Replotting the Nyquist Plot: A New Visualization Proposal

Predrag Pejović

Introduction

- ▶ Nyquist stability criterion ...
- ▶ started as a fix to Barkhausen “criterion”
- ▶ original derivation complicated ...
- ▶ nowadays taught using Cauchy’s argument principle
- ▶ fundamental, fairly esoteric, deep math ... but elegant!
- ▶ highly abstract topological criterion, reduces to:
 1. CW encirclement of $-1 + j0$ adds one unstable pole
 2. CCW encirclement of $-1 + j0$ removes one unstable polewhile closing the loop; topological and relative
- ▶ hard to teach, requires focused (and competent) students
- ▶ frequently hard to visualize due to imaginary axis poles, “closed” curve is not closed, but it “encloses” ...
- ▶ and this is **the** point where our story begins ...

Nyquist Criterion Revisited: the tracking system

Nyquist Criterion Revisited: assumptions

the transfer function

$$\underline{W}(\underline{s}) = \frac{\underline{N}(\underline{s})}{\underline{D}(\underline{s})}$$

let $N(s)$ and $D(s)$ be polynomials such that

$$\deg(N(\underline{s})) \leq \deg(D(\underline{s}))$$

which is satisfied for systems without algebraic degeneration

the problem is whether

$$\underline{H}(\underline{s}) = \frac{\underline{W}(\underline{s})}{1 + \underline{W}(\underline{s})}$$

is stable or not?

A Word on Barkhausen ...

poles at

$$1 + \underline{W}(\underline{s}) = 0$$

i.e.

$$-\underline{W}(\underline{s}) = 1 + j0$$

WRONG generalization and a **NONSENSE**: stable if

~~$$-\underline{W}(\underline{s}) \leq 1 + j0$$~~

or

$$|\underline{W}(\underline{s})| \geq 1$$

BTW, which $\underline{s}$? For oscillators $\underline{s} = j\omega_0$, where $\Im(\underline{W}(\underline{s})) = 0 \dots$

Nyquist Criterion Revisited: the first disaster,

$$\underline{W}_0(\underline{s}) = \frac{1}{\underline{s}}, \text{ stable for sure}$$

$$\underline{W}_0(\underline{s}) = \frac{1}{\underline{s}}$$

$$\frac{\underline{W}_0(\underline{s})}{1 + \underline{W}_0(\underline{s})} = \frac{1}{1 + \underline{s}}$$

pole at $\underline{s} = -1 + j0$, definitely stable

Nyquist Criterion Revisited: the first disaster,

$\underline{W}_0(\underline{s}) = \frac{1}{\underline{s}}$, straightforward

Nyquist Criterion Revisited: the first disaster,

$$\underline{W}_0(\underline{s}) = \frac{1}{\underline{s}}, \text{ escape contour}$$

Nyquist Criterion Revisited: the first disaster,
 $\underline{W}_0(\underline{s}) = \frac{1}{\underline{s}}$, here is what we need

What Do We Really Plot?

$$\underline{W}(\underline{s}) = W_r(\underline{s}) + j W_i(\underline{s})$$

where $W_r(\underline{s}) = \Re(\underline{W}(\underline{s}))$, and $W_i(\underline{s}) = \Im(\underline{W}(\underline{s}))$

in polar form

$$r(\underline{s}) = \sqrt{(W_r(\underline{s}))^2 + (W_i(\underline{s}))^2}$$

$$\varphi(\underline{s}) = \text{atan2}(W_i(\underline{s}), W_r(\underline{s}))$$

and $r(\underline{s})$ is the problem!

idea: compress $r(\underline{s})$ somehow?

Compression Function Requirements

1. $\rho(r)$ is monotonic, to preserve topological properties of the critical point encirclements,
2. $\rho(0) = 0$, to keep the same base point where the phase is irrelevant,
3. $\rho(1) = 1$, to keep the critical point and visualization of the phase margin,
4. $\lim_{r \rightarrow \infty} \rho(r)$ is finite, to confine the diagram in a finite space.

Compression Function

$$\rho(r) = \frac{4}{\pi} \arctan(r)$$

“amplitude angle”

A Family of Compression Functions

$$\rho(r) = \frac{r_{max} r^k}{r_{max} - 1 + r^k}$$

r_{max} is the radius of the circle the plot is confined into, this degree of freedom might be of some value

$k > 1$ is a parameter

$r_{max} = 2$ and $k = \frac{4}{\pi}$ approximates the compression function applied in this paper the best, almost the same function, within 1.5% of r_{max} (0.03)

Bode Plot, $\underline{W}_0(\underline{s}) = \frac{1}{\underline{s}}$

Amplitude Compression, r , r in decibels, and ρ

r	r [dB]	ρ
1	0	1
10	20	1.8731
100	40	1.9873
1,000	60	1.9987
10,000	80	1.9999
100,000	100	2.0000

Bode Plot Alternative, $\underline{W}_0(\underline{s}) = \frac{1}{\underline{s}}$

Alternative Nyquist Plot, $\underline{W}_0(\underline{s}) = \frac{1}{\underline{s}}$,
strange, you've seen this graph before

$\frac{1}{2}$ of the Contour ...

Argument Increase, $\underline{W}_0(\underline{s}) = \frac{1}{\underline{s}}$

Example 1, $\underline{W}_1(\underline{s}) = \frac{1}{(\underline{s} + 1)^2}$

Example 1, $\underline{W}_1(\underline{s}) = \frac{1}{(\underline{s} + 1)^2}$

Example 1, $\underline{W}_1(\underline{s}) = \frac{1}{(\underline{s} + 1)^2}$

Example 1, $\underline{W}_1(\underline{s}) = \frac{1}{(\underline{s} + 1)^2}$

Example 1, $\underline{W}_1(\underline{s}) = \frac{1}{(\underline{s} + 1)^2}$

Example 2, $\underline{W}_2(\underline{s}) = \frac{1}{\underline{s}(\underline{s} + 1)^2}$

Example 2, $\underline{W}_2(\underline{s}) = \frac{1}{\underline{s}(\underline{s} + 1)^2}$

Example 2, $\underline{W}_2(\underline{s}) = \frac{1}{\underline{s}(\underline{s} + 1)^2}$

Example 2, $\underline{W}_2(\underline{s}) = \frac{1}{\underline{s}(\underline{s} + 1)^2}$

Example 2, $\underline{W}_2(\underline{s}) = \frac{1}{\underline{s}(\underline{s} + 1)^2}$

Example 2, $\underline{W}_2(\underline{s}) = \frac{1}{\underline{s}(\underline{s} + 1)^2}$

closed loop, stable

Example 2a, $\underline{W}_{2a}(\underline{s}) = 3 \underline{W}_2(\underline{s}) = \frac{3}{\underline{s}(\underline{s} + 1)^2}$

Example 2a, $\underline{W}_{2a}(\underline{s}) = 3 \underline{W}_2(\underline{s}) = \frac{3}{\underline{s}(\underline{s} + 1)^2}$

Example 2a, $\underline{W}_{2a}(\underline{s}) = 3 \underline{W}_2(\underline{s}) = \frac{3}{\underline{s}(\underline{s} + 1)^2}$

Example 2a, $\underline{W}_{2a}(\underline{s}) = 3 \underline{W}_2(\underline{s}) = \frac{3}{\underline{s}(\underline{s} + 1)^2}$

Example 2a, $\underline{W}_{2a}(\underline{s}) = 3 \underline{W}_2(\underline{s}) = \frac{3}{\underline{s}(\underline{s} + 1)^2}$

closed loop, unstable

Example 3, $\underline{W}_3(\underline{s}) = \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3, $\underline{W}_3(\underline{s}) = \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3, $\underline{W}_3(\underline{s}) = \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3, $\underline{W}_3(\underline{s}) = \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3, $\underline{W}_3(\underline{s}) = \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3, $\underline{W}_3(\underline{s}) = \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

bypass ambiguity

Example 3, $\underline{W}_3(\underline{s}) = \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

closed loop, on the stability boundary

Pole-Zero Map

Example 3a, $\underline{W}_{3a}(\underline{s}) = 2 \underline{W}_3(\underline{s}) = 2 \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3a, $\underline{W}_{3a}(\underline{s}) = 2 \underline{W}_3(\underline{s}) = 2 \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3a, $\underline{W}_{3a}(\underline{s}) = 2 \underline{W}_3(\underline{s}) = 2 \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3a, $\underline{W}_{3a}(\underline{s}) = 2 \underline{W}_3(\underline{s}) = 2 \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3b, $\underline{W}_{3b}(\underline{s}) = \frac{1}{2} \underline{W}_3(\underline{s}) = \frac{1}{2} \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3b, $\underline{W}_{3b}(\underline{s}) = \frac{1}{2} \underline{W}_3(\underline{s}) = \frac{1}{2} \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3b, $\underline{W}_{3b}(\underline{s}) = \frac{1}{2} \underline{W}_3(\underline{s}) = \frac{1}{2} \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3b, $\underline{W}_{3b}(\underline{s}) = \frac{1}{2} \underline{W}_3(\underline{s}) = \frac{1}{2} \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

Example 3b, $\underline{W}_{3b}(\underline{s}) = \frac{1}{2} \underline{W}_3(\underline{s}) = \frac{1}{2} \frac{\underline{s} + 1}{\underline{s} (0.1 \underline{s} - 1)}$

closed loop, unstable

Example 4, $\underline{W}_4(\underline{s}) = \frac{(\underline{s} + 10)^2}{\underline{s}^3}$

Example 4, $\underline{W}_4(\underline{s}) = \frac{(\underline{s} + 10)^2}{\underline{s}^3}$

Example 4, $\underline{W}_4(\underline{s}) = \frac{(\underline{s} + 10)^2}{\underline{s}^3}$

Example 4, $\underline{W}_4(\underline{s}) = \frac{(\underline{s} + 10)^2}{\underline{s}^3}$

Example 4, $\underline{W}_4(\underline{s}) = \frac{(\underline{s} + 10)^2}{\underline{s}^3}$

Example 4, $\underline{W}_4(\underline{s}) = \frac{(\underline{s} + 10)}{s^3}$

closed loop, unstable

Example 4a, $\underline{W}_{4a}(\underline{s}) = 10 \underline{W}_4(\underline{s}) = 10 \frac{(\underline{s} + 10)^2}{\underline{s}^3}$

Example 4a, $\underline{W}_{4a}(\underline{s}) = 10 \underline{W}_4(\underline{s}) = 10 \frac{(\underline{s} + 10)^2}{\underline{s}^3}$

Example 4a, $\underline{W}_{4a}(\underline{s}) = 10 \underline{W}_4(\underline{s}) = 10 \frac{(\underline{s} + 10)^2}{\underline{s}^3}$

Example 4a, $\underline{W}_{4a}(\underline{s}) = 10 \underline{W}_4(\underline{s}) = 10 \frac{(\underline{s} + 10)^2}{\underline{s}^3}$

Example 4a, $\underline{W}_{4a}(\underline{s}) = 10 \underline{W}_4(\underline{s}) = 10 \frac{(\underline{s} + 10)}{s^3}$

closed loop, stable

Conclusions

- ▶ an alternative visualization for Nyquist plots
- ▶ affects only magnitude of the polar plot, not phase
- ▶ topological properties kept
- ▶ a compression function $\rho(r)$...
- ▶ ... and a family of compression functions
- ▶ closed contour mapped into a closed contour
- ▶ problems with imaginary axis poles removed
- ▶ the plot confined into a finite space
- ▶ dynamics of the amplitude plot reduced
- ▶ a program that counts encirclements, $\Delta\#$ of unstable poles
- ▶ three zones of loop gain identified
- ▶ numerous examples ...
- ▶ this presentation is posted at Zenodo ...