A Pre-BSC Model for Distributed Turbo Codes

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Abstract—**A model that effectively captures the relay decoding error is proposed for a distributed turbo code (DTC) over a cooperative relay channel. The model consists of a memoryless binary symmetric channel (BSC) placed before the relay encoder, hence called pre-BSC. Accordingly the required iterative decoder is derived that consists of an extra update function to be applied on extrinsic information before their exchange. The derived update function is in form of a smooth clipping and therefore the overall decoder is called clipped iterative decoder (clipped ID). Clipped ID is robust to the relay decoding error and is able to significantly improve the final performance, especially when source-to-relay link becomes erroneous.**

Index Terms—**Relay Channels, cooperative, DF, distributed turbo code, robustness**

I. Introduction

One of the common strategies used in a cooperative communication is decode and forward (DF) [1] that is a form of regenerative relaying where the helping relay node decode, re-encode, and then forward the received message to the final destination. By interleaving the decoded information bits prior to encoding at the relay node a distributed turbo code (DTC) will be realized [2]-[3]. The resulted code will be parallel concatenation of the two component codes: one at the source and the other at the relay node. Similar to its collocated counterpart a distributed turbo code is shown to be effective in exploiting the potential capacity of a cooperative communication system. However, as pointed out in [3], the effectiveness of DTC relies on perfect decoding of the source data at the relay node. This off course cannot be guaranteed and as a result the system will suffer from serious error propagation due to errors occurred at the relay decoder. Concerned with this issue [3] proposes to use soft information relaying. The scheme is only proposed for binary phase shift keying (BPSK). Generally relay decoding error is an issue for any DF scheme weather or not a DTC is employed. Reference [4] extends the soft information relaying concept to high order quadrature amplitude modulations (MQAM). Another approach proposed in [5] is instead of soft signal relaying a single relay data quality that is either an attributed signal to noise ratio (SNR) or a reliability factor is signaled from the relay node to destination. This quality factor is used to properly combine the information received via direct and indirect links and therefore endows system with robustness to relay decoding errors. Given that the relay data reliability is available at the destination node [6] addresses the optimal decoding at the destination receiver and shows that the optimal decoder should carry out a global search over the joint code space of the source and the relay nodes taking into account the fact that due to relay decoding error source and relay codewords are not necessarily in one-to-one correspondence. The reference [7] takes over the optimal decoding concept and proposes a novel joint state space viterbi decoder that jointly searches the state space of the both source and relay codes. This decoder incorporates the relay data reliability as a coupling factor between the associated finite state machines of the two codes. Here we focus on improving the performance of a DTC given that the relay data reliability is available at the final receiver at the destination node. A pre-BSC model is proposed that captures relay decoding error by placing a memoryless binary symmetric channel (BSC) before the relay encoder. The proposed model is also compared with another one called post-BSC model used in [5] and [8] that instead places a BSC after the relay encoder. The pre-BSC model is used to drive a new iterative decoder that exercises a smooth clipping of the extrinsic information prior to their exchange between the two component decoders. This decoder achieves significant robustness to erroneous source-to-relay links especially when system is operating under static fading condition. The new decoder is generic and can be used with any combination of modulation at the source and relay nodes. However for the sake of brevity the simulation results provided here are only for BPSK.

The rest of the paper is organized as follows. The system model is presented in Section II. Section III briefly describes the collocated turbo code, DTC, and its two pre and post-BSC models. Then the modified iterative decoder is described in IV. Section V provides numerical and simulation results and section VI concludes the paper.

II. System Model

Let's assume a simple cooperative communication system composed of three nodes: source node S, relay node R, and destination node D. Let's further assume that nodes transmission-reception is based on a simple protocol composed of two phases. In the first phase of this protocol S broadcasts its signal to R and D, and in the second phase only R transmits to D. Even though more efficient approach is to allow S and R jointly transmit in the second phase, for the convenience of introduction of the proposed approach we adhere to this simple protocol. Let the links S-R, S-D, and R-D are enumerated with 0, 1, and 2, respectively. The processing to be performed at each node is as follows:

- *Source:* The information bits $\underline{\boldsymbol{u}}_1$ are encoded by a binary recursive convolutional code (RCC) called RCC1. The

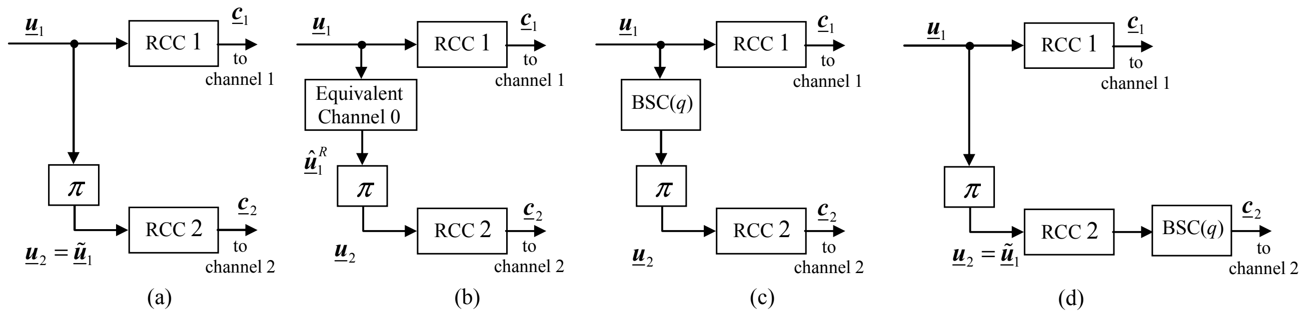


Figure 1 Block diagram of (a) collocated, (b) distributed, (c) pre-BSC, and (d) post-BSC turbo code.

encoded bit sequence denoted by $\underline{\boldsymbol{c}}_1$ can be optionally punctured, and interleaved. Then the resulted bit sequence is modulated and broadcasted over the channels 0 and 1.

- *Relay:* The received signal from S in the first phase is demodulated and the resulted bit reliabilities after necessary de-interleaving and de-puncturing are fed to a maximum likelihood decoder (for example Viterbi algorithm) of the RCC1. Let $\hat{\underline{\boldsymbol{u}}}_1^R$ denotes the outcome of the decoding. This bit sequence is further interleaved and then encoded by another RCC called RCC2. The input and output bit sequences of this encoder are denoted by $\underline{\boldsymbol{u}}_2$ and $\underline{\boldsymbol{c}}_2$, respectively. Then similar to the transmitter of S, $\underline{\boldsymbol{c}}_2$ after necessary puncturing and interleaving is modulated and forwarded to D over the channel 2.
- *Destination:* demodulates the signals received from S and R over the channels 1 and 2, respectively. Then after necessary de-interleaving and de-puncturing feeds the respective bit reliabilities to an iterative decoder. The decoder iteratively exchanges the extrinsic soft information between the two soft-input soft-output (SISO) component decoders corresponding to RCC1 and RCC2.

III. Distributed Turbo Code Models

Let's aggregate all the processing carried out on the encoding, transmission, reception, and decoding chain of S-R by an equivalent channel between bit sequences $\underline{\boldsymbol{u}}_1$ and $\hat{\underline{\boldsymbol{u}}}_1^R$. This channel called "equivalent channel 0" is off course could be far more reliable than the original channel 0 due to the incorporated error correction functionality. This equivalent channel is binary-input binary-output. Let's assume that (if necessary) a scrambling function is employed to symmetrize the system; therefore the equivalent channel can be safely assumed to be symmetric. However this channel is not memoryless as error events happened at the relay decoder will cause burst of errors.

Using this equivalent channel the comparison between a collocated turbo code, which is a parallel concatenation of two convolutional codes, and its distributed version, will become clearer. Figure 1 shows the block diagrams of the original collocated turbo code along with distributed one. Also shown are the two proposed models for the distributed code. The two models are called pre-BSC and post-BSC models as a binary symmetric memoryless channel (BSC) is placed before or after the second component code, respectively. The two models will have different implications on the type of the required processing to be carried out at the destination receiver. Concerned with improving the combining of the directly and indirectly received signals when distributed turbo coding is not employed, [5] and [8] was able to obtain an improved performance using a post-BSC model. However, as it can be seen from Figure 1, for our considered distributed turbo code system the pre-BSC model makes a natural candidate. Thanks to the interleaver π, the memory of the equivalent channel 0 can be broken and hence the equivalent channel can be replaced by a simple BSC before the second code RCC2.

IV. Proposed Iterative Decoding

Figure 3 illustrates the Bayesian graphs [9] of the considered turbo code models of Figure 1. Just to exemplify the two constituent convolutional codes are assumed to have an associated trellis diagram with one information bit and two coded bits per each state transition. The state sequences of the two component codes are denoted by $\underline{\boldsymbol{s}}_1$ and $\underline{\boldsymbol{s}}_2$ in this figure. The existence of the equivalent channel 0 between the variable nodes $\underline{\boldsymbol{u}}_1$ and $\hat{\underline{\boldsymbol{u}}}_1^R$ implies that the message propagation from $\underline{\boldsymbol{u}}_1$ nodes to $\hat{\underline{\boldsymbol{u}}}_1^R$ nodes and vice versa has to be jointly carried out. Assuming that the size of the two sets is *K*, this means that the message updates from $\underline{\boldsymbol{u}}_1$ to $\hat{\underline{\boldsymbol{u}}}_1^R$ and vice versa will have a complexity order of 2^K. The pre-BSC model simplifies this process and significantly reduces its complexity as with this model the message update will be per node. Lets' denote the *k*-th (*k*=1,…,*K*) bit of $\underline{\boldsymbol{u}}_1$ and $\hat{\underline{\boldsymbol{u}}}_1^R$ with $u_{1,k}$ and $\hat{u}_{1,k}^R$, respectively. Let also q denote the probability of error of the BSC. The required message update for a message propagating from $u_{1,k}$ to $\hat{u}_{1,k}^R$ will be the marginalization over $u_{1,k}$:

$$f_{out}\left(\hat{u}_{1,k}^R\right) = \sum_{u_{1,k}=0}^{1} f_{in}\left(u_{1,k}\right) p_{BSC}\left(\hat{u}_{1,k}^R \middle| u_{1,k}\right), \text{ where} \qquad (1)$$

$$p_{BSC}\left(\hat{u}_{1,k}^R \middle| u_{1,k}\right) = \begin{cases} 1-q & \hat{u}_{1,k}^R = u_{1,k} \\ q & \hat{u}_{1,k}^R \neq u_{1,k} \end{cases}$$

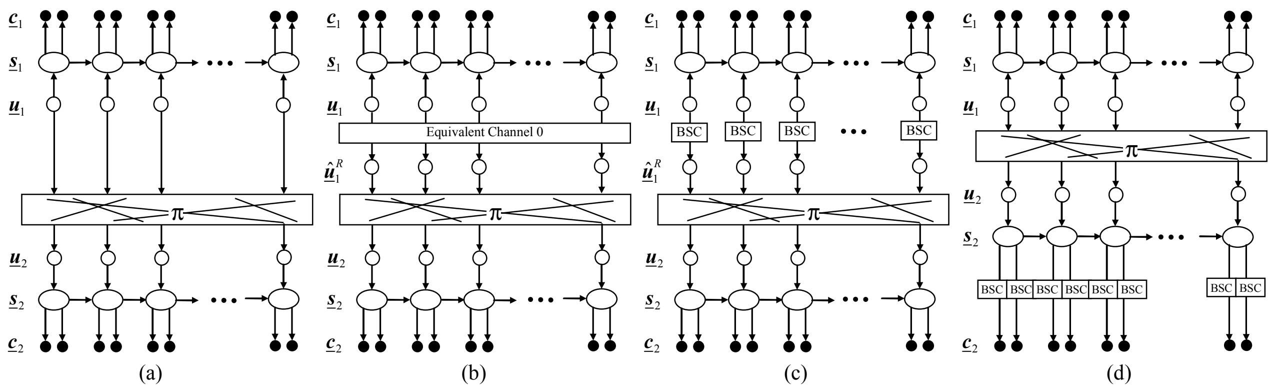


Figure 3 Bayesian graphs of the considered block diagrams of Figure 1: (a) collocated, (b) distributed, (c) pre-BSC, (d) post-BSC models.

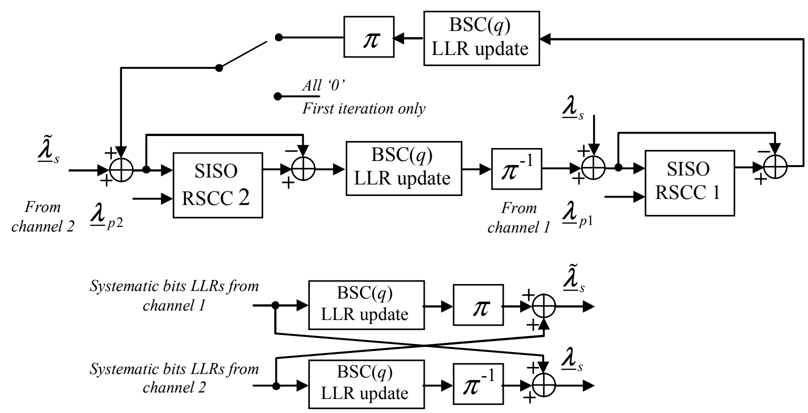


Figure 2 Block diagram of an iterative decoder for a distributed systematic turbo code based on pre-BSC model.

$f_{in}\left(u_{1,k}\right)$ and $f_{out}\left(\hat{u}_{1,k}^{R}\right)$ are corresponding input and output binary messages. Using logarithm of likelihood ratio (LLR) values the above message update can be written as follows:

$$\lambda_{out}=\log\left(\frac{e^{\frac{1}{2}\left(L_q+\lambda_{in}\right)}+e^{-\frac{1}{2}\left(L_q+\lambda_{in}\right)}}{e^{\frac{1}{2}\left(L_q-\lambda_{in}\right)}+e^{-\frac{1}{2}\left(L_q-\lambda_{in}\right)}}\right)\approx Clip\left(\lambda_{in};L_q\right), \qquad (2)$$

where $\lambda_{in}=\log\frac{f_{in}(1)}{f_{in}(0)}$, $\lambda_{out}=\log\frac{f_{out}(1)}{f_{out}(0)}$, and

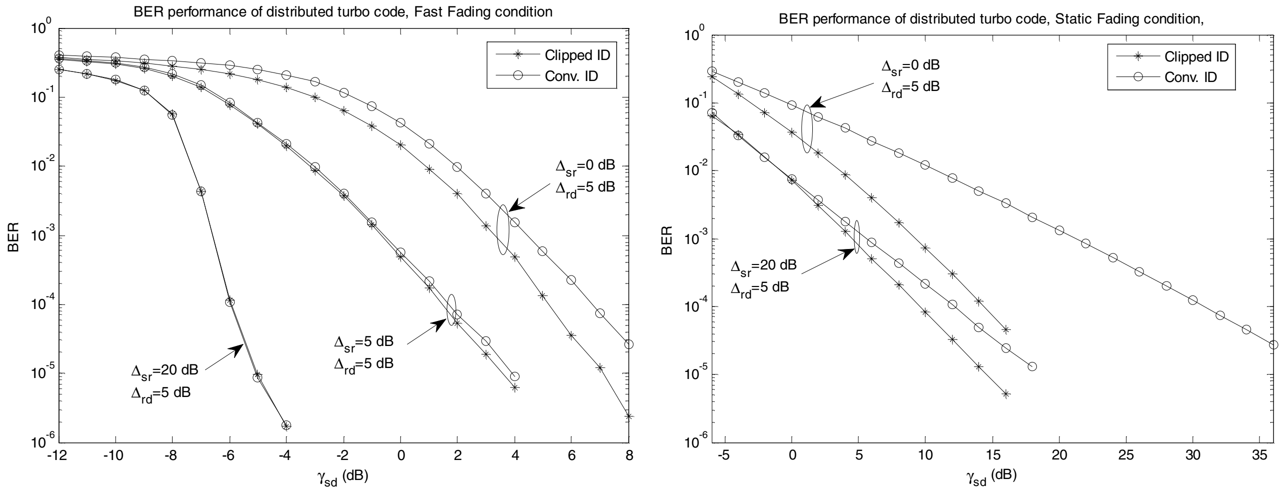
$L_q=\log(1-q)/q$

The parameter L_q is in fact indicating the reliability of the data hold at the relay node and thus is called relay data reliability. Message propagation from $\hat{u}_{1,k}^{R}$ to $u_{1,k}$ will lead to the same LLR update. It should be noted that the above LLR update is based on an input-output relationship defined by a BSC and therefore is the same as derived in [5] and [8]. There the post-BSC model is used to update the LLR values of the indirectly received signal before combining. The same message update but with properly chosen q can be used for both pre and post-BSC models presented in Figure 1 and Figure 3. The difference is that for post-BSC the LLR values extracted from the output of channel 2 only needs to be updated once and then used throughout the whole iterations, while for pre-BSC model the update should be exercised for every half-iteration.

Figure 2 depicts the block diagram of a new iterative decoder (ID) based on pre-BSC model for a DTC with two constituent recursive systematic convolutional codes (RSCC). This ID applies BSC LLR update on the exchanged extrinsic information between the two component decoders and as the LLR update is approximately performing clipping hence we call the new decoder as clipped ID. In this figure the parity bit LLRs received from channels 1 and 2 are denoted by $\underline{\lambda}_{p1}$ and $\underline{\lambda}_{p2}$, respectively. The systematic bit LLRs received from channel 2 is first BSC LLR updated and then after being de-interleaved are added to systematic bit LLRs of channel 1. The result is denoted by $\underline{\lambda}_s$. Similar procedure is performed to generate the second code systematic input LLRs denoted by $\tilde{\underline{\lambda}}_s$. Each component SISO decoder accepts two inputs: one for systematic bits and one for parity bits, however soft output is only required for systematic bits.

A. Relay Data Reliability Estimation and Signaling

The BSC LLR update suggested by (2) requires knowledge of q or the relay data reliability L_q at the destination node receiver. The L_q itself depends on the exploited error correction coding used over channel 0 as well as the average and instantaneous



(a) (b)

Figure 4, BER performance of the considered cooperative distributed turbo code with conventional and proposed clipped iterative decoders. Results are shown for different SNR offsets Δ_{sr} and Δ_{rd}=5 dB under (a) fast fading, and (b) static fading conditions.

conditions of this channel. As suggested in [5] this parameter can be estimated at the relay node and then piggy forwarded to the destination node every frame or every multiple frames depending on the time varying condition of the channel 0. The estimation will require using of a SISO decoder at the relay node and then q can be estimated by assuming a Gaussian distribution for the decoded soft values. Using the Gaussian model each LLR value λ_k for $k=1,\dots,K$ is approximated by a BPSK modulated signal plus an additive white Gaussian noise:

$$\lambda_k = a x_k + e_k \text{, for } k=1,\dots,K \tag{3}$$

where a is the signal gain, $x_k \in \{\pm 1\}$ is BPSK signal, and e_k is the real-valued, zero-mean stationary Gaussian noise with variance σ_e^2. Based on this model, the SNR of the LLR values will be $\gamma_\lambda = a^2/\sigma_e^2$. The SNR γ_λ can be estimated from the first and second time series moments of the absolute values of λ_k as used in [10]:

$$\gamma_\lambda \approx \hat{\gamma}_\lambda = f_Z^{-1}\left(\sum_{k=1}^{K}\lambda_k^2 \Big/ \sum_{k=1}^{K}|\lambda_k|\right)\text{, where} \tag{4}$$

$$f_Z(\gamma) = \frac{1+\gamma}{\left(\sqrt{\gamma}\,erf\left(\sqrt{\gamma}\right) + e^{-\gamma}/\sqrt{\pi}\right)^2},$$

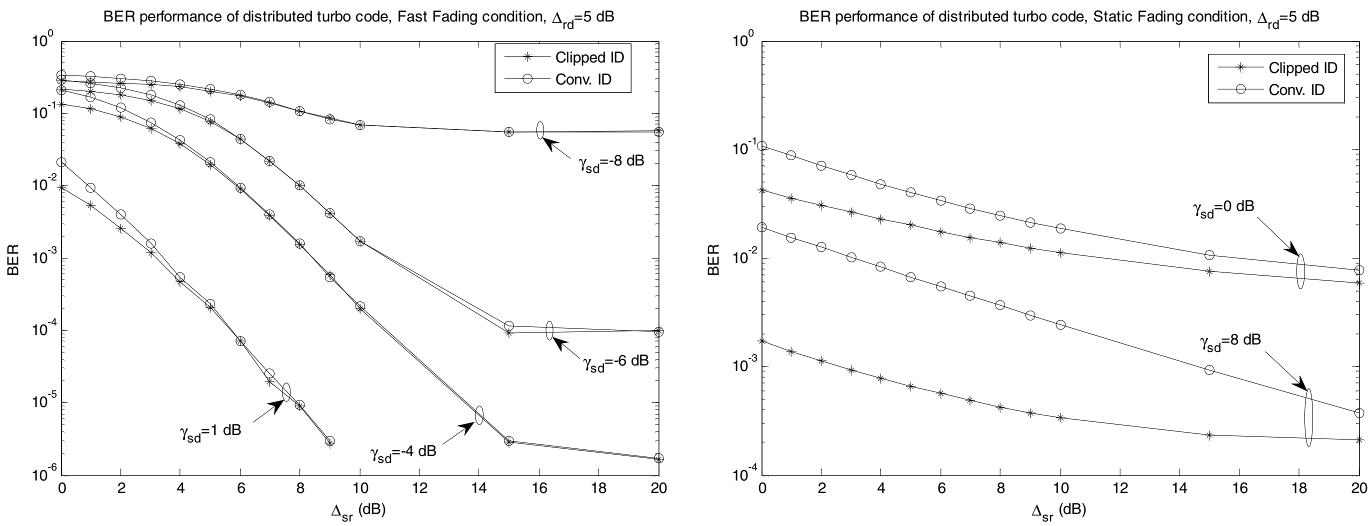
$erf(x) = 2/\sqrt{\pi}\int_0^x e^{-t^2}dt$ is the erf function, and f_Z^{-1} is the inverse function of f_Z. $f_Z(\gamma)$ is monotonically decreasing function, and therefore its inverse can be numerically calculated and stored in a look up table (LUT). Now based on the model of (3) and the estimated SNR $\hat{\gamma}_\lambda$, the BER q and its corresponding reliability value L_q can be estimated as follows:

$$q \approx \hat{q} = Q\left(\sqrt{2\gamma_\lambda}\right)\text{, and. } L_q \approx \hat{L}_q = \log\left((1-\hat{q})/\hat{q}\right), \tag{5}$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}dt$ is the Q function.

V. Numerical and Simulation Results

Here we provide some simulation results to demonstrate the effectiveness of the proposed iterative decoder. The constituent channels' SNRs are adjusted relative to the channel 1 (S-D) SNR: $\gamma_0=\Delta_{sr}\cdot\gamma_1$, $\gamma_2=\Delta_{rd}\cdot\gamma_1$, where Δ_{sr} and Δ_{rd} represent SNR offsets of the S-R and R-D channels. All the nodes are single antenna. An identical 1/2 rate 4-state systematic recursive convolutional code with octal generator (7,5) is used for both constituent codes used at S and R. Information block size is 598 bits and both codes are terminated with 2 zero tail bits that results in a coded block size of 1200 bits for both phases of transmissions. A pseudo random interleave pattern is generated once and used for the distributed turbo code internal interleaver throughout the whole simulation. BPSK modulation is used for both S and R. Fast and static fading conditions are assumed where in fast fading case all the constituent channels are assumed to be frequency non-selective Rayleigh fading with independent and identically distributed (iid) fading coefficients for each channel and each time index. In the static fading case the three channels are still assumed to be independent and fading coefficients are generated independently from frame to frame but kept fixed during each frame. Figure 4 shows the simulated BER performance for both the conventional iterative decoder, and the modified clipped ID. The BER performance is depicted versus direct link SNR γ_{sd} with different values of Δ_{sr} and Δ_{rd}=5 dB as parameters. As it is observed in fast fading condition (Figure 4a) only when Δ_{sr} is low the clipped ID provides gain otherwise its performance is identical with the conventional ID. For example for Δ_{sr}=0 dB clipped ID brings about 1.7 dB gain at 10^{-5} BER. For static fading case (Figure 4b) situation is totally different and clipped ID has much more impact on the performance. It is able to recover the diversity that the



(a) (b)

Figure 5, BER performance sensitivity to source-to-relay reliability of the considered cooperative distributed turbo code with conventional and proposed clipped iterative decoders. Results are shown for different SNR conditions γ_{sd} and Δ_{rd}=5 dB under (a) fast fading, and (b) static fading conditions.

original DF scheme fails to obtain. In this case gains are much higher and are around 16 and 5 dB at 10^{-5} BER for Δ_{sr}=0 and 20 dB, respectively. The intuition behind this achievement is that in static fading conditions when the S-R link is in outage the clipping level used in clipped ID becomes very small and effectively the two component SISO decoders are decoupled. This avoids the erroneous forwarded data contaminate the directly received data.

As the main shortcoming of a DTC is its sensitivity to relay decoder errors, a sensitivity analysis is conducted by simulating the system BER for different values of Δ_{sr} ranging from 0 to 20 dB while the direct link SNR (γ_{sd}) and Δ_{rd} are kept fixed. Figure 5 shows the obtained results. In fast fading case similar to the previous results at medium to large Δ_{sr} the clipped and conventional decoders' performance are identical. However at lower values of Δ_{sr} the clipped ID becomes beneficial and is more robust to S-R link errors. In static fading condition the advantage gained by the clipped ID is eye-catching. The system is awarded with much higher level of robustness such that for achieving a same level of BER the clipped ID requires much less Δ_{sr}. For example to obtain 4×10^{-4} BER with γ_{sd}=8 dB the clipped ID needs 11 dB less Δ_{sr}. Simulations are also carried out to check the effectiveness of the post-BSC model but the obtained results were not as promising as the clipped ID derived from pre-BSC model.

VI. Conclusion

In this paper an effective model is proposed to capture the relay decoding error of a distributed turbo code. The model is placing a memoryless binary symmetric channel before the relay encoder and thus is called pre-BSC model. Accordingly the required modifications on iterative decoding are derived from this model. The new decoder attains significant level of robustness to erroneous source-to-relay link especially under static fading condition where each link of a cooperative communication can easily undergoes an outage condition. The derived decoder is generic and can be used in conjunction with any combination of modulations to be used at the source and relay nodes.

VII. Acknowledgment

This work was performed in ROCKET project, funded by the European Commission Framework Program (FP7).

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