MATHEMATICAL AND COMPUTER MODELS IN ESTIMATION OF DYNAMIC PROCESSES OF VEHICLES

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ABSTRACT

A great role in the evaluation of questions of the interaction of the rolling stock and the path, along with full-scale experiment and laboratory research, belongs to the methods of mathematical modeling. The proposed method allows us to consider the behavior of the rolling stock without large expenditures, in a wide range of dynamic processes.

Keywords: *Experiment, Mathematical Model, Rolling stock, Running Gear, Speed*

1. INTRODUCTION

In recent years, the dynamics of machines is actively developing in a number of areas that are associated with the extensive use of automatic control theory methods [1]. This is reflected in solving theoretical and practical issues in robotics mechatronics, vibration diagnostics, developing methods and means of ensuring the reliability and safety of operating complex technical objects [2-6]. Considerable attention is paid to protecting machines, equipment and equipment from vibration external influences, which stimulates the development of scientific and applied research in the field of system analysis and the development of dynamic synthesis methods in systems with a complex structure and variety of dynamic interactions between elements of different physical nature [7].

Development of computer technology has created the conditions for controlling the dynamic state using mechatronic devices or active control devices that combine the ability to quickly process information from sensor systems and their implementation by servo drives [8].

In this regard, it would be possible to note the growing interest in the formation of mathematical models of management processes for the state of technical objects, taking into account detailed ideas about the properties of systems and the conditions of their functioning, the influence of features of the structural-technical forms and requirements for dynamic quality. Traditional approaches to solving

these problems are based on the implementation of a rather complex system of interconnected stages of studying and constructing design schemes of objects that reflect their characteristic properties and features; building mathematical models of various forms: from systems of differential equations to structural diagrams and chains of system analogs; development of methods and means for solving various problems of estimating dynamic properties, appropriate selection of parameters, etc. [9-12]

The construction of mathematical models for technical objects is partially connected to the consideration of various mechanical oscillatory systems [13]. In this regard, of great interest are the issues of adequate mapping of mathematical features of the structure, the nature of relationships between elements, the formation of a set of typical elements and methods of their connection in mathematical models. Engineering practice focuses on the traditionally established ideas about the elements of technical objects in the form of parts and assemblies consisting of solid bodies of a certain shape, connecting devices (springs, dampers, shock absorbers, dampers, etc.) providing a certain kind of relative movements [14].

Previous studies on the evaluation of the accumulation of residual deformations of the railway track show that the main reason for them is the vibrations arising from the passage of the rolling stock. At present, in the study of the dynamics of railway rolling stock, methods of mathematical modeling are widely used. They make it possible to

give a comprehensive assessment of the effect of a multitude of parameters over a wide range of changes in their values with insignificant outlays of resources and time.

2. RELATED WORKS

In solving the problems of the dynamics of machines, methods of the theory of chains are found, based on the characteristics of the construction of chain structures characteristic of many technical objects of mechanical, electrical and electromechanical nature. Many problems of the dynamics of machines are solved in an interdisciplinary space, formed by various approaches and methods of theoretical mechanics, the theory of mechanisms and machines, the theory of oscillations and applied system analysis. Development of methodological foundations for solving problems of research, design, and calculation of modern machines is reflected in the works [15-19]; focused on the analytical apparatus of system analysis, control theory. Various applications of theoretical developments are associated with the dynamics of robotic systems [20], the tasks of the dynamics of various mechanisms and machines [21-23], the creation of technology for calculating and ensuring the reliability of technical objects in conditions of vibration effects [24-27].

The peculiarity of the work of many machines and mechanisms in comparison with control objects in the theory of automatic control is the actual coincidence of physical and informational representations [28]. This is due to the fact that at a certain level of consideration mechanical systems are perceived at the level of transformation of specific force or kinematic parameters of the dynamic state, which is quite understandable and follows from the accepted form of perception, defined by the laws of mechanics. At the same time, external power and kinematic parameters can be perceived as input and output signals, under the assumption that any technical object can be considered as some kind of converter [29]. Such a converter provides a certain connection between the input and output signal. Taking into account the parameters of such ratios, one can quite definitely assess the dynamic state of the machine, its components, and parts. Such an approach is quite legitimate but requires its own forms, methods, and methods for the adequate presentation of mathematical models [30–32].

Mechanical oscillatory systems, consisting of mass-inertial, elastic (springs) and dissipative (dampers) links, in many problems of dynamics are considered as design diagrams. Mathematical models in the approaches, if we bear in mind the preliminary stages of the study, are usually reduced to systems of ordinary linear differential equations with constant coefficients. With respect to such systems, a sufficiently detailed methodological base has been developed, as reflected, for example, in [33-36]. A number of sections of the theory of oscillations found application in the theory of circuits, in particular, when using electromechanical analogies to study the general properties of mechanical and electrical systems [36-40]. The theory of mechanical circuits is developed in close cooperation with the theory of electrical circuits, which is predetermined by the notions of elementary type units [41-42]. The development of concepts about the properties of elements and devices of various mechanisms and machines in those aspects that are associated with the analysis of the integration possibilities of elastic-dissipative mass-inertial properties in real constructive forms were considered in the first section. Note that, as in electrical circuits, mechanical elements are only mathematical abstractions. Real physical elements usually have several properties simultaneously attributed to these idealized elements. Bodies with mass usually also have elasticity, and springs usually have mass. Both in those and in other bodies, internal energy losses occur, which are characteristic of elements with viscous friction and cause damping of oscillations. Finally, the damping elements are also not without masses. The whole question is only in the relative significance of these minor properties of physical elements. The question of the appropriateness of the inclusion in the study of those or other secondary properties of the elements should be resolved independently in each specific case.

The natural development of such representations is the possibility of constructing certain technologies of mathematical models, which, as mentioned, lead to systems of linear ordinary differential equations with constant coefficients [43-44]. The solution of such equations in applications to the theory of mechanical circuits often relies on the use of operational calculus, which gives results in a convenient form for both the theory of mechanical circuits and the theory of automatic control, where operator methods are widely used [45-47]. The fundamentals of operational calculus as applied to chain structures are described in [48-52].

Managed mechanical systems in their various manifestations, connected, in particular, with robotics, mechatronics, active protection against noise and vibrations, can be attributed to such complex objects for which the construction of devices for controlling the state (or motion) requires taking into account many features of the task. This requires an assessment not only of the specifics of the movement of inertial objects but also of the processes of realization of forces, taking into account the reactions of elements to influences, as well as the use of certain technologies for collecting and processing information [53]. In such a multi-sided integration, tasks are solved when building walking machines, active vibration-protective systems, modern robotic, and flexible production complexes. If the problems of external forces, kinematic effects, distribution of displacement elements, velocities and forces are related to the problems of mechanics, in its various applications, then the passage of signals and their transformations relate mostly to the theory of communication and control [54]. The latter relates to the issues of information processing, although many aspects are common for mechanical systems, which in the abstracted form consist of a limited number of typical elements or links [55]. Indicative in this regard are electrical circuits that have a close relationship with mechanical circuits based on electromechanical analogies [56-58].

Thus, at a certain stage of assessing the dynamic properties of technical objects, it is quite reasonable to use system approaches based on generalized ideas about the interactions of a mechanical system with the external environment, the elements between themselves and the conditions for the passage of signals through a dynamic system.

3. PROBLEM STATEMENT. CALCULATION SCHEMES OF VERTICAL OSCILLATIONS OF THE LOCOMOTIVE

In the study of dynamic processes characteristic of technical objects, the selection and construction of mathematical models are of great importance. At the first stages, the design scheme is formed as a result of abstraction from irrelevant concreteness and the transition to the definition of possible relationships between elements. The next step in detailing ideas about the processes in

selecting and specifying a model is taking into account the possibilities of compact description, convenient use of source data, facilitating the receipt of necessary information, etc. Models of continuous processes for which the creation of sequences can be considered. In this case, the domain of definition forms a discrete set [59].

There are different approaches and, at the same time, various mathematical models of the process of spatial oscillations of rolling stock.

Consider the calculated scheme of vertical oscillations of the locomotive, not connected with the train. The features of various design schemes that can be used to study the oscillations of the supers orbed structure are presented in table 1 [60].

The calculation schemes are conditionally divided into six levels (I-VI), possible idealizations of which are described below. Indirect studies, these schemes are assembled from a set of concrete realizations of the reduced levels.

3.1 Level 1

One of the tasks of optimal PS design is reliable vibration protection of people (service personnel and passengers) and equipment. For this purpose, amortization of cabins and power equipment of locomotives is widely used, and vibration isolation seats in special wagons. The use of idealizations presented at level 1 has little effect on the overall picture of PS variations.

Scheme 1.1 is used in the calculation of oscillations of locomotives, especially for the detection of the spectrum of forces in the locations of installation of vibration dampers:
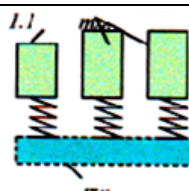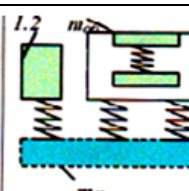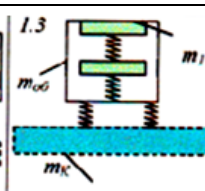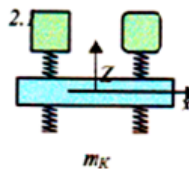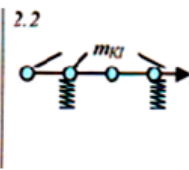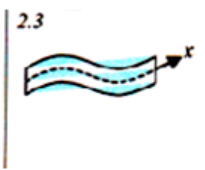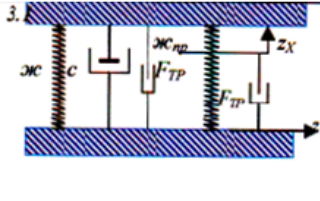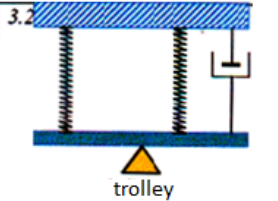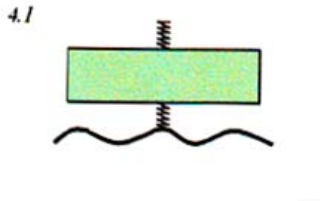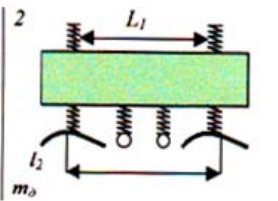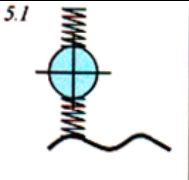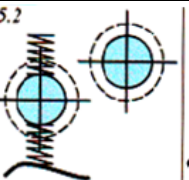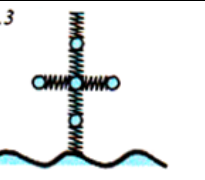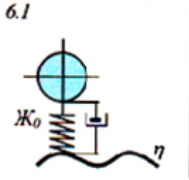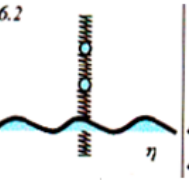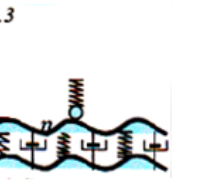
$$\frac{m_{tm}}{m_{bm}} << 1 \qquad (1)$$

Where m_{tm} is total mass of the equipment, m_k is the body weight.

Scheme 1.2 is widely used in the automotive industry. It makes it possible to take into account the frequency characteristics of the human body. In these cases, there is no need to apply a "physiological filter" frequencies. For the objective function, the energy supplied to the mass can be accepted m1 – head of a person.

Scheme 1.3 is used when it is necessary to take into account the elasticity of the cabin walls for a more accurate estimation of the spectrum of sound frequencies. In transport mechanics, she has not yet applied the application.

Table 1: Calculation scheme of vertical oscillations of the locomotive

N		
1	Cushioning body component	
2	Body	
3	Connection of the body with trolleys	
4	Trolleys	
5	Pair of wheels	
6	Way	

3.2 Level 2

Scheme 2.1 was most widely used in solving problems of analyzing and synthesizing spring suspension. It well reflects the qualitative characteristics of the PS, especially locomotives, in the low-frequency region of the perturbation spectrum. In order to take into account the specifics of the design schemes of wagons with liquid cargo, we can consider the value of m1 as a variable.

Scheme 2.2 more accurately reflects the real properties of the system for medium- and high-frequency disturbances. This scheme is mandatory in the analysis of the stressed state of bodies, as well as the dynamics of high-speed cars. For the analysis of natural frequencies, schemes without energy dissipation are widely used. Internal inelastic resistances are described according to the hypothesis of E.S. Sorokin or Voigt. The first hypothesis more accurately reflects the physics of the process. The analytical determination of the matrices of the coefficients of inelastic resistances is currently difficult. For the bodies already built, it is necessary to identify their elastic-dissipative characteristics, especially since the information is available on the nonlinearity of dissipation characteristics of similar systems. The discrete

masses mKi can depend on the time when modeling vertical oscillations of a special substation (for example, tanks with liquid cargo).

Scheme 2.3 is used to calculate the vertical oscillations of the supers orbed structure. Inelastic resistance is described by the hypothesis of E.S. Sorokin or Voigt. In principle, calculations under this scheme can provide greater reliability in comparison with the calculations of scheme 2.2, however, in our opinion, its use complicates the algorithm of the computing process for a computer.

3.3 Level 3

Schemes 3.1 and 5.1 are fairly general idealizations of the elements of spring suspension. The strength in these elements is determined by an expression

$$F = R_{pr} \delta' + R\delta$$

$$+ \psi R [\Delta_{ST} - \delta] sign \, \dot{\delta} + c \, \dot{\delta}^{\beta} \qquad (2)$$

Here, R is the rigidity of the elements indicated in the diagram

$\delta = z - z_0$ – current deflection of spring suspension elements

$\delta' = z_x - z_0$ – current deflection of an elastic element with R_{pr} rigidity

Δ_{cm} - static deflection of the suspension
c is the coefficient of proportionality

The last three components of formula (2) describe the friction in the system: Coulomb with the R_{pr} force modulus; Coulomb with a modulus of force proportional to the deflection of an elastic element having stiffness R and a coefficient of relative friction ψ, viscous friction. In the general case, the values of x, c, R_{pr} of the functional dependence on δ.

The presence in the elastic element with a rigidity of the Coulomb friction R_{pr} leads to the necessity of introducing an additional degree of freedom. The kinematic control for the additional generalized coordinate z_x has the form

$$\delta' R_{pr}$$

$$= R(\delta - \delta') + R'_{pr} \, sign(\dot{\delta} - \delta') \qquad (3)$$

Equation (3) to subject to formal transformations, it is more convenient to present in the following form:

$$\delta' R_{pr}$$

$$= -\delta' (R_{pr} - R_p) + F'_{tp} \, sign(\dot{\delta} - \delta') \qquad (4)$$

Graphically, the dependence of the force on the element with stiffness Z_x on the force in the element with stiffness R_{pr} according to equation (4) is a nonlinear function of the type of backlash.

Graphically, the dependence of the force on the element with stiffness xp on the force in the element with stiffness xpl according to equation (4) is a nonlinear function of the type of backlash.

Schemes 3.2 and 5.2 are characteristic for spring suspension having balancers. Studies show that taking inertial masses into account is particularly important in determining the coefficients of dynamics of spring suspension of locomotives.

3.4 Level 4

Scheme 4.1 is most common in studies of vertical oscillations of the supers orbed structure. For non-self-propelled wagons and a number of locomotives, hardly any further complication is required.

Scheme 4.2 is designed to account for the suspension of traction motor mD locomotives and self-propelled cars with a two-stage suspension. As a result of the research, it was found that neglecting the traction drive can lead to significant errors in a certain frequency range.

3.5 Level 5

Scheme 5.1. most common in studies of oscillations of the supers orbed structure of a non-self-moving PS. In this case, the mass of the wheel pair is given the path mass, which in general is a variable quantity.

Scheme 5.2, like the scheme described above, is typical for a self-propelled PS.

Scheme 5.3 is applicable for the detailing of the elements of a wheel pair and is necessary mainly for the analysis of loads on these elements.

3.6 Level 6

According to scheme 5.1, the path is considered as an elastic-dissipative input, the perturbation on which is formed as a result of a change in time in the coordinate η or the stiffness parameters z_0. In a number of cases, when analyzing the oscillations of the supers orbed structure, it can be assumed without special errors that perturbations are applied directly to the wheel. The scheme is greatly

simplified, especially when calculating the multi-axis PS. When calculating according to this scheme, you can abandon the time-consuming, and sometimes, if $\eta(t)$ – discontinuous function, and impossible work by definition $d\eta/dt$, if accept damping in transit-dependent on the absolute speed of wheel movement. In addition, in the high-frequency part of the perturbation spectrum, the effect of path oscillations is taken into account. The elastic-dissipative parameters of circuit 5.2, as well as the weight of the path given to the wheel, can be taken as variables, depending on the nature and magnitude of the load in the path.

According to the scheme 5.2, the path is detailed, considering it as a set of discrete masses, which makes it possible to abandon some of the assumptions of scheme 5.1, for example, the stiffness constant, the quenching coefficients, and the reduced path mass. Such detailing is only necessary for path computations and is less significant in analyzing the dynamics of the FS superstructure.

Scheme 6.1 becomes more and more widely used in the analysis of the interaction between the SS and the path. According to this scheme, the railway is viewed as a continuum. The main advantage of the scheme is the possibility of taking into account the influence of oscillations of neighboring wheelsets.

Thus, depending on the purpose of analyzing the PS as an oscillating system or optimizing certain design parameters, its design schemes (mathematical models of the process) can be modified in a wide range. Note that an unjustified complication of the design scheme increases not only the difficulties of its analysis and synthesis but also the probability of occurrence of increased errors in numerical methods of analysis.

4. MATHEMATICAL MODEL OF VERTICAL OSCILLATIONS OF A SERIAL LOCOMOTIVE VL60

The study of the vertical dynamics of locomotives poses, basically, two problems - the analysis of the influence of parameters on the dynamic quality and the evaluation of the stability of motion.

For theoretical studies of locomotive spatial oscillations, we use a mechanical system similar to the VL60 electric locomotive. The calculation scheme is shown in Figure 1.
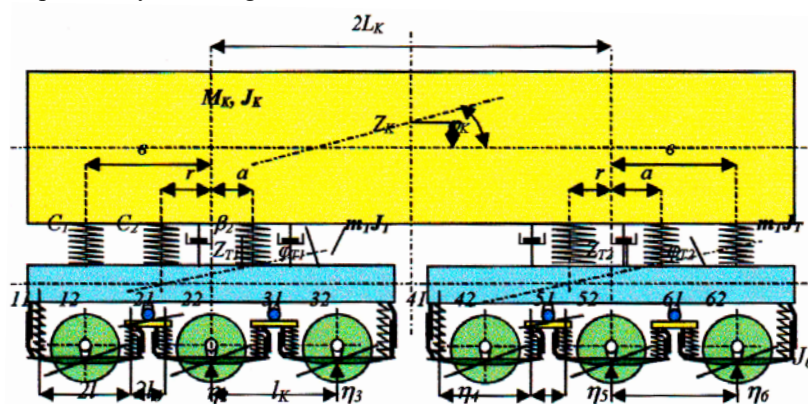
Figure 1: Calculation Scheme

The following symbols are used in the scheme:
M_K – body weight;
M_T – the weight of the curb parts of the trolley;
J_K, J_T – moments of inertia of the body and the trolley relative to the horizontal transverse central axes respectively;
J_B – a moment of inertia of the rocker relative to the axis of rotation;
C_I – stiffness of the side support of the central suspension;

β_2 – coefficient of inelastic resistance of rubber elements of the central suspension;
l_K – the distance between axes of wheel sets;
d – displacement of the center of gravity of the curb parts of the trolley;
$2l$ – the base of the rocker;
$2L_K$ – the base of the body of an electric locomotive;
a,b,r – distance from the elastic elements of the central suspension to the transverse plane of symmetry of the trolley;

Z_K – vertical translational movement of the center of gravity of the body;

$\varphi_к$ – the angle of rotation of the body with respect to the transverse horizontal axis passing through the center of gravity;

Z_{T1}, Z_{T2} – vertical translational movements of the center of gravity of the sprung buggy parts;

φ_{T1}, φ_{T2} – angles of rotation of the clamped parts of bogies relative to the horizontal transverse axes passing through the centers of gravity;

$\varphi_{\delta i}$ – angles of turns of balancers ($i = 1,2,3, ….6$);

η_i – geometric unevenness.

The investigated electric locomotive VL60 is a spatial system with many degrees of freedom, the number of which is determined by the design of the crew and the nature of the imposed links.

It is generally accepted to decompose complex dynamic processes into simpler ones [61-64]. For example, the electric locomotive oscillations are considered separately along the planes. Introduced simplifications are determined by the objectives of the study and depend on what features of the behavior of the system we are interested in.

As can be seen from figure 1 [65], the accepted design scheme has 12 degrees of freedom under the following assumptions:

- the inertial characteristics of the elastic elements are negligibly small in comparison with the inertial characteristics of the most important parts of the crew and are therefore not taken into account;

- The main bearing elements of the crew under study (body, the frame of trolleys, wheel pairs) are represented by absolutely rigid bodies;

- the stiffness of rubber cones of central supports constant;

- The locomotive moves at a constant speed along a straight section of the track;

- The horizontal stiffness and the coefficient of inelastic resistance of the path do not depend on the magnitude and rate of its deformation;

- The wear of the shroud and rail is not taken into account;

- it is considered that the wheel does not detach from the rail, i.e. is considered without impact movement at the butt jagged surfaces.

The features of the presented design scheme are:

- the presence of elastic bonds: vertical - in the pedestrian suspension stage and horizontal - between the body and the trolleys;

- allowance for the elastically dissipative properties of the path as a function of the magnitude and rate of its deformation;

- non-equal-elasticity along the length of the rail link;

- the introduction of gaps in the pedestrian knot, as well as between the crest of the shroud and the head of the rail in order to account for the free run-up of wheelsets.

The construction of mathematical models describing the fluctuations of the railway crews is carried out in two stages. On the first, a fairly simple and adequate structure of the model is chosen, and the second is the estimation of the parameters of the latter [66-69].

When choosing the structure of the mathematical model of the crew under study, it is necessary to fulfill two contradictory requirements: on the one hand, it is as accurate as possible to describe the qualitative features of the crew structure and the physical aspects of its oscillations, which leads to obtaining complex and nonlinear mathematical models, and on the other, to obtain a model that allows , at least, hardware implementation, which makes it necessary to build fairly simple mathematical models.

When drawing up the schemes it is taken into account that each wheel pair undergoes different effects received from the side of the path at the considered moment of time (Figure 2).

The body of the locomotive is symmetrical with respect to the longitudinal and transverse planes. The oscillations of bouncing and galloping occur separately. The centers of gravity of the carriages are offset relative to the transverse plane of symmetry. Because of this, the oscillations of bouncing and galloping of the trolleys are interconnected.

The problem of choosing the structure of the mathematical model of vertical oscillations of a locomotive is solved by methods of classical Newtonian mechanics [70] with the help of the Lagrange control of the second kind:

$$\frac{d\left(\dfrac{\partial T}{\partial \dot{r}_s}\right)}{dt} - \frac{dt}{\partial r_s} + \frac{\partial U}{\partial r_s} + \frac{\partial \phi}{\partial \dot{r}_s} = Q_S \qquad (5)$$

where   T- kinetic energy of the system at the considered moment;

U – potential energy at the same time;

$r_s, \dot{r}_s, Q_s$ - generalized coordinates, speed, force;

Φ – scattering function;

S – a number of degrees of freedom (S=12 for the locomotive calculation scheme on pneumatic suspension).

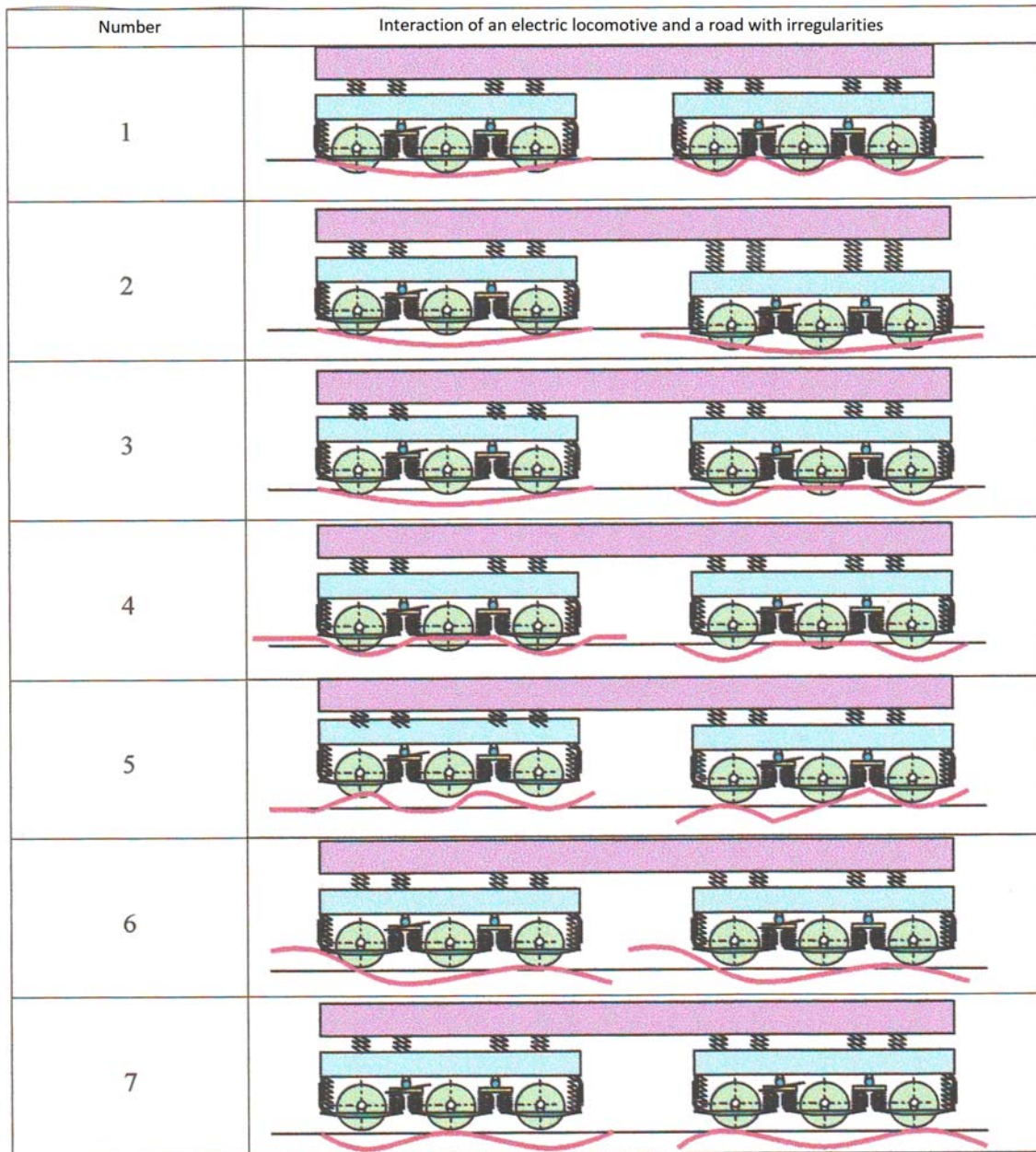| Number	Interaction of an electric locomotive and a road with irregularities
1	
2	
3	
4	
5	
6	
7	

Figure 2: Example of an image with the acceptable resolution

The expression of the kinetic energy of oscillations of an electric locomotive according to Koenig's theorem will be written in the form [71-72]:

$$T = \frac{1}{2}\left[M_K \dot{Z}_K^2 + J_K \dot{\varphi}_K^2 + m_T\left(\dot{Z}_{T1}^2 + \dot{Z}_{T2}^2\right) + J_T\left(\dot{\varphi}_{T1}^2 + \dot{\varphi}_{T2}^2\right)\right] + J_\delta \sum_{i=1}^{6} \dot{\varphi}_{\delta i}^2 \quad (6)$$

The potential energy of the system:

$$U = \frac{1}{2}\left[C_2 \delta'^2_{Ki} + C_2 \delta^2_{Ki}\right]$$
$$+ \sum_{i=1}^{6}\sum_{K=1}^{2}\int_0^{\Delta iK} Q_{iK}\, d\Delta_{iK} \tag{7}$$

The scattering function has the form:

$$\phi = \frac{1}{2}\beta_2 \sum_{i=1}^{4}\dot\delta_{Ki} + \sum_{i=1}^{6}\sum_{K=1}^{2}\int_0^{\Delta iK} F_{iK}\, d_{\Delta iK} \tag{8}$$

The mathematical model of vertical oscillations of an electric locomotive VL60 on a metal suspension is as follows:

$$M_k Z_k + \sum_{i=1}^{4}\left[C_1\delta_i + C_2\delta_i + \beta_2\,\dot\delta_{yi}\right] = 0$$

$$_k\ddot\varphi_k + \sum_{i=1}^{4}\left[C_1 D_i\delta_i + C_2 E_i\delta_i + \beta_2\,\dot\delta_{yi}\right] = 0$$

$$m_T \ddot Z_{T1} - \sum_{i=1}^{4}\left[C_1\delta_i + C_2\delta_i + \beta_2\,\dot\delta_{yi}\right] - \sum_{k=1}^{2}\sum_{i=1}^{3}\left(\theta_{ik}+F_{ik}\right) = 0$$

$$_T\ddot\varphi_{T1} + \sum_{i=1}^{2}\left[B_i C_1\delta_i + k_i C_2\delta_{yi} + k_i\beta_2\,\dot\delta_{yi}\right] + \sum_{k=1}^{2}\sum_{i=1}^{3}\left[N_{ik}\left(\theta_{ik}+F_{ik}\right)\right] = 0$$

$$m_T \ddot Z_{T2} - \sum_{i=1}^{4}\left[C_1\delta_i + C_2\delta_i + \beta_2\,\dot\delta_{yi}\right] - \sum_{k=1}^{2}\sum_{i=1}^{3}\left(\theta_{ik}+F_{ik}\right) = 0$$

$$J_T\ddot\varphi_{T2} + \sum_{i=1}^{2}\left[B_i C_1\delta_i + k_i C_2\delta_{yi} + k_i\beta_2\,\dot\delta_{yi}\right] + \sum_{k=1}^{2}\sum_{i=4}^{6}\left[N_{ik}\left(\theta_{ik}+F_{ik}\right)\right] = 0$$

$$J_\delta\varphi_{\delta j} + \sum_{i=1}^{2}\left[G_{ik}\left(\theta_{ik}+F_i\right)\right] = 0, \quad J = 1,\dots,6 \tag{9}$$

$$J\ddot\varphi_j + H_{J2}\left(\theta_{ik}+F_{ik}\right) + H_{J=1,1}\left(\theta_{J+1,1}+F_{J+1,1}\right) = 0, \quad J = 1,\dots,5$$

$$H_{11} = H_{32} = H_{41} = H_{62} = 0$$

Here, $H_{12} = H_{22} = H_{42} = H_{52} = l_\delta$

$$H_{21} = H_{31} = H_{51} = H_{61} = -l_\delta$$

$2l_\delta$ – base of rocker

φ_J – angles of swivel movements (J = 1,..., 5)

Elastic and dissipative forces in the spring suspension stage

$$\theta_{ik} = C_p\Delta_{ik} = F_{Tp}\,sign\left(\dot\Delta_{ik}\right) \tag{10}$$

Deflections of springs are determined by the formula

$$\Delta_{ik} = \eta_i - z_{T1} + G_{ik}\varphi_{\delta1} + N_{ik} + H_{ik}\varphi_J,$$
$$\left(i = 1,2,3; k = 1,2\right)\begin{cases}J = 1\, for\, i, k = 1,2,\cdot2,1\\ J = 2\, for\, i, k = 2,2,\cdot3,1\end{cases}$$
$$\Delta_{ik} = \eta_i - z_{T2} + G_{ik}\varphi_{\delta1} + N_{ik} + H_{ik}\varphi_J, \tag{11}$$
$$\left(i = 4,5,6; k = 1,2\right)\begin{cases}J = 4\, for\, i, k = 4,2,\cdot5,1\\ J = 5\, for\, i, k = 5,2,\cdot6,1\end{cases}$$

The impact on the wheel was set in the form of an unevenness

$$\eta_i = A\sin(wt - \gamma_i) \tag{12}$$

Here i is the number of the wheel pair
A is the amplitude of the unevenness, cm

Initial phases characterizing the lag of the vertical displacements γ_i - of the second, third, etc. wheel pairs in relation to the first in the direction of movement of the locomotive

$$\gamma_i = \frac{2\pi d_i}{l_H} \tag{13}$$

where l_i is the distance from the axis of the first wheel pair to the axis of the sixth wheel pair;

l_H - length of unevenness

We rewrite the systems of differential equations [10] in expanded form. For this, instead of δi, we substitute expression

$$\left. \begin{array}{l} \delta_i = z_k - z_{T1} + D_i \varphi_k + B_i \varphi_{T1} \quad i=1,2 \\ \delta_i = z_k - z_{T2} + D_i \varphi_k + B_i \varphi_{T2} \quad i=3,4 \end{array} \right\} (14)$$

Then the first equation of the system is transformed into the form

$$M_k \ddot{z}_k$$
$$= \sum_{i=1}^{2} C_1 (z_k - z_{T1} + D_i \varphi_k + B_i \varphi_{T1})$$
$$+ \sum_{i=1}^{2} C_1 (z_k - z_{T2} + D_i \varphi_k + B_i \varphi_{T2})$$
$$+ \sum_{i=1}^{2} C_2 (z_k - z_{T1} + D_i \varphi_k + k_i \varphi_{T1}) \ (15)$$
$$+ \sum_{i=1}^{2} C_2 (z_k - z_{T2} + D_i \varphi_k + k_i \varphi_{T2})$$
$$+ \sum_{i=1}^{4} \beta_2 (\dot{z}_k - \dot{z}_{T1} + D_i \dot{\varphi}_k + B_i \dot{\varphi}_{T1})$$
$$+ \sum_{i31}^{2} C_2 (\dot{z}_k - \dot{z}_{T2} + E_i \dot{\varphi}_k + k_i \dot{\varphi}_{T2})$$

Or

$$M_k \ddot{z}_k + (4C_1 + 4C_2) z_k$$
$$- 2(C_1 + C_2) z_{T2} + C_1 \sum_{t=1}^{4} D_i \varphi_k$$
$$+ \left(C_1 \sum_{i=1}^{2} B_i + C_2 \sum_{i=1}^{2} E_i \right) \varphi_{T1}$$
$$+ \left(C_1 \sum_{i=1}^{3} E_i \right) \varphi_{T2} - 2\beta_2 \dot{z}_{T1} - 2\beta_2 \dot{z}_{T2}$$
$$+ \beta_2 \sum_{i=1}^{4} E_i \dot{\varphi}_k + \beta_2 (k_1 + k_2) \dot{\varphi}_{T1}$$
$$+ \beta_2 (k_3 + k_4) \dot{\varphi}_{T2} + 4\beta_2 \dot{z}_k$$

$$(16)$$

Transform the sums

$$I = \sum_{k=1}^{2} \sum_{i=1}^{3} (\theta_{ik} + F_{ik}):$$
$$\sum_{k=1}^{2} \sum_{i=1}^{3} C_p \left(\begin{array}{l} \eta_i - z_{T1} + G_{ik} \varphi_{\delta i} \\ + N_{ik} \varphi_{T1} + H_{ik} \varphi_j \end{array} \right)$$
$$+ \sum_{k=1}^{2} \sum_{i=1}^{3} F \cdot sign \left(\dot{\Delta}_{ik} \right)$$

$$(17)$$

$$= -6 Cp z_{T1} + \sum_{k=1}^{2} \sum_{i=1}^{2} CpG_{ik} \varphi_{\delta i}$$
$$+ \sum_{k=1}^{2} \sum_{i=1}^{3} \begin{array}{l} CpN_{ik} \varphi_{T1} \\ + \sum_{k=1}^{2} \sum_{i=1}^{2} H_{ik} Cp \varphi_{Ji} \end{array}$$
$$+ 2Cp \sum_{i=1}^{3} \eta_i + \sum_{k=1}^{2} \sum_{i=1}^{3} F \cdot sign \left(\dot{\Delta}_{ik} \right)$$

Introduce the notation

$$a_{11} = 4C_1; \ a_{13} = -2(C_1 + C_2) - 6Cp;$$
$$a_{15} = -2(C_1 + C_2); \ a_{12} = C_1 \sum_{i=1}^{4} D_i,$$
$$a_{14} = C_1 (B_1 + B_2) + C_2 (E_1 + E_2) + \sum_{k=1}^{2} \sum_{i=1}^{3} CpN_{ik};$$
$$a_{16} = C_1 (B_3 + B_4) + C_2 (E_3 + E_4);$$
$$a_{17} = \sum CpG_{ik} \varphi_{\delta 1}; \ a_{18} = \sum_{k=1}^{2} CpG_{ik} \varphi_{\delta 1};$$
$$a_{19} = \sum_{k=1}^{2} CpG_{3k} \varphi_{\delta 3};$$
$$a_{1,13} = cp(H_{21} + H_{12}); \ a_{1,14} = Cp(H_{21} + H_{12});$$
$$f_1 = -2Cp \sum_{i=1}^{3} \eta_i - \sum_{k=1}^{2} \sum_{i=1}^{3} F \cdot sign \left(\dot{\Delta}_{ik} \right)$$

$$(18)$$

Introduce the vector

$$\left. \begin{array}{l} x = \left(\begin{array}{l} z_k, \varphi_k, z_{T1}, \varphi_{T1}, z_{T2}, \varphi_{T2}, \varphi_{\delta 1}, \varphi_{\delta 2}, \\ \varphi_{\delta 3}, \varphi_{\delta 4}, \varphi_{\delta 5}, \varphi_{\delta 6}, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5 \end{array} \right) \\ b_{11} = 4\beta_2, b_{13} = -2\beta_2, b_{15} = -2\beta_2, b_{14} = \beta_2 (k_1 + k_2) \\ b_{16} = \beta_2 (k_3 + k_4), b_{12} = \beta_{12} \sum_{i=1}^{4} E_i \end{array} \right\}$$

$$(19)$$

Then (16) and (19) can be rewritten in the form

$$\ddot{x}_1 = \sum_{J=1}^{16} a_{1J} x_J + \sum_{J=1}^{16} b_{1J} \dot{x}_J = f_1 \qquad (20)$$

$$a_{1J} = a_{1J} / M_k,$$

Here, $b_{1J} = b_{1J} / M_k$

$$f_1 = f / M_k$$

Similarly, transforming the remaining equations of the system (9), we obtain

$$\ddot{x} = Ax + B\dot{y} = f(t, x) \qquad (21)$$

5. EXPERIMENT RESULTS

When performing calculations of the dynamic and running properties of the trolley, digital numerical realizations of the geometrical irregularities of the rail threads were reasonably accepted. For computer simulation, oscillograms of vertical and horizontal irregularities of the railway track, taken from full-scale railway lines are shown in Figures 3-6. Several tools (Matlab, C++, Fortran) have been used for dynamic simulation of the developed mathematical model. The model studied in Section 4 is implemented in Matlab/Simulink platform which is a flexible dynamic simulation environment for multi-domain simulation and model-based design. A simple flow chart of the program is illustrated in Figs. 3-6.
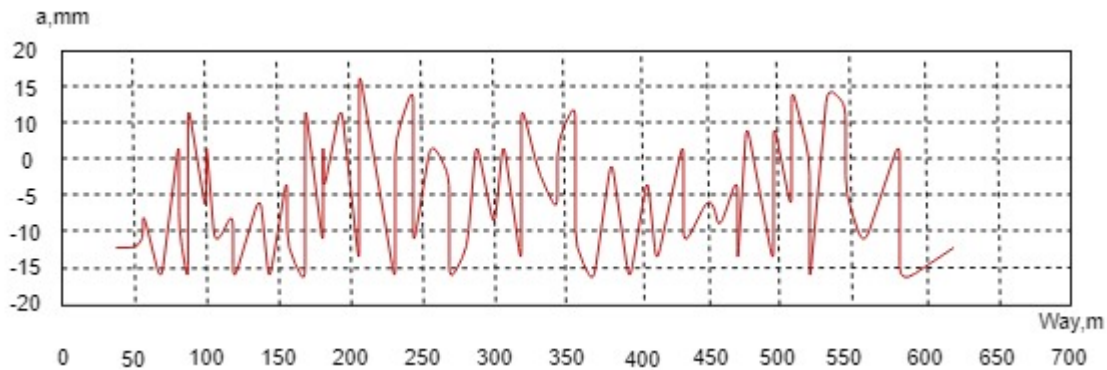

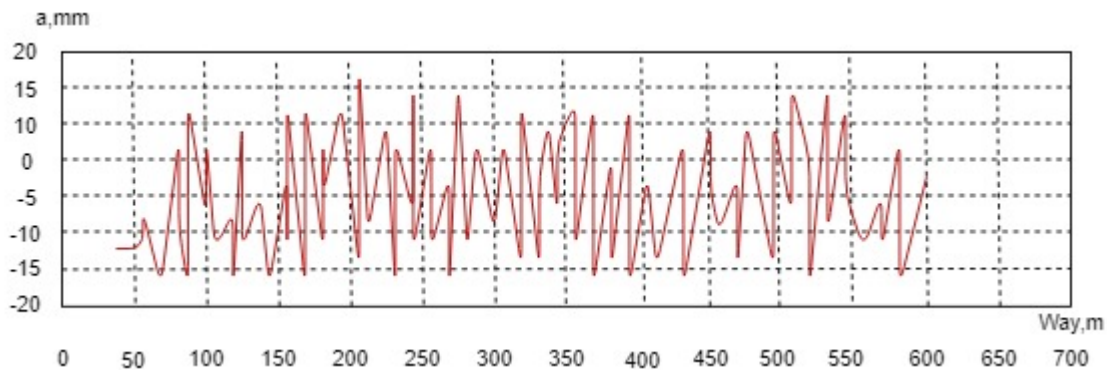*Figure 3. Vertical irregularities of the right rail thread*

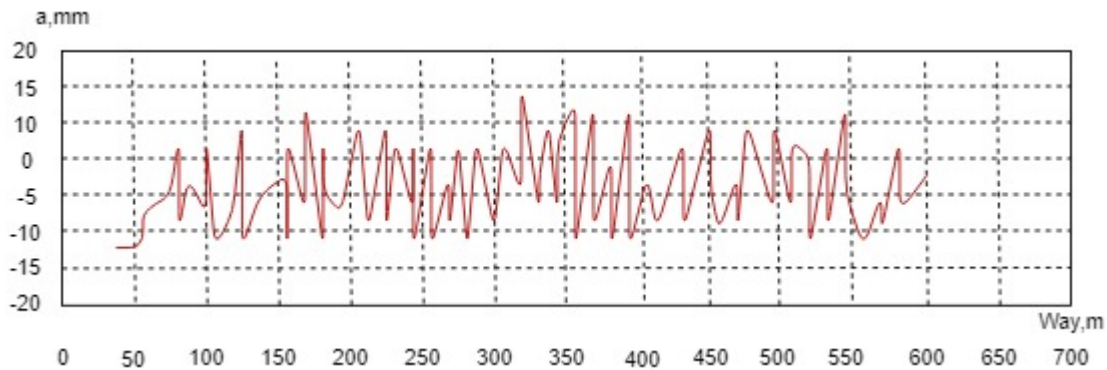*Figure 4. Vertical irregularities of the left rail thread*

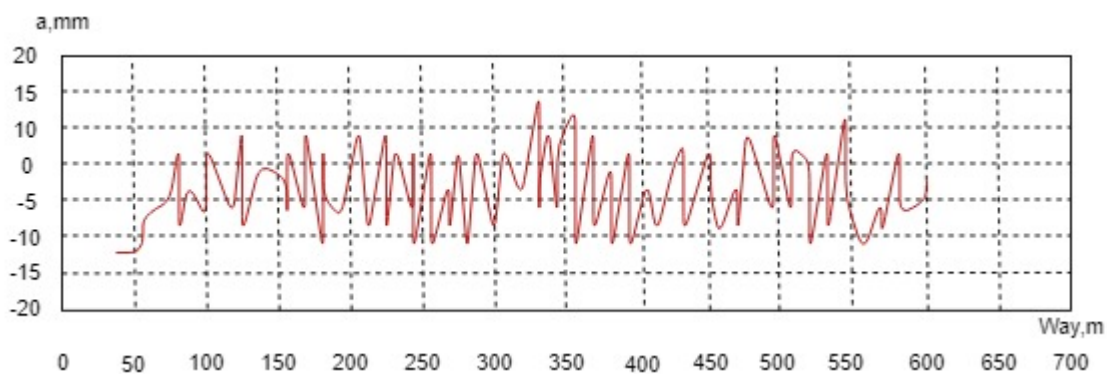*Figure 5. Horizontal irregularities of the right rail thread*

Figure 6. Horizontal irregularities of the left rail thread

Due to the fact that the considered sections of the railway track with the presence of deviations of degrees III-IV are considered unsatisfactory, the level of amplitudes of digital realizations of irregularities is reduced by 50% from the original amplitudes of digital realizations.

Comparison of the recorded experimental data of dynamic processes was carried out with the results of computer simulation: in a curve with a radius of 350 m, with a rail head elevation of 100 mm (when driving speed between 5 km/hour and 80 km/hour) in a curve with a radius of 500 m, with a rail head elevation of 100 mm (while driving with speeds of 20–100 km/hour) in the empty and loaded driving modes with axle loads of 8.5 tf and 23.65 tf, respectively.

The paper compares the following dynamic indicators:

- wheel safety factors against derailment;
- values of frame forces in shares of axial load;
- coefficients of vertical dynamics on the frame.

Graphic displays of the compared experimental values and calculated data for the empty and loaded wagon when moving in curves with a radius of 350 m and 500 m are shown in Figures 7-12.
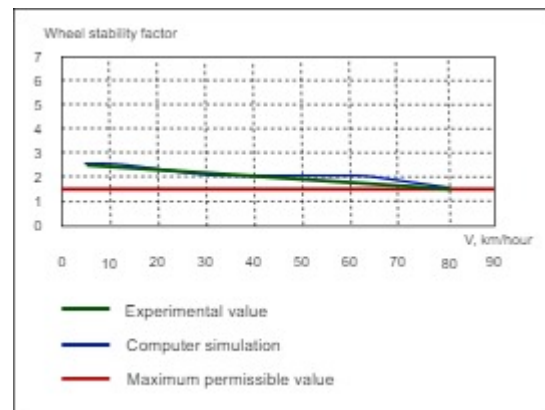

Figure 7. Comparison of experimental and calculated data when driving an empty car in a curve of a radius of 350 meters in terms of the safety factor
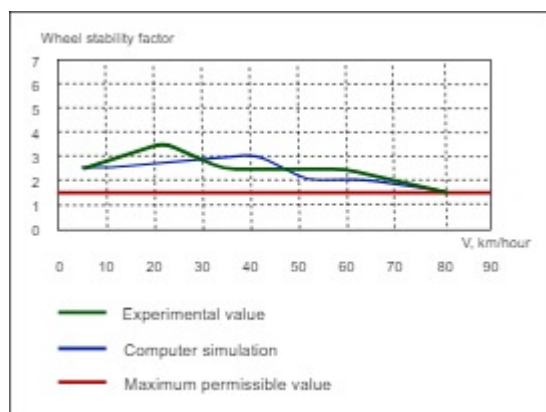
Figure 8. Comparison of experimental and calculated data when driving an empty car in a radius curve of 500 meters in terms of the safety factor
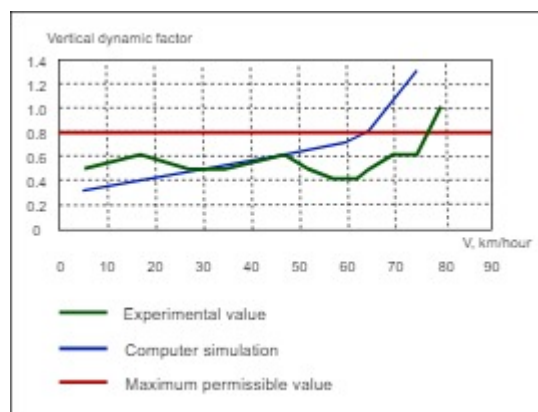

Figure 11. Comparison of experimental and calculated data when driving an empty car in a curve of a radius of 350 meters according to the values of the dynamics of the frame

Figure 9. Comparison of experimental and calculated data for the movement of an empty car in a curve of a radius of 350 meters, the values of frame forces in axial load fractions

Figure 12. Comparison of experimental and calculated data when driving an empty car in a curve of radius of 500 meters according to the values of the dynamics of the frame
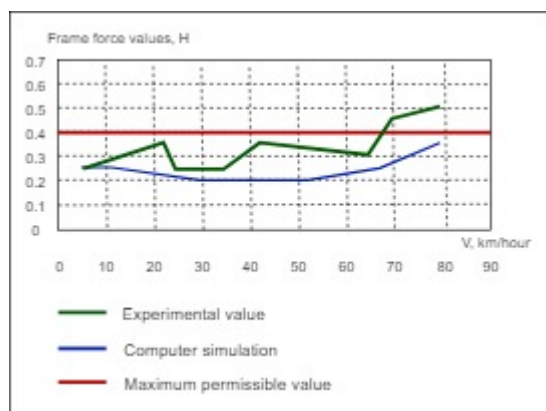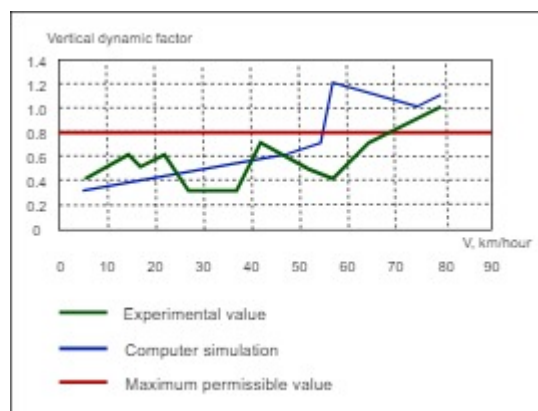
Thus, obtained using computer simulation, the results show satisfactory agreement with the experimental data. This indicates the reliability of the results obtained on the proposed mathematical model, which provides the basis for the application of the proposed computer model for solving practical problems.

6. DISCUSSION AND CONCLUSION

In solving the problems of the dynamics of machines, more and more attention is paid to the variety of forms of dynamic interactions of the elements of the systems, as well as to the relationships that determine the functional features of technical objects. A comparative review of methodological positions in the consideration of tasks related to the assessment and control of the dynamic state shows that mechanical oscillatory
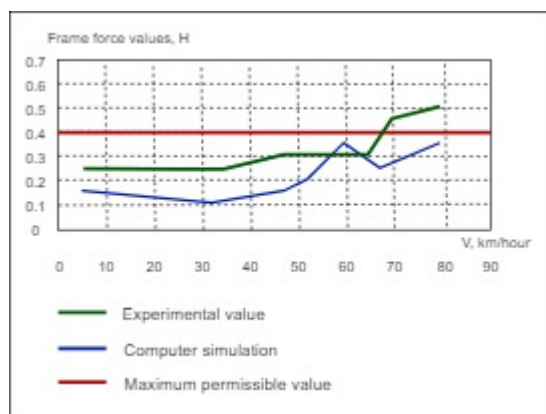

Figure 10. Comparison of experimental and calculated data for the movement of an empty car in the curve of a radius of 500 meters, the values of frame forces in axial load fractions

systems are used as design schemes. A variety of systems and features of the goals and objectives of the calculation determines the diversity of research methods and the choice of state parameters.

At the same time, it can be reasonably assumed that the majority of mechanical systems are described by differential equations that take into account the mass-inertial and elastic-dissipative properties of elements, links, or devices with lumped parameters [8-14, 32, 35-40, 43-46, 73-75]. If systems have nonlinear characteristics, as well as the distribution of mass-inertia and elastic properties, then in such cases simplified models are developed and applied, ultimately based on the linearization of the properties of the elements and their connections. The tendency to consider the integral properties of the elements forming the system is quite obvious, which manifests itself in taking into account several joint properties, such as spring mass, elastic-dissipative connections, which can be viewed as manifestations of emerging ideas about the possibilities of creating some generalized approaches dynamic systems in the processes of signal transmission and conversion. In this sense, the tasks of communication theory and control theory may coincide, which in certain conditions allows the transformation of signals and external influences on dynamic systems to be considered from a unified position.

One of the directions of forming a methodological basis in technologies for solving dynamic problems is the use of the analytical apparatus of the theory of systems, systems analysis, and modeling methods, in which structural representations of technical objects occupy a large place, which is based on the use of operator research methods. The introduction of techniques and technologies used in control theory and circuit theory is focused on the construction of mathematical models that reflect typical situations, standard signals and external influences, causing appropriate reactions, which creates a good basis for assessing the properties of systems and comparative analysis.

In the present work, an attempt was made to systematically analyze the existing methodological positions in solving problems of a mathematical model of vertical locomotive oscillations, which are characterized by problems of reducing the dynamic influence of external disturbing factors. Such an approach predetermines, first of all, attention to the methods and methods of constructing mathematical models and the connection of these models with the features of computational models, as well as with the possibilities of simplifying models with the subsequent use of structural methods of analysis and synthesis characteristic of control theory.

The study of the vertical dynamics of locomotives mainly involves two tasks: analysis of the influence of dynamic parameters on the railway track and evaluation of stability and traffic safety. The adopted design scheme has 12 degrees of freedom. The problem of the mathematical model of vertical oscillations of a locomotive is solved by methods of Newton's classical mechanics using the Lagrange equation of the second type.

The stability of the computational process shows that the vertical oscillations of the locomotive strictly depend on the form of the function c (Δc).

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