4, 24, 25]. It has been proposed by many authors as an adequate substitute to weighted arithmetic mean (WAM) or ordered weighted averaging (OWA) operator to aggregate interacting criteria [10, 14, 19]. The Choquet integral identifies with a WAM (resp. OWA) as soon as the fuzzy measure is additive (resp. cardinality-based).

4.2. Interval neutrosophic numbers Choquet integral operator

Definition 4.3. *Let $\tilde{f} : X \to L$ be an interval neutrosophic number function on X, and μ be a fuzzy measure on X. The interval neutrosophic number Choquet integral ($INNCI$) of f with respective to μ is defined as*

$$(C)\int \tilde{f} d\mu = \sum_{i=1}^{n}{}^{\oplus} \left[\mu(X_{(i)}) - \mu(X_{(i+1)})\right] \tilde{f}(x_{(i)}) \quad (5)$$

where $(\cdot)$ indicates a permutation on X according to a given total order $\leq$ such that $\tilde{f}(x_{(1)}) \leq \tilde{f}(x_{(2)}) \cdots \leq \tilde{f}(x_{(n)})$ and $X_{(\cdot)} = \{x_{(i)}, \cdots, x_{(n)}\}$, $X_{(n+1)} = \emptyset$.

Example 4.1. *Let $X = \{x_1, x_2\}$, an interval neutrosophic number function $\tilde{f} : X \to L$ is given by $\tilde{f}(x_1) = \langle[0.59, 0.68], [0.05, 0.15], [0.10, 0.18]\rangle$, $\tilde{f}(x_2) = \langle[0.65, 0.70], [0.25, 0.35], [0.15, 0.30]\rangle$. Since $H(\tilde{f}(x_1)) = 0.5682$, $H(\tilde{f}(x_2)) = 0.5686$, then $\tilde{f}(x_1) <_H \tilde{f}(x_2)$.*

Assume that fuzzy measure $\mu : X \to [0, 1]$ is given by $\mu(\emptyset) = 0$, $\mu(\{x_1\}) = 0.5$, $\mu(\{x_2\}) = 0.3$, $\mu(\{x_1, x_2\}) = 1$. By Eq. (5), we have

$$\begin{aligned}(C)\int \tilde{f} d\mu &= \left[\mu(X) - \mu(x_{(2)})\right] \tilde{f}(x_{(1)}) + [\mu(\{x_2\}) - \mu(\emptyset)]\, \tilde{f}(x_{(2)}) \\ &= \langle[0.6090, 0.6861], [0.0810, 0.1934], [0.1129, 0.2098]\rangle\end{aligned}$$

Theorem 4.1. *Let $\tilde{f} : X \to L$ be an interval neutrosophic number function on X, and μ be a fuzzy measure on X. Then their aggregated value by using the $INNCI$ operator is also an interval neutrosophic number, and*

$$\begin{aligned}(C)\int \tilde{f} d\mu = \Bigg\langle &\left[1 - \prod_{i=1}^{n}\left(1 - T^L_{\tilde{f}(x_{(i)})}\right)^{\mu(X_{(i)})-\mu(X_{(i+1)})}, 1 - \prod_{i=1}^{n}\left(1 - T^U_{\tilde{f}(x_{(i)})}\right)^{\mu(X_{(i)})-\mu(X_{(i+1)})}\right], \\ &\left[\prod_{i=1}^{n}(I^L_{\tilde{f}(x_{(i)})})^{\mu(X_{(i)})-\mu(X_{(i+1)})}, \prod_{i=1}^{n}(I^U_{\tilde{f}(x_{(i)})})^{\mu(X_{(i)})-\mu(X_{(i+1)})}\right], \quad (6) \\ &\left[\prod_{i=1}^{n}(F^L_{\tilde{f}(x_{(i)})})^{\mu(X_{(i)})-\mu(X(i+1))}, \prod_{i=1}^{n}(F^U_{\tilde{f}(x_{(i)})})^{\mu(X_{(i)})-\mu(X_{(i+1)})}\right]\Bigg\rangle\end{aligned}$$

where $(\cdot)$ indicates a permutation on X according to a given total order $\leq$ such that $\tilde{f}(x_{(1)}) \leq \tilde{f}(x_{(2)}) \cdots \leq \tilde{f}(x_{(n)})$ and $X_{(\cdot)} = \{x_{(i)}, \cdots, x_{(n)}\}$, $X_{(n+1)} = \emptyset$.

Proof. The first result can be directly obtained from Definition 4.3 and Theorem 2.1. Eq. (6) is easily proved by using mathematical induction on n.

Definition 4.4. *Let $\tilde{f}$ and $\tilde{g}$ be two interval neutrosophic number functions on X. $\tilde{f}$ and $\tilde{g}$ are said to be comonotonic about a given order relation $\leq$ if*

$$\tilde{f}(x_i) \leq \tilde{f}(x_j) \text{ iff } \tilde{g}(x_i) \leq \tilde{g}(x_j) \ \forall i, j \in \{1, 2, \cdots, n\}$$

The following propositions show some properties of the INNCI operator.

Proposition 4.1. (Idempotency). *Let $\tilde{f}$, $\tilde{g}$ be interval neutrosophic number functions on X, and $\tilde{a}$ be an INN. If $\tilde{f}(x_i) = \tilde{a}$ for $x_i \in X$, then $(C)\int \tilde{f} d\mu = \tilde{a}$.*

Proposition 4.2. *Let $\tilde{f}$, $\tilde{g}$ be interval neutrosophic number functions on X and μ, ν be fuzzy measures on X, then*

$$(C)\int \tilde{f} d\mu \oplus (C)\int \tilde{f} d\nu = (C)\int \tilde{f} d(\mu + \nu).$$

Proposition 4.3. *Let $\tilde{f}$ be interval neutrosophic number function on X, $\lambda > 0$, and μ be fuzzy measures on X, then*

$$C)\int \tilde{f} d(\alpha\mu) = \alpha(C)\int \tilde{f} d\mu.$$

Remark 4.1. *According to Definition 4.3 and statement (4) of Proposition 2.1, it is easy to obtain Proposition 4.1, Proposition 4.2 and Proposition 4.3. The set functions $\mu + \nu$ and $\alpha\mu$, $\alpha > 0$, (in the Proposition 4.2 and 4.3), are not normal fuzzy measures (see Definition 4.1) since their ranges are the interval $[0, 2]$ and $[0, \alpha]$.*

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However, we can also calculate the two expressions, $(C)\int \tilde f d\mu \oplus (C)\int \tilde f dv$ *and* $(C)\int \tilde f d(\alpha\mu)$, *by Eqs.* (5) *and* (6).

Proposition 4.4. (monotonicity). *Let* $\tilde f$, $\tilde g$ *be interval neutrosophic number functions on* X. *For a* $\leq_L$ *implied operation-invariant total order* $\dot\leq$, *if* $\tilde f(x_i)\dot\leq\tilde g(x_i)$ *for all* $x_i \in X$, *then*

$$(C)\int \tilde f d\mu \dot\leq (C)\int \tilde g d\mu.$$

Proof. Let $(\pi(1), \pi(2), \cdots, \pi(n))$ be the permutations on $(1, 2, \cdots, n)$ such that $\tilde f(x_{\pi(1)}) \dot\leq \tilde f(x_{\pi(2)}) \dot\leq \cdots \dot\leq \tilde f(x_{\pi(n)})$.

If $\tilde f$ and $\tilde g$ are comonotonic about the permutations $(\pi(1), \pi(2), \cdots, \pi(n))$, then the conclusion is completed.

In the following, we consider the case that If $\tilde f$ and $\tilde g$ are not comonotonic about the permutation $(\pi(1), \pi(2), \cdots, \pi(n))$. Let

$$\tilde g(x_{\pi(i_0)}) = \min_{1\leq i\leq n}\left\{\tilde g(x_{\pi(i)})\right\} \quad 1 \leq i_0 \leq n$$

The interval neutrosophic numbers function $\tilde f^{(1)}$ is defined as:

$$\tilde f^{(1)}(x_{\pi(j)}) = \begin{cases} \tilde f(x_{\pi(i_0)}), & j \leq i_0, \\ \tilde f(x_{\pi(j)}), & \text{otherwise.} \end{cases}$$

Let $\pi^{(1)} = (\pi^{(1)}(1), \cdots, \pi^{(1)}(n))$ is a permutation such that $\tilde g(x_{\pi^{(1)}(1)})\dot\leq\tilde g(x_{\pi^{(1)}(2)})\dot\leq\cdots\dot\leq\tilde g(x_{\pi^{(1)}(i_0)})$ for $1 \leq j \leq i_0$, and $\pi^{(1)}(j) = \pi(j)$ for $i_0 + 1 \leq j \leq n$.

We will consider two cases.

Case 1. If $\tilde g(x_{\pi^{(1)}(i_0+1)})\dot\leq\tilde g(x_{\pi^{(1)}(i_0+2)})\dot\leq\cdots\dot\leq\tilde g(x_{\pi^{(1)}(n)})$, then the function $\tilde f^{(1)}$ and $\tilde g$ are comonotonic about the permutation $\pi^{(1)} = (\pi^{(1)}(1), \cdots, \pi^{(1)}(n))$. We can obtain $(C)\int \tilde f^{(1)} d\mu \dot\leq (C)\int \tilde g d\mu$.

By the definition of $\tilde f^{(1)}$,

$$(C)\int \tilde f d\mu \dot\leq (C)\int \tilde f^{(1)} d\mu \dot\leq (C)\int \tilde g d\mu.$$

Case 2. If there exists at least one point x_{j_0} such that $\tilde g(x_{\pi^{(1)}(j_0+1)})\dot\geq\tilde g(x_{\pi^{(1)}(j_0+1)})$, we define the interval neutrosophic numbers function $\tilde f^{(2)}$ as

$$\tilde f^{(2)}(x_{\pi(j)}) = \begin{cases} \tilde f(x_{\pi(j_0)}), & i_0 + 1 \leq j \leq j_0, \\ \tilde f^{(1)}(x_{\pi(j)}), & \text{otherwise.} \end{cases}$$

Let $\pi^{(2)} = (\pi^{(2)}(1), \cdots, \pi^{(2)}(n))$ is a permutation such that $\tilde g(x_{\pi^{(2)}(i_0+1)})\dot\leq\tilde g(x_{\pi^{(2)}(i_0+2)})\dot\leq\cdots\dot\leq\tilde g(x_{\pi^{(2)}(j_0)})$ for $1 \leq j \leq i_0$, and $\pi^{(2)}(j) = \pi(j)$ for $1 \leq j \leq i_0$ and $j_0 + 1 \leq j \leq n$.

If $\tilde g(x_{\pi^{(2)}(j_0+1)})\dot\leq\tilde g(x_{\pi^{(2)}(j_0+2)})\dot\leq\cdots\dot\leq\tilde g(x_{\pi^{(2)}(n)})$, then the function $\tilde f^{(1)}$ and $\tilde g$ are comonotonic about the permutation $\pi^{(2)} = (\pi^{(2)}(1), \cdots, \pi^{(2)}(n))$. We can obtain $(C)\int \tilde f^{(2)} d\mu \dot\leq (C)\int \tilde g d\mu$.

By the definition of $\tilde f^{(2)}$,

$$(C)\int \tilde f d\mu \dot\leq (C)\int \tilde f^{(1)} d\mu \dot\leq \int \tilde f^{(2)} d\mu \dot\leq (C)\int \tilde g d\mu.$$

If there exists at least one point x_{k_0} such that $\tilde g(x_{\pi^{(2)}(k_0+1)}) \dot\geq \tilde g(x_{\pi^{(2)}(k_0+1)})$, repeat the above steps until $k_0 = n$. Then we will define a series interval neutrosophic numbers functions $\tilde f^{(k)}$ $(k \leq n)$ and permutations $\pi^{(k)}$, then the function $\tilde f^{(k)}$ and $\tilde g$ are comonotonic about the permutation $\pi^{(k)} = (\pi^{(k)}(1), \cdots, \pi^{(k)}(n))$. We can obtain $(C)\int \tilde f^{(k)} d\mu \dot\leq (C)\int \tilde g d\mu$.

By the definition of $\tilde f^{(k)}$,

$$(C)\int \tilde f d\mu \dot\leq (C)\int \tilde f^{(1)} d\mu \dot\leq \cdots \dot\leq (C) \int \tilde f^{(k)} d\mu \dot\leq (C)\int \tilde g d\mu.$$

Proposition 4.5. *Let* $\tilde f$, $\tilde g$ *be interval neutrosophic number functions on* X. *For a* $\leq_L$ *implied operation-invariant total order* $\dot\leq$, *if* $\tilde f$ *and* $\tilde g$ *are comonotonic about order relation* $\dot\leq$, *then*

$$(C)\int (\tilde f \oplus \tilde g) d\mu = (C)\int \tilde f d\mu \oplus (C)\int \tilde g d\mu.$$

Proof. According to Definition 4.4 and Definition 3.4, it is easy to get the conclusion.

Proposition 4.6. *Let* $\tilde f$ *be interval neutrosophic number function on* X. *For a* $\leq_L$ *implied operation-invariant total order* $\dot\leq$ *and* $\lambda \geq 0$,

$$(C)\int \lambda \tilde f d\mu = \lambda (C)\int \tilde f d\mu.$$

Proof. According to Definition 3.4, it is easy to get the conclusion.

From Proposition 4.5 and 4.6, we can obtain the following corollary.

Corollary 4.1. *Let* $\tilde f$, $\tilde g$ *be interval neutrosophic number functions on* X, $\tilde a$ *be an INNs. For a* $\leq_L$ *implied operation-invariant total order* $\dot\leq$ *and* $\lambda \geq 0$,

$$(C)\int \lambda(\tilde f \oplus \tilde a) d\mu = \lambda\left((C)\int \tilde f d\mu \oplus \tilde a\right).$$

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It is not hard to see that the aggregation properties of the INNCI greatly depend on the given order relation. Based on the order relations $\leq_H$ and $\leq_P$, the INNCI only has the Properties $4.1-4.2$. However, based on a $\leq_L$ implied operation-invariant total order, the INNCI has all the properties presented above.

Zhang et al. [34] have developed interval neutrosophic number weighted averaging (INNWA) operator and interval neutrosophic number weighted geometric (INNWG) operator, which are respectively defined as follows.

Definition 4.5. *Let $\tilde{a}_j$ $(j=1,2,\cdots,n)$ be a collection of INNs, and let $INNWA: INN^n \to INN$, INNWA is called an interval neutrosophic number weighted averaging operator if*

$$INNWA_\omega(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\sum_{i=1}^{n}{}^{\oplus}\ \omega_i\tilde{a}_i$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$ is the weight vector of $\tilde{a}_i$ $(i=1,2,\cdots,n)$, with $\omega_i\geq 0$ $(i=1,2,\cdots,n)$ and $\sum\limits_{i=1}^{n}\omega_i=1$.

Definition 4.6. *Let $\tilde{a}_j$ $(j=1,2,\cdots,n)$ be a collection of INNs, and let $INNWG: INN^n \to INN$, INNWG is called an interval neutrosophic number weighted geometric operator if*

$$INNWG_\omega(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\sum_{i=1}^{n}{}^{\oplus}\ \tilde{a}_i^{\omega_i}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$ is the weight vector of $\tilde{a}_i$ $(i=1,2,\cdots,n)$, with $\omega_i\geq 0$ $(i=1,2,\cdots,n)$ and $\sum\limits_{i=1}^{n}\omega_i=1$.

The following theorems show that INNCI operator is a generalization of INNWA and INNWG.

Theorem 4.2. *Let $\tilde{f}$ be an interval neutrosophic number function on X, μ be a fuzzy measure on X.*

(1) If μ is additive, then there exists $\omega\in[0,1]^n$, such that $(C)\int\tilde{f}d\mu = INNWA$.

(2) If μ is cardinality-based, then there exists $\omega\in[0,1]^n$, such that $(C)\int\tilde{f}d\mu = INNWG$.

Proof. (1) If μ is additive, let $\omega_i=\mu(\{i\})$, then the conclusion is obvious.

(2) If μ is cardinality-based, ω is defined as follows:

$\omega_n=\mu(\{i\})$ for any $\{i\}\subseteq X$,

$\omega_{n-1}=\mu(\{i,j\})-\omega_n$ for any $\{i,j\}\subseteq X$,

$\cdots$

$\omega_{n-k}=\mu(\{i+1,i+2,\cdots,i+k\})-\omega_{n-k+1}$ for any $\{i+1,i+2,\cdots,i+k\}\subseteq X$,

$\cdots$

$\omega_1=1-\omega_2$,

then the proof is completed.

Remark 4.2. *The relationship between fuzzy measure and weight vector can be given as follows:*

(1) The additive fuzzy measure μ associated to an INNWA is given by $\mu(A)=\sum_{x_i\in A}\omega_i$ for $A\subseteq X$.

(2) The cardinality-based fuzzy measure μ associated to an INNWG is given by $\mu(A)=\sum_{i=n-|A|+1}^{n}\omega_i$ for any non-empty subset $A\subseteq X$.

5. Multicriteria decision-making method based on interval neutrosophic numbers Choquet integral operator

This section presents a new method for MCDM, in which the partial evaluations of the alternatives are given by INNs and the interaction among the criteria are allowed.

For a MCDM problem, let $Y=\{y_1,y_2,\cdots,y_m\}$ be a set of alternatives, $X=\{x_1,x_2,\cdots,x_n\}$ be a set of criteria. To get the best alternative, the MCDM procedure based on INNCI operator is proposed as follows.

Step 1. Construct the INNs decision matrix. Assume that the partial evaluation of the alternative y_i $(i=1,2,\cdots,m)$ on the criteria x_j $(j=1,2,\cdots,n)$ is measured by INN $\tilde{d}_{ij}=\langle T_{ij},I_{ij},F_{ij}\rangle$, where T_{ij} indicates the degree to which the alternative y_i satisfies the criterion x_j, I_{ij} indicates the indeterminacy degree to which the alternative y_i satisfies or does not satisfy the criterion x_j, F_{ij} indicates the degree to which the alternative y_i does not satisfy the criterion x_j, and $T_{ij}\subseteq[0,1]$, $I_{ij}\subseteq[0,1]$, $F_{ij}\subseteq[0,1]$. Then we can obtain a decision making matrix as follow:

$$\tilde{D}=\begin{pmatrix}\tilde{d}_{11},\tilde{d}_{12},\cdots,\tilde{d}_{1n}\\ \tilde{d}_{21},\tilde{d}_{22},\cdots,\tilde{d}_{2n}\\ \cdots\ \cdots\ \cdots\\ \tilde{d}_{m1},\tilde{d}_{m2},\cdots,\tilde{d}_{mn}\end{pmatrix}$$

Step 2. Reorder the partial evaluation $\tilde{d}_{ij}$ of the alternative y_j such that $\tilde{d}_{i(j)}\leq\tilde{d}_{i(j+1)}$ for a given order relation. For example, by order relation $\leq_H$ of Definition 3.2, we calculate $H(\tilde{d}_{ij})$ of the partial evaluation $\tilde{d}_{ij}$ of the alternative y_j $(j=1,2,\cdots,m)$ on the criteria x_i $(i=1,2,\cdots,n)$, and then rank the partial evaluation $\tilde{d}_{ij}$ such that $\tilde{d}_{i(j)}\leq_H\tilde{d}_{i(j+1)}$.

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Step 3. Identify the fuzzy measure on the criterion set $X = \{x_1, x_2, \cdots, x_n\}$. There are several methods for the determination of the fuzzy measure, such as linear methods [18], quadratic methods [6, 7], heuristic-based methods [8], genetic algorithms [28] and so on.

Step 4. Choose the INNCI operator to aggregate the partial evaluations of each alternative and get the overall evaluations.

Step 5. Rank those overall evaluations according to the given total order relation on INNs, and select the best one(s).

6. An illustrative example

This section presents an application of the proposed method to select the third party logistics providers. Suppose that there are four providers (y_1, y_2, y_3, y_4) whose core competencies are evaluated by means of the following four criteria (x_1, x_2, x_3, x_4):

(1) the cost of service (x_1);
(2) the operational experience in the industry (x_2);
(3) customer satisfaction (x_3);
(4) market reputation (x_4).

6.1. Procedures of decision making based on three order relation $\leq_H$, $\leq_P$ and $\leq_S$

Step 1. The INNs decision matrix of the third party logistics providers is made up according to the four evaluating criteria. The evaluation of a provider $y_i (i = 1, 2, 3, 4)$ with respect to a criteria $x_j (j = 1, 2, 3, 4)$ is obtained from the experts. Suppose that the INNs decision matrix is constructed as shown in Table 1.

Step 2. According to Table 1, by Definition 3.2, Definition 3.5 and Definition 3.6, the partial evaluation $\tilde{d}_{ij}$ of the candidate y_j is reordered such that $\tilde{d}_{i(j)} \leq_* \tilde{d}_{i(j+1)}$ $(i = 1, 2, 3, 4)$, where $\leq_*$ denotes the order relations $\leq_H$, $\leq_P$ and $\leq_S$, respectively. The results are shown in Table 2.

Therefore

$$\tilde{d}_{1(1)} = \tilde{d}_{12}, \tilde{d}_{1(2)} = \tilde{d}_{13}, \tilde{d}_{1(3)} = \tilde{d}_{11}, \tilde{d}_{1(4)} = \tilde{d}_{14};$$
$$\tilde{d}_{2(1)} = \tilde{d}_{24}, \tilde{d}_{2(2)} = \tilde{d}_{21}, \tilde{d}_{2(3)} = \tilde{d}_{23}, \tilde{d}_{4(4)} = \tilde{d}_{22};$$
$$\tilde{d}_{3(1)} = \tilde{d}_{31}, \tilde{d}_{3(2)} = \tilde{d}_{34}, \tilde{d}_{3(3)} = \tilde{d}_{33}, \tilde{d}_{3(4)} = \tilde{d}_{32};$$
$$\tilde{d}_{4(1)} = \tilde{d}_{42}, \tilde{d}_{4(2)} = \tilde{d}_{43}, \tilde{d}_{4(3)} = \tilde{d}_{44}, \tilde{d}_{4(4)} = \tilde{d}_{41}.$$

Step 3. Suppose that the fuzzy measures of criteria of X and subsets of X are shown in Table 3.

Table 1
The INNs decision matrix of the third party logistics providers

	x_1
y_1	⟨[0.60, 0.76], [0.22, 0.33], [0.23, 0.30]⟩
y_2	⟨[0.66, 0.71], [0.26, 0.31], [0.25, 0.33]⟩
y_3	⟨[0.66, 0.71], [0.26, 0.31], [0.25, 0.33]⟩
y_4	⟨[0.60, 0.76], [0.22, 0.33], [0.23, 0.30]⟩
	x_2
y_1	⟨[0.70, 0.72], [0.31, 0.37], [0.28, 0.31]⟩
y_2	⟨[0.68, 0.75], [0.19, 0.29], [0.28, 0.35]⟩
y_3	⟨[0.68, 0.75], [0.19, 0.29], [0.28, 0.35]⟩
y_4	⟨[0.70, 0.72], [0.31, 0.37], [0.28, 0.31]⟩
	x_3
y_1	⟨[0.71, 0.73], [0.21, 0.27], [0.35, 0.39]⟩
y_2	⟨[0.55, 0.73], [0.20, 0.25], [0.23, 0.30]⟩
y_3	⟨[0.55, 0.73], [0.20, 0.25], [0.23, 0.30]⟩
y_4	⟨[0.71, 0.73], [0.21, 0.27], [0.35, 0.39]⟩
	x_4
y_1	⟨[0.71, 0.77], [0.23, 0.30], [0.29, 0.37]⟩
y_2	⟨[0.65, 0.70], [0.31, 0.32], [0.25, 0.27]⟩
y_3	⟨[0.65, 0.72], [0.25, 0.32], [0.22, 0.35]⟩
y_4	⟨[0.61, 0.74], [0.28, 0.30], [0.20, 0.32]⟩

Table 2
Ranking order of $\tilde{d}_{ij}$ based on the order relation $\leq_*$.

ranking order	$\leq_*$
y_1	$\tilde{d}_{12} \leq_* \tilde{d}_{13} \leq_* \tilde{d}_{11} \leq_* \tilde{d}_{14}$
y_2	$\tilde{d}_{24} \leq_* \tilde{d}_{21} \leq_* \tilde{d}_{23} \leq_* \tilde{d}_{22}$
y_3	$\tilde{d}_{31} \leq_* \tilde{d}_{34} \leq_* \tilde{d}_{33} \leq_* \tilde{d}_{32}$
y_4	$\tilde{d}_{42} \leq_* \tilde{d}_{43} \leq_* \tilde{d}_{44} \leq_* \tilde{d}_{41}$

Table 3
Every subset A of X and its corresponding fuzzy measure value

A	$\mu(A)$	A	$\mu(A)$	A	$\mu(A)$
∅	0	{3}	0.10	{1, 3}	0.20
{1}	0.10	{4}	0.10	{1, 4}	0.50
{2}	0.10	{1, 2}	0.30	{2, 3}	0.4
A	$\mu(A)$	A	$\mu(A)$		
{2, 4}	0.40	{1, 3, 4}	0.60		
{3, 4}	0.20	{2, 3, 4}	0.30		
{1, 2, 3}	0.85	{1, 2, 3, 4}	1		

Table 4
Results of aggregation based on INNCI operator

	Results of aggregation
y_1	⟨[0.6657, 0.7428], [0.2523, 0.3354], [0.2656, 0.3187]⟩
y_2	⟨[0.6308, 0.7189], [0.2391, 0.2901], [0.2466, 0.3130]⟩
y_3	⟨[0.6335, 0.7193], [0.2338, 0.2878], [0.2498, 0.3207]⟩
y_4	⟨[0.6582, 0.7362], [0.2461, 0.2965], [0.2624, 0.3430]⟩

Step 4. By Eq. (6), utilizing the INNCI operator to aggregate the partial evaluations of each alternative y_i $(i = 1, 2, 3, 4)$. The results are shown in Table 4.

Step 5. According to the overall evaluations of the third party logistic providers, by Definition 3.2,

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Table 5
Ranking order of the third party logistics providers.

Order relation	Ranking order
$\leq_H$	$y_1 \succ y_2 \succ y_3 \succ y_4$
$\leq_P$	$y_2 \succ y_3 \succ y_4 \succ y_1$
$\leq_S$	$y_1 \succ y_4 \succ y_3 \succ y_2$

Definition 3.5 and Definition 3.6, the ranking order based on the order relation $\leq_*$ of providers is shown in Table 5.

6.2. Comparison analysis of results

In Section 6.1, we have given the ranking order based on three order relations $\leq_H$, $\leq_P$ and $\leq_S$, but from the results shown in Table 5, the raking order calculated based on three order relations is different. Therefore, it is very difficult to decide which alternative is the best choice and whether an alternative is definitely better than another one.

From Table 1, we have that

$$\tilde{d}_{11} = \tilde{d}_{41}, \tilde{d}_{12} = \tilde{d}_{42}, \tilde{d}_{13} = \tilde{d}_{43},$$

$$\tilde{d}_{14} >_S \tilde{d}_{44},$$

$$\tilde{d}_{14} >_H \tilde{d}_{44} \text{ (since } H(\tilde{d}_{14}) = 0.7496 > H(\tilde{d}_{44}) = 0.7460),$$

$$\tilde{d}_{14} >_P \tilde{d}_{44} \text{ (since } p(s(\tilde{d}_{14}) > s(\tilde{d}_{44})) = 0.5417).$$

Which mean that, on each criterion, the partial evaluation of y_1 is larger or at least equal to that of y_4 (denoted as $y_1 \succ y_4$). Hence, the overall evaluation of y_1 should be larger or at least equal to that of y_4. But unfortunately, aggregation function INNCI produce the contrary results based on the order relation $<_P$:

$$\text{INNCI}(y_1) <_P \text{INNCI}(y_4).$$

It is obvious that the above result is unreasonable. Table 6 shows the unreasonable results in this illustrative example based on the order relation $\leq_H$ and $\leq_P$. The main reason for such unreasonable results is that, as mentioned in Section 3.3, the order relation $\leq_H$ and $\leq_P$ are not order-preserving for operations, that is , these two order relations are not an operation-invariant total order on INNs. The increasingness property of an aggregation function strongly depends on the order-preserving property of the given order relation. Therefore, the interval neutrosophic numbers

Table 6
The unreasonable results based on the order relation$\leq_H$ and $\leq_P$.

	Corresponding unreasonable ranking order
$y_1 >_Z y_4$	$y_1 <_Z y_4$
$y_2 <_Z y_3$	$y_2 >_Z y_3$
$y_2 <_H y_3$	$y_2 >_H y_3$

Choquet integral operator, INNCI, is not increasing with respect to these two order relation.

The key to avoiding such unreasonable results is to adopt a $\leq_L$ implied operation-invariant total order (see Definition 3.5) in the aggregation process. Based on the $\leq_L$ implied operation-invariant total order $\leq_S$, we can generate the overall evaluations of the alternatives by using the INNCI, as shown in Table 5, and then we will select provider y_1.

7. Conclusions

Interval neutrosophic set (INS) is a subclass of a neutrosophic set, which can be applied in the problems with uncertain, imprecise, incomplete, and inconsistent information existing in real applications. As an aggregation function, the Choquet integral with respect to fuzzy measures is able to flexibly describe the relative importance of decision criteria as well as their interactions. In this paper, we combined the Choquet integral and the INS theory to propose INNCI operator for MCDM problem with netrosophic information and investigated their aggregation properties, such as idempotency and monotonicity. INNCI operator can represent INNWA and INNWG. Therefore, INNCI operator is superior to existing operators.

Increasingness is a natural requirement for an aggregation function in MCDM. The increasingness property of INNCI aggregation function strongly depends on the order-preserving property of the given order relation. In this paper, we proposed two approaches to compare two INNs. According to its geometrical structure, the ranking index is developed according to some geometrical quantities which reflect the inferiors and superiors of INNs. Based on the ranking index, an order relation, denoted by $\leq_H$, is proposed. Examples shown that order relations $\leq_H$ and $\leq_P$ have some limitations, but they can be viewed as mutually complementary if the comparing results is obviously irrational. Because order relation $\leq_H$ and $\leq_P$ are not order-preserving for operations, we furthermore proposed a $\leq_L$ implied operation-invariant

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total order to ensure the increasingness of INNCI operator, and it is more superior than the order relations $\leq_H$ and $\leq_P$.

We only have proposed a kind of a $\leq_L$ implied operation-invariant total order (Definition 3.5), which does not adequately reflect the characteristics of INNs. Therefore, it is of great interest to find a $\leq_L$ implied operation-invariant total order which can better reflect the characteristics of INNs. Furthermore, how to find a good aggregation operation is also an important key issue in netrosophic MCDM problems.

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