Why Unorthodox Analytic Philosophy

GUILLERMO E. ROSADO HADDOCK

ABSTRACT

So–called analytic philosophy is a product of early XXth century philosophy, resulting both as a response to the revolutions in logic, physics and mathematics of that period, as well as a reaction to the uncontrolled metaphysical systems that flourished in most of the XIXth century. But so–called analytic philosophy was from the very beginning biased by an ideological empiricism that blinded the understanding both of the history of philosophy, as well as that of physics and mathematics. Their grotesque division of philosophy in analytic and continental philosophy served only the purpose of arbitrarily excluding important contributions to rigorous philosophy not based on the gigantic meta–theoretic dogma of empiricism. Unorthodox analytic philosophy, on the contrary, though by no means ignoring the results of the physical sciences, is not based on any empiricist dogma, being perfectly conscious of the difficulties of empiricism to understand both physical and deductive sciences. Some of those difficulties of the empiricist ideology, both of general epistemological nature, and more specifically in the assessment of physical science and very especially in its misguided attempts to deal with the deductive sciences, will be examined.

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G. E. Rosado Haddock (✉)
University of Puerto Rico at Rio Piedras, Puerto Rico
e–mail: gerosadohaddock@gmail.com

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GUILLERMO E. ROSADO HADDOCK

§1. Introduction: Post–Kantian Philosophy and Scientific Revolutions

IMMANUEL KANT IS CERTAINLY ONE OF THE GREATEST PHILOSOPHERS ever. He was not only the culmination of the so–called modern philosophy, but also a seminal philosopher for the generations to come. In his *opus magnum*, *Kritik der reinen Vernunft*[1] —and not only there— Kant offered the most serious attempt up to his date at an explanation of how our scientific knowledge is possible, at the same time by demarcating it from metaphysical pseudo–knowledge. Having been influenced, on the one hand, by Leibniz and, on the other by Hume, and taking into account the three fundamental sciences, namely, logic, mathematics and physics, he tried to answer the question: How is scientific knowledge possible? Seen from our perspective, the status of such sciences represented a limitation for Kantian theoretical philosophy: for him logic was basically Aristotelian logic, physics Newtonian classical mechanics, while mathematics was basically Euclidean geometry, arithmetic, elementary algebra and non–rigorous intuitive infinitesimal calculus of Newton and Leibniz. Things will begin to change very fast both in philosophy and in the fundamental sciences, but in opposite directions.

In philosophy in the first half of the nineteenth century, and especially in Germany, the presumed "overcoming" of Kant consisted in the construction of metaphysical systems of a new sort by Kant's student Fichte —who "overcame Kant" in the same sense in which more than a century later Husserl's assistant, Heidegger "overcame Husserl"—, by Schelling and Hegel. Hegel's metaphysical system, which was by far the most prominent, dominated the philosophical scene

1 *Kritik der reinen Vernunft* 1781, revised second edition 1787, reprint of both versions, Felix Meiner 1930, third edition 1990. See also *Prolegomena zu einer jeden künftigen Metaphysik, die als Wissenschaft wird auftreten können* 1783, reprint, Felix Meiner 2001, and *Metaphysische Anfangsgründe der Naturwissenschaften* 1786, reprint, Felix Meiner 1997.

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in Germany for a great part of that century, and his *Wissenschaft der Logik*[2] —that is *Science of Logic*—, which has nothing to do either with logic or with science, and is metaphysics in its pristine sense, has served to inspire some theoretical nonsense, like the so–called "scientific" method of dialectical materialism of the so–called "science" of Marxism.

In contrast with those metaphysical excesses, there was in the domain of rigorous science in the nineteenth century and the beginning of the twentieth an almost total overcoming of the sciences that served Kant as basis for his seminal philosophy. In mathematics, already a few years after Kant's death began two different trends in the areas of infinitesimal calculus and geometry that would revolutionize mathematics. On the one hand, Cauchy, Bolzano and others began the process of building a rigorous foundation of the infinitesimal calculus, a trend that continued during the whole nineteenth century and culminated with Weierstrass' foundation of infinitesimal calculus on the real number system and Dedekind's foundation of the real number system on that of the natural numbers.[3] On the other hand, already in Kant's time some mathematicians questioned shyly Euclid's parallels postulate, and in the first decades of the nineteenth century Gauß, Bolyai and Lobatchevsky developed the first non–Euclidean geometry, in which the sum of the angles of a triangle is less than 180 degrees, that is, the space has negative curvature and in which a point P not on a line L is intersected by more than one —in fact, by infinitely many—, lines that do not intersect L.[4]

Moreover, at the beginning of the second half of that century, one of Gauß' last students, Bernhard Riemann, not only introduced a second sort of non–Euclidean geometry, namely, one of positive curvature, in which the sum of the angles of a triangle exceeds 180 degrees and in which there are no lines intersecting a point P not on line L, which do not also intersect L. More importantly, the great Riemann revolutionized our concepts of space and

2 Georg W. F. Hegel, *Wissenschaft der Logik I* 1812, *II* 1813, second edition 1834, reprint, Felix Meiner (*I*) 1967 and (*II*) 1969.

3 For the foundation of infinitesimal calculus on the real number system, see any good textbook on real analysis, for example, Stephen Abbott's *Understanding Analysis*, Springer 2001 For the foundation of the real number system on the natural numbers, see, for example, Elliot Mendelson's *Number Systems and the Foundations of Analysis*, Academic Press 1973

4 For non–Euclidean geometry and its history, see, for example, Marvin J. Greenberg, *Euclidean and Non–Euclidean Geometries: Development and History*, second edition, W. H. Freeman 1980, or Judith N. Cederberg, *A Course in Modern Geometries*, second edition, Springer 2001, or B. A. Rosenfeld, *A History of Non–Euclidean Geometry*, Springer 1988.

geometry, firstly, by separating the study of pure geometry from any considerations about the geometry of physical space, considering geometry as a formal discipline studying all sorts of Euclidean and non–Euclidean geometries, not only of two or three dimensions, but of n dimensions, for any finite $n \geq 2$. On the other hand, Riemann considered that the problem of the nature of physical space, whether it has three, four or whatever dimensions, and whether it has zero curvature, negative curvature or positive curvature, can only be decided empirically. The only restriction he put on the nature of space is that it is locally Euclidean, that is, that the space immediately surrounding us should have curvature indistinguishable from that of Euclidean manifolds. In fact, Riemann went much further, when he generalized the notion of geometrical manifold to one simply of manifolds and considered not only continuous but also discrete manifolds, like that of the natural numbers; and ended his extraordinary monograph 'Über die Hypothesen, welche der Geometrie zu Grunde liegen'[5] with the up to now ignored observation that space at the infinitesimal level could very well not be continuous but discrete. It would be interesting to see what happens if some physicists take seriously that last Riemannian hypothesis and apply it to the microphysical world.[6]

Though less relevant for this paper, it should be pointed out that during the nineteenth century notions of abstract algebra began to emerge, as well as generalizations of geometrical notions, culminating in the early twentieth century with the development of general topology and a few decades later of universal algebra and finally of category theory. Set theory, which began its development as a discipline at the end of the nineteenth century with Georg Cantor and culminated its development in the twentieth century, should also be mentioned as part of this decisive trend towards generalization and abstraction.

But not only mathematics changed substantially during the nineteenth and twentieth century. Something even more radical happened in logic. Already in the mid–nineteenth century George Boole and Augustus de Morgan developed logic beyond the traditional Aristotelian logic, and such a trend was developed even more by Charles Sanders Peirce and Ernst Schröder. But beginning in 1879 the mathematician turned philosopher Gottlob Frege revolutionized logic[7],

[5] 'Über die Hypothesen, welche der Geometrie zu Grunde liegen', 1867, third edition 1923, reprint Chelsea 1973.

[6] According to Sunny Y. Auyang —see her outstanding 'How is Quantum Field Theory Possible?', p. 137—, there are already some first steps in this direction.

[7] *Begriffsschrift* 1879. See also his *Grundgesetze der Arithmetik I* 1893 and *II* 1903.

developing that discipline far more than the most optimist Aristotelian could have dreamt. Frege was followed by Whitehead and Russell[8], both in the development of logic and in the commitment to the philosophical view that the whole of mathematics could be obtained by definitions and derivations from the most fundamental logical statements. In some sense, such a thesis was a sort of culmination of the Weierstrass–Dedekind program, but it faced difficulties that the more modest program of the arithmetization of analysis did not face. In fact, besides the difficulties that the logic system of Frege faced —namely, it was inconsistent—, as well as those faced by the Whitehead–Russell system —namely, it included as axioms statements that by no means belonged to logic—, precisely the development of general topology, universal algebra and category theory makes such a logicist program nothing more than an utopia. Finally, it should be pointed out that logic itself had an extraordinary development after Frege, Whitehead and Russell, to the point that their specific logical systems have nowadays mostly a historical importance.

Last, but not least, physics also had an extraordinary development since Kant, beginning in the nineteenth century and especially at the beginning of the twentieth century. Already, in the mid–nineteenth century the study of electromagnetic phenomena caused some tensions in physics. Classical mechanics was supposed to be the fundamental physical discipline on which all physical phenomena should be grounded and explained, but electromagnetic phenomena did not easily fit into that scheme. The tensions were not sufficient to produce what some people call a "scientific revolution"[9], but at the beginning of the twentieth century three new theories were to change the course of physics radically producing a sort of triple–headed scientific revolution. Firstly, there were the two revolutionary theories originated by Einstein, the special theory of relativity of 1905 and the general theory in 1915 —to which Hilbert also arrived some two weeks after Einstein. Both theories deal with macrophysical phenomena, the latter one even with cosmological phenomena. Almost parallel to the development of general relativity were the first steps in the development of the third revolutionary physical theory of the twentieth century, dealing this time with microphysical phenomena, namely, quantum mechanics, which was systematized in the 1920s and 1930s, and whose nature and foundations are still the objects of vivid philosophical debate.

[8] *Principia Mathematica* 3 vols. 1910–1913, revised edition 1925–1927, reprint Cambridge 1963.

[9] See Thomas Kuhn's *The Structure of Scientific Revolutions* University of Chicago Press 1962, expanded second edition 1970.

§2. On Logical Empiricism

Though during the nineteenth century some philosophers differed fundamentally from the predominant metaphysical trend, for example, the great philosopher–mathematician Bernard Bolzano and Hermann Lotze, on the one hand, who followed the Leibnizian tradition of conceiving logic as inseparably linked to mathematics[10], and, on the other hand, the school of Ernst Mach and Richard Avenarius, more concerned with physics and advocating some sort of empiricism or positivism[11], it was not until the twentieth century that such isolated trends consolidated and even momentarily tried to coexist. At the end of the second decade of the twentieth century, so–called logical positivism or logical empiricism was born basically at the hands of Moritz Schlick —a student of Max Planck— and Hans Reichenbach —a student of Albert Einstein. The movement consolidated both in Berlin in the group built around Hans Reichenbach and Kurt Grelling, and especially in Vienna as Moritz Schlick moved to Vienna and joined forces with the mathematician Hans Hahn and the economist and staunch empiricist with some Marxist leanings Otto Neurath, both of which had already been working on some of the issues dear to the movement with the physicist Philip Frank, who in the meantime had moved to Prague. The movement, especially what later became known as the Vienna circle, was not only impressed by the recent developments in physics, mostly by Albert Einstein[12], and in a lesser degree by the nascent revolution in microphysics, but by Ludwig Wittgenstein's *Logisch–philosophische Abhandlung*, published in 1921[13]. Thus, was born this moderate sort of empiricism, which acknowledged the independence of logic and mathematics from experience, committed to the logicism of Frege and Russell, while pretending to ground all laws of the empirical sciences basically on experience. Interestingly enough, the best–known defender of logical empiricism —especially in Anglo–American circles—, namely, Rudolf Carnap arrived at the scene much later, in fact, in 1926. Contrary to the very questionable historical renderings by Anglo–American scholars, Carnap had been up to then

[10] See Bernard Bolzano's *Grundlegung der Logik*, which is a selection of volumes 1 and 2 of his monumental *Wissenschaft der Logik*. Lotze was a logicist before Frege and could have influenced the latter, and as Frege was also blind with respect to the non–Euclidean geometries.

[11] On this point, see Mach's *Erkenntnis und Irrtum: Skizzen zur Psychologie der Forschung* 1905, third edition 1917, reprint Darmstadt 1980.

[12] Both Moritz Schlick and Hans Reichenbach wrote books on the philosophical implications of general relativity almost immediately after Einstein's groundbreaking paper. See the references.

[13] In particular, Moritz Schlick was impressed by Wittgenstein's *Tractatus* –as it is popularly called– to a point difficult to accept by other members of the Vienna Circle.

a follower of Edmund Husserl. Carnap's dissertation, *Der Raum* was written from the perspective of a follower of Husserl, and after he finished it, he began to visit Husserl's seminars and did that during four semesters, while writing his Husserlian professorship's thesis *Der logische Aufbau der Welt*, which he had already completed before moving to Vienna and after Husserl did not accept to sponsor such a Husserlian work.[14] Carnap made some hurried changes to the book — most surely deleting some references to Husserl— and presented it in Vienna as professorship's thesis sponsored by Moritz Schlick. The result was plagiarism, and from then on Carnap tried to ignore Husserl's works as much as possible, what did not free him from other similar difficulties.[15]

Leaving those issues aside, logical empiricism's doctrine was based on a dichotomy between the formal and the so–called empirical sciences. Logic and mathematics say nothing about the world, they are analytic, mathematics being merely an extension of logic that can be derived completely from the logical axioms. Thus, the logical empiricists' views of logic and mathematics were basically similar to those of Frege and Russell, with the small difference that for Carnap, who soon became the principal figure of the movement, there were other analytic statements, besides those of logic and mathematics, namely, statements like "No bachelor is married", which are known to be true presumably by means of the analysis of the concepts involved. Hence, Carnap's conception of analyticity was different from that of Frege. It did not mean 'derivable from logical axioms' but 'true on the basis of the pure conceptual analysis. One can very well say that Carnap's notion of analyticity is an extension of Kant's official notion of analyticity, expounded at the beginning of the *Kritik der reinen Vernunft*, whereas Frege's notion of analyticity is a generalization of Kant's second unofficial notion of analyticity, expounded in the section of Kant's masterpiece on the highest principle of analytic judgements —as Kant used to say, 'statements' in our present parlance.[16]

14 On this issue, see our book *The Young Carnap's Unknown Master*, our papers 'On the Interpretation of the Young Carnap's Philosophy' and 'The Old Husserl and the Young Carnap', as well as Verena Mayer's pioneering papers 'Die Konstruktion der Erfahrungswelt: Husserl und Carnap' and 'Carnap und Husserl', and her more recent and far more extensive and detailed 'Der logische Aubau als Plagiat'. Finally, see also Carnap's letter of 26 September 1925 to Jonas Cohn.

15 See the references mentioned in the foregoing footnote. By the way, besides the non–academic personal attacks in the Carnap Blog, the Carnap fans have still to answer Mayer's and my interpretation of the *Aufbau*. In any case, they have the most powerful publishers at their service.

16 See *Kritik der reinen Vernunft*, Meiner edition, pp. 45f. (A7–10, B11–14) for the official definition and pp. 207f. (A150–153, B191–193) for the second characterization by means of the highest principle of analytic judgements.

On the other hand, the statements of the empirical sciences are all empirical. They are based on very concrete elementary statements, called by the logical empiricists ‘protocol sentences’, which were —in one version (the so–called ‘phenomenalist’)— very concrete and simple statements about our sensations, and, in the other (the so–called ‘physicalist’), very concrete and simple statements about the physical world. All laws of the empirical sciences were based by means of some sort of induction on such concrete and simple statements. The logical empiricists developed criteria of empirical significance and pretended to show that all laws of physics could be empirically tested that way. Thus, they developed first a criterion of verification, which was easily refuted by means of elementary logic, then a criterion of falsification, which also was easily refuted by means of elementary logic, and then a criterion of cognitive significance, which also suffered the same destiny.[17] In fact, such a project, even in its third version, was from the very beginning preposterous. Even the Newtonian law of gravitation and the three basic laws of his mechanics are certainly not reducible to an accumulation of any finite number of protocol–like statements.

Finally, some of the logical empiricists, especially Reichenbach, tried to add the component of conventionalism to the understanding of physical science, but it was insufficient.[18] Logical empiricism was a complete failure beyond repair. However, what replaced it in the empiricist world was even less defendable, namely, the radical empiricism of Quine and its various disciples. But before examining Quine’s empiricism, let us briefly consider a few points brought by philosophers before the advent of logical empiricism that, if carefully examined, should have deterred, Schlick, Carnap and their friends from propounding such simplistic and clearly false views on physical science.

§3. On Duhem, Husserl and beyond

Some years before the advent of logical empiricism the important French historian of science and philosopher Pierre Duhem had advanced a fundamental thesis with respect to physics, while at the same time contrasting that discipline with other natural sciences. Duhem argued that in physics, contrary to biology and other empirical sciences, one cannot isolate hypotheses.[19] The reason for it

17 One can find expositions of these refutations probably in any serious account of logical empiricism.

18 See, for example, Reichenbach’s *The Theory of Relativity and a priori Knowledge* and *Philosophy of Space and Time*.

19 See Pierre Duhem’s classic book *La Théorie Physique, son Objet, sa Structure* 1914, English translation, *The Aim and Structure of Physical Theory*, Princeton University Press 1955, reprint 1991, as well as his

is the following: while when observing, for example, microorganisms in biology through a microscope, the biologist can isolate a biological hypothesis from any other law or hypothesis in biology, presupposing here only that some physical laws —in this case of optics— are correct, in the case of physics that is not possible. When doing an experiment to contrast any physical hypothesis the physicist presupposes the correctness of other physical laws on which the instruments used in the experiment are based. Thus, if the result of the experiment is what was expected, it was not obtained exclusively from the hypothesis under discussion, but from the conjunction of that hypothesis with the laws that had been presupposed; and if the result of the experiment is not what one expected, it is not the hypothesis which has been proved false, but the conjunction of the hypothesis with all the laws presupposed in the experiment. Duhem correctly concluded that one cannot isolate hypotheses in physics —even though one can very well do it in other not so fundamental empirical sciences.

This very sound result of Duhem has been smeared by irresponsibly associating it with a clearly preposterous thesis of Quine, according to which all our beliefs are connected in a giant nexus of beliefs and do not differ essentially in the certitude of their truth or falsity.[20] This absurd thesis —on which more will be said below— has been dubbed the "Duhem–Quine Thesis", granting an undeserved honour to Quine, while tergiversating one of Duhem's most important contributions to philosophy and staining his memory.

Duhem, by the way, had also made an important observation about physical laws that, had it been taken seriously by the logical empiricists, would have prevented them from some of their errors. Duhem considered —as well as Husserl— that the most important laws of physical science cannot be obtained by induction from simple observations, leaving a margin of empirical indeterminacy for physical laws.[21]

Such an issue brings us to Husserl, one of the greatest philosophers in the history of mankind, and one almost completely ostracized by the ignorance of orthodox analytic philosophers. Husserl's extensive work has probably no parallel in the history of philosophy, but we will be concerned here only with two important but very little known Husserlian conceptions, namely with some of his views on physics and later with those on categorial intuition.

older paper of 1894, translated in his *Essays in the History and Philosophy of Science*, 'Some Reflections on the Subject of Experimental Physics'.

20 See § 4 below, in which we examine Quine's views.

21 See Duhem's book referred to in footnote 18.

In the first part of the last chapter —Chapter XI— of the first volume of his *opus magnum*, *Logische Untersuchungen*[22], Husserl distinguishes between purely descriptive sciences and what he called "explanatory sciences", which include the three fundamental disciplines of logic, mathematics and physics. Although Husserl's interest in that chapter is mainly to expound his conception of logic, mathematics and their bond in his version of the *mathesis universalis* and the theory of all deductive theories, he begins by considering physical science. Firstly, he distinguishes between three different nexus that are present in physical science[23], namely, the subjective nexus in the consciousness of the researcher, the objective ontological nexus of the objects of study of the science, and the also objective nexus of the truths of the science. He leaves aside the subjective nexus —about which he is not going to say anything more in that context— and considers only the two objective ones. The ontological nexus, that is, the coming together of things ontologically related, is present in all sciences. Thus, the same discipline, history, studies the historical events that occurred before Christ, in the middle ages and in the twentieth century, be it in Greece, in China, in Germany or in Brazil, while zoology studies tigers and dogs, elephants and flies, and classical mechanics studies masses, forces, velocities and accelerations. Finally, there is the nexus of truths, which is objective but different from the ontological nexus. The second law of classical mechanics, namely, force = mass × acceleration belongs to the nexus of truths. Another certainly incorrect presumed law could have combined the same ontologically related concepts in a different way. Thus, even in this very simple example one can see not only that the nexus of truths and the ontological nexus neither coincide nor determine each other, and that even in classical mechanics laws are not simply a picture of observed objects or events.

Husserl went, however, much further, arguing that the general laws of physics are in no way obtained by induction. They are what he called "*hypotheses cum fundamento in re*"[24], that is, general hypotheses thinly related or compatible with experience, which serve to explain the low–level laws obtained more or less by induction. The relation between the *hypotheses cum fundamento in re* and the low–level laws is no other than that described much later by Popper, Hempel and others as the deductive–nomological and the probabilistic–nomological schemes of explanation of laws. But since the low–level laws cannot determine the higher–

22 *Logische Untersuchunge I*, §§62–66.

23 Ibid., §62.

24 Ibid., §§65–66.

level laws, this opens the door, as Husserl stated some years before Duhem and probably more forcefully, for the indeterminacy of the high–level laws.[25] They are only *hypotheses cum fundamento in re*, but other such hypotheses compatible with the low–level laws are possible. Hence, before Duhem and a half–century before Quine, Husserl had acknowledged the indeterminacy of physical laws in a sober and rational way without any fanfare.

Finally, concerning Husserl and physical science, we would like to bring a point here that is certainly ignored even by the most ardent Husserlians. Contrary to other distinguished philosophers of his time, especially Lotze, Frege, Brentano, Natorp and in a less radical way Poincaré and Duhem, already in 1892 the mathematician turned philosopher Edmund Husserl —student and later assistant of Karl Weierstrass— had accepted Riemann's conception of geometry in all its aspects. Thus, for example, in a letter to his philosophy professor Brentano at the end of 1892 Husserl pointed out that he had accepted both Riemann's view of geometry as a mathematical science being a theory of manifolds of n dimensions, for $n\geq 2$ and curvatures $c=0$, $c>0$ or $c<0$, all equally justified from a mathematical standpoint; and also that the dimensionality and curvature of physical space cannot be decided *a priori*, but only empirically.[26] Similar remarks are present in letters to Paul Natorp of 1897 and 1902, and in the last part of his *Studien zur Arithmetik und Geometrie*, all written many years before the advent of Einstein's theory of general relativity, in which Riemann's views received their due acceptance.[27]

§4. Empiricism as Ideology: Quine and beyond

The demise of logical empiricism because it was incapable of accounting for physical laws of higher level should have produced either a still more liberal version of empiricism or a complete abandonment of empiricism. That is the normal rational course taken in the history of philosophy from Locke to Hume, and from Hume, on the one hand, and Leibniz, on the other hand, to Kant. However, contrary to all judicious expectations, the demise of logical empiricism gave rise to much more radical sorts of empiricism, very especially, to Quinean empiricism, one that could certainly be even more incapable of doing justice to

[25] Ibid.

[26] The letter to his teacher Franz Brentano is from 29 December 1892. See Husserl's *Briefwechsel I*, pp. 8–11.

[27] See Husserl's posthumous *Studien zur Arithmetik und Geometrie*, Hua. XXI, 1983. For the letters to Paul Natorp, see Husserl's *Briefwechsel V*, pp. 59–64 and 80–86.

physical laws of higher level.[28] Such a course of events is a clear indication that at least beginning with Quine and up to this date Anglo–American empiricism is not simply a usual philosophical trend, but an ideology, based only on prejudices not on scientific grounds or on philosophical argumentation, and very similar to the historical and dialectical materialism ideology that hindered so much the natural course of rigorous philosophical work in the ancient Soviet Union and the other former so–called socialist countries of Eastern Europe. Let us then say a few things about Quine's views.

In his famous 'Two Dogmas of Empiricism'[29], Willard van Orman Quine examined very critically Carnap's conception of analyticity, and in some sense brought to an end the search for analytic statements in the empiricist tradition —what certainly at first sight seems to be a great achievement and has played a decisive role in Quine's recognition as a sort of dean of Anglo–American empiricists. One should not forget, however, that Carnap's conception of analyticity, according to which a statement is analytic if its truth can be obtained by the mere conceptual analysis of the given statement, combined two very different sorts of statements, namely, logical and mathematical statements, on the one hand, and statements like 'All bachelors are not married', on the other. In fact, Quine concentrates his efforts in showing that the statements of the latter group are not analytic.[30] And certainly he is right: those statements are not analytic but empirical. They are empirical because their truth depends not on any convention or whatsoever, but on the historic evolution of natural languages —in this case English—, which is without any doubt an empirical evolution. Nonetheless, to conclude that because statements like 'A bachelor is not married' are not analytic, logical statements like '$\neg(p\wedge\neg p)$' or arithmetical statements like '2+2=4' are also not analytic is clearly a *non sequitur*.

But even in case, on other grounds, Carnap's definition of analyticity applied only to logical and mathematical statements were not adequate, Quine could not conclude, as he does, that there are no analytic statements. As a matter of fact,

28 To put it as simply as possible: if you cannot derive δ from α, β and γ, you cannot derive δ from α alone. (And you can replace 'derive from' with 'grounded on' or 'based on' in the preceding statement and it remains true.) Quine and all his radical empiricists followers ignored such a logical triviality that disqualified from the start all their attempts at accounting for physical science.

29 'Two Dogmas of Empiricism' 1951, reprint in Quine's *From a Logical Point of View*, Harvard University Press 1953, pp. 20–46.

30 On p. 24 he makes this perfectly clear: "Our problem, however, is analyticity, and here the major difficulty lies not in the first class of analytic statements, the logical truths, but rather in the second class, which depends on the notion of synonymy".

there existed other very different definitions of analyticity, like that of Frege —briefly: a statement is analytic if it is derivable from logical axioms by means of logical rules and definitions— or that of Husserl —briefly: a statement is analytic if it is completely formalizable *salva veritate*. Hence, Quine is guilty of a second *non sequitur*.

Moreover, from his unwarranted conclusion Quine goes on to argue[31] that there is no essential difference with respect to the grounding of truth between any statements, but that they are all immersed in a web of belief, and we only pragmatically decide out of convenience to protect the truth of logical statements —like: '$\neg(p\wedge\neg p)$'— and mathematical statements —like: '2+2=4'— instead of that of more worldly statements like 'Newton had a toothache when he discovered the law of gravitation'. Here we have a third *non sequitur*, since even if all the above mentioned definitions of analyticity were shown to be false, that would not exclude other possible correct definition of analyticity[32], nor does it exclude that logical and mathematical statements were synthetic *a priori*, as Kant thought, and hence essentially different from empirical statements.

Quine's last–mentioned thesis, namely, that all our statements are connected in a sort of web of belief, and that no statement has the privileged status of being analytical, being all essentially on the same level, lies at the origin of Quine's caricature of Duhem's thesis. According to Quine, all statements in our web of belief face experience together and, as already pointed out, it is a purely pragmatic decision to protect a statement from being refuted, for example, a logical law or a physical law, instead of a more mundane statement. You have to be really a staunch ideological empiricist to swallow this pill!

But Quine's ideological empiricism does not stop there. In his *Word and Object*[33] and other writings Quine has tried to develop a sort of naturalized

31 Op. cit., pp. 42–43.

32 On this issue, see the definition of analyticity given in our paper 'On Analytic *a posteriori* Statements: are they Possible?', which is certainly immune not only to Quine's criticisms, but to other criticisms directed against Frege's or even against Husserl's definition. By the way, in the meantime —at the end of 2018— we have modified a little our definition in order to guarantee that there are analytic *a posteriori* statements and not merely *a posteriori* instances of analytic laws. In the future any reprinting of the paper will include such modification as an *addenda*.

33 *Word and Object*, MIT Press 1960, especially Chapter II. See also his *The Roots of Reference*, Open Court 1973, *From Stimulus to Science*, Harvard University Press 1995, second printing 1998, and his Theories *and Things,* Harvard University Press 1981, as well as his 'Epistemology Naturalized', in his *Ontological Relativity and other Essays*, Columbia University Press 1969, pp. 69–90. See also some of the papers included in his posthumous *Confessions of a Confirmed Extensionalist and Other Essays*, Harvard University Press 2008.

epistemology based on the already discredited psychological behaviourism, also a child of grotesquely radical empiricism. According to Quine, both linguistics and epistemology are reducible to the now discredited behaviouristic psychology, which for him is reducible to theoretical physics. In fact, Quine pretends that even logic and mathematics should be reduced to theoretical physics.[34] According to Quine, who, by the way, never attempted to refute Kant's —not to say Husserl's— or others' rival views, all our knowledge begins in our acquaintance with the physical world: we passively receive some stimulations in our sensory receptors from the outer world, some sort of irradiations that originate some neural impulses in the brain, were they are somehow processed producing an object. In this way we perceive the object.[35] It seems pertinent to quote Quine here:

> Impacts of molecules and light rays upon our nerve endings set off neural impulses that travel to the brain, where they get processed in extravagantly complex and unexpected ways…. Stimulations and neural impulses build up speech habits and other habits in us that continue to be brought into play by subsequent sensory stimulation, through elaborate processes of association and reinforcement.[36]

> Among the myriad manifestations of these largely unfathomed processes, one outcome is a neat correlation between certain ranges of sensory stimulation and certain sentences…. We can limit our consideration of neural events to the superficial ones that answer directly to the impacts and input from the external world. [37]

In a recent paper[38] we called such a Quinean conception of our knowledge of objects of the physical world "bugs epistemology", a designation that would seem at first sight somewhat too severe. However, the fact of the matter is that such a description of our process of getting acquainted with objects has not only too many loose points —see, e.g. the phrases "processed in extravagantly complex and unexpected ways" and "elaborate processes of association and

34 See his 'On Empirically Equivalent Systems of the World' 1975, reprinted in *Confessions of a Confirmed Extensionalist*, pp. 228–243, in particular, p. 229.

35 See, e.g., 'Epistemology Naturalized', pp. 71, 75–76, 82–83 and 84, as well as his relatively late (1986) 'The Sensory Support of Science', p. 329, in *Confessions of a Confirmed Extensionalist and Other Essays*, pp. 327–337.

36 'The Sensory Support of Science', p. 329, in *Confessions of a Confirmed Extensionalist and Other Essays*.

37 Ibid.

38 See our 'Husserl and Kant: voilà la différence', in our *Unorthodox Analytic Philosophy*, London 2018, pp. 115–141.

reinforcement"—, but more importantly is based on the behaviouristic conception of psychology and language, whose alleged scientific nature and grounding on physics could provoke a heart attack to any respectable physicist. In any case, leaving human beings aside, about which we will say a little below, it is extremely questionable that mammals, reptiles and birds have so primitive forms of knowledge of the objects in the physical world as Quine believes that we have. If what Quine says about our way of getting acquainted with the physical world were true of such animals, smaller animals whose habitat is, for example, the Indian jungle would probably have been extinguished. Imagine if one of them receives some stimulation in his nerves of some elongated extension, and some stimulations that he is going to process as being of something yellow, as well as other stimulations that he will process as being of brown stripes. Before he finishes processing the different stimulations and finds out that the object is a tiger, he is already partly in the mouth of the tiger. The fact of the matter, however, is that mammals, reptiles and birds see objects directly and immediately —without any noticeable intervening processing of stimulations, irradiations or whatever—, and after seeing the object they either run away, go after it or remain indifferent. The case of human beings is certainly even less Quinean, and we will say a little about it below. But before we return to that issue from a by far more serious perspective, a few words seem pertinent on another influential Quinean thesis.

In his early paper 'On What there is'[39] Quine introduces his famous thesis completely accepted as a dogma in orthodox analytic circles that to be is to be the value of a quantified variable. Hence, since in first–order logic one quantifies only over individual variables, that is, variables that range over individuals, first–order logic is committed only to individuals. On the other hand, since second–order —and, in general, higher–order— logic quantifies over sets or relations, second–order —and, of course, higher–order— logic is committed to the existence of sets and relations, that is, of abstract entities. This new dogma has

[39] 'On what there is' 1951, reprinted in *From a Logical Point of View*, pp. 1–19. See especially, though not exclusively, pp. 12–13, where he states: "But this is, essentially, the *only* way we can involve ourselves in ontological commitments: by our use of bound variables…. To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable…. The variables of quantification, 'something', 'nothing', 'everything', range over our whole ontology, whatever it may be; and we are convicted of a particular ontological presupposition if, and only if, the alleged presuppositum has to be reckoned among the entities over which our variables range in order to render one of our affirmations true". Quine's selection of words above is especially illuminating. You are usually convicted of crimes, but now you are also "convicted of a particular ontological presupposition", and in case you presuppose classes or relations, you are certainly guilty of a major felony.

been accepted by all later empiricists, who feel free to use first–order logic, but block out any of its natural extensions. In fact, it has been accepted as an article of faith even by philosopher–logicians that cannot be accused of being empiricists.

Quine's paper was published before the development of model theory, which was born in the early 1950s thanks to Alfred Tarski, Abraham Robinson and their collaborators. And though there are important extensions of model theory to the natural —that is, the hierarchy of n–order logics— and less natural extensions of first–order logic, classical model theory, that is, model theory of first–order logic, is by far the most developed.[40] And in classical model theory, though we quantify only over individuals, we speak constantly about structures, which include a universe of objects, sets and relations. Thus, in some sense the commitment to first–order logic in order to avoid talking about sets and relations loses its force and seems hypocritical. To make an analogy with everyday objects, my computer is on the table. It makes no sense that I should acknowledge the existence of the computer but try not to acknowledge the existence of the table nor the fact that the computer and the table are in the relation of the first one being on the second one. But that brings us to the next issue we want to discuss here.

§5. Some Husserlian Epistemological Medicine for a Quinean Disease

Let us consider once more the situation in which the computer is on the desk and there is a notebook at its side. What does a human being see when he enters the room? Of course, he does not have sensations of the gray of the computer, of its rectangular extension, of the redness on the surface of the notebook and it's also rectangular shape, and of the brown colour of the desk, its form, and so on, and then bring them together by multiple steps of a patching procedure. Moreover, the person that enters the room does not first see the isolated notebook, the isolated computer and the isolated desk, and then brings them together in the particular fashion that the computer and the notebook are on the desk side by side —and not, for example, in such a way that the computer may be under the desk and the notebook over the computer. In fact, what the person sees are not single objects but the state of affairs that the computer and the notebook are side by side on the desk. What we see in this and other cases

40 A classic text on classical model theory is that of Chang and Keisler included in the references. More recent textbooks are those of Wilfrid Hodges also included in the references.

are not single objects but states of affairs, in this case, the state of affairs that the computer is on the desk at the side of the notebook. And likewise when Charles and Peter are at the door, we do not see the isolated Charles, the isolated Peter and the isolated door, and put them together. We see the state of affairs that Charles and Peter are at the door. Moreover, we also see the state of affairs that Charles is taller than Peter. Finally, when we enter a conference room at the university we see a collection —or group in usual parlance— of persons, a set of persons, not first the single objects that we then bring together to form the collection.

All those examples illustrate the basis of Husserl's epistemology of mathematics. What humans perceive in sense perception are not single objects but states of affairs. And in those states of affairs that we perceive in sense perception not all components correspond to, or are the correlates of, sensations. There are already in all sense perceptions categorial components. There are no sensuous correlates of the words 'on' and 'at the side of' when we perceive the computer on the desk at the side of the notebook. There are no sensuous correlates —and certainly no intervening sensory stimulations or irradiations— for the conjunction 'and' and the word 'at' when we see Charles and John at the door, nor is there any sensuous correlate of the relational word 'taller than' when we say that Charles is taller than Peter. Nor is there any sensuous correlate for the word 'group' or 'collection' when we say that we see a group —or collection— of persons in the conference room. To all those words without sensuous correlates in sense perception —which certainly do not originate in any sort of sensory stimulation— there correspond categorial components of sense perception. Thus, to say it briefly, we do not merely perceive objects but perceive states of affairs, and in the perception of states of affairs some non–sensuous categorial components are perceived. This is the point of departure of Husserl's conception of categorial intuition and, as we will see below, of mathematical intuition, being intuition in Husserl's parlance either perception or imagination.[41] In fact, for Husserl's purpose from this point on imagination plays the more decisive role.

We can bring together two states of affairs and compare them, for example, two collections of persons, and pair them one to one in order to determine which collection is larger; or bring them together to form a larger collection. Moreover,

[41] For Husserl's epistemology of mathematics, the classic text is *Logische Untersuchungen II*, second part of the Sixth Logical Investigation, especially the Sixth and Eight Chapters. See also his *Erfahrung und Urteil*, especially its second and third parts.

we can consider many different relations between perceived or imagined objects of the same or different collections (or sets), for example, being taller than or shorter than, or larger than between persons. But we do not need to stop there, and with the aid of the notions of set and relation, we can build relations between sets of concrete individuals or sets of relations of concrete individuals, sets of sets of concrete individuals and relations of relations between concrete individuals, and then consider, for example, sets of relations between sets of concrete individuals, or relations of sets of relations between concrete individuals, and so forth. In that way, we can in principle obtain an unlimited, in fact, countably infinite hierarchy of categorial objects.

That infinite hierarchy of categorial objects, however, is still not a hierarchy of mathematical objects, since its basis is contaminated with sensibility. Nonetheless, at any stage of the hierarchy of categorial objects we can introduce the process that Husserl calls "formal abstraction", which is simply what we would call a process of formalization. Thus, for example, on the basis of our intuition that Charles is taller than Peter, we can proceed to replace Charles and Peter with variables, as well as the relation 'taller than' by the mathematical symbol '>', since 'taller than' has exactly the formal properties that characterize the symbol '>'. And in the statement, 'the collection of people outside the conference room is larger than the collection of people inside the conference room' we could replace the two collections by set variables 'M' and 'N' and the relation of being larger than between concrete collections once more with the relation '>', and obtain the statement 'M>N'. It is through this process of formalization that the hierarchy of mathematical entities is constituted —to use a terminology both neutral and very dear to Husserl. Thus, mathematical entities are formalized categorial entities, and mathematical intuition is for Husserl categorial intuition plus formal abstraction —that is, categorial intuition plus formalization.[42]

§6. Empiricism's Sibling: Nominalism in the Philosophy of Mathematics

Empiricism of all sorts, even the most liberal ones of David Hume and the logical empiricists, is incapable of doing justice to the theories in physical science. Though physics, in contrast to logic and mathematics, certainly is an empirical science in the specific sense that it has to take experience into account, it is from

[42] See *Logische Untersuchungen II*, U. VI, Second Part, especially §60. See also *LU I*, §67, as well as *Ideen I*, §10.

top to bottom theoretical even in the design of experiments. In fact, it is a basically rational explanatory science concerned with a fundamental part of our physical world, and though contrary to logic and mathematics it has to take observation and, thus, experience into account, the more sophisticated theories of the last century, namely, quantum mechanics, special relativity, general relativity and quantum field theory —which has brought together quantum mechanics and special relativity— require a by far more sophisticated notion of observability than any admitted by empiricism of all sorts.[43]

As a matter of fact, after the demise of logical positivism there have been multiple attempts to produce more adequate analyses of physical science. Leaving Popper's attempt aside, which was almost contemporary to logical empiricism and did not survive much longer, there was Kuhn's conception of revolutions in science, which caught the imagination of a few, but was really a deviation from the central problems in the philosophy of science, and there was Lakatos attempt to combine Popperian and Kuhnian components. Probably the most interesting and wide ranging of all attempts is the variety of analyses of physical science baptized as "semantic". It is perfectly clear that logical empiricism's "logical component" was exclusively syntactic. Thus, beginning in the 1950s and up to this date a variety of semantic analyses of science have flourished like mushrooms, all being based more or less on some rudimentary notions of model theory. Thus, we have, on the one hand, Patrick Suppes and his collaborators already in the late 1950s with the first semantic approach, which was followed already in the 1960s in Poland by Marian Przelecki's semantic approach introduced in his *The Logic of Empirical Theories*[44], and almost simultaneously by the work of the Sneed–Stegmüller school centred in Munich, and beginning in the 1970s by van Fraassen's views[45]. Such a multi–headed trend is certainly much more promising than its forerunners, and without doubt model theory is the area of logic with more philosophical relevance[46]. However, without neglecting the possible importance of a semantic approach to science, that

[43] See on this point, e.g., Chapter 5, especially, §15 of Sunny Y. Auyang's excellent book *How is Quantum Field Theory Possible?*, Oxford 1995.

[44] *The Logic of Empirical Theories*, London 1968.

[45] We have certainly not mentioned all variants, but for our purposes that is far from necessary. For more information, the reader can consult, e.g., Frederick Suppe's, *The Semantic Conception of Theories and Scientific Realism*, Urbana & Chicago 1989.

[46] See on this issue, e.g., our paper 'The Interplay between Logic, Mathematics and Philosophy', originally published in the South–American Journal of Logic and reprinted in our book *Unorthodox Analytic Philosophy*, London 2018.

approach alone is insufficient, and in order to be adequate it would need to incorporate in some way the observations of Duhem and Husserl expounded in §3 above, and would also have to include a new and much wider conception of observability, as mentioned in the preceding paragraph.

Leaving empiricism and physical science aside, we would like to briefly examine here empiricism's sibling nominalism and other variants of non–Platonist philosophies of mathematics. In fact, radical empiricism in the empirical sciences has its counterpart in nominalism with respect to mathematics, that is, in the view that there do not exist any mathematical entities. Nominalists abhor abstract mathematical entities and panic when they are merely mentioned, as if they were ghosts or devils. Thus, for them all existential mathematical statements are false, while all universal mathematical statements are vacuously true. But as we already mentioned, contrary to Quine's logical dogma, if you want to avoid presupposing mathematical entities, you cannot accept first–order logic. Propositional logic is the only part of classical logic free of ontological commitments —as it is also the only part of classical logic in which Frege's version of the sense–referent distinction, with the referents of statements being truth—values, could work. The fact of the matter is that first–order logic, with its rich semantics of classical model theory, presupposes the existence of abstract entities, namely, structures, sets and relations. But still more decisively, there is a theorem of classical model theory that refutes nominalism, namely, Abraham Robinson's Model–Completeness Test. To put it very simply, a first–order logical theory T is model–complete when for any two models M and M^* of T, if M is a substructure of M^* —in symbols: $M \subseteq M^*$—, then M is an elementary substructure of M^*[47] —in symbols: $M \prec M^*$. According to the Model–Completeness Test[48], among other things, a theory is model–complete if and only, if for any existential statement φ, there is a universal statement ψ logically equivalent to it. Thus, if there exist model–complete theories —and certainly they exist—, any existential

[47] Given two structures M and M^* for a first–order language L, $M \subseteq M^*$ if and only if for any atomic formula φ (or, in fact, any Boolean combination of atomic formulas) of L with n variables and any n–tuple $m_1, \ldots m_n$ of members of the universe of M, the given n–tuple satisfies φ in M if and only if it satisfies φ in M^*. If that happens also for quantified formulas, then M is an elementary substructure of M^*, in symbols: Mp*. By the way, two structures, e.g., M and M^* for a first–order language L are elementary equivalent if and only if any closed formula —that is, formula without free variables— σ of L is true in M if and only if it is true in M^*.

[48] For the formulation of the Model Completeness Test, see any textbook on model theory. By the way, in our paper 'The Fine Structure of Sense–Referent Semantics', we included the formulation of the Model Completeness Test and of Morley's Theorem, mentioned below.

mathematical statement is logically equivalent to a universal one. Now, according to nominalism, all existential mathematical statements are false and all universal ones are true, thus, no existential statement could be logically equivalent to a universal one. Hence, on the basis of the Model–Completeness Test and the existence of model–complete theories, one can conclude that nominalism is false. By the way, the Model Completeness Test also refutes conventionalism in the philosophy of mathematics, since it forbids us to conventionally declare all existential statements false, or true, or all universal statements true, or false.

Other less radical anti–Platonist philosophies of mathematics do not fare much better than nominalism. One of them is constructivism. The first problem that confronts constructivism is making its central notion precise, since there are many different and non–equivalent sorts of constructivism, for example, those of Kant, Brouwer, Griss, Markov and Bishop, and one would have to offer arguments to be able to choose between them.[49] Secondly, absolutely all sorts of constructivism are unable to account not only for the whole of existing mathematics but even for the mathematics used in physical theories, for example, that of the set of all real–valued functions. Finally, as pointed out by the distinguished English logician Wilfrid Hodges[50] in a critique of Putnam[51], constructivism is incompatible with the Löwenheim–Skolem–Tarski Theorem that says that if a first–order theory has an infinite model, it also has models of any infinite cardinality. In fact, constructivism is incompatible with many other results in model theory, like Skolem's Theorem on the existence of non–standard models, which combined with the Löwenheim–Skolem–Tarski Theorem give us non–standard models of any infinite cardinality, or Morley's Theorem[52] that states that, under certain conditions, categoricity in an uncountable model κ entails categoricity in any uncountable model λ, being an immediate consequence of this last theorem that, under such conditions $\aleph_1$–categoricity implies λ–categoricity for any uncountable λ. About fictionalism of all sorts it is not even worth writing anything at all. That the laws of arithmetic valid under any circumstances —or the laws of any other part of mathematics— are pure fictions like novels or poetry is simply preposterous.

[49] Griss went so far as to defend a negationless mathematics, which none of the remaining above–mentioned constructivists would have dared to defend.

[50] See Hodges' 'Elementary Logic', in D. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic I*, Reidel 1983, pp. 1–131.

[51] See Hilary Putnam's 'Models and Reality', *Journal of Symbolic Logic 45*, 1980, pp. 464–482.

[52] See footnote 38 above.

The last attempt to save anti–Platonism is less radical than nominalism or fictionalism, but is also the most grotesque fruit of the imagination of the empiricists in the realm of the philosophy of mathematics, namely, the so–called indispensabilism, which originated also in the minds of none others than Quine and Putnam. According to such a view, mathematical entities exist only in case they are applicable to physics, and their existence is similar to that of physical objects. Thus, according to indispensabilists, the presumed objects of mathematical studies are divided into mathematical entities that exist, for example, real numbers and differential equations, and those that do not exist, for example the fifth uncountable cardinal and ultraproducts. Besides the already preposterous attempt at attributing mathematical entities, which are practically *per definitionem* non–physical entities —and the problem is even worse if you take into account their simplistic view of physical entities—, a sort of physical existence, the distinction between existing and non–existing mathematical entities seems arbitrary and gratuitous. The situation for the indispensabilists, however, is even worse if you take into account that some mathematical theories found their physical applications many years after been discovered, and that others could be abandoned as mathematical tools in physics in case the physical theories that guaranteed their existence are superseded or simply abandoned. Hence, a consequence of the indispensabilist view is that some mathematical theories could be born and some living ones could die. According to this view, tensors were born the moment in which the midwife general relativity brought them into existence, and Hilbert spaces were born when the midwife quantum mechanics helped them to be born. On the other hand, some unlucky mathematical entities have still not been born —though mathematicians prove theorems about them— and some, like ultrapowers or higher infinite cardinalities, have very little hope of ever finding a physical midwife that would bring them into existence. Hence, for indispensabilists, mathematical entities begin to look even more similar to human beings than the gods in Mount Olympus, having similar fundamental experiences of birth and (potential) death as those of human beings.

§7. Some Miscellaneous Consequences of the Empiricist Ideology

The empiricist ideology not only has completely failed in its attempt to produce any serious philosophy of science and even more a philosophy of mathematics, but its grotesque distinction between analytic and so–called continental philosophy has served to arbitrarily exclude the serious study of important philosophers classified by them as continental philosophers, as well as to distort the philosophy of others in order to make their views, as the Germans would say "*salonfähig*", that is, presentable in society, in this case, empiricist society. We have already mentioned probably the most glaring case, that of Husserl, certainly one of the greatest philosophers ever and the greatest at least since Kant. But we could also had mentioned Leibniz and Bolzano, to mention other two philosophical giants exiled from the "philosophical paradise" on the basis of a salad of prejudice and ignorance.[53] We had referred to Husserl on a few points that serve to illustrate both the ignorance and the prejudices involved.

Husserl's epistemology of mathematics, as presented in the Sixth Logical Investigation of his *opus magnum* and in *Erfahrung und Urteil*[54], the work he was finishing when he died in 1938 and that was published a year later, is probably the most serious attempt ever to explain how is it that we have mathematical knowledge. And as we pointed out above, his relatively brief remarks on physical theories in *Logische Untersuchungen* certainly made the logical empiricist's criteria of empirical significance a non–starter. Thirdly, his acceptance of Riemann's conception both of geometrical manifolds and of the geometry of physical space already by 1892 contrasts him favourably to all other philosophers of his time dearer to the empiricists. Moreover, though we have not dealt with this issue in this paper, Husserl's philosophy of logic and mathematics, presented for the first time also in Chapter XI of *Logische Untersuchungen*, is certainly more relevant for present mathematics than Frege's logicist program. Husserl conceived logic as a syntactic–semantic sister discipline of mathematics, being the latter a sort of formal ontology. Thus, Husserl was a structuralist Platonist and, though mathematics and logic were for him strictly related, he was not a logicist. By the way, Husserl's conception of mathematics as a theory of structures anticipated very strongly the Bourbaki school.[55]

The other point briefly mentioned is Husserl's unacknowledged influence on the work of Carnap. As we have argued more than once[56], Carnap's *Der Logische Aufbau* der Welt was written under the strong influence of Husserl, which was by far the most important philosophical influence on that work —as it also had happened with Carnap's dissertation, *Der Raum*. The notion of constitution (*Konstitution* in German) —not construction (*Konstruktion* in German)— fundamental to the whole book is taken from Husserl; the notion of grounding

[53] But see the end of this § for an "extremely valiant" attempt to rescue them from the continental prison.

[54] See footnote 34 above.

[55] See, e.g., our forthcoming paper 'Husserl and Bourbaki'.

[56] See footnote 13 above.

(*Fundierung*) is also taken from Husserl; the autopsychological basis is also taken from Husserl's *Ideen zu einer reinen Phänomenologie und einen phänomenologischen Philosophie I*[57]; the whole project of constitution is also taken from that work —by the way, Kant used the term 'constitution' basically in the opposite direction—; and the whole constitution of the so–called heteropsychological is taken from Husserl's manuscripts for the second volume of that work, which Husserl's assistant Landgrebe was preparing for publication in the time in which he was befriended with Carnap and used to discuss with him about philosophy. Thus, Quine's[58] interpretation of Carnap's *Aufbau* is clearly false, and Friedman's[59] interpretation is also equally false. Furthermore, in some later works Carnap also used other Husserlian views without any recognition of the source. In his paper 'Die Überwindung der Metaphysik durch logische Analyse der Sprache' the distinction between two sorts of nonsense is taken from Husserl's *Logische Untersuchungen*, but Carnap does not even include that book in the references.[60] The same happens in Carnap's *Logische Syntax der Sprache*, in which Carnap takes the distinction made by Husserl between the laws that protect against nonsense and the laws of formal consequence that protect against countersense, changes the terminology and presents it as his, namely, as the distinction between the formation rules and the transformation rules of sentences[61] —a distinction nowadays accepted in all rigorous logic introductory books.

In fact, in great part because of the prejudices, but in many cases also because of the epidemic ignorance of foreign languages in Anglo–American circles, the presumed specialists in different philosophers offer completely preposterous renderings even of their chosen philosophers. For example, empiricism from

57 *Ideen zu einer reinen Phänomenologie und einen phänomenologischen Philosophie I*, 1913, Hua. III 1950, revised edition 1976.

58 See 'From a Logical Point of View', p. 39, as well as his 'Epistemology Naturalized', in *Ontological Relativity and Other Essays*, p. 74 for Quine's clearly false interpretation of Carnap's *Aufbau*, based on his empiricist prejudices and —since Carnap and Quine were good friends— probably also on Carnap's dishonesty.

59 See Michael Friedman's *Reconsidering Logical Positivism*, Cambridge 1999 for a Kantian, also incorrect, interpretation of the *Aufbau*, based on the incapability to distinguish Husserl's views from those of Kant. On this last point, see our paper 'Husserl and Kant: voilà la différence' in our *Unorthodox Analytic Philosophy*, London 2018, pp. 115–141.

60 It is interesting that Husserl's *Logische Untersuchungen* were included in the references of the *Aufbau*, in which Carnap took from that work basically the notion of *Fundierung*, but not included in the references of the paper and of *Logische Syntax*, where the influence of that specific work of Husserl is much more decisive and which were written some years after the *Aufbau*. This points to Carnap's conscious attempt to cover up his appropriation of Husserl's ideas.

61 See especially *Logische Untersuchungen I*, Chapter XI, but also *II*, U. IV.

Russell and Carnap up to the present has had to deal with the fact that the father of contemporary logic, the great logician and very important philosopher, Gottlob Frege, was a staunch rationalist. In fact, the —like Husserl and Whitehead— mathematician turned philosopher of Jena, who in comparison with most philosophers really wrote very little philosophy, was mostly a philosopher of mathematics and a philosopher of logic. As a philosopher of mathematics, he was a logicist and a non–stucturalist Platonist. Though he has made famous the distinction between sense and referent, he did not write much about the philosophy of language. By the way, that distinction, to which Bolzano almost arrived many years before, was obtained by Frege and Husserl independently of each other at about the same time. Husserl obtained it around 1890 and is present in a paper of his, 'Zur Logik der Zeichen: Semiotik'[62] written in 1890 and published only posthumously. The first publication of Husserl's containing that distinction is his review[63] of the first volume of Ernst Schröder's *Vorlesungen über die Algebra der Logik*, which appeared in March of 1891 and was already in press in January of the same year when Frege's paper 'Funktion und Begriff'[64] was published, in which Frege presented the distinction for the first time. Though Frege himself in a letter to Husserl of 24 May 1891[65] acknowledged that both had independently arrived at the distinction, authors like Evert W. Beth[66], Alonzo Church[67] and Dagfinn Føllesdal[68] have argued that the distinction was obtained by Frege and that Husserl took it from Frege. After the publication of 'Zur Logik der Zeichen: Semiotik' as an appendix to the Husserliana edition of *Philosophie der Arithmetik*[69] in 1970 and the publication of Frege's

[62] 'Zur Logik der Zeichen: Semiotik' (1890), published as Appendix B.1 of *Philosophie der Arithmetik*, Hua. XII, 1970, pp. 340–373.

[63] 'Besprechung von E. Schröders *Vorlesungen über die Algebra der Logik I*' 1891, reprinted in Edmund Husserl, *Aufsätze und Rezensionen 1890–1910*, Hua. XXII 1979, pp. 3–43.

[64] 'Funktion und Begriff' 1891, reprinted in his *Kleine Schriften* 1967, second edition, Hildesheim 1990, pp. 125–142.

[65] See Frege's *Wissenschaftlicher Briefwechsel*, pp. 94–98.

[66] See Beth's *The Foundations of Mathematics*, Amsterdam 1965, p. 353.

[67] See his 'Review of M. Farber, *The Foundation of Phenomenology*', *Journal of Symbolic Logic 9* 1944, pp. 63–65.

[68] See his *Husserl und Frege, ein Beitrag zur Beleuchtung der phänomenologischen Philosophie* 1964, English translation in L. Haaparanta (ed.), *Mind, Meaning and Mathematics*, Dordrecht 1994, pp. 3–47. The monograph is essentially Føllesdal's Master Thesis from 1958.

[69] See footnote 54.

Wissenschftlicher Briefwechsel[70] in 1976 the problem should have been solved for any rational being —namely, both philosophers obtained the sense–referent distinction independently of each other and almost simultaneously. Nonetheless, still most analytic philosophers believe in the myth based on ignorance propagated by Beth, Church and Føllesdal. By the way, there were two important differences between Frege's and Husserl's sense–referent theories. Contrary to Frege —for which the referent of what he called a conceptual word was the concept, being the extension of the concept a step further away, for Husserl the referent of the conceptual word was the extension, whereas the concept is its sense. Frege never explained what was for him the sense of a conceptual word and how can one differentiate it from the conceptual word, on the one hand, and from the concept, on the other. The other distinction concerned the referent of statements. Whereas for Frege the referents of statements are truth–values, for Husserl the referents of statements are states of affairs (Sachverhalt[en]), which are based on what he called situations of affairs (Sachlage[n]). Thus, the road from the senses of the statements, that is, the thoughts (in Frege's parlance) or propositions (in Husserl's usual parlance) to the truth–values is in Husserl's semantics longer than in Frege, there being two very important intermediate steps. Such a difference between the two sense–referent theories is fundamental, being Husserl's much more fruitful than Frege's.[71]

Returning to Frege, he wrote little about philosophy of language and certainly there is no reason to believe with the Dummett of his *Frege: Philosophy of Language*[72] that Frege was foremost a philosopher of language. Nonetheless, Frege wrote much more about philosophy of language than about epistemology. There is not a single piece of Frege's writings before 1923 that one could classify as purely epistemological. There are simply remarks on epistemological issues scattered in 'Der Gedanke' and elsewhere[73], and then more openly in some minor and very short writings almost at the end of his life. Nonetheless, beginning with Hans Sluga[74] and followed by Joan Weiner[75], some Frege so–called specialists

[70] See footnote 57.

[71] See on this issue our paper 'The Fine Structure of Sense–Referent Semantics' referred to in footnote 47.

[72] See M. Dummett's *Frege: Philosophy of Language*, London 1974.

[73] 'Der Gedanke' 1918, reprinted in *Kleine Schriften*, pp. 342–362. See also his posthumously published paper presumably written in 1897 under the title 'Logik', which seems to be a forerunner of 'Der Gedanke', in *Nachgelassene Schriften*, pp. 137–163.

[74] See, e.g., his book *Gottlob Frege* London 1980.

[75] See, e.g., her book *Frege in Perspective*, Ithaca and London 1990.

have developed an unfounded rendering of Frege as an epistemologist. But things could be much worse. In fact, they are. There is an author, namely, Jamie Tappenden[76], who has tried to link Frege's views to those of Riemann. The great reason behind Tappenden's incredible view is that Frege studied mathematics in Göttingen, where Riemann had taught more than a decade before. According to Tappenden, there were two schools in German universities, the Riemannian in Göttingen and the Weierstraßian in Berlin, and those who studied in Berlin followed Weierstraß, while those who studied in Göttingen followed Riemann. Hence, since Frege studied in Göttingen (and Jena), he must have been a Riemannian. Such a cowboys versus Indians theory is wrong for many reasons. Firstly, after Riemann died, the most influential mathematics professors in Göttingen were neither Riemannian nor Weierstraßian, but belonged to a third school, namely, to that of projective geometry, and it was with such professors that Frege was schooled. The projective geometricians were isolated from the Riemannian trend until the great German mathematician Felix Klein, contemporary of Frege, brought the two schools together in a series of important papers.[77] Moreover, Tappenden has completely ignored the fact that great minds do not follow blindly their teachers. Cantor, Husserl and Minkowski were students of Kronecker and, especially, of Weierstraß, but each in a different sense was later nearer to Riemann than to their master. In the case of Husserl, who was also Weierstraß' assistant, the commitment to Riemann's views on geometric manifolds and on the geometry of physical space was complete. In fact, to put it bluntly, Husserl's philosophy of mathematics combines a generalization of Riemann's notion of geometric manifolds to all of mathematics, with a much more clear distinction —anticipating Bourbaki— between fundamental structures and derived structures, as well as a new version of Leibniz' *mathesis universalis.*[78] Thus, in Frege's case, even if he had studied with followers of Riemann, it does not mean that he should be a Riemannian. In any case, as a matter of fact, Frege not only did not study with any follower of Riemann but seems not to have had any acquaintance with his views. Neither in Frege's works

[76] See his 'The Riemannian Background of Frege's Philosophy', in José Ferreiros and Jeremy Gray (eds.), *The Architecture of Modern Mathematics*, pp. 97–132. See also Tappenden's 'Geometry and Generality in Frege's Philosophy of Arithmetic', *Synthese 102 (3)*, 1995, pp. 319–361.

[77] See Klein's *Le Programme d'Erlangen: Considérations Comparatives sur les Recherches Géométriques Modernes*, Paris 1974.

[78] For Husserl's philosophy of mathematics, besides *Logische Untersuchungen I*, Chapter XI, see *Formale und transzendentale Logik*, *Einleitung in die Logik und Erkenntnistheorie* or *Logik und allgemeine Wissenschaftstheorie*, all included in the references.

published during his lifetime nor in his posthumous writings is there any reference of Frege to Riemann. Only in his *Wissenschatlicher Briefwechsel*[79] are there references to Riemann, though none of them by Frege himself. Thus, on pp. 51–53, the editors mention some lost letters of the mathematician Robert Haußner to Frege, in which Haußner referred to a course of Riemann on analysis —not on geometry—; and on pp. 256–257 in a letter of the philosopher Carl Stumpf to Frege the former mentions the Riemann–Helmoltz conception of geometry. As a matter of fact, there is no evidence that Frege was acquainted with Riemann's views on geometry. Finally, Frege's remarks in his posthumous writing 'Über Euklidische Geometrie'[80], in which he considers non–Euclidean geometry on the same level as alchemy and astrology, besides being preposterous, is completely incompatible with the view that Frege was after all a Riemannian, like the incredible Tappenden wanted us to believe.

Tappenden is not necessarily the worst Frege interpreter. There is Danielle Macbeth with her "three levels of articulation" in her 'Striving for Truth in the Practice of Mathematics'[81], who is without doubt a serious rival of Tappenden in that competition, and we suppose they are not the only two candidates. But it seems unnecessary to continue to talk about such extraordinarily imaginative commentators of the great Frege —especially since some Anglo–American so–called specialists in other philosophers, like Carnap, do not fare much better. Nonetheless, we want to point out that such grotesque interpretations in order to save so–called "continental" philosophers from being "condemned as guilty of being continental" is not limited to Anglo–American authors talking about Frege or Kant. It extends to some Austrian authors that revere the Vienna Circle. Thus, we found this beauty written by Friedrich Stadler in a book he edited together with Rudolf Haller:

> Mit dem Wirken von *Franz Brentano (1838–1917)* in Wien beginnt der Diskurs einer empirischen Philosophie in der Tradition von Leibniz und Bolzano mit Abgrenzung zum deutschen Idealismus.[82]

79 See *Wissenschaftlicher Briefwechsel*, Hamburg 1974.

80 'Über Euklidische Geometrie', in his *Nachgelassene Schriften*, pp. 182–184.

81 See Dirk Greimann's anthology, *Essays on Frege's Conception of Truth*, Hildesheim 2003.

82 In English, for those that do not read German: "With the impact of *Franz Brentano (1838–1917)* in Vienna begins the talk of an empiricist philosophy in the tradition of Leibniz and Bolzano while distancing from German idealism". The quote is taken from Friedrich Stadler's introductory essay 'Wien–Berlin–Prag: Zum Aufstieg der wissenschaftlichen Philosophie, p. 13 of Rudolf Haller and Friedrich Stadler (eds.), *Der Aufstieg der wissenschaftlichen Philosophie*, Hölder–Pichler–Tempsky 1993.

Now, the ultimate rationalist Leibniz and the also rationalist Bolzano are considered by Stadler as empiricists. That is so preposterous as to consider John Stuart Mill and John Locke rationalists, and deserves no more comments.

§8. Analytic Philosophy without the Meta–Dogmas of Ideological Empiricism

As we have stated, more than once, the most adequate name for the present endeavours would be 'critical rationalism'. But since the important Austrian philosopher Karl Popper[83] already used that nomenclature to designate his philosophical views, we have opted to use the names 'unorthodox analytic philosophy' or 'heterodox analytic philosophy'. The fact of the matter is that there is no respectable philosophical endeavour that does not take seriously the results of the deductive and the natural sciences, very especially, the three fundamental ones of logic, mathematics and physics. Hence, we reject with the same force as the empiricists the uncontrolled metaphysics of philosophers in the mould of German idealists, without denying that they could have isolated interesting insights. We certainly also reject existentialism with the same force, very especially the philosophy of Martin Heidegger, whose metaphysics borders with irrationalism and even has racist undertones.[84] But even more strongly we reject all sorts of irrationalism, be it of Nietzsche or of whoever, and certainly the more recent charlatanry of post–modernism. To put it bluntly, for us post–modernism is the philosophy of those who cannot understand or have made no serious effort to understand rigorous philosophy.[85]

[83] See, e.g., his *Conjectures and Refutations*, where he explicitly refers to his own philosophical views —and also Kant's— as critical rationalism. By the way, we fully coincide with Popper that the opposite of rationalism is irrationalism, not empiricism. Critical rationalism is perfectly compatible with moderate critical —not ideological— empiricism. An important point in which we radically differ is with respect to Husserl, whom Popper seems to never have seriously studied or understood.

[84] Martin Heidegger sustained that the *Sein* (the being) manifested itself only in Ancient Greek and Modern German. He was trying not so subtly to underscore not only that those languages are the only adequate vehicles for doing true philosophy —and are, hence, superior languages—, but that the people that have such languages as mother tongue are in some philosophical sense superior to the mere mortals whose native tongue is not German —since nobody nowadays has Ancient Greek as mother tongue.

[85] Some post–modernists and other philosophical dilettantes sometimes refer, to justify their relativism, to the phrase 'If God does not exist, everything is allowed', which they attribute to their hero Nietzsche but is at least as old as Dostoievsky. That conditional statement, however, if true, would serve, purely by means of propositional logic and elementary school arithmetic, to prove the existence of God. The phrase has the logical form: $\neg\alpha\rightarrow\beta$. By propositional logic, that is logically equivalent to $\neg\neg\alpha\vee\beta$, which is logically equivalent to $\alpha\vee\beta$. However, β is false, that is, it is not the case that everything is allowed, for

Now, the three fundamental sciences of logic, mathematics and physics are rational enterprises and explanatory sciences in Husserl's sense. Moreover, logic and mathematics are completely rational deductive sciences that do not have to take into account absolutely anything obtained from experience. Of course, most people learn to count and to add using small blocks or other sort of empirical devices, but the results of such counting and of such addition are valid independently of the empirical devices with whose help the child learnt those arithmetical operations. Logic and mathematics are in no way based on experience and do not need to take experience into consideration. Moreover, logic and mathematics do not need either to devaluate by means of a sort of nominalism or fictionalism, respectively, justify by means of indispensabilism or whatever the truth of their statements nor the existence of abstract mathematical entities.

At first sight, the case of physics seems to be completely different, namely, it is classified as an empirical science. Such an assertion is correct in one sense and false in another sense. Contrary to logic and mathematics, which are purely deductive formal sciences, dealing with abstract entities and their relations, physics is concerned with our natural world, more precisely, with the most general aspects of our natural world. Thus, physics has to take into account experience, it cannot go against experience. But that physics cannot go against experience does not mean that it is fundamentally an empirical science like biology or psychology. In fact, physics is also an explanatory and fundamentally rational enterprise, as logic and mathematics. As Husserl already pointed out before the advent of all the revolutions in physics in the twentieth century, physical laws of higher level are *hypotheses cum fundamento in re*, compatible with low–level physical laws only thinly connected to them, and such that the latter could admit other higher level laws incompatible with the presently accepted higher level laws. Thus, there exists an empirical indeterminacy in usual physical laws: they are in no way determined or even conditioned by experience, but simply need to be compatible with the sophisticated experiments designed in physical science and have explanatory power.

As a matter of fact, after the revolutions in physics in the first quarter of the twentieth century, the relation between physical theory and observation has become much more tenuous. To understand the physical processes both at the cosmological level and at the microphysical one, physicists have needed to widen

example, $2^2=17$ is not allowed: it is impossible. Thus, the second member of the disjunction $\alpha\vee\beta$ is false. Hence, α is true, that is, God exists.

considerably their concept of observability, going much farther than Duhem anticipated in the theoretical "contamination" of the physicists' observations.[86]

The task of the rigorous philosopher is not to put empiricist blinkers to the scientific enterprise, but to assess and reflect on the three fundamental scientific disciplines and use them as point of departure of his meta–theoretical and, in general, rational way of philosophizing.[87]

86 We refer once more to Auyang's important book mentioned in footnotes 6 and 43.

87 We are very grateful to our lifelong friend and colleague for more than three decades Miguel A. Badía Cabrera, who read this paper thoroughly, detected some printing errors and made more than a dozen very valuable suggestions to improve the paper, mostly but not always of a purely stylistic nature.

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NOTES ON CONTRIBUTOR

GUILLERMO E. ROSADO HADDOCK is a retired Professor in the Department of Philosophy, University of Puerto Rico at Rio Piedras, Puerto Rico. PhD at the Department of Logic of the Friedrich Wilhelm's University of Bonn (Germany) in 1973, with a dissertation on Husserl's philosophy of logic and mathematics. His work is mainly in the Philosophy of Logic and the Philosophy of Mathematics, mostly, though not exclusively, on the views of Husserl and Frege. He has published papers or critical studies in some twenty international journals as well as books such as *Unorthodox Analytic Philosophy* (College Publications, 2018); *Against the Current* (Ontos, 2012); *The Young Carnap's Unknown Master* (Ashgate, 2008); *A Critical Introduction to the Philosophy of Gottlob Frege* (Ashgate 2006), as well as the book of essays *Husserl and Analytic Philosophy* (de Gruyter, 2016), of which he is both editor and co-author; and *Husserl or Frege?: Meaning, Objectivity and Mathematics* (Open Court 2000, 2003) co-authored with Claire Ortiz Hill.

CONTACT INFORMATION

Oficina Central de Correos, Apartado 367893. San Juan, Puerto Rico 00936-7893 e-mail (✉): gerosadohaddock@gmail.com.

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