Flexible Amplify and Forward Relaying Protocol with optimized duplexing

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***Abstract*— In this paper we propose a novel flexible amplify and forward (AF) relaying protocol with different duplexing ratios. We show that the average throughput can be considerably improved by properly adjusting the duplex ratio. Here it is considered that the transmitted and received channel state information (CSI) are available at the relay. We introduce a new processing technique at relay for above case in order to achieve higher throughput and lower bit error rate (BER) than traditional schemes. Also BER upper bounds for our presented cooperative model are derived.**

Keywords-component; amplify and forward, duplexing ratio, average mutual information, linear dispersion codes

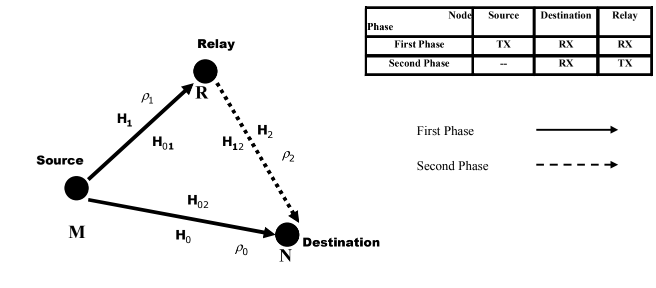
I. INTRODUCTION

The performance of wireless communication will be improved significantly as well as diversity and system robustness by utilizing cooperation between nodes distributed in space especially when direct link between involved nodes fail to provide reliable connection [1]. This cooperative diversity is exploited through the independent source-destination channels and the virtual antenna arrays [2]-[3].

The performance of the linear transceiver design for AF scheme where multiple input multiple output (MIMO) channel state information are available at relay has been investigated in [4] and [5]. It has shown that lower bound of the cooperative mutual information is the mutual information of the relaying path. They have provided a linear transceiver design for amplify and forward scheme using transmitter CSI at relay. In above works length of time slots allocated to relay and source are considered to be equal (duplexing ratio equals to 0.5). Here we propose a novel scheme to provide different duplexing ratios for AF cooperative scheme using linear dispersion Codes (LDC). LDC scheme disperses sub-streams of data stream in linear combination over space and time. LDC codes are designed to maximize mutual information between the received and transmit signals [6]. By applying LDC codes and distributing data on the time and space, another dimension is opened on the design of linear processing at relay and degree of freedom for the AF scheme is increased by considering different time slot length for source and relay transmission. Here it is considered that the relay is aware about its input and output channel. For this case we introduce new processing techniques at relay for our flexible AF schemes with different duplexing ratios. We show that our flexible scheme provides better throughput as well as lower BER in many signal-to-noise-ratio (SNR) configurations. We also look at BER analysis for our flexible AF model and we derive upper bound for the presented model. The remainder of this paper is organised as follows. Section II describes the system model for LDC based AF concept. Section III discusses in detail the proposed processing technique at relay. Then the BER upper bound for our LDC based cooperative AF model is derived in section IV and throughput and BER performance results of the proposed scheme are compared in section V. Finally section VI provides conclusion remarks.

II. SYSTEM MODEL

A simple TDD/TDMA one way relay channel scenario with one source node and only one relay has been used here (Figure 1). It is assumed that nodes transmission/reception is based on a simple protocol composed of two phases. The source Node with M transmit antennas sends its message to the relay which has R receive and transmit antennas and the destination node with N receive antennas in the channel phase 1, P_1 (solid line). Relay combines the received signal from source through a linear combiner and sends in broadcast to destination in the channel phase 2, P_2 (doted line). The channel phases P_1 and P_2 are orthogonal in time and thus, have no interference. We assume that input data are encoded and modulated and then pass through LDC encoder. LDC scheme disperses sub-streams of data stream in linear combination over space and time [6]. At first phase during kth LDC time slot with size



Node / Phase	Source	Destination	Relay
First Phase	TX	RX	RX
Second Phase	--	RX	TX

Figure 1. **Used cooperative scenario for AF.**

T_1 , transmit signal $s_0(k)$ $k = 1,...,P_1 / T_1$ is LDC coded with dispersion matrices $\{\mathbf{A_q}, \mathbf{B_q}\}$ $q = 1,...,Q_0$. The design of the

LDC codes depends on the selection of the parameters T_1, Q_0, M and dispersion matrices[6]. LDC coded signal with size $T_1 \times M$ is transmitted through random complex normal MIMO channels $\mathbf{H}_{01} \in \mathbb{C}^{R\times M}$ and $\mathbf{H}_{02} \in \mathbb{C}^{N\times M}$ to relay and destination respectively. During second phase, relay transmits its desired signal through MIMO channel $\mathbf{H}_{12} \in \mathbb{C}^{N\times R}$ to destination. $\mathbf{H}_{01}$, $\mathbf{H}_{02}$ and $\mathbf{H}_{12}$ are defined as random channel matrix with variance ρ_1, ρ_0 and ρ_2 respectively. ρ_0, ρ_1 and ρ_2 are defined as SNR of the Links between source and destination, source and relay and relay to destination respectively. $\mathbf{H}_{01}$, $\mathbf{H}_{02}$ are assumed fixed during each LDC time slot (T_1) and $\mathbf{H}_{12}$ fixed during LDC Time slot T_2. At relay and destination, white Complex Gaussian distributed noises with zero mean, unit-variance are added to received signals. Because of the linear structure of dispersion matrices $\{\mathbf{A_q}, \mathbf{B_q}\}$ and original channels $\mathbf{H}_{01}$, $\mathbf{H}_{02}$ and $\mathbf{H}_{12}$, the equivalent channel matrices can be calculated based on them [6], all known to the destination. It is assumed that source does not have any information about channel matrices.

In this paper hereinafter for simplicity we continue our work with equivalent channel matrices between source and relay $\mathbf{H}_1$ with size $2RT_1 \times 2Q_0$, source and destination $\mathbf{H}_0$ with size $2NT_1 \times 2Q_0$ and relay and destination $\mathbf{H}_2$ with size $2NT_2 \times 2RT_2$. Factor 2 is for considering real and imaginary values separately. Now all the complex valued equations can be converted to equivalent real valued equations. So for example for each LDC time slot T_1 we have:

$$\begin{aligned} \ddot{\mathbf{y}}_0 &= \sqrt{\frac{1}{M}}\mathbf{H}_0\ddot{\mathbf{s}}_0 + \ddot{\mathbf{v}}_0 \\ \ddot{\mathbf{y}}_1 &= \sqrt{\frac{1}{M}}\mathbf{H}_1\ddot{\mathbf{s}}_0 + \ddot{\mathbf{v}}_1 \end{aligned} \tag{1}$$

where $\ddot{\mathbf{s}}_0$ is real valued transmit signal and $\ddot{\mathbf{y}}_0$ and $\ddot{\mathbf{y}}_1$ received signal at destination and relay during LDC time slot T_1. $\ddot{\mathbf{v}}_0$ and $\ddot{\mathbf{v}}_1$ are real valued additive white Gaussian noise signal. Generally $\ddot{\mathbf{x}}$ represents a real valued column vector consists of real parts and imaginary parts of any vector $\mathbf{x}$, i.e. $\ddot{\mathbf{x}} = \begin{bmatrix} real(\mathbf{x}) \\ imag(\mathbf{x}) \end{bmatrix}$. During second time slot T_2, relay applies a linear matrix $\mathbf{\Gamma}$ with size $2RT_2 \times 2RT_1$ over received signal and transmits it through equivalent channel $\mathbf{H}_2$ channel to destination. Then by assuming $\ddot{\mathbf{v}}_2$ as a real valued additive white Gaussian noise signal, we have:

$$\begin{aligned} \ddot{\mathbf{y}}_2 &= \mathbf{H}_2\mathbf{\Gamma}\ddot{\mathbf{y}}_1 + \ddot{\mathbf{v}}_2 \\ &= \mathbf{H}_2\mathbf{\Gamma}\left(\sqrt{\frac{1}{M}}\mathbf{H}_1\ddot{\mathbf{s}}_0 + \ddot{\mathbf{v}}_1\right) + \ddot{\mathbf{v}}_2 \end{aligned} . \tag{2}$$

Now we can derive an equivalent model for whole system as $\ddot{\mathbf{Y}} = \mathbf{H}\ddot{\mathbf{X}} + \ddot{\mathbf{V}}$, where $\mathbf{Y} = \begin{bmatrix} \ddot{\mathbf{y}}_0 \\ \ddot{\mathbf{y}}_2 \end{bmatrix}$ is real valued received vector at destination during both LDC time slots T_1 and T_2 and $\ddot{\mathbf{X}} = \left[\sqrt{\frac{1}{M}}\ddot{\mathbf{s}}_0\right]$ is real value transmitted signal from source node. Overall equivalent channel matrix is $\mathbf{H} = \begin{bmatrix} \mathbf{H_0} \\ \mathbf{H_2\Gamma H_1} \end{bmatrix}$ and noise matrix $\ddot{\mathbf{V}}$ is $\ddot{\mathbf{V}} = \begin{bmatrix} \mathbf{I}_{2NT_1\times 2NT_1} & \mathbf{0}_{2NT_1\times 2RT_1} & \mathbf{0}_{2NT_1\times 2NT_2} \\ \mathbf{0}_{2NT_2\times 2NT_1} & \mathbf{H}_2\mathbf{\Gamma} & \mathbf{I}_{2NT_2\times 2NT_2} \end{bmatrix}\begin{bmatrix} \ddot{\mathbf{v}}_0 \\ \ddot{\mathbf{v}}_1 \\ \ddot{\mathbf{v}}_2 \end{bmatrix}$.

Matrix $\mathbf{R}_{\ddot{\mathbf{x}}}$ is defined as the covariance matrix of signal transmitted from the source $\mathbf{R}_{\ddot{\mathbf{x}}} = \mathcal{E}\{\ddot{\mathbf{X}}\ddot{\mathbf{X}}^\dagger\} = (1/2M)\mathbf{I}_{2Q_0}$ and so $trace(\mathbf{R}_{\ddot{\mathbf{x}}}) = T_1$. The letter $\mathcal{E}$ represents expectation and $\dagger$ is used for hermitian function. The factor 2 is applied regarding to real valued vector $\ddot{\mathbf{X}}$. It is assumed that total transmit powers of source and also relay are equal one. $\mathbf{R}_{\ddot{v}_0}, \mathbf{R}_{\ddot{v}_1}, \mathbf{R}_{\ddot{v}_2}$ are defined as the noise covariance matrices. $\mathbf{R}_{\ddot{v}_0} = \mathcal{E}\{\ddot{\mathbf{v}}_0\ddot{\mathbf{v}}_0^\dagger\} = \frac{1}{2}\mathbf{I}_{2NT_1}$, $\mathbf{R}_{\ddot{v}_1} = \mathcal{E}\{\ddot{\mathbf{v}}_1\ddot{\mathbf{v}}_1^\dagger\} = \frac{1}{2}\mathbf{I}_{2RT_1}$ and $\mathbf{R}_{\ddot{v}_2} = \mathcal{E}\{\ddot{\mathbf{v}}_2\ddot{\mathbf{v}}_2^\dagger\} = \frac{1}{2}\mathbf{I}_{2NT_2}$. Then overall noise covariance matrix $\mathbf{R}_{\ddot{V}}$ is calculated as following:

$$\mathbf{R}_{\ddot{\mathbf{V}}} = \frac{1}{2}\begin{bmatrix} \mathbf{I}_{2NT_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{H_2\Gamma\Gamma^\dagger H_2^\dagger} + \mathbf{I}_{2NT_2} \end{bmatrix}. \tag{3}$$

The covariance matrix of the signal transmitted by the relay is $\mathbf{R}_{\Gamma\ddot{\mathbf{y}}_1} = \mathcal{E}(\mathbf{\Gamma}\ddot{\mathbf{y}}_1\ddot{\mathbf{y}}_1^\dagger\mathbf{\Gamma}^\dagger) = \frac{1}{2}\mathbf{\Gamma}(\frac{1}{M}\mathbf{H}_1\mathbf{H}_1^\dagger + \mathbf{I}_{2RT_1})\mathbf{\Gamma}^\dagger$ with constraint $trace(\mathbf{R}_{\Gamma\ddot{\mathbf{y}}_1}) = T_2$.

III. Lower Bound for LDC Based Cooperative Mutual Information

Based on the overall equivalent $\mathbf{H}$ and covariance matrix of overall noise $\mathbf{R}_{\ddot{V}}$, the cooperative mutual information will be:

$$\begin{aligned} I(\ddot{\mathbf{Y}};\ddot{\mathbf{X}}) &= \frac{1}{2}\frac{1}{T_1+T_2}\log_2\det\left(\boldsymbol{R}_{\ddot{\mathbf{Y}}}\mathbf{R}_{\ddot{V}}^{-1}\right) \\ &= \frac{1}{2}\frac{1}{T_1+T_2}\log_2\det\left(\frac{\mathbf{H}\mathbf{R}_{\ddot{\mathbf{x}}}\mathbf{H}^\dagger + \mathbf{R}_{\ddot{V}}}{\mathbf{R}_{\ddot{V}}}\right) \end{aligned} , \tag{4}$$

where the factor 1/2 is for real valued effective channel and each LDC matrix spans over T_1+T_2 channel use. The notation det represents determinant function. Duplexing ratio is defined as $\alpha = T_1/(T_1+T_2)$. Similar to the basic MIMO scenario [4] we can show that it is not possible to calculate the upper bound for $I(\tilde{\mathbf{Y}};\tilde{\mathbf{X}})$. The mutual information of the relaying (multihop) channel when direct link is not existed can be considered as the lower bound for the cooperative system. The mutual information of Multihop channel $I_{Multihop}$ can be written as:

$$I_{Multihop} = \frac{1}{2}\frac{1}{T_1+T_2}\log_2 \det\left(\frac{\mathbf{I}_{(2NT_2)} + \mathbf{H_2\Gamma\Gamma^\dagger H_2^\dagger} + \frac{1}{M}\mathbf{H_2\Gamma H_1 H_1^\dagger \Gamma^\dagger H_2^\dagger}}{\mathbf{I}_{(2NT_2)} + \mathbf{H_2\Gamma\Gamma^\dagger H_2^\dagger}}\right), \quad (5)$$

subject to $trace(\frac{1}{2}\mathbf{\Gamma}(\frac{1}{M}\mathbf{H_1H_1^\dagger} + \mathbf{I}_{2RT_1})\mathbf{\Gamma}^\dagger) \le T_2$.

Here we assume that relay knows about its input equivalent channel $\mathbf{H}_1$ and forward equivalent channel $\mathbf{H}_2$. We can prove that matrix $\mathbf{\Gamma}$ that maximizes multihop mutual information (5) is given by: $\mathbf{\Gamma} = \tilde{\mathbf{V}}_2\tilde{\mathbf{\Gamma}}\mathbf{U}_1^\dagger$ where $\mathbf{U_1}$ is left eigenvector of the equivalent channel matrix $\mathbf{H_1} = \mathbf{U_1\Lambda_1^{1/2}V_1^\dagger}$ and $\tilde{\mathbf{V}}_2$ is permutated right eigenvector of the equivalent channel matrix $\mathbf{H_2} = \mathbf{U_2\Lambda_2^{1/2}V_2^\dagger}$. $\tilde{\mathbf{\Gamma}}$ is a $2RT_2 \times 2RT_1$ matrix. Here we can see that $\tilde{\mathbf{\Gamma}}$ can have different column and row size depending to value of T_1 and T_2 .Here we consider $\tilde{\mathbf{\Gamma}} = \begin{cases} \begin{bmatrix}\hat{\mathbf{\Gamma}} & 0\end{bmatrix} & T_2 < T_1 \\ \hat{\mathbf{\Gamma}} & T_2 = T_1 \\ \begin{bmatrix}\hat{\mathbf{\Gamma}} \\ 0\end{bmatrix} & T_2 > T_1 \end{cases}$

where $\hat{\mathbf{\Gamma}}$ is a diagonal matrix with size $2R\min(T_1,T_2)\times 2R\min(T_1,T_2)$. Considering $\tilde{\mathbf{\Lambda}}_2^{1/2} = \mathbf{P\Lambda}_2^{1/2}$ where $\mathbf{P}$ is a permutation matrix and by some modification over equation (5) we obtain following equation:

$$I_{Multihop} = \frac{1}{2}\frac{1}{T_1+T_2}\sum_{k=1}^{K}\log_2\left(\frac{\frac{1}{2}(1+\tilde{\gamma}_{2k}\tilde{\lambda}_{2,k}) + \gamma_{1k}\lambda_{1k}\tilde{\gamma}_{2k}\tilde{\lambda}_{2,k}}{\frac{1}{2}(1+\tilde{\gamma}_{2k}\tilde{\lambda}_{2,k})}\right). \quad (6)$$

where K is minimum of number of eigen values of channels $\mathbf{H}_1$ and $\mathbf{H}_2$. $\gamma_{1k} = 1/2M$. Also we have the constraint $\sum_{k=1}^{K}\left(\tilde{\gamma}_{2k}(\frac{1}{2}+\gamma_{1k}\lambda_{1,k})\right) \le T_2$. $\tilde{\gamma}_{2k}$ is assumed as the kth diagonal element of matrix $\hat{\mathbf{\Gamma}}\hat{\mathbf{\Gamma}}^\dagger$ and λ_{1k} and $\tilde{\lambda}_{2k}$ as the squared of kth eigen values of $\mathbf{\Lambda_1^{1/2}}$ and $\tilde{\mathbf{\Lambda}}_2^{1/2}$ respectively. The optimization problem will be Maximizing $\sum_{k=1}^{K}\log_2\left(\frac{\frac{1}{2}(1+\tilde{\gamma}_{2k}\tilde{\lambda}_{2,k}) + \gamma_{1k}\lambda_{1k}\tilde{\gamma}_{2k}\tilde{\lambda}_{2,k}}{\frac{1}{2}(1+\tilde{\gamma}_{2k}\tilde{\lambda}_{2,k})}\right)$, subject to $\sum_{k=1}^{K}\left(\tilde{\gamma}_{2k}(\frac{1}{2}+\gamma_{1k}\lambda_{1,k})\right) \le T_2$.

The optimum solution for $k=1,...,K$ using Lagrange method is as following

$$(\frac{1}{2}+\gamma_{1k}\lambda_{1,k})\tilde{\gamma}_{2k} = \max\left(0,\left[\sqrt{\mu\frac{\gamma_{1k}\lambda_{1k}}{\lambda_{2k}} + \left(\frac{\gamma_{1k}\lambda_{1k}}{2\lambda_{2k}}\right)^2} - \left(\frac{\gamma_{1k}\lambda_{1k}}{2\lambda_{2k}}\right) - \frac{1}{2\lambda_{2k}}\right]\right), \quad (7)$$

where μ is the Lagrange multiplier and its starting point is $\mu_{\min} = \max(\frac{1}{2}(\frac{1}{2}+\gamma_{1k}\lambda_{1k})/(\gamma_{1k}\lambda_{1k}\lambda_{2k})$ for $k=1,...,K)$ and in order to update μ the Newton-Raphson algorithm can be used. Ordering of squared Eigen values $\lambda_{1,k}$ and $\lambda_{2,k}$ (permutation matrix P) also needs to be optimized. From the simulation result we found that sorting eigen values in decreasing order maximize mutual information.

IV. BER ANALYSIS FOR COOPERATIVE BICM AF

Encoder at source uses binary code $\mathbf{C}$ and generates code sequence $\underline{\mathbf{c}} \in \mathbf{C}$. Bit interleaving π is done at source. A bit interleaver π in the bit interleaved coded modulation (BICM) scheme establishes a one-to-one correspondence $\pi: k \to (k',i)$, where k denotes the time ordering of the bit sequence before interleaving, k' denotes the time ordering of the modulated signals, and i indicates the position of bit in the label of the signal. Modulator (μ,χ) is done at source. It is memory less and are over signal set $\chi \in \mathbb{C}^L$ with size $|\chi| = 2^m$ and uses one-to-one binary labelling map $\mu:\{0,1\}^m \to \chi$. $\mathbb{C}^L$ is the complex L -dimensional Euclidean space. The ML criterion for decoding the observations $\tilde{\mathbf{Y}}$ given CSIs $\mathbf{H}_0$, $\mathbf{H}_1$ and $\mathbf{H}_2$ at destination is written as $(\hat{\underline{\mathbf{c}}}) = \arg\max_{\underline{c} \in C} p_{\mathbf{H}}(\tilde{\mathbf{Y}}|\underline{\mathbf{c}})$ where $\mathbf{H}$ is the equivalent channel of whole cooperative system. Let $P(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}})$ denote the pairwise error probability (PEP) as the probability that the decoder at destination prefers $\hat{\underline{\mathbf{c}}}$ to $\underline{\mathbf{c}}$. d denotes the Hamming distance between the corresponding component code-words of $\underline{\mathbf{c}}$ and $\hat{\underline{\mathbf{c}}}$ .The PEP will be a function of d , μ and χ , $P(\underline{\mathbf{c}} \to \hat{\underline{\mathbf{c}}}) = f(d,\mu,\chi)$. The PEP can be used in conjunction with usual union bound to upper bound the BER. It means for convolutional codes rate k_c/n_c we have: $P_b \le \frac{1}{k_c}\sum_{d=1}^{\infty} \mathrm{W}_I(d) f(d,\mu,\chi)$ where $\mathrm{W}_I(d)$ is total input weight of error events at Hamming distance d [7]. Now, we focus on the calculation of the PEP $f(d,\mu,\chi)$. For the derivation of the BICM union bound $f_{up}(d,\mu,\chi)$:

$$f_{up}(d,\mu,\chi) = \frac{1}{2\pi j}\int_{\alpha-j\infty}^{\alpha+j\infty} [\psi(s)]^d \frac{ds}{s}, \quad (8)$$

where $\psi(s)=\dfrac{1}{mQ_0 2^{mQ_0}}\sum_{i=1}^{mQ_0}\sum_{b=0}^{1}\sum_{\mathbf{X}\in\chi(i,b)}\sum_{\mathbf{Z}\in\chi(i,\bar{b})}\Phi_{\Delta(\mathbf{X},\mathbf{Z})}(s)$.

$\mathbf{X}=\sqrt{M}\ddot{\mathbf{X}}$ and $\mathbf{Z}=\sqrt{M}\ddot{\mathbf{Z}}$ are signal vectors corresponding to the paths $\underline{\mathbf{c}}$ and $\hat{\underline{\mathbf{c}}}$. Equation (8) can be calculated using scheme mentioned in [8]; similar to [9] removing irrelevant error events in (8) will result in the cooperative BICM Expurgated Bound. $\Phi_{\Delta(\mathbf{X},\mathbf{Z})}(s)$ is the Laplace transforms of the probability density function of the metric difference $\Delta(\mathbf{X},\mathbf{Z})$. It means: $\Phi_{\Delta(\mathbf{X},\mathbf{Z})}(s)=\mathcal{E}_{\ddot{\mathbf{Y}},\mathbf{H}}\left[e^{-s\Delta(\mathbf{X},\mathbf{Z})}\right]$ where

$$\Delta(\mathbf{X},\mathbf{Z})=\log(p_{\mathbf{H}_0,\mathbf{H}_1,\mathbf{H}_2}(\ddot{\mathbf{Y}}|\mathbf{X}))-\log(p_{\mathbf{H}_0,\mathbf{H}_1,\mathbf{H}_2}(\ddot{\mathbf{Y}}|\mathbf{Z})). \qquad (9)$$

Here we can separate direct transmission and relay transmission: $\Delta(\mathbf{X},\mathbf{Z})=\Delta_{direct}(\mathbf{X},\mathbf{Z})+\Delta_{relay}(\mathbf{X},\mathbf{Z})$. By considering $(\mathbf{X-Z})^{\dagger}\mathbf{H}_0^{\dagger}\mathbf{H}_0(\mathbf{X-Z})=\mathbf{h}_0^{\dagger}Q(\mathbf{X},\mathbf{Z})\mathbf{h}_0$ where $Q(\mathbf{X},\mathbf{Z})=(\mathbf{X-Z})(\mathbf{X-Z})^{\dagger}\otimes\mathbf{I}_{2NT_1}$ ($\otimes$ is used for Kronecker tensor product) and $\mathbf{h}_0=vec(\mathbf{H}_0)$ is a column vector consist of all elements of $\mathbf{H}_0$ and also $\mathbf{C}_{\mathbf{h}_{0,norm}}=\frac{1}{2}\mathcal{E}(\mathbf{h}_{0,norm}\mathbf{h}_{0,norm}^{\dagger})$ ($\mathbf{C}_{\mathbf{h}_{0,norm}}$ depends to specification of used LDC code where $\mathbf{h}_{0,norm}=\sqrt{1/\rho_0}\mathbf{h}_0$) we have:

$$\begin{aligned}\Phi_{\Delta_{direct}(\mathbf{X},\mathbf{Z})}(s)&=\mathcal{E}_{\mathbf{H}_0}\left[e^{(-s+s^2)\mathbf{h}_0^{\dagger}Q(\mathbf{X-Z})\mathbf{h}_0}\right]\\&=\left[\det(\mathbf{I}+2\frac{\rho_0}{M}(s-s^2)Q(\mathbf{X-Z})\mathbf{C}_{\mathbf{h}_{0,norm}})\right]^{-1}\end{aligned}. \qquad (10)$$

For relay link, by considering $\tilde{\mathbf{H}}_{2,\mathrm{norm}}=\mathbf{H}_{2,\mathrm{norm}}\mathbf{\Gamma}\mathbf{H}_{1,\mathrm{norm}}$ and $\tilde{\mathbf{V}}=\mathbf{H}_2\mathbf{\Gamma}\ddot{\mathbf{V}}_1+\ddot{\mathbf{V}}_2$ and assuming $\mathbf{R}'_{\tilde{V}}=(\rho_2\mathbf{H}_{2,norm}\mathbf{\Gamma}\mathbf{\Gamma}^{\dagger}\mathbf{H}_{2,norm}^{\dagger}+\mathbf{I}_{2NT_2})$ then after getting expectation over noise vectors we will have:

$$\Phi_{\Delta_{relay}(\mathbf{X},\mathbf{Z})}(s)=E_{\mathbf{H}_1,\mathbf{H}_2}\left[e^{\frac{\rho_1\rho_2}{M}(-s+s^2)(\mathbf{X-Z})^{\dagger}\tilde{\mathbf{H}}_{2,norm}^{\dagger}\mathbf{R}_{\tilde{\mathbf{V}}}'^{-1}\tilde{\mathbf{H}}_{2,norm}(\mathbf{X-Z})}\right]. \qquad (11)$$

Recall that $\mathbf{\Gamma}$ is the function of $\rho_1,\rho_2,\mathbf{H}_{1,norm}$ and $\mathbf{H}_{2,norm}$. By considering $\mathbf{H}_1=\sqrt{\rho_1}\mathbf{H}_{1,norm}$ and $\mathbf{H}_2=\sqrt{\rho_2}\mathbf{H}_{2,norm}$, for relay link we need to do expectation over channel matrices numerically. Then finally we can calculate $\Phi_{\Delta(\mathbf{X},\mathbf{Z})}(s)=\Phi_{\Delta_{Direct}(\mathbf{X},\mathbf{Z})}(s)\Phi_{\Delta_{relay}(\mathbf{X},\mathbf{Z})}(s)$ numerically for different values of s.

V. PERFORMANCE RESULT

The performance of our proposed flexible LDC based AF scheme has been examined for different number of transmit /receive antennas as well as different SNR configuration. Due to the lack of space in this section we provide performance result for a system with two transmit antennas at source, two receive/transmit antennas at relay and two receive antennas at destination for different SNR configurations. Here we use LDC code from [6] for 2x2 MIMO system. One tap complex independent and identically distributed (i.i.d) Rayleigh fading channel coefficients are multiplied with transmitted signals. Coherency time of the channel has been set to $\max(T_1,T_2)$. Figure 2 shows average mutual information for different duplexing ratios for $\rho_0=4dB$ and $\rho_2=15dB$. As we can see during ρ_1 from $6dB$ to $9dB$, the curve with $\alpha=8/10$ has better average throughput than direct transmission ($\alpha=1$) and $\alpha=5/10$. During ρ_1 from $9dB$ to $16dB$, the curve with $\alpha=6/10$ has better average throughput. But for $\rho_1\geq 16dB$ the curve with $\alpha=5/10$ has the best throughput. Also the area that we can achieve throughput gain compared with $\alpha=5/10$ and $\alpha=1$ has been highlighted. Here we can see clearly that by choosing optimum duplex ratio we can increase average throughput. Figure 3 shows amount of achieved SNR gain in ρ_1 compare to $\alpha=1$ and $\alpha=5/10$ for $\rho_0=4dB$ and $\rho_2=15dB$ if we consider fixed average throughput. Near 3dB gain can be achieved at rate near 3bps/Hz. Figure 4 compares the average mutual information for $\alpha=1/2$ ($T_1=2$ and $T_2=2$) and $\alpha=2/3$ ($T_1=2$ and $T_2=1$) for $\rho_0=5dB$ and $\rho_2=15dB$. As we can see during ρ_1 from $8dB$ to $16dB$, the curve with $\alpha=2/3$ has better average throughput than $\alpha=1$ and $\alpha=1/2$. Now we look at BER performance result. For $\alpha=2/3$ at the source node, the information bit sequences are encoded with an optimum 1/2 rate systematic recursive convolutional code with generator polynomial (13, 15) in octal representation. Then the interleaved coded bits are QPSK modulated and LDC encoded and transmitted to other nodes. Coherency time of the channel has been set to $\max(T_1,T_2)=2$ .For $\alpha=1/2$ in order to provide result for same total rate, we apply puncturing after the 1/2 rate convolutional code to convert total coding rate to 2/3 which converts total rate of $\alpha=1/2$ equal to total rate of $\alpha=2/3$. As it is seen in Figure 5 the system with $\alpha=2/3$ is performing better than $\alpha=1/2$ in all considered SNRs for $\rho_1=5dB$. In Figure 5 we compare BER performance for $\alpha=1/2$ ($T=2_1,T_2=2$) and $\alpha=2/3$ ($T=2_1,T_2=1$) in a SISO system. The coding and modulation schemes are same as above. As it is seen system with $\alpha=2/3$ is performing better than $\alpha=1/2$ in considered SNRs for $\rho_2=5dB$ .Expurgated BER upper bounds for $\alpha=2/3$ have been provided using BER analysis from section IV.

VI. CONCLUSION

A novel method has been proposed to increase average throughput of cooperative AF scheme with choosing optimum duplexing ratio. Duplexing ratio changes with increase or decrease of time slot length for source and relay transmission. For each SNR region the optimum duplexing ratio can be different. The processing techniques at relay for this new AF scheme have been presented for the case when relay has knowledge about its input and output channel. It was shown that with applying this new flexible scheme and choosing optimum duplexing ratio, much better average throughput can be achieved as well as improvement in BER performance in many SNR configurations.

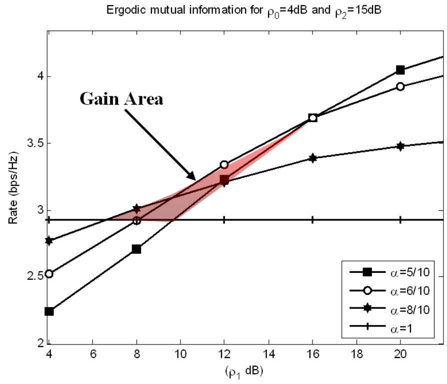


Figure 2. Average throughput comparison for different duplexing ratios for $\rho_0 = 4dB$ and $\rho_2 = 15dB$.

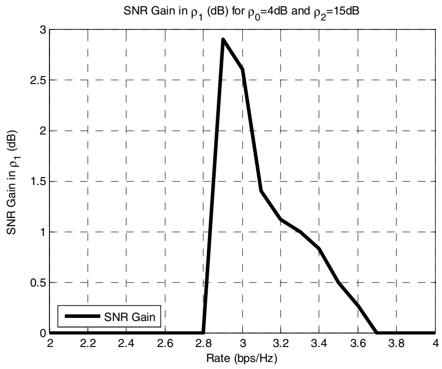


Figure 3. Achieved SNR gain for different rates for $\rho_0 = 4dB$ and $\rho_2 = 15dB$.

ACKNOWLEDGMENT

This work was partly supported by the framework of the European ICT-215282 ROCKET project.

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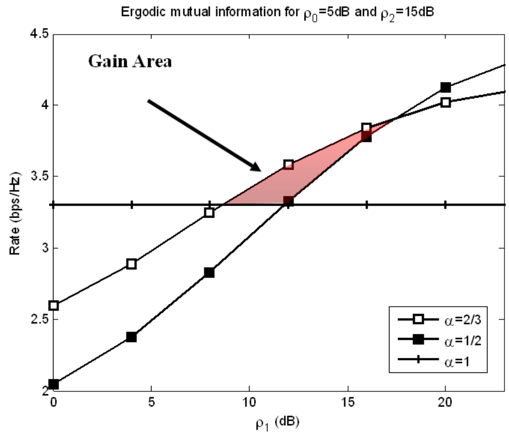

Figure 4. Average throughput comparison for different duplexing ratios for $\rho_0 = 5dB$ and $\rho_2 = 15dB$.

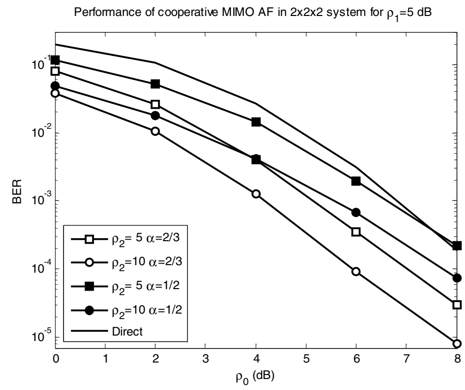

Figure 5. BER performance comparison for duplexing ratios $1/2$ and $2/3$ for $\rho_1 = 5dB$.

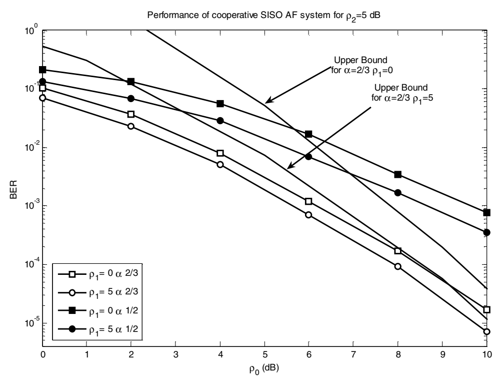

Figure 6. BER performance comparison for duplexing ratios $1/2$ and $2/3$ for $\rho_2 = 5dB$.