Effects of normal and through-thickness shear stresses on the forming limit curves of AA3104-H19 using advanced yield criteria

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Abstract

In the present study, the effects of normal and through-thickness shear (TTS) stresses are simultaneously considered to predict the generalized forming limit diagram (GFLD) of AA3104-H19 using Yld96 [1], Yld2004 [2] and Yld2011 [3] anisotropic yield criterion. For this purpose, considering the general stress state, including the normal and through-thickness shear stresses on the basis of modified Marciniak–Kuczynski (M-K) model is employed. The Newton-Raphson numerical method is applied to solve the nonlinear equations set and then the forming limit diagram (FLD) and forming limit stress diagrams (FLSD) are computed. At first, the forming limit curves are presented by considering the through-thickness shear stress and then the generalized forming limit curves are presented by applying the normal and through-thickness shear stresses simultaneously. The results show that the through-thickness shear stress has positive sensitivity on the limit strains. Also, the normal stress is more efficient than of through-thickness shear stress on the formability of the plate. Comparison between the results indicates that the predicted limit strains by Yld2011 anisotropic yield function are lower than the Yld96 ones.

Keywords: Generalized Forming Limit Diagram, Normal Stress, Through-Thickness Shear Stress, Anisotropic Yield Function, Modified Marciniak–Kuczynski Model.

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1-Introduction

Considering the effects of normal and through-thickness shear (TTS) stresses in the incremental sheet forming (ISF) is one of the most important parameters to predict the forming limit diagram (FLD) and forming limit stress diagram (FLSD). The forming limit curve (FLC) can be presented in terms of the principal strains to estimate the strain state, the localized instability analysis, fracture forming limit by experimental or numerical investigation. Many theoretical and analytical methods, including linearized theory of perturbations, bifurcation methods, necking theory, Marciniak-Kuczynski (M-K) model and strain gradient approach have been proposed and utilized to estimate the forming limit curve by scientists. The idea of FLC was initially indicated by Keeler and Backofen [4] to predict the beginning of plastic instability, and it was developed by Goodwin [5], Marciniak and Kuczynski [6, 7]. The different isotropic and anisotropic yield criteria to detect the yield surface along with an associated flow rule scheme to describe the plastic behavior of metal play the critical roles in the plastic material modeling [8]. In some metal forming procedure such as cold rolling, hydroforming and incremental sheet forming process, the material anisotropy characteristics were implicated to employ the anisotropic yield functions. For this reason, many studies were concentrated to examine the effects of different yield functions on the FLCs for sheet metals.

Butuc et al. [9] were applied Yld96 and BBC2000 [10] and Voce hardening law along with the M-K theory for orthotropic sheet metals under plane stress condition to predict the FLCs. Ganjiani and Assempour [11] investigated the M-K model into the Hosford [12] and BBC2000 yield criteria to predict the FLD. They concluded that Hosford yield criterion with the exponent of 6 can predict the FLDs for AK steel and both BBC2000 and Hosford yield functions with the exponent of 8 can predict limit strains for AA5XXX alloy which have good agreement with the experimental data. Based on the M-K model, the influence of different yield criteria such as BBC2000, BBC2002 and BBC2003 [13-14] functions and Voce and Swift hardening law on the AA3003-O aluminum alloy were studied by Ahmadi et al. [15]. They found that the theoretical obtained results are in reasonably good agreement with the experimental ones. Dasappa et al. [16] examined the FLD of the AA5754 aluminum sheet metal by employing five yield criteria, including Hill's 48 [17], Hill's 90 [18], Hill's 93 [19], Barlat89 [20] and Plunkett [21]. The FLD dependence of the yield surface shape and the method of determining the material parameters are the main results of their investigation. Li et al. [22] used von Mises, Hill'48, Hosford and Barlat89 yield functions in M-K model to investigate the FLD of AA-Li2198 aluminum alloy experimentally and theoretically. The capability of different yield criteria, including von Mises, Hill's 48 and Yld2000 [23] with Voce and Swift hardening law based on the M-K theory to prediction of FLD and FLSD was

experimentally and numerically investigated by Panich et al. [24]. Ozturk et al. [25] applied Hill's 48, Barlat89, and YLD2000 to M-K model to predict the effect of different yield criteria on the FLD in comparison with experimental data. They found that the theoretical results have satisfactory accuracy with experimental data based on Yld2000 yield criterion. Sheng and Mallick [26] proposed the new ductile failure criterion under plane stress condition to predict the forming limit sheet metal. Critical damage as were defined as a function of strain path and initial sheet thickness and treated localized necking as a failure. The function can be used to predict FLC under linear and bilinear strain paths and failure in strong nonlinear strain path condition.

Due to high levels of the fluid pressure in industrial metal forming such as hydroforming, considering the influence of normal stress is necessary to compute the strain based forming limit diagram. The material response subjected to very high hydrostatic pressure was experimentally examined by Bridgman [27]. The experimental data show that the ductility of material increases under hydrostatic pressure. The double-sided hydraulic pressure to a tubular hydroforming operation was introduced by Fuchs [28]. He found that in the expansion of tube forming the ratio of stress is efficient.

Assempour et al. [29] studied the effect of normal stress on FLD based on the M-K model for AA6011 and STKM-11A, which the compared with experimental data. Their results indicate that the normal compressive stress has the positive effect on the formability of sheet metal. Considering the two types of pre-straining and through-thickness normal stress, the influence of strain path was investigated by Hashemi and Abrinia [30] to predict the FLC based on modified M-K model. Nurcheshmeh and Green [31] considered the effects of strain path non-linearity and the normal stress based on the M-K theory to predict the FLC of AISI-1012, AA6011, and STKM-11A. They observed a satisfactory agreement between the numerical and experimental data. Zhang et al. [32] studied numerical analysis to investigate the influence of through-thickness normal stress and material anisotropy on FLD of AA5XXX and AA6011 aluminum alloy using Barlat's vield function, Barlat 1989, into M-K method. They showed a formability improvement of AA5XXX and AA6011 aluminum alloy sheet metal by increasing the through-thickness normal stress. The effect of different plane stress yield criteria, including Hill's 48, Barlat89 and Yld2003 yield criteria into M-K method was carried out by Zhang et al. [33] to predict the FLC for AA6111-T3 aluminum alloy. Moreover, FLC, effective strain forming limit curve (eFLC), stressbased forming limit curve (FLSC) and extended stress-based forming limit curve (XSFLC) prediction by considering the effect of through-thickness normal stress was numerically calculated by considering a modified M-K model combined. The influence of through-thickness normal stress on FLDs of the 5A06-O aluminum alloy with ductile fracture criterion was carried out experimentally and compared to the Abaqus/Explicit simulation results by Yang et al. [34]. Mirfalah Nasiri et al. [35] investigated the effect

of different yield function including Karafillis-Boyce (K-B), Yld96 and Yld2011 on the FLD, directional normalized uniaxial yield stresses, directional r-value and for AA3104-H19 aluminum alloy under plane stress condition. Moreover, the effect of normal stress on forming limit curve was investigated based on Yld2011 yield criterion. The influence of out of plane compressive stresses on the prediction of strain localization was described by using the bifurcation theory and the initial imperfection approach by Ben Battaieb and Abed-Meraim [36]. The numerical predictions of limit strains were shown that the impact of out of plane compressive stresses was highlighted on the enhancement of sheet metal formability.

Although plane strain and equibiaxial stretching states are the most commonly seen when manufacturing parts of different geometrical shapes, the studies have clarified the improved formability and deformation stability by using experimental and FEM analysis in the incremental sheet forming deformation [37]. The effect of through-thickness shear stress state on the sheet metal formability in some sheet metal forming such as incremental sheet forming processes are most important to investigate the instability and the onset of localized necking. Few researches were focused to study the influence of through-thickness shear stress state on the FLC. Allwood and Shouler [38] considered all six components of the stress tensor in sheet metal forming and proposed a generalized forming limit diagram (GFLD). The M-K model was modified by Eyckens et al. [39, 40] to investigate the effect of through-thickness shear (TTS) stress on the localized necking in sheet metal forming operations and FLD. They found that the formability of sheet metal can be increased by considering the TTS. Fatemi and Dariani [41] proposed a modified M-K model to predict the FLC of AA3003 by considering the normal and through thickness shear stresses on the formability of metal sheets.

Based on the literature review and the published experimental results, considering of classic yield criteria such as von Mises and Hill's 48 are not proper anisotropic criteria to predict the anisotropy plastic behavior and FLCs of the anisotropic aluminum alloy sheet. Also, in some industrial sheet metal process such as hydroforming, incremental sheet forming and spinning processes, the normal and through-thickness shear stress state should be considered to predict the FLD of sheet metal. For this reason, in this study, based on the modified M-K theory the Yld96, Yld2004 and Yld2011 anisotropic yield criteria are implemented to predict the FLD of AA3104-H19 under general stress state including normal stress and through-thickness shear stress simultaneously at first. In other words, sensitivity analysis of FLD and FLSD subjected to normal and through-thickness shear stress state using new anisotropic criteria, including Yld96, Yld2004, and Yld2011 anisotropic yield functions into the modified M-K model for AA3104-H19 sheet metal is the innovation of this work. FLD and FLSD are computed by solving of nonlinear resulting equations set using Newton-Raphson numerical method. The effects of TTS and TTS along with normal stress on the FLC of AA3104-H19 sheet metal are investigated. It is observed that by

increasing the TTS, the limit strains increase. Moreover, the results show that the FLC of AA3104-H19 sheet metal under normal stress is more sensitive than the TTS.

2-Theoretical approach and solution methodology

The in-plane stress states are not appropriate assumption in some sheet metal forming process such as incremental sheet forming. However, the traditional M-K theory, as shown in figure (1), can be modified for general stress state, including the normal and through-thickness shear stresses.

Considering the anisotropic advanced yield functions along with the modified M-K theory, the formability of AA3104-H19 sheet metal under proportional loading is investigated. For this purpose, the stress tensor in the general state at homogeneous region can be considered as follows:

$$\boldsymbol{\sigma}^{a} = \begin{bmatrix} 1 & 0 & \alpha_{13} \\ 0 & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \sigma_{11}^{a}$$
(1)

where σ_{11}^{a} is the stress in rolling direction (RD) and the coefficients α_{ij} are defined in terms of the tensor of stress.

$$\alpha_{ij} = \frac{\sigma_{ij}}{\sigma_{11}} \tag{2}$$

Throughout the deformation, it can be assumed that these coefficients are constant and the stress in rolling direction increases steadily up to desirable deformation. The increment of strain also can be considered as linearly proportional as stress state [38].

$$d\varepsilon_{ij}{}^{a} = \begin{bmatrix} 1 & 0 & \beta_{13} \\ 0 & \beta_{22} & \beta_{23} \\ \beta_{13} & \beta_{23} & \beta_{33} \end{bmatrix} d\varepsilon_{11}{}^{a}$$
(3)

where

$$\beta_{ij} = \frac{d\varepsilon_{ij}}{d\varepsilon_{11}} \tag{4}$$

 β_{33} can be alternatively calculated based on the volume constancy as follows:

$$\beta_{33} = -(1 + \beta_{22}) \tag{5}$$

An algorithmic chart of the solution procedure to compute of FLC based on the modified M-K model is similar to the traditional ones [35, 42]. At first step, by considering zero value for all strains and employing an appropriate yield function and hardening law, the effective strain increment can be assumed in the homogeneous region to calculate the strain and stress states. The advanced yield criterions including Yld96, Yld2004, and Yld2011 are proposed to satisfy the plastic behavior of aluminum alloys

based on the experimental investigations. Barlat proposed a yield criterion using the weight factors socalled Yld96 [1]:

$$\bar{\sigma} = \frac{1}{2} [\alpha_1 | S_1 - S_2 |^M + \alpha_2 | S_2 - S_3 |^M + \alpha_3 | S_3 - S_1 |^M]^{1/M}$$
(6)

In Eq(6) α_1, α_2 and α_3 are the weight factors which are related to the anisotropy of materials. *M* is a coefficient which is equal to 8 for FCC and 6 for BCC material. S_1 , S_2 and S_3 are the principal value of the isotropic plasticity equivalent stress **S** which is defined by the following linear transformation operator.

$$S = L: \sigma \tag{7}$$

where σ and L are the Chauchy stresses and a fourth order tensor respectively in which is calculated as follows:

$$\boldsymbol{L} = \begin{bmatrix} \frac{c_2 + c_3}{3} & -\frac{c_3}{3} & -\frac{c_2}{3} & 0 & 0 & 0 \\ -\frac{c_3}{3} & \frac{c_3 + c_1}{3} & -\frac{c_1}{3} & 0 & 0 & 0 \\ -\frac{c_2}{3} & -\frac{c_1}{3} & \frac{c_2 + c_1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_6 \end{bmatrix}$$
(8)

In Eq(8) c_i are the material parameters. α_1, α_2 and α_3 are defined as follows:

$$\alpha_k = \alpha_x p_{1k}{}^2 + \alpha_y p_{2k}{}^2 + \alpha_z p_{3k}{}^2 \tag{9}$$

where p is the transformation matrix between the principal directions of stress tensor S and principal axes of anisotropy. α_x , α_y and α_z are the anisotropic variable quantities. β_i are the angle between the principal directions of S and the anisotropic axes which are defined as:

$$\alpha_{x} = \alpha_{x0} \cos^{2} 2\beta_{1} + \alpha_{x1} \cos^{2} 2\beta_{1}$$

$$\alpha_{y} = \alpha_{y0} \cos^{2} 2\beta_{2} + \alpha_{y1} \cos^{2} 2\beta_{2}$$
(10)

$$\alpha_x = \alpha_{z0} \cos^2 2\beta_3 + \alpha_{z1} \cos^2 2\beta_3$$

$$\begin{cases} \beta_i = 0 \quad \to \quad \alpha_{i0} = \alpha_i \\ \pi \end{cases}$$
(11)

$$\left\{\beta_i = \frac{\pi}{2} \to \alpha_{i1} = \alpha_i\right\}$$
(11)

$$\cos^{2} 2\beta_{1} = \begin{cases} y.1, |S_{1}| \ge |S| \\ y.3, |S_{1}| < |S_{3}| \\ z.3, |S_{1}| < |S_{3}| \\ z.3, |S_{1}| < |S_{3}| \\ z.3, |S_{1}| < |S_{3}| \end{cases}$$
(12)
$$\cos^{2} 2\beta_{3} = \begin{cases} x.1, |S_{1}| \ge |S_{3}| \\ x.3, |S_{1}| < |S_{3}| \\ x.3, |S_{1}| < |S_{3}| \end{cases}$$

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The Yld2004 yield criterion can be indicated as [2, 43]:

$$\bar{\sigma} = \frac{1}{4} [|S_1' - S_1''|^M + |S_1' - S_2''|^M + |S_1' - S_3''|^M + |S_2' - S_1''|^M + |S_2' - S_2''|^M + |S_2' - S_3''|^M + |S_3' - S_1''|^M + |S_3' - S_1''|^M + |S_3' - S_3''|^M]^{1/M}$$
(13)

In Eq (13), *M* is a coefficient which is equal to 8 for FCC and 6 for BCC material and S' and S'' are the linear transformation on the stress deviator in which are defined as:

$$S' = C': S, S'' = C'': S$$
 (14)

where the linear transformation matrices *C*' and *C*'' can be presented as:

$$\begin{bmatrix} C' \end{bmatrix} = \begin{bmatrix} 0 & -C'_{12} & -C'_{13} & 0 & 0 & 0 \\ -C'_{21} & 0 & -C'_{23} & 0 & 0 & 0 \\ -C'_{31} & -C'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C'_{66} \end{bmatrix}$$

$$\begin{bmatrix} C'' \end{bmatrix} = \begin{bmatrix} 0 & -C''_{12} & -C''_{13} & 0 & 0 & 0 \\ -C''_{21} & 0 & -C''_{23} & 0 & 0 & 0 \\ -C''_{31} & -C''_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C''_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C''_{66} \end{bmatrix}$$
(15)

The anisotropic Yld2011-18p yield criterion is produced with 18 calibrated parameters to experimental data [3]. The Yld2011-18p yield criterion with two linear transformations is indicated as:

$$\bar{\sigma} = \left\{ \frac{1}{\xi} \left[\sum_{i=1}^{3} \sum_{j=1}^{3} \left| S'_{i} + S''_{j} \right|^{M} \right] \right\}^{1/M}$$
(16)

where S'_i and S''_j are the linear transformation of the stress deviator, and *M* is the yield function exponent. In Eq. (16), the scalar quantity ξ is defined according to:

$$\xi = \left(\frac{4}{3}\right)^a + 4\left(\frac{2}{3}\right)^a + 4\left(\frac{1}{3}\right)^a, a \ge 1$$
(17)

The linear transformations in Eq. (16) are defined in the following form:

$$\begin{bmatrix} S'_{11} \\ S'_{22} \\ S'_{33} \\ S'_{32} \\ S'_{31} \\ S'_{12} \end{bmatrix} = \begin{bmatrix} 0 & -C'_{12} & -C'_{13} & 0 & 0 & 0 \\ -C'_{21} & 0 & -C'_{23} & 0 & 0 & 0 \\ -C'_{31} & -C'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C'_{55} \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{22} \\ S_{33} \\ S_{32} \\ S_{31} \\ S_{21} \end{bmatrix}$$
(18)

Considering the uniaxial tension test along rolling direction, a type of law relationship is used to define the work hardening law for AA3104-H19 as follows [3, 44]:

$$\bar{\sigma} = A + (B + C\bar{\varepsilon}) (1 - exp(-\bar{\varepsilon}[D/B]))$$

$$= A + (B + C(\bar{\varepsilon} + d\bar{\varepsilon})) (1 - exp(-(\bar{\varepsilon} + d\bar{\varepsilon})[D/B]))$$
(19)

where A=276.0, B=43.6, C=116.2, D=2213.0 MPa.

Substituting Eqs (19) and (1) into Eq (6), Eq (13) or (16), the six components of stress in the first step can be calculated in the homogeneous region. Then the six components of strain increment can be computed by considering the flow rule, $d\varepsilon_{ij} = d\overline{\varepsilon} (\partial f / \partial \sigma_{ij})$, and Yld96, Yld2004 or Yld2011 yield functions in the homogeneous region. Applying the rotation matrix, *T* the stress and strain tensor in regions (a) can be calculated in the groove coordinates.

$$T = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0\\ -\sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1 \end{bmatrix}$$
(20)

$$[\sigma^{a}]_{nt3} = \begin{bmatrix} \sigma_{nn}{}^{a} & \sigma_{nt}{}^{a} & \sigma_{n3}{}^{a} \\ \sigma_{nt}{}^{a} & \sigma_{t3}{}^{a} & \sigma_{t3}{}^{a} \\ \sigma_{n3}{}^{a} & \sigma_{t3}{}^{a} & \sigma_{33}{}^{a} \end{bmatrix} = T \begin{bmatrix} \sigma_{11}{}^{a} & 0 & \sigma_{13}{}^{a} \\ 0 & \sigma_{22}{}^{a} & \sigma_{23}{}^{a} \\ \sigma_{13}{}^{a} & \sigma_{23}{}^{a} & \sigma_{33}{}^{a} \end{bmatrix} T^{T}$$
(21)

$$[d\varepsilon^a]_{nt3} = T[d\varepsilon^a]_{123}T^T$$
(22)

The angle of groove can be derived in any step of loading regarding strain increments in the homogeneous region according to [42, 45]:

$$\tan(\theta + d\theta) = \tan(\theta) \frac{1 + d\varepsilon_1{}^a}{1 + d\varepsilon_2{}^a}$$
(23)

In this step, seven unknown parameters including effective strain increment and six components of stress in the inhomogeneous region should be calculated. For this purpose, as depicted in figure (2) the force equilibrium over the interface between homogeneous and groove region can be indicated as:

$$F^b - F^a = 0 \tag{24}$$

$$\left(F_{nn}{}^{b} - F_{nn}{}^{a}\right)^{2} + \left(F_{nt}{}^{b} - F_{nt}{}^{a}\right)^{2} + \left(F_{n3}{}^{b} - F_{n3}{}^{a}\right)^{2} = 0$$
(25)

The Eqs (24) and (25) can be written in terms of stress components as:

$$\sigma_{nn}{}^b t^b = \sigma_{nn}{}^a t^a \tag{26}$$

$$\sigma_{nt}{}^{b}t^{b} = \sigma_{nt}{}^{a}t^{a} \tag{27}$$

$$\sigma_{n3}{}^b t^b = \sigma_{n3}{}^a t^a \tag{28}$$

Eqs (26) to (28) can be rewritten in term of imperfection coefficient as:

$$\sigma_{nn}{}^{b} = \frac{\sigma_{nn}{}^{a}}{f} \tag{29}$$

$$\sigma_{nt}{}^{b} = \frac{\sigma_{nt}{}^{a}}{f} \tag{30}$$

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$$\sigma_{n3}{}^b = \frac{\sigma_{n3}{}^a}{f} \tag{31}$$

where

$$f = \frac{t^b}{t^a} = f_0 \exp\left(\varepsilon_3^b - \varepsilon_3^a\right) \tag{32}$$

In Eq (32), $f_0 = t_0^b/t_0^a$ is the initial imperfection. In this study the initial imperfection coefficient was considered as $f_0 = 0.992$ for AA3104-H19 alloy [46, 47]. The strain compatibility condition in the homogeneous and inhomogeneous regions across the groove direction can be indicated as:

$$d\varepsilon_{tt}{}^{b} = d\varepsilon_{tt}{}^{a} \tag{33}$$

The surface traction on the surface in the homogeneous and inhomogeneous regions has been considered to be uniform as:

$$\sigma_{33}{}^b = \sigma_{33}{}^a \tag{34}$$

$$\sigma_{t3}{}^b = \sigma_{t3}{}^a \tag{35}$$

To calculate the stress and strain increment in the groove region, one can be combined the work hardening law, Eq (19) and one of the yield functions, Eq (6), Eq (13) or (16) in which one nonlinear equation in terms of $\sigma_{tt}{}^{b}$ and $d\bar{\varepsilon}^{b}$ is derived. Another nonlinear equation in terms of $\sigma_{tt}{}^{b}$ and $d\bar{\varepsilon}^{b}$ can be derived by considering the flow rule, $d\varepsilon_{ij} = d\bar{\varepsilon}(\partial\bar{\sigma}/\partial\sigma_{ij})$, and Yld96, Yld2004 or Yld2011 yield functions along with the strain compatibility condition, Eq (33). The numerical Newton-Raphson method is used to solve the nonlinear set of equations. Based on the M-K model, it is assumed that the onset of plastic instability in sheet metal occurs when the $d\bar{\varepsilon}^{b}/d\bar{\varepsilon}^{a} \ge 10$ [48] and limiting strains are saved. This analysis process is carried out for different strain path between uniaxial tensile, ($\alpha_{22} = 0$), and equibiaxial tensile, ($\alpha_{22} = 1$), to compute the FLC.

3-Results and discussion

In this section firstly the effect of general stress state on the FLCs prediction, including FLD, FLSD, and XFLSD (Extended Forming Limit Stress Diagram) is studied by using of Yld2011 into the modified M-K theory for AA3104-H19 sheet metal. Then the GFLD predictions of AA3104-H19 sheet metal using Yld2011, Yld2004, and Yld96 are compared. In the previous work [35] has been shown that in the plane stress condition the Yld96 and Yld2011 yield criterion gives satisfactory accuracy corresponding to experimental data. For this reason, in this work, these two yield functions are considered to predict the GFLD of AA3104-H19 sheet metal.

3-1- Effect general stress state on the FLCs

The experimental data of single point incremental forming (SPIF) is available for limit material such as AA5754. As shown in figure (3) the limit strain of the AA5754 sheet metal is computed for tension-tension strain state (right-hand side of the FLD) in two kinds of sheet metal forming processes including general sheet metal forming process (plane stress state) and incremental sheet forming (out-of-plane stress) by considering the through-thickness shear stresses. As depicted in figure (3) a good agreement can be found between the present work to predict the FLD and experimental data [49]. The out-of-plane stress ratios for this validation test were extracted $\alpha_{33} = -0.43, 0.02 < \alpha_{13} = \alpha_{23} < 0.1$.

The effect of σ_{13} TTS Stress on the FLCs including FLD, FLSD, and XFLSD for AA3104-H19 sheet metal is depicted in figure (4a) to (4c) respectively. As shown in figure (4a), the effect of σ_{13} TTS stress on the increasing of strain limit in the left-hand side of FLD including the tension-compression strain states is more considerable than of tension-tension strain states in the right-hand side ones. Moreover, by increasing the σ_{13} TTS stress, the stress limit decreases as depicted in figure (4b). Figure (4c) illustrates that the effective stress in terms of the TTS depends on the strain path which TTS increases the XFLSD in the equi-biaxial tension and decreases in the uniaxial tension. As illustrated in figure (5a) to (5c), the influence of σ_{23} TTS stress on the FLD, FLSD, and XFLSD for AA3104-H19 is similar to the effect of σ_{13} TTS stress qualitatively. The formability of sheet metal increases by considering this type of TTS. Simultaneous effects of TTS and normal stress on the FLD, FLSD, and XFLSD of AA3104-H19 aluminum alloy sheet metal is shown in figure (6a) to (6c). Increasing of the strain limit along with an upward shift in the strain based limit diagram and decreasing of stress limit with a downward shift in the stress based limit diagram by considering the TTS and normal stress simultaneously can be observed.

The influence of stress component on the FLD and FLSD for AA3104-H19 sheet metal is compared with together in figure (7a-b). One can be found that by applying the σ_{23} TTS stress has more considerable effect than the σ_{13} TTS stress to decrease of stress limit and increase the formability of sheet metal. In addition, the normal stress is more efficient than of TTS stresses to increase the formability of sheet metal as depicted in figure (7a-b).

Figure (8) shows the sensitivity of the yield surface to stress component, including the TTS and the normal stresses. According to these results, one can conclude that the shear stress decreases the yield surface and normal stress shifts yield surface along the equi-biaxial compression direction and principal in-plane stress in the tension-tension region would reduce which results in the sheet formability increase. Moreover, minimum and maximum of tensile yield stress correspond to normal stress and plane stress conditions respectively. Adding the σ_{13} TTS stress, σ_{23} TTS stress and normal compressive stress on the plane stress condition independently, the yield stress in tension-tension region decreases which increases the formability of sheet metal respectively. By increasing the formability of sheet metal, the limit strain

decreases. For this reason by applying the σ_{13} TTS stress, σ_{23} TTS stress and normal compressive stress on the plane stress condition independently and respectively the limit strain increases in which these important results concluded latter in figure (7).

3-2-Effect of Yld96, Yld2004, and Yld2011 on the GFLDs

Figure (9a) presents to compare the predicted FLC by Yld2011, Yld2004 and Yld96 with the available experimental results [46] for plane stress condition. Based on these new results it can be concluded that for the different path loading the accuracy of predicted FLD are different. For example, the predicted FLD using Yld2011 in the left-hand side of FLC (tension-compression strain states), both Yld2011 and Yld2004 in the plane strain state and the Yld96 yield criterion for equi-biaxial tension have good accuracy to experimental data to predict the limit strain. Figure (9b) presents to illustrate the comparison between the FLSD predicted by Yld96, Yld2004 and Yld2011 yield function at plane stress condition.

The effects of Yld96, Yld2004 and Yld2011 yield functions on the FLD and FLSD for AA3104-H19 alloy under σ_{13} TTS stress condition is computed and compared in figure (10). Compared with the Yld2011 yield criterion, the Yld96 and Yld2004 yield criteria predicts more strain limit especially in the right-hand side of FLD in the tension-tension region in which estimate the more formability for AA3104-H19. The effect of σ_{23} TTS stress on the FLD and FLSD for AA3104-H19 is similar to the effect of σ_{13} TTS stress qualitatively, as shown in figure (11a-b).

Simultaneous effects of TTS and normal stresses on the FLD, FLSD for AA3104-H19 sheet metal based on the modified M-K model along with the Yld96, Yld2004, and Yld2011 yield functions are depicted in figure (12a-b). From figure (12a-b), it is observed that the predicted strain limit by using of Yld2011 and Yld2004 are smaller than of Yld69 yield criterion.

The increased percentage of FLD_0 for different loading condition by applying of Yld96, Yld2004 and Yld2011 yield functions are displayed in Table 1. As the table illustrates by applying the normal compressive stress along with the TTS, one can be increased the limit strain up to 25% for AA3104-H19 aluminum alloy.

4-Conclusion

In this investigation, based on the modified M-K theory along with the two advanced anisotropic yield functions including the Yld96, Yld2004, and Yld2011, the simultaneous effects of TTS and normal stresses on the FLD, FLSD, and XFLSD of AA3104-H19 aluminum alloy were numerically investigated. The set of nonlinear equations was solved by employing the Newton-Raphson numerical method to calculate the stress component and effective strain increment. A force equilibrium condition and

compatibility conditions have been introduced to develop the M-K theory. The effects of out of plane TTS and normal stresses on the FLCs prediction for different loading condition were investigated independently and simultaneously. The FLCs prediction of AA3104-H19 sheet metal by Yld2011 yield criterion was compared with Yld2004 and Yld96 yield criteria in the different loading conditions.

The following notation can be concluded of the present investigation.

- 1- Applying the TTS and normal stresses increases the limit strain and formability of AA3104-H19 alloy by considering the Yld96, Yld2004, and Yld2011 yield criteria.
- 2- In the out of plane shear stress condition, the σ_{23} TTS stress is more efficient than the σ_{13} TTS stress to decrease of stress limit and increase the formability of sheet metal by using of Yld96, Yld2004, and Yld2011 yield functions.
- 3- The normal stress has more considerable effect than of out of plane shear stress condition to increase the formability of sheet metal for both Yld96, Yld2004, and Yld2011 yield functions.
- 4- The GFLD analysis of AA3104-H19 indicates that the predicted limit strain by implementing of Yld96 and Yld2004 yield criteria are larger than of Yld2011 ones.
- 5- The percentage of FLD₀ increase for different of loading GFLD by using of Yld2011 yield criterion is larger than of Yld96 ones.

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References

[1] Barlat, F., Maeda, Y., Chung, K., Yanagawa, M., Brem. J.C., Hayashida, Y., Lege, D.J., Matsui, K., Murtha, S.J., Hattori, S., Becker R.C. and Makosey, S.: Yield function development for aluminum alloy sheets, Journal of the Mechanics and Physics of Solids, 45, 1727-1763, 1997.

[2] Barlat, F., Aretz, H., Yoon, J.W., Karabin M.E., Brem, J.C. and Dick R.E.: Linear transformationbased anisotropic yield functions, International Journal of Plasticity, 21, 1009-1039, 2005.

[3] Aretz, H. and Barlat, F.: New convex yield functions for orthotropic metal plasticity, International Journal of Non-Linear Mechanics, 51, 97-111, 2013.

[4] Keeler, S. P. and Backofen, W. A.: Plastic instability and fracture in sheets stretched over rigid punches, Transactions of the American Society for Metals, 56, 25-48, 1963.

[5] Goodwin, G. M.: Application of strain analysis to sheet metal forming problems, metal italienisch, 8, 1, 767-771, 1968.

[6] Marciniak, Z. and Kuczynski, K.: Limit strains in the processes of stretch forming sheet metal, International Journal of Mechanical Sciences, 9, 609-620, 1967.

[7] Marciniak, Z., Kuczynski, K. and Pokora, T.: Influence of the plastic properties of a material on the forming limit diagram for sheet metal in tension, International Journal Mechanical Sciences, 15, 789-805, 1973.

[8] Cardoso, R. P. R. and Adetoro O. B.: A generalisation of the Hill's quadratic yield function for planar plastic anisotropy to consider loading direction, International Journal Mechanical Sciences, 128-129, 253-268, 2017.

[9] Butuc, M. C., Banabic, D., Barata da Rocha, A. and Gracio, J. J.: The performance of Yld96 and BBC2000 yield functions in forming limit prediction, Journal of Materials Processing Technology, 125, 281-286, 2002.

[10] Barlat, F. and Lian, J.: Plastic behavior and stretchability of sheet metals (Part I) : A yield function for orthotropic sheet under plane stress conditions, International Journal of Plasticity, 5, 51-56, 1989.

[11] Ganjiani, M. and Assempour, A.: An improved analytical approach for determination of forming limit diagrams considering the effects of yield functions, Journal of Materials Processing Technology, 182, 598-607, 2007.

[12] Hosford, W. F.: A generalised isotropic yield criterion, Journal of Applied Mechanics, 39, 607-609, 1972.

[13] Banabic, D., Kuwabara, T., Balan, T., Comsa, D. S. and Julean, D.: Non-quadratic yield criterion for orthotropic sheet metals under plane stress conditions, International Journal Mechanical Science, 45, 797-811, 2003.

[14] Banabic, D., Aretz, H., Comsa, D.S. and Paraianu, L.: An improved analytical description of orthotropy in metallic sheets, International Journal of Plasticity, 21, 493-512, 2005.

[15] Ahmadi, S., Eivani, A. R. and Akbarzadeh, A.: An experimental and theoretical study on the prediction of forming limit diagrams using new BBC yield criteria and M-K analysis, Computational Materials Science, 44, 1272-1280, 2009.

[16] Dasappa, P., Inal, K. and Mishra, R.: The effects of anisotropic yield functions and their material parameters on prediction of forming limit diagram, International Journal of Solids and Structures, 49, 3528-3550, 2012.

[17] Hill, R.: A theory of the yielding and plastic flow of anisotropic metals, Proceedings of the Royal Society of London, 193, 281-297, 1948.

[18] Hill, R.: Constitutive modeling of orthotropic plasticity in sheet metals, Journal of the Mechanics and Physics of Solids, 38, 405-417, 1990.

[19] Hill, R.: A user-friendly theory of orthotropic plasticity in sheet metals International Journal Mechanical Science, 35, 19-25, 1993.

[20] Barlat, F. and Richmond, O.: Prediction of tricomponent plane stress yield surfaces and associated

flow and failure behaviour of strongly textured FCC polycrystalline sheets, Materials Science and Engineering, 91, 15-29, 1987.

[21] Plunkett, B., Cazacu, O. and Barlat, F.: Orthotropic yield criteria for description of the anisotropy in tension and compression of sheet metals, International Journal of Plasticity, 24, 847-866, 2008.

[22] Li, X., Song, N., Guo, G. and Sun, Zh.: Prediction of forming limit curve (FLC) for Al-Li alloy 2198-T3 sheet using different yield functions, Chinese Journal of Aeronautics, 26, 1317-1323, 2013.

[23] Barlat, F., Brem, J. C., Yoon, J. W., Chung, K., Dick, R. E., Lege, D. J., Pourgoghrat, F., Choi, S. H. and Chu, E.: Plane stress yield function for aluminum alloy sheets (Part 1) : Theory, International Journal of Plasticity, 19, 1297-1319, 2003.

[24] Panich, S., Barlat, F., Uthaisangsuk, V., Suranuntchai, S. and Jirathearanat, S.: Experimental and theoretical formability analysis using strain and stress based forming limit diagram for advanced high strength steels, Materials and Design, 51, 756-766, 2013.

[25] Ozturk, F., Toros, S. and Kilic, S.: Effects of anisotropic yield functions on prediction of forming limit diagrams of DP600 advanced high strength steel, Procedia Engineering, 81, 760-765. 2014.

[26] Sheng, Z. Q. and Mallick, P. K.: A ductile failure criterion for predicting sheet metal forming limit, International Journal Mechanical Sciences, 128-129, 345-360, 2017.

[27] Bridgman, P. W.: Studies in large plastic flow and fracture, McGraw-Hill, New York, 1952.

[28] Fuchs, P. J.: Hydrostatic pressure: its role in metal forming, Mechanical Engineering, 34-40, 1966.

[29] Assempour, A., Khakpour Nejadkhaki, H. and Hashemi, R.: Forming limit diagrams with the existence of through-thickness normal stress, Computational Materials Science, 48, 504-508, 2010.

[30] Hashemi, R. and Abrinia, K.: Analysis of the extended stress-based forming limit curve considering the effects of strain path and through-thickness normal stress, Materials and Design, 54, 670-677, 2014.

[31] Nurcheshmeh, M. and Green, D. E.: The effect of normal stress on the formability of sheet metals under non-proportional loading, International Journal of Mechanical Sciences, 82, 131-139, 2014.

[32] Zhang, F., Chen, J., Chen, J. and Zhu, X.: Forming limit model evaluation for anisotropic sheet metals under through-thickness normal stress, International Journal of Mechanical Sciences, 89, 40-46, 2014.

[33] Zhang, F., Chen, J. and Chen, J.: Effect of through-thickness normal stress on forming limits under Yld2003 yield criterion and M-K model, International Journal of Mechanical Sciences, 89, 92-100, 2014.

[34] Yang, X., Lang, L., Liu, K. and Guo, C.: Modified MK model combined with ductile fracture criterion and its application in warm hydroforming, Transactions Nonferrous Metals Society of China, 25, 3389-3398, 2015.

[35] Mirfalah-Nasiri, S.M., Basti, A. and Hashemi, R.:
 Forming limit curves analysis of aluminum alloy considering the through thickness normal stress, anisotropic yield functions and strain rate, International Journal of Mechanical Sciences, 117, 93-101, 2016.

[36] Ben Bettaieb, M. and Abed-Meraim, F.: Theoretical and numerical investigation of the impact of out-of-plane compressive stress on sheet metal formability, International Journal of Mechanical Sciences, In Press, 2017.

[37] Ai, S., Lu, B., Chen, J., Long, H. and Ou, H.: Evaluation of deformation stability and fracture mechanism in incremental sheet forming, International Journal of Mechanical Sciences, 124-125, 174-184, 2017.

[38] Allwood, J.M. and Shouler, D.R.: Generalised forming limit diagrams showing increased forming limits with non-planar stress states, International Journal of Plasticity, 25, 1207-1230, 2009.

[39] Eyckens, P., Van Bael, A., and Van Houtte, P.: Marciniak–Kuczynski type modelling of the effect of through-thickness shear on the forming limits of sheet metal, International Journal of Plasticity, 25, 2249-2268, 2009.

[40] Eyckens, P., Van Bael, A., and Van Houtte, P.: An extended Marciniak–Kuczynski model for anisotropic sheet subjected to monotonic strain paths with through-thickness shear, International Journal of Plasticity, 27, 1577-1597, 2011.

[41] Fatemi, A. and Mollaei Dariani, B.: Forming limit prediction of anisotropic material subjected to normal and through thickness shear stresses using a modified M-K model, Internatinal Journal of Advanced Manufacturing Technology, 80, 1497-1509, 2015.

[42] Ma, B. L., Wan, M., Li, X. J., Wu, X. D. and Diao, K.S.: Evaluation of limit strain and temperature history in hot stamping of advanced high strength steels (AHSS), International Journal of Mechanical Science, 128-129, 607-613, 2017.

[43]., Aretz, H. and Barlat, F.: in: Proceedings of the 8th International Conference on Numerical Methods in Industrial Forming Processes-NUMIFORM 2004, AIP Conference Proceedings, 712, 147-156, 2004.
[44] Lebensohn, R. A. and Tome, C. N.: A self-consistent anisotropic approach for the simulation of plastic deformation and texture development of poly crystals: application to zirconium alloys, Acta Metal lurgica et Materialia, 41, 2611-2624, 1993.

[45] Sowerby, R. and Duncan, D. L.: Failure in sheet metal in biaxial tension, International Journal of Mechanical Science, 13, 3, 217-229, 1971.

[46] Wu, P. D., Jain, M., Savoie, J., MacEwen, S. R., Tugcu, P. and Neale, K. W.: Evaluation of anisotropic yield functions for aluminum sheets, International Journal of Plasticity, 19, 121-138, 2003.

[47] Soare, S. C.: Theoretical considerations upon the MK model for limit strains prediction: the plane strain case, strain-rate effects, yield surface influence, and material heterogeneity, European Journal of Mechanics/A Solids, 29, 938-950, 2010.

[48] Barata da Rocha, A., Barlat, F. and Jalinier, J. M.: Prediction of the forming limit diagrams of Anisotropic sheets in linear and non-linear loading, Materials Science and engineering, 68, 2,151-164,1985.

[49] Ham, M. and Jeswiet, J.: Forming Limit Curves in Single Point Incremental Forming, Annals of the CIRP, 56, 277-280, 2007.

List of captions:

Figure caption

Fig. 1. Schematic of initial geometrical M-K model a) undeformed, b) deformed.

Fig. 2. Force equilibrium between homogeneous and groove region: a) the sheet normal direction 3, b) along the groove direction t.

Fig. 3. FLD for AA5754 alloy in two kinds of sheet metal forming processes.

Fig. 4. Effect of σ_{13} TTS Stress on the FLCs of AA3104-H19 a) FLD, b) FLSD, c) XFLSD.

Fig. 5. Effect of σ_{23} TTS stress on the FLCs of AA3104-H19 a) FLD, b) FLSD, c) XFLSD.

Fig. 6. Simultaneous effect of TTS and normal stresses on the FLCs of AA3104-H19 a) FLD, b) FLSD, c) XFLSD.

Fig. 7. Comparison of the effect of stress components on the FLCs of AA3104-H19 a) FLD, b) FLSD.

Fig. 8. Effect of stress components on the yield surface of AA3104-H19.

Fig. 9. Comparison of experimental [46] and theoretical FLC under plane-stress condition for AA3104-H19 alloy a) FLD, b) FLSD.

Fig. 10. Effect of Yld96, Yld2004 and Yld2011 criteria on the GFLD by applying the σ_{13} TTS stress a) FLD, b) FLSD.

Fig. 11. Effect of Yld96, Yld2004 and Yld2011 criteria on the GFLD by applying the σ_{23} TTS stress a) FLD, b) FLSD.

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Fig. 12. Effect of Yld96, Yld2004 and Yld2011 criteria on the GFLD by applying the TTS and normal stresses a) FLD, b) FLSD.

Table caption

Table 1. Percentage of FLD₀ increase for different loading condition.

Highlights:

- 1- Generalized Forming Limit Diagram (GFLD) for AA3104-H19 alloy is investigated by employing Three advanced anisotropic yield criteria, including Yld96, Yld2004 and Yld2011.
- 2- The M-K model is modified by considering the normal and through-thickness shear (TTS) stresses.
- 3- The GFLD analysis of AA3104-H19 indicates that the predicted limit strain by implementing of Yld96 yield criterion is the largest.
- 4- The normal stress has more considerable effect than of out of plane shear stress condition to increase the formability of sheet metal.





