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PRELIMINARY DISPLACEMENT-BASED ASSESSMENT PROCEDURE FOR BUILDINGS ON LIQUEFIED SOIL

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ABSTRACT

This paper provides a simple preliminary procedure for estimating the performance of a building on liquefiable soil. The procedure directly accounts for damage related to ground shaking and in-directly accounts for settlements. Additionally, it also considers the change in shaking demand and changes to the natural vibration modes of the systems due to liquefaction. The proposed procedure makes use of a displacement-based assessment procedure that considers nonlinear soil-foundation-structure interaction and extends it to include the effects of liquefaction. The extensions rely on several assumptions about the behaviour of the soil, site response and the structure, which require further research to improve the robustness of the assessment.
Two small studies are conducted: one explores at what time the peak displacement of a system occurs during shaking, and the second explores the potential changes in site amplification due to liquefaction, which provide some justification to the proposed assumptions for the procedure. The procedure is applied to a six-storey two-bay case study reinforced concrete frame building to demonstrate the influence of various effects of liquefaction. For the case study building, the role of shaking damage was large and the estimated reduction in shaking demand was important to the estimated level of ductility demand, highlighting the importance of quantifying the expected site response for the assessment of building performance.

Keywords: Liquefaction, Shallow foundations, Performance-assessment, soil-foundation-structure interaction

1. INTRODUCTION

Current approaches to the performance assessment of buildings on liquefied soil have a strong focus on damage related to soil and foundation deformation. The disregard of damage associated to strong ground shaking has been justified by the natural isolation that can occur due to the weakening of the soil during liquefaction. However, complete liquefaction does not occur instantly at the beginning of shaking (e.g. Wildlife record from the 1987 Superstition Hills earthquake (Kramer et al., 2011)), and therefore the building can be exposed to intense shaking prior to liquefaction or while the soil is in a semi-liquefied state. The partial development of liquefaction under a building causes modification to the dynamic properties of both the soil deposit and soil-foundation-structure system and could potentially amplify the response beyond the non-liquefied conditions. Centrifuge experiments (e.g. Dobry and Liu, 1994) and numerical simulations (e.g. Karamitros et al., 2013) have also highlighted that high vertical stress from the foundation limits the build-up of pore pressure to the extent that negative pore pressures can even develop directly under the foundation. The limitation of pore pressure build-up under high vertical stress can result in buildings being subjected to strong shaking even though liquefaction occurs in nearby free-field conditions. The strong shaking response is seen in the centrifuge experiments by (Dashti et al., 2010) shown in Figure 1), for the centrifuge

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experiment titled SHD02-04. The results show that even after pore pressure build up, the building still had a strong shaking response as seen in Figure 1 (a).

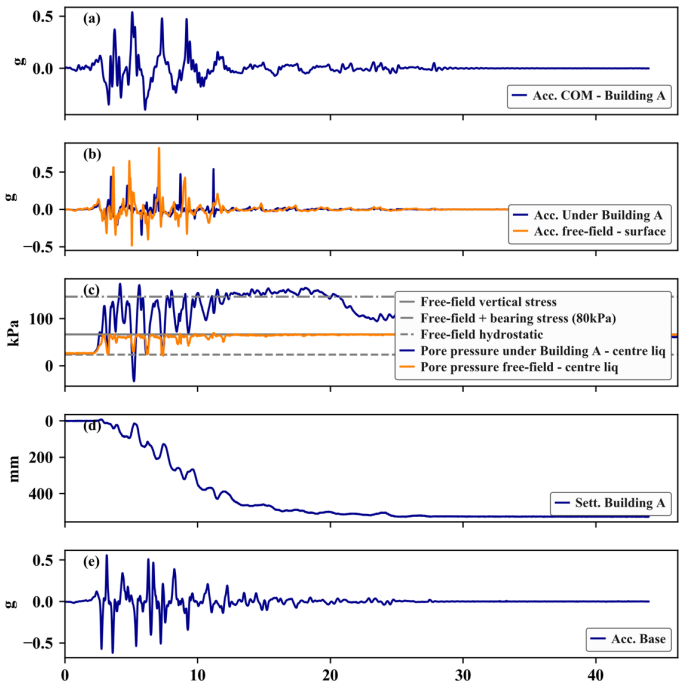

Figure 1. Soil, foundation and structure response from centrifuge experiment SHD02-04 from Dashti et al. (2010)

Clearly it is not acceptable to disregard strong shaking when liquefaction occurs and there is a need to better understand soil-liquefaction-foundation-structure interaction, as well as to develop a framework to consider the combined damage of both soil-foundation deformation and ground shaking. Furthermore, the development of liquefaction mitigation techniques that focus only on limiting pore pressure development (e.g. several methods in MBIE (2016)), should be re-assessed in regards to both soil/foundation deformation and ground shaking.

The numerical simulation of buildings on liquefiable soil that can simulate both the fully-coupled soil-fluid effective stress behaviour of the soil and the degradation and collapse of the structure are still beyond the capabilities of available software. However, simple analytical and empirical techniques can provide useful insights into the expected level of damage from soil-foundation deformation and damage from ground shaking, which can help the engineer focus on the most critical parts of the building. This paper presents the first efforts to develop a simplified procedure to include both liquefaction-related damage and ground shaking damage for use in preliminary building assessment. The proposed procedure makes use of existing literature and experimental works and attempts to highlight the missing research that is needed to increase the robustness of this approach.

2. OVERVIEW OF DISPLACEMENT-BASED ASSESSMENT

The displacement-based assessment (DBA) procedure considers the performance of a building under a level of displacement demand (Sullivan et al., 2014). The procedure allows the consideration of soil-foundation-structure interaction (SFSI) through the use of effective stiffness properties of the structure and the soil and through displacement reduction factors (or equivalent viscous damping) to capture energy loss through hysteretic and radiation damping in the foundation.

The DBA procedure from Millen et al. (2016) is outlined here and in Figure 2, which includes nonlinear SFSI:

1. Assess the pushover response of the structure to determine the yielding and the ultimate force and displacement
2. Determine the displacements from the foundation at the point of structural failure
3. Convert the soil-foundation-structure system to an SDOF with an equivalent mass, height, stiffness and a factor to reduce the elastic displacement spectrum to account for energy dissipation
4. Reduce the spectrum and assess whether the displacement capacity of the SDOF is greater than the spectral demand.

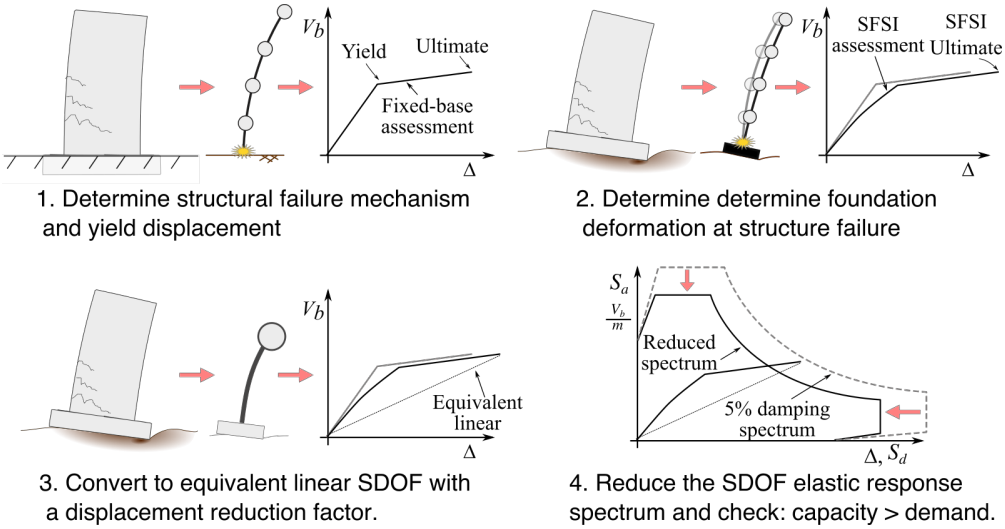

Figure 2. Displacement-based Assessment with SFSI

3. EFFECTS OF LIQUEFACTION

To account for liquefaction in this procedure, the influence of liquefaction compared to a conventional SFSI problem can be considered through three effects:

1. Changes in the ground shaking hazard (modify the displacement spectrum and displacement reduction factors)
2. Changes in the soil-foundation-structure system (modification to the effective stiffness properties of the soil-foundation interface)
3. Increases in the soil-foundation permanent deformations (modification to local damage and the structural yield and ultimate displacements due to differential settlement, changes in overall performance due to rigid body tilt and settlement)

3.1 Changes in ground shaking hazard

The displacement-based assessment procedure is concerned with the maximum ductility demand of the structure. If the strongest shaking occurs prior to the development of liquefaction, it could be expected that the building performance in terms of maximum ductility demand would be similar to assessing the building in non-liquefied conditions. However, the strongest shaking (at the base of the deposit) may occur after liquefaction, meaning that the liquefied soil would modify the surface shaking. Liquefaction tends to reduce high frequency ground shaking and can potentially increase low frequency shaking. Deterministically, it is impossible to accurately determine the maximum shaking demand on the building as the development of pore pressures is highly sensitive to the soil conditions, and the soil properties after liquefaction are poorly understood. However, two simple studies can highlight the relative importance of these

two concepts (peak response before liquefaction occurs, and amplified low frequency content). The first study uses the second set of 40 ground motions from Millen (2016) that were selected from site with Vs,30 values of between 120-360m/s from the ground motion data from Ancheta et al. (2013). A series of elastic SDOF analyses were conducted at various periods to determine when the peak displacement would occur in relation to the significant duration of the record, determined using the cumulative acceleration according to Trifunac and Brady (1975). The maximum response for two periods (0.5 seconds and 4.0 seconds) and a critical damping of 20% for the ground motion RSN3317_2 are shown in Figure 3 (a), and the corresponding input acceleration and significant duration are shown in Figure 3 (b).

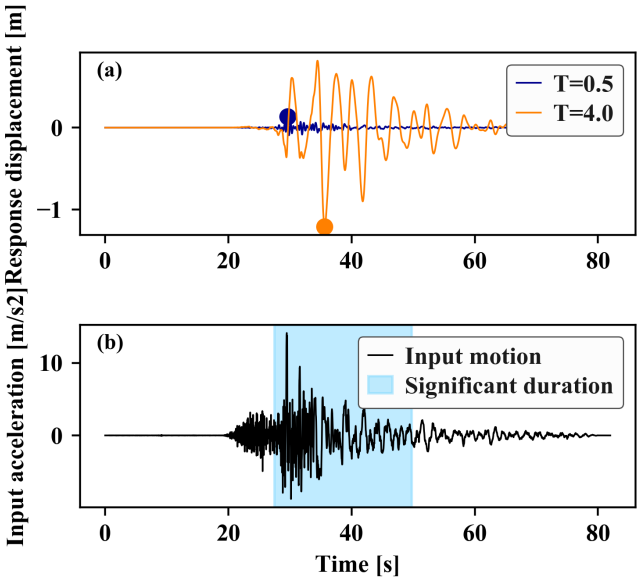

Figure 3. RSN3317_2 motion (a) Response of Elastic systems (b) Time series and significant duration

The results of the 40 ground motions for SDOF periods between 0.1-5 seconds and critical damping ξ of 20% are shown in Figure 4. It can be seen that short period structures typically experience their peak displacement earlier in the motion, while for longer period structures the peak displacement occurs later. Figure 4 also highlights that for short period structures (less than 1.5 seconds), the peak response typically occurs in the first 30% of the strong shaking.

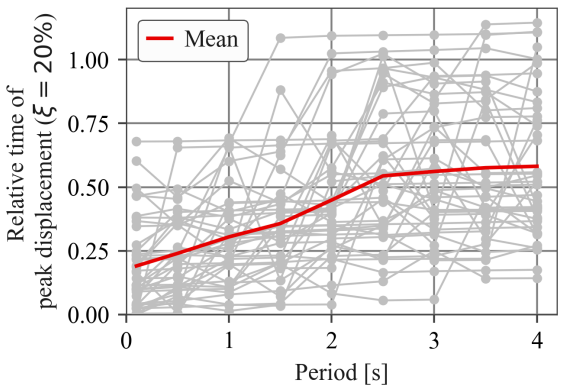

Figure 4. Occurrence of response displacement of elastic systems

While this study uses only elastic systems and therefore may only be appropriate for low-ductility systems, a study by Gazetas (2012) investigated the response of nonlinear systems. Gazetas (2012) showed that linear systems typically develop their peak response through the

cumulative excitation of shaking, while the peak response of rigid-inelastic (sliding-block) systems develop their peak response due to the excitation of a single pulse. Therefore, it could be expected that the peak response of a nonlinear system would occur earlier in a record than for an equivalent linear system, since the cumulative excitation is less important if the nonlinearity increases. However, the characteristics of the individual ground motion in terms of the occurrence of pulses and the rate of pore pressure development in soil would govern the actual peak response of the structure.

The second study simulated the site response using simple assumptions of the change in soil stiffness and energy dissipation due to liquefaction and modelled the response using linear elastic analysis. Liquefaction is a highly nonlinear phenomenon; however, the frequency content of the surface motion is largely dependent on two parameters: the shear wave velocity and energy dissipation (or viscous damping).

Ground motions are modified as they travel up through a soil deposit, and some frequency content is amplified while other frequencies are de-amplified, largely based on the natural period of the site and the standing waves that develop. The natural period of a site (T_{site}) can be determined through Equation 1, where $H_{profile}$ is the height of the soil profile and $V_{s,av.}$ is the average shear wave velocity of the profile.

$$T_{site} = \frac{4H_{profile}}{V_{s,av.}} \tag{1}$$

As the shear stiffness of the soil deteriorates, the site period increases and subsequently can amplify longer period motion (Bouckovalas et al., 2016). In this study, a 20m soil profile is modelled over an elastic bedrock (Vs=800m/s). In the first analysis, the soil is modelled with a shear wave velocity of 120m/s, a unit weight of 18kN/m^3 and a critical damping ratio of 5% to simulate non-liquefied conditions. In the second study, the shear wave velocity is reduced to 30% of the original value and the critical damping ratio is increased to 25% over the lower 10 metres of the deposit to simulate liquefaction. The first five ground motions from the previous study (motion codes: RSN3271_1, RSN3317_2, RSN3512_1, RSN3663_1, RSN3670_1) were first scaled to match the design spectrum with a hazard factor of 0.3 and a soil class C and then were input at the base of the soil profile. The response spectra of the surface shaking compared to the original scaled motions are shown in Figure 5. It can clearly be seen that for the non-liquefied case the soil deposit amplifies the response around the period range of 0.8 seconds and is relatively unchanged over the remainder of the spectrum. The liquefied deposit shows a reduction in response in the low period range, due to the increase in damping, however, there is strong amplification in the period range around 3 seconds. This analysis is extremely simplistic and an elastic analysis is not suitable for simulating the highly nonlinear liquefaction phenomenon, the main drawback being that an elastic analysis means that the standing waves are at a constant frequency through the whole motion and therefore a strong amplification develops at these frequencies. In a profile that is liquefying, the natural frequency of the deposit is constantly changing so amplification does not develop at a single period. However, a recent proposal by Bouckovalas et al. (2017) suggests the elastic design spectra can be obtained from the envelope of two equivalent linear analyses. The first analysis considers the response of pre-liquefaction ground motion and site conditions and the second analysis considers the ground motion after liquefaction using post-liquefaction site conditions.

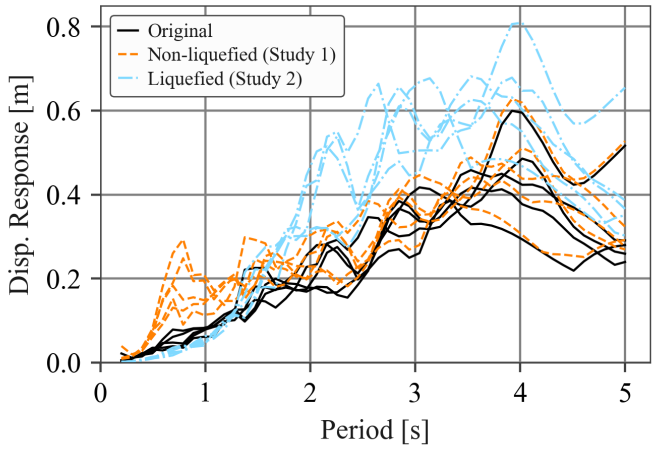

Figure 5. Shift in response spectra due to site effects

3.2 Changes in the soil-foundation-structure system

Once liquefaction has occurred, the soil has softened considerably, which alters the foundation impedance. Karatzia et al. (2017) developed expressions to quantify the small strain foundation impedance (stiffness and damping) for circular and equivalent circular surface foundations on liquefied soil deposits with a clay crust. The results showed a decrease in the rocking stiffness of almost 40% for a shallow crust (foundation width to crust depth ratio of 0.5). The nonlinear stiffness in terms of uplift behaviour and soil yielding would also be expected to change. The nonlinear response at large strains is also expected to change as the strength of the liquefied layer has also decreased.

3.3 Increases in the soil-foundation permanent deformations

Liquefaction produces a dramatic reduction in stiffness and strength which often results in settlement and tilting of the foundation (See Figure 1 (d) settlement results from Dashti et al. (2010)). The level of deformation depends on numerous factors ranging from pore water flow rates, to soil heterogeneity or stress fields from adjacent buildings. Some of this deformation can occur in a uniform manner such as rigid-body settlement and rigid-body tilting (Figure 6), which can cause health issues for building occupants (Keino and Kohiyama, 2012). However, when the deformation happens in a non-uniform manner it cannot only cause health-related effects but can also introduce additional stresses and strains in the superstructure (Figure 6).

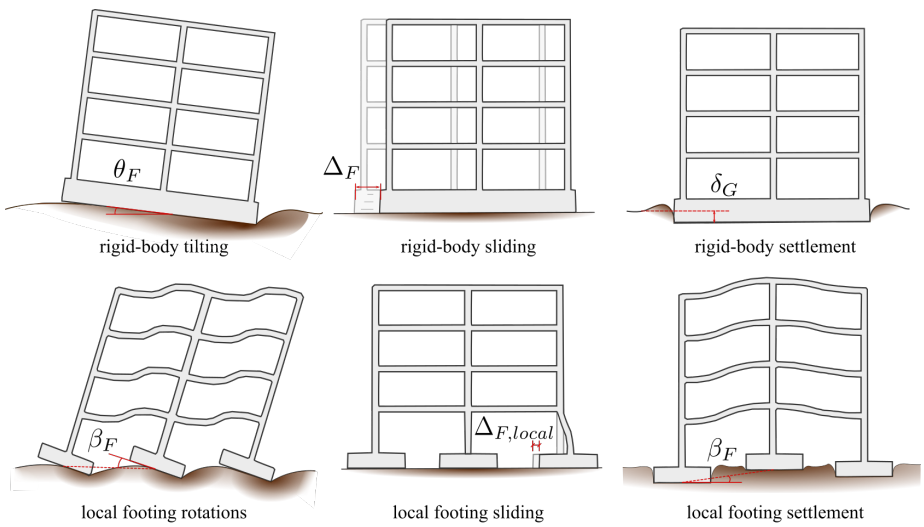

Figure 6. Rigid-body and differential movements of the foundation *(taken from Millen (2016))*

The additional stresses in the superstructure result in an earlier onset of yielding failure of members. Figure 7 shows the conceptual change in the push over response of a structure due to differential settlements, where the yield response is smoother due to earlier yielding of some members, while others are delayed until the stresses are redistributed and eventually failure occurs earlier due to the higher strains in the earlier yielding members.

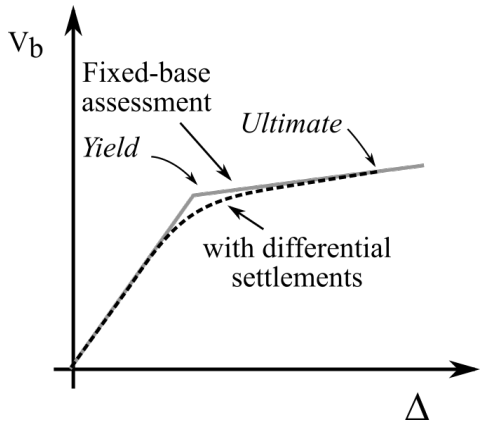

Figure 7. Expected influence of differential settlements on the pushover response of a building

The level of shear demand on the beams due to the complete loss of bearing under one footing compared to the demands of seismic action can be estimated using the Equation 2 from Gomez et al. (2018), where n_s is the number of storeys, h is the inter-storey height, S_a is the seismic acceleration (or spectral acceleration), and L_b is the average distance between columns. This value ranges from 0.1 - 0.3 for typical reinforced concrete frame buildings.

$$\sum V_{beam,eq} \approx \frac{2(0.7 \cdot n_s - 0.6) \cdot h \cdot S_a}{L_b} \tag{2}$$

Equation 2 does not correspond directly to a reduction in yield and ultimate displacement capacity, and requires a nonlinear analysis of individual buildings to assess how stresses and strains would be redistributed within the structure. However, it could be expected that a strong correlation does exist and that differential displacements could be incorporated into a displacement-based procedure in a rational manner for preliminary performance assessments.

3.4 Assumptions to extend DBA for liquefaction

The purpose of this paper is to propose an extension of the displacement-based assessment procedure to highlight the current deficiencies in the procedure and to examine the magnitude of their influence. To achieve this initial extension several broad assumptions have been applied as outlined below, and can be considered as areas of further research. Additionally, the assumptions provide exact values for phenomena that are inherently variable to allow the building to be assessed deterministically and thus highlight significant trends. However, it is recognised that a probabilistic framework would be more appropriate, even for a preliminary assessment.

To account for liquefaction in DBA the following assumptions have been made:

1. Two separate assessments will be conducted: the first represents the loads and the system prior to liquefaction, and the second represents the system after liquefaction
2. The spectral demand of the pre-liquefaction analysis will be 90% of the demand for a non-liquefiable site, to reflect that the peak response often occurs in the earlier part of shaking and may occur before the onset of liquefaction

3. The pre-liquefaction analysis would be conducted with no reduction in the initial foundation impedances and no effects of differential settlement
4. The post-liquefaction analysis would use a 20% increase in the corner spectral period and a reduction to 30% of the non-liquefied spectral corner acceleration to capture the change in the site response of the liquefied soil deposit
5. The post-liquefaction analysis will assume the small strain foundation rotational stiffness has reduced to 60% of the original value and the friction angle has reduced to 70% to represent a reduction in foundation impedance and bearing capacity due to the build-up of pore pressure
6. The post-liquefaction analysis will assume the development of differential settlements results in no change to the yield displacement but in a 10% reduction in ultimate displacement capacity
7. The performance of the building will be considered as the envelope of the member responses from the pre- and post-liquefaction analyses
8. Uniform displacements (rigid-body settlement and rigid body tilting) have been ignored in the study of the building performance

4. CASE STUDY

The above extensions were applied to a six-storey two-bay case study reinforced concrete frame building to determine the influence of liquefaction on its performance (Figure 8 and Table 1).

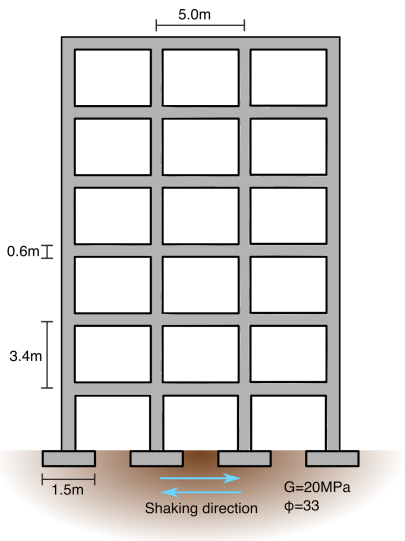

Figure 8. Schematic of the case study building

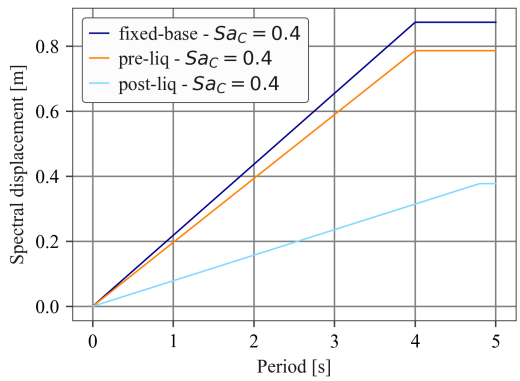

Figure 9. Response spectra for case study building

The seismic demand for the case study was defined in terms of the spectral displacement (Sd_c) using a corner period (T_c) and corner spectral acceleration (Sa_c) determined from Equation 3, shown in Figure 9 for Sa_c = 0.4.

$$Sd_C = \frac{Sa_C \cdot 9.8}{\left(\frac{2\pi}{T_c}\right)^2} \tag{2}$$

Table 1. Case study inputs

Parameter	Value
Foundation axial load ratio	6.6
Small rotation foundation rotational stiffness	6100MNm
Foundation shear stiffness	670MN
Soil bearing strength	1900kPa
Yield drift	0.0069
Effective mass (per frame)	220T
Effective stiffness	1520kN/m
Effective height (from foundation base)	15.4m
Effective period	2.4s
Base shear (per frame)	450kN
Foundation rotation	0.0015rad
Displacement from foundation	0.024m
Foundation pseudo uplift angle	0.0034rad
Displacement reduction factor	0.63
Ductility capacity	4.26
Ductility demand	2.77
Assessed drift	0.023rad
Displacement of SDOF	0.30m
Yield displacement of superstructure	0.99m

The assessment results and intermediate values for the pre-liquefaction case of Sa_c = 0.4 are shown in Table 2. For the post-liquefaction triggering case, the axial load ratio was reduced to 1.6, and the small rotation foundation rotational stiffness reduced to 3600MNm, while the structural ductility demand from shaking was only 0.89.

Table 2. Pre-liquefaction assessment results for Sac=0.4

Parameter	Value
Foundation axial load ratio	6.6
Small rotation foundation rotational stiffness	6100MNm
Foundation shear stiffness	670MN
Soil bearing strength	1900kPa
Yield drift	0.0069
Effective mass (per frame)	220T
Effective stiffness	1520kN/m
Effective height (from foundation base)	15.4m
Effective period	2.4s
Base shear (per frame)	450kN
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Displacement reduction factor	0.63
Ductility capacity	4.26
Ductility demand	2.77
Assessed drift	0.023rad
Displacement of SDOF	0.30m
Yield displacement of superstructure	0.99m

The building was analysed at increasing Sa_C values between 0.1-0.6 in increments of 0.05, for fixed-base conditions, normal SFSI conditions, pre-liquefaction conditions and post-

liquefaction conditions. The results of the desktop assessment are presented in Figure 10 in terms of the ductility demand and the displacement of the equivalent SDOF. It can be seen that the SFSI analysis and the fixed-base analyses are almost identical, this is because of the large foundation rotational stiffness that is typical of frame structures. It can also be noted that failure is expected at a Sa_C value of 0.45, which is reflected in the displacements plot where the results plateau. The displacements from foundation deformations are also shown in Figure 10 (b) and as expected, the values are very small, approximately 0.02m. The reduction in ductility and displacement for the pre-liquefaction case is a reflection of the reduction in spectral acceleration to 90% of the SFSI case. This results in an expected failure at approximately Sa_C=0.42. The post-liquefaction triggering case showed a considerable decrease in expected ductility and displacement due to the considerable decrease in seismic demand. Even though the system had a lower bearing capacity and lower foundation rotational stiffness, the foundation deformation was still very small and contributed only about 0.03m to the total SDOF displacement. The low foundation rotation is due to the yield of the frame, which essentially caps the moment demand that is applied to the foundation (as seen by the constant value of the foundation rotation for increasing demand after yield has occurred in Figure 10 (b)). It could be expected that structures with higher yield strength would increase the expected foundation rotation and increase the expected contribution from soil-foundation-structure interaction. It can also be noted that the post-liquefaction case did not result in failure of the structure, even with the reduced capacity due to differential settlements, in fact it only just yielded at a Sa_c value of about 0.5.

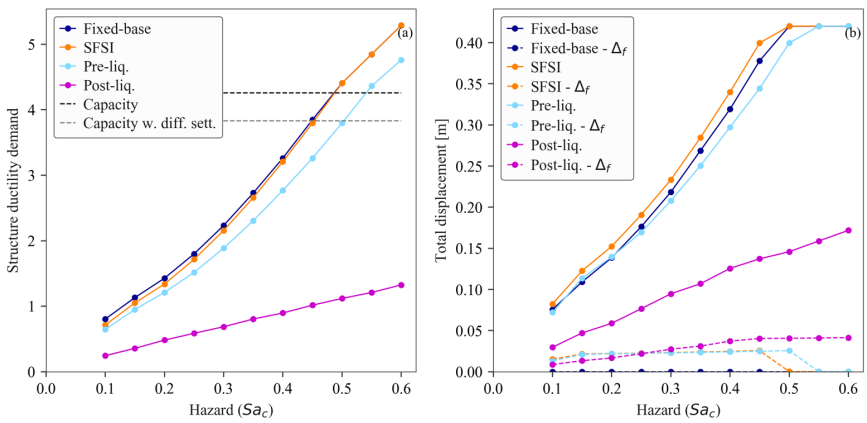

Figure 10. Performance assessment results from case study building

5. DISCUSSION

Given the minor role of SFSI in this case study, the major influence on the building performance can be attributed to the ground motion characteristics. Conceptually, this highlights an important issue with current mitigation techniques that focus solely on reducing settlements, since the reduction in liquefaction may result in higher shaking demands and eventual collapse of the structure. The rigid-body deformations were ignored in this preliminary assessment, however, and the relative importance of differential settlements and rigid-body movements on the continued use of the building needs to be quantified to better assess the holistic performance of the building. Conceivably a modern building that has been designed with a low-damage philosophy for the superstructure would be able to handle the strong shaking but large settlements would be intolerable and therefore soil mitigation would be appropriate. However, for other buildings the opposite may be true.

The preliminary procedure outlined here has highlighted the relative contributions of soil-foundation-structure-interaction, site effects and differential settlements. The major benefits of this procedure are:

1. Direct consideration of SFSI
2. Intuitive step-by-step procedure that highlights the relative contributions of individual mechanisms (e.g. direct calculation of expected foundation rotation)
3. Extends the well-established displacement-based assessment procedure
4. Updatable - new expressions to quantify site effects, SFSI and settlements/tilt can easily be incorporated
5. Easily extensible to include other liquefaction effects such as lateral spreading
6. Procedure can be applied at to individual buildings and in regional loss modelling as does not required detailed data of the soil and building properties
7. Can be used to assess the expected success of soil and structural mitigation techniques

However, the procedure as it stands is limited by the introduction of the unfounded assumptions in Section 3.4 and an accurate quantification of these relationships is needed before the procedure could be applied for practical benefit.

6. CONCLUSION

This paper developed an extension to the displacement-based assessment procedure to include the effects of liquefaction. The effects of liquefaction were quantified through modifications to the soil-foundation-structure system, the site response, the inclusion of differential settlements. The development of the procedure highlighted key shortfalls in the understanding of soil-liquefaction-foundation-structure interaction, and therefore several assumptions were made to allow the proposed procedure to be implemented. Given the extreme difficulty of assessing buildings on liquefiable soil deposits using numerical time history simulations, the proposed procedure may provide a cost-effective screening process to preliminarily assess building performance and any proposed mitigation techniques.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

Ancheta, T. D., Darragh, R., Stewart, J. P., Emel, S., Silva, W., Chiou, B., Wooddell, K. E., Graves, R. W., Kottke, A. R., Boore, D. M., Kishida, T., and Donahue, J. (2013). PEER NGA-West2 Database. Technical Report 03.

Bouckovalas, G. D., Tsiapas, Y. Z., Theocharis, A. I., and Chaloulos, Y. K. (2016). Ground response at liquefied sites: seismic isolation or amplification? *Soil Dyn and Earthq Eng*, DOI: 10.1016/j.soildyn.2016.09.028.

Bouckovalas, G. D., Tsiapas, Y. Z., Zontanou, V. A., and Kalogeraki, C. G. (2017). Equivalent Linear Computation of Response Spectra for Liquefiable Sites: The Spectral Envelope Method. *Journal of Geotechnical and Geoenvironmental Engineering*, 143(4):04016115–12, DOI: 10.1061/(ASCE)GT.1943-5606.0001625.

Dashti, S., Bray, J. D., Pestana, J. M., Riemer, M., and Wilson, D. (2010). Mechanisms of Seismically Induced Settlement of Buildings with Shallow Foundations on Liquefiable Soil. pages 1–14, DOI: 10.1061/ ASCE GT.1943-5606.0000179.

Dashti, S., Bray, J. D., Riemer, M., and Wilson, D. (2007). NEESR-II Project: "Towards Developing an Engineering Procedure for Evaluating Building Performance on Softened Ground" Centrifuge Test Plan for Test Series SHD02. pages 1–38.

Dobry, R. and Liu, L. (1994). Centrifuge modeling of soil liquefaction. In *10th World Conference of Earthquake Engineering*.

Gazetas, G. (2012). Should Elastic Response Spectra be the Basis of Seismic Design of Strongly Inelastic and Soft-Soil–Structure Systems? In *Proceedings of the 3rd International Symposium on Advances in Urban Safety*, pages 1–12, Nadjing.

Gomez, F., Millen, M. D. L., Romão, X., Costa, P. A., and Viana da Fonseca, A. (2018). Potential relevance of differential settlements in earthquake-induced liquefaction damage assessment. In *16th European Conference on Earthquake Engineering*, Thessaloniki.

Karamitros, D. K., Bouckovalas, G. D., and Chaloulos, Y. K. (2013). Insight into the Seismic Liquefaction Performance of Shallow Foundations. 139(4):599–607, DOI: 10.1061/(ASCE)GT.1943-5606.0000797.

Karatzia, X. A., Mylonakis, G., and Bouckovalas, G. D. (2017). Equivalent-linear Dynamic Stiffness of Surface Footings on Liquefiable Soil. In *COMPDYN*, pages 1–15.

Keino, C. and Kohiyama, M. (2012). Health disturbance of residents in a house with liquefaction damage in Mihama Ward, Chiba City. *Proceedings of the International Symposium on Engineering Lessons Learned from the 2011 Great East Japan Earthquake*.

Kramer, S. L., Hartvigsen, A. J., Sideras, S. S., and Ozener, P. T. (2011). *Site response modeling in liquefiable soil deposits*. 4th IASPEI/IAEE international.

MBIE (2016). Module 3: Identification, assessment and mitigation of liquefaction hazards. pages 1–44.

Millen, M. D. L. (2016). *Integrated Performance-based Design of Building-foundation Systems*. PhD thesis.

Millen, M. D. L., Pampanin, S., Cubrinovski, M., and Carr, A. J. (2016). A performance assessment procedure for existing buildings considering foundation deformations. In *2016 NZSEE Conference*, pages 1–8.

Sullivan, T. J., Welch, D. P., and Calvi, G. M. (2014). Simplified seismic performance assessment and implications for seismic design. *Earthquake Engineering and Engineering Vibration*, 13(S1):95–122, DOI: 10.1007/s11803-014-0242-0.

Trifunac, M. D. and Brady, A. G. (1975). A study on the duration of strong earthquake ground motion. *Bulletin of the Seismological Society of America*, 65(3):581–626.