SUPPLEMENTARY MATERIAL: DEGREES OF FREEDOM FOR PIECEWISE LIPSCHITZ ESTIMATORS

FREDERIK RIIS MIKKELSEN AND NIELS RICHARD HANSEN

1. Computational Time and Number of Selected Predictors

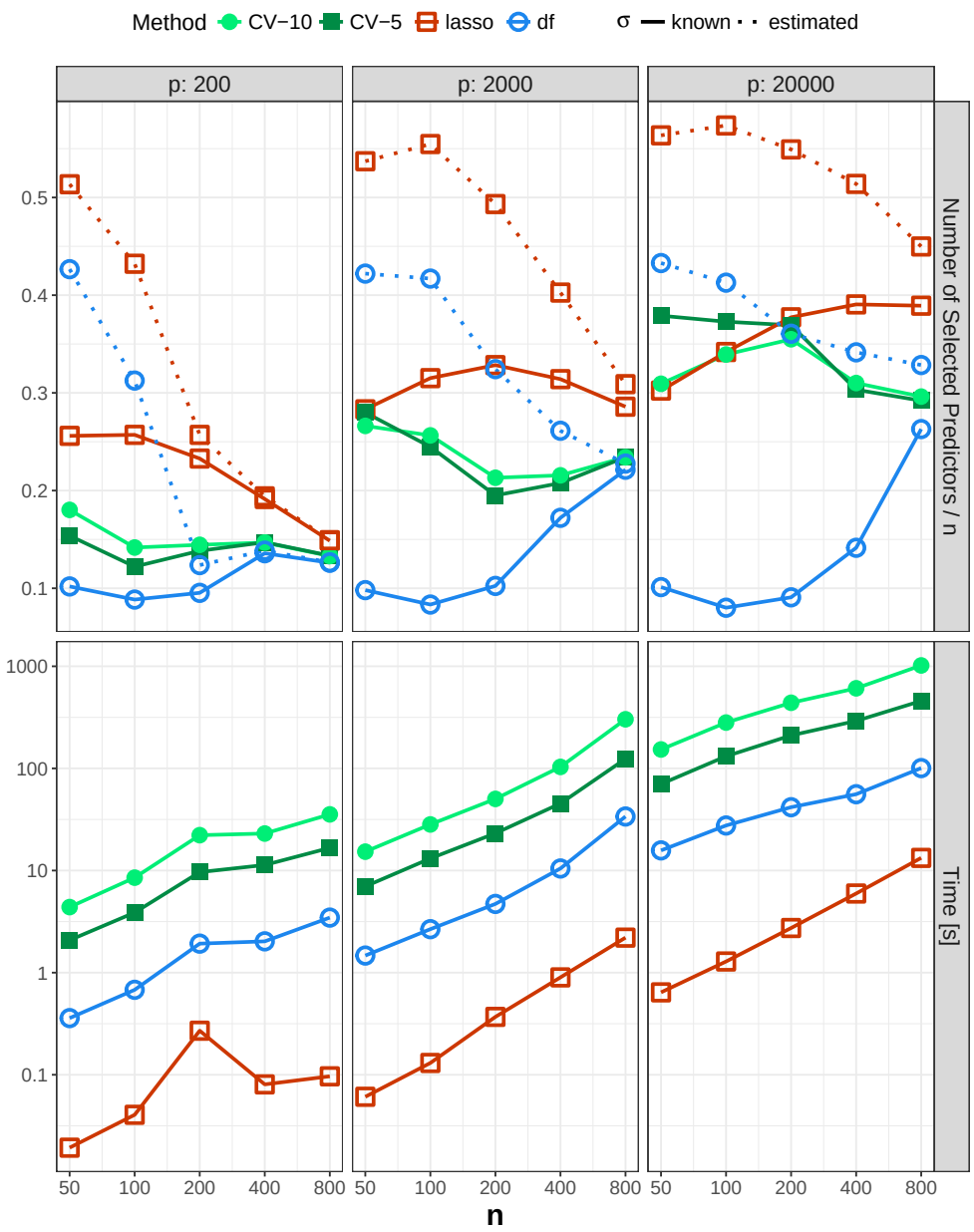
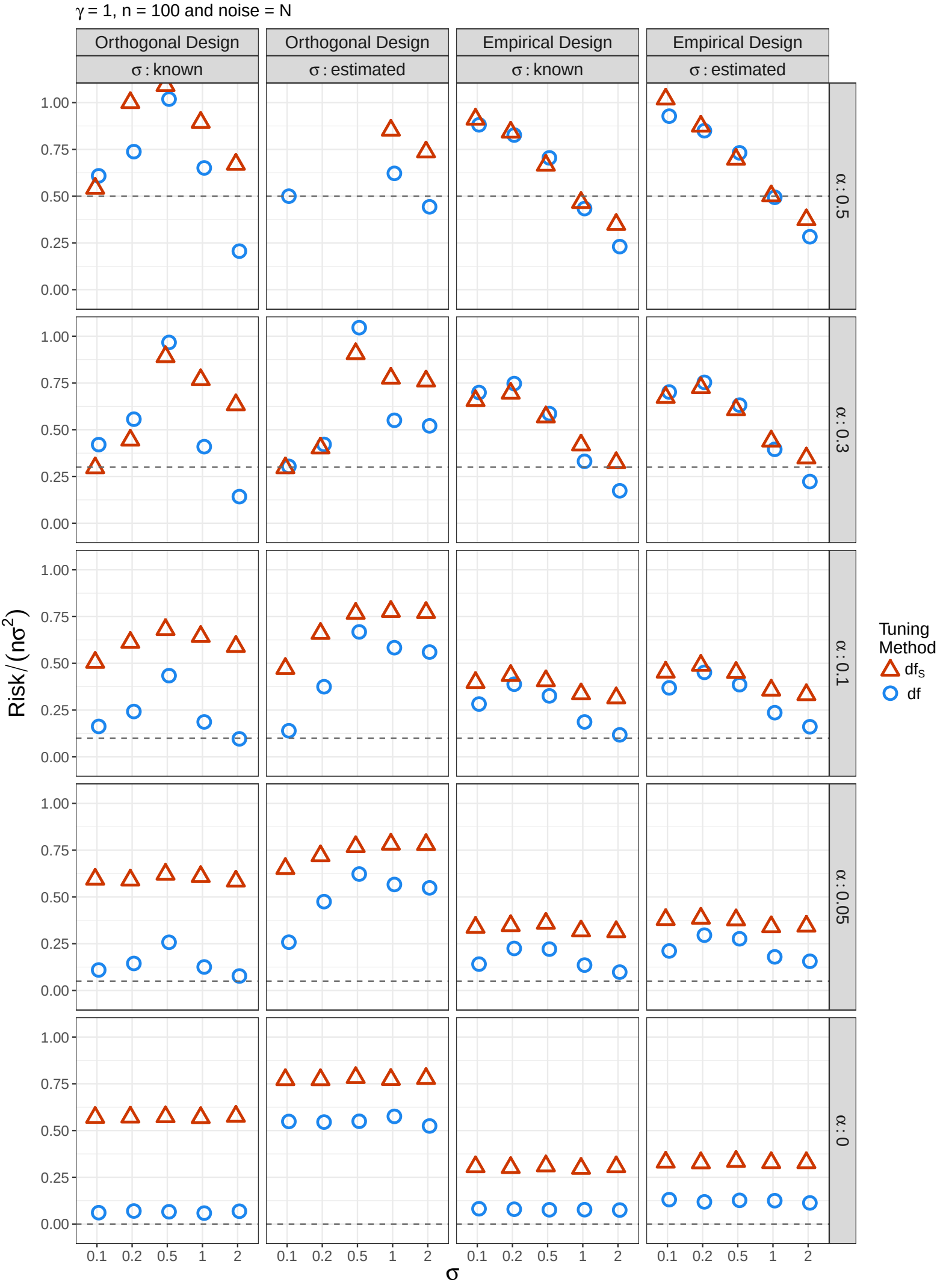
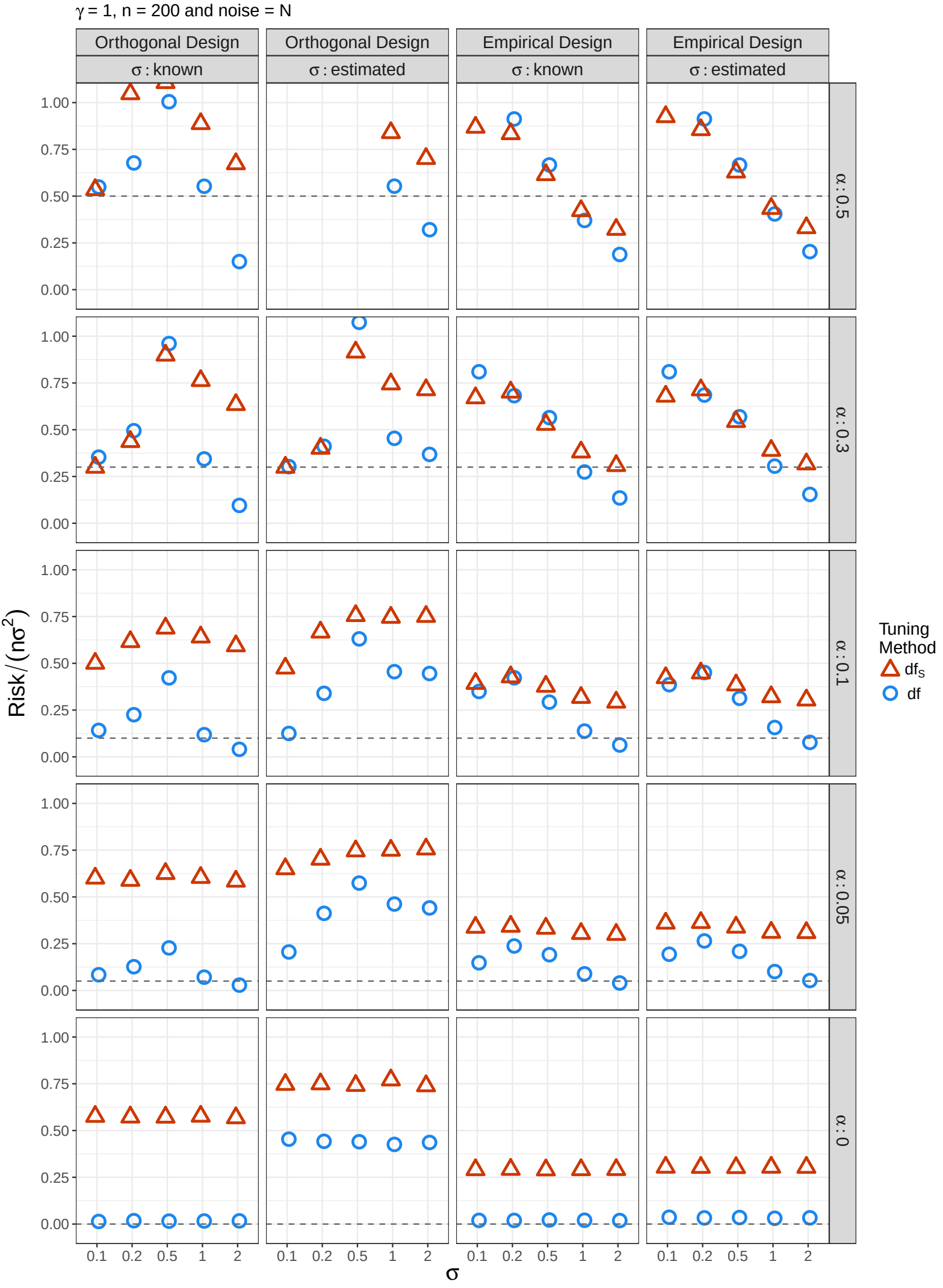


FIGURE 1. The number of selected predictors divided by n (top), along with computational time of evaluating the estimator and tuning the λ-parameter using the different methods (bottom). The design parameters were: $\sigma = 0.5$, $\gamma = 1$, $\alpha = 0.1$, and the design type was (S) with a constant correlation of $\rho = 0.1$ (see Section 4)

2. Risk estimates

Plots of the risk estimates relative to $n\sigma^2$ for the estimators $\hat{\mu}^{\hat{\lambda}_{\mathrm{df}_{\mathbb{S}}}}_{\mathrm{OLS.l}}$ and $\hat{\mu}^{\hat{\lambda}_{\mathrm{df}}}_{\mathrm{OLS.l}}$. The dashed lines are the relative risks for the oracle-OLS estimator.


γ = 1, n = 100 and noise = N
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
df_S
df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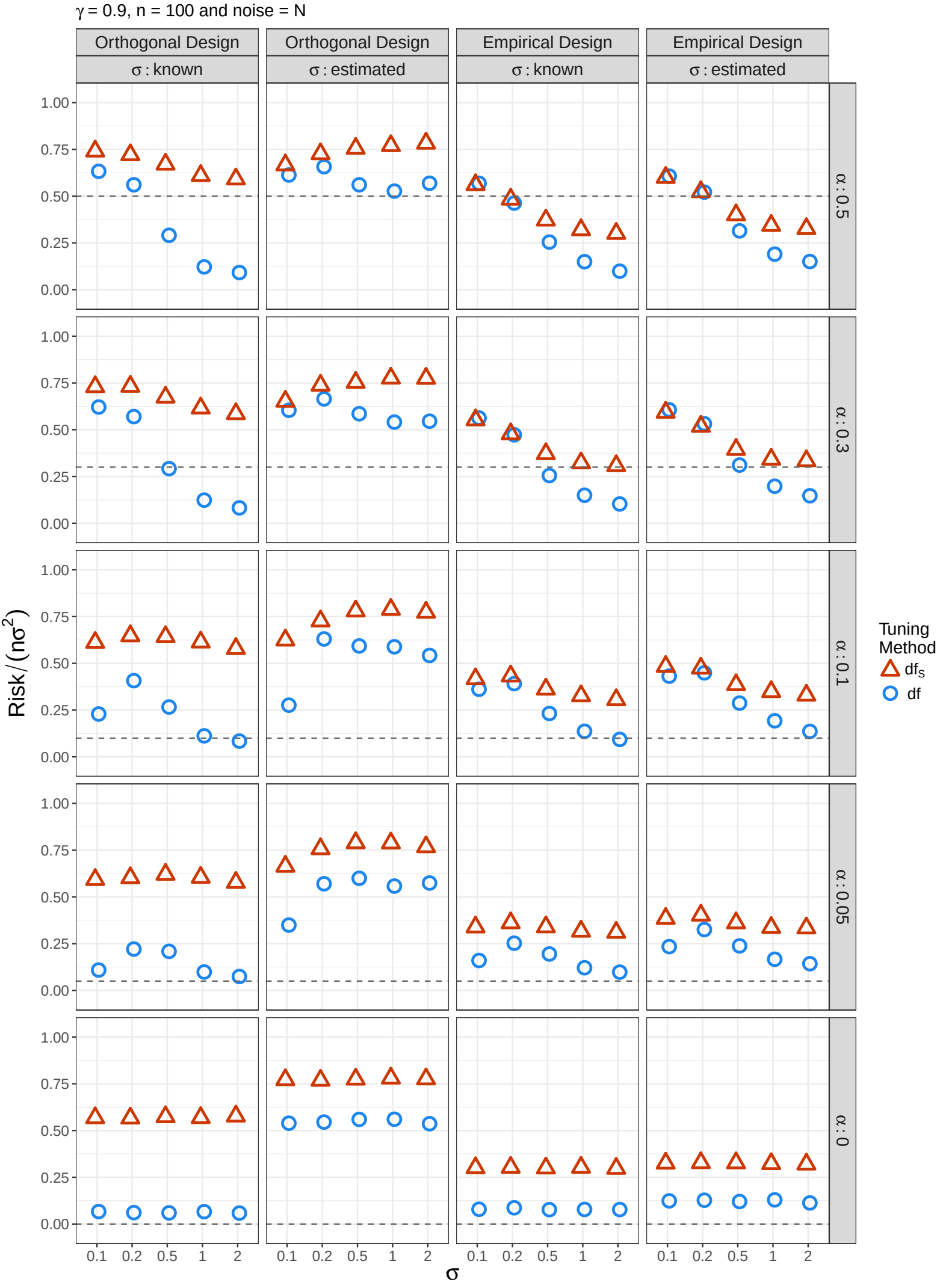
γ = 0.9, n = 100 and noise = N
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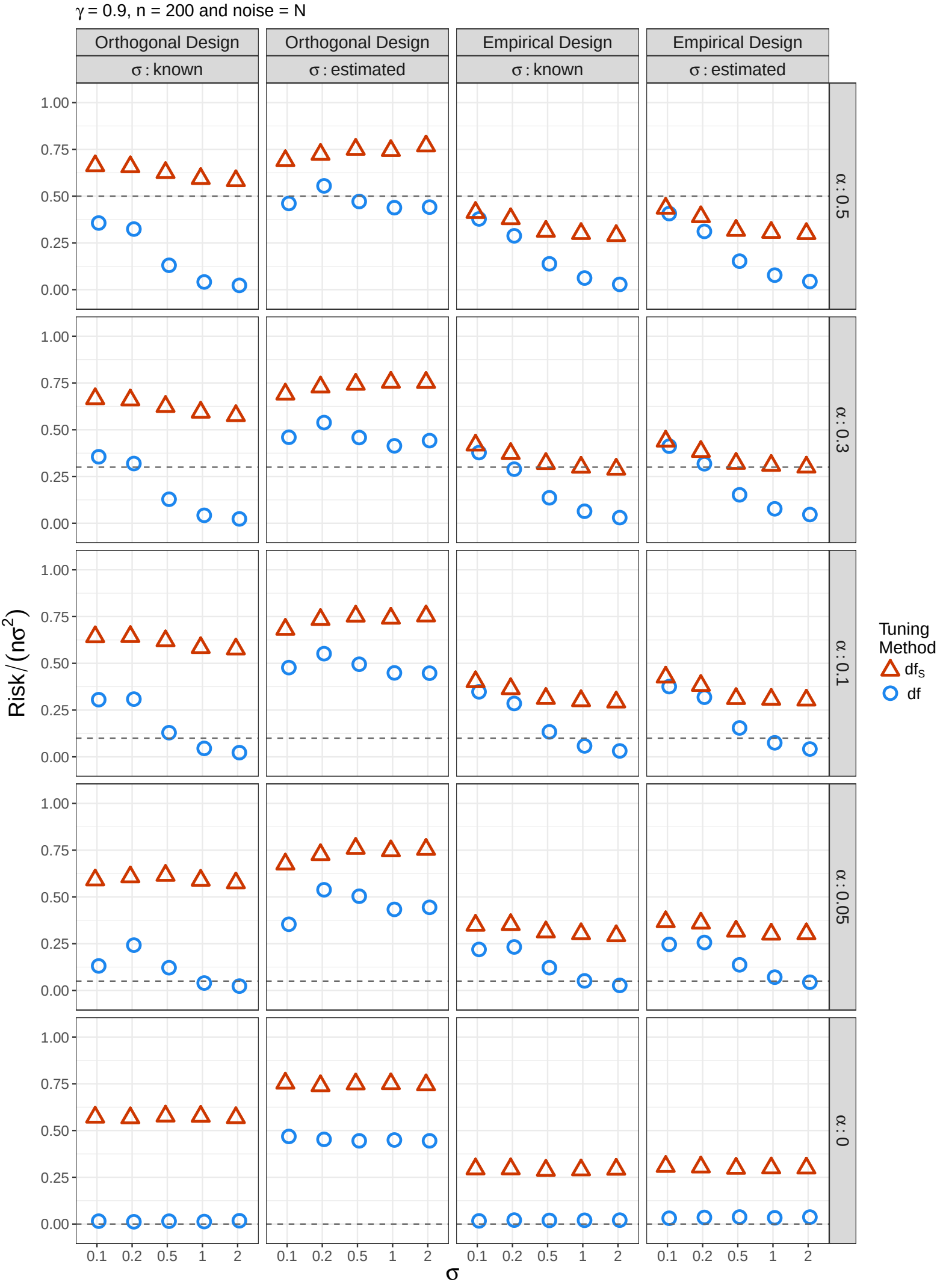
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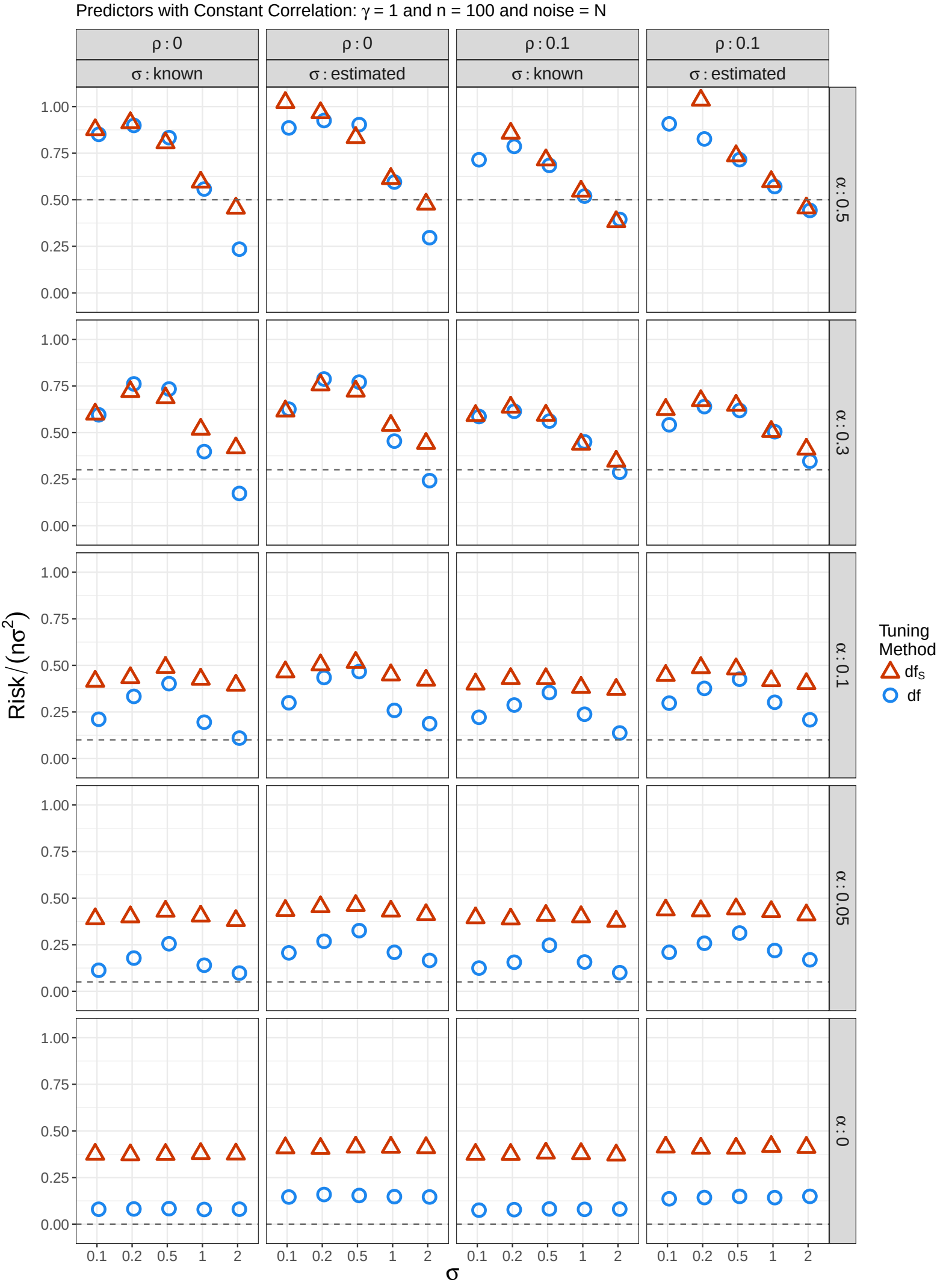
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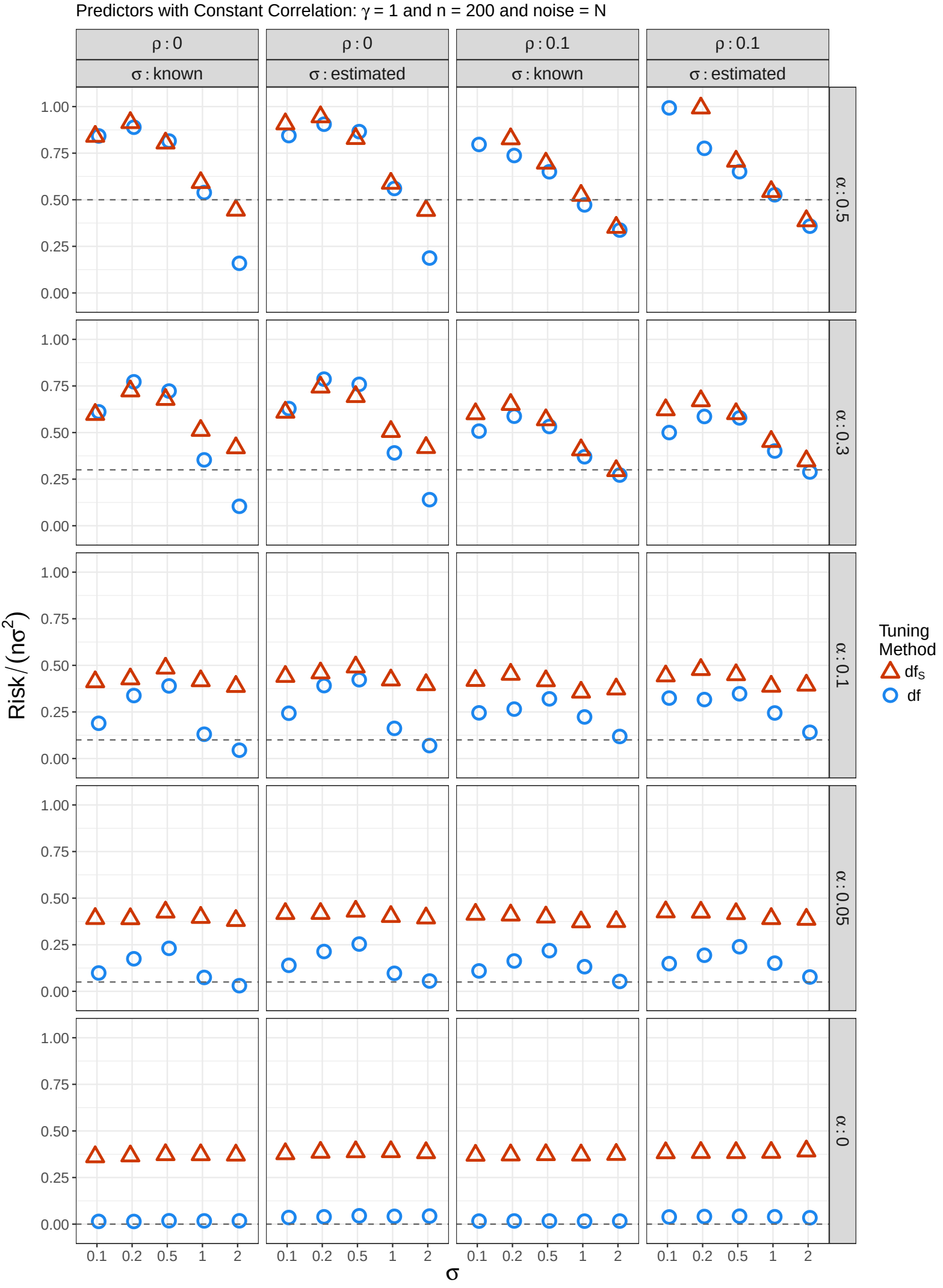
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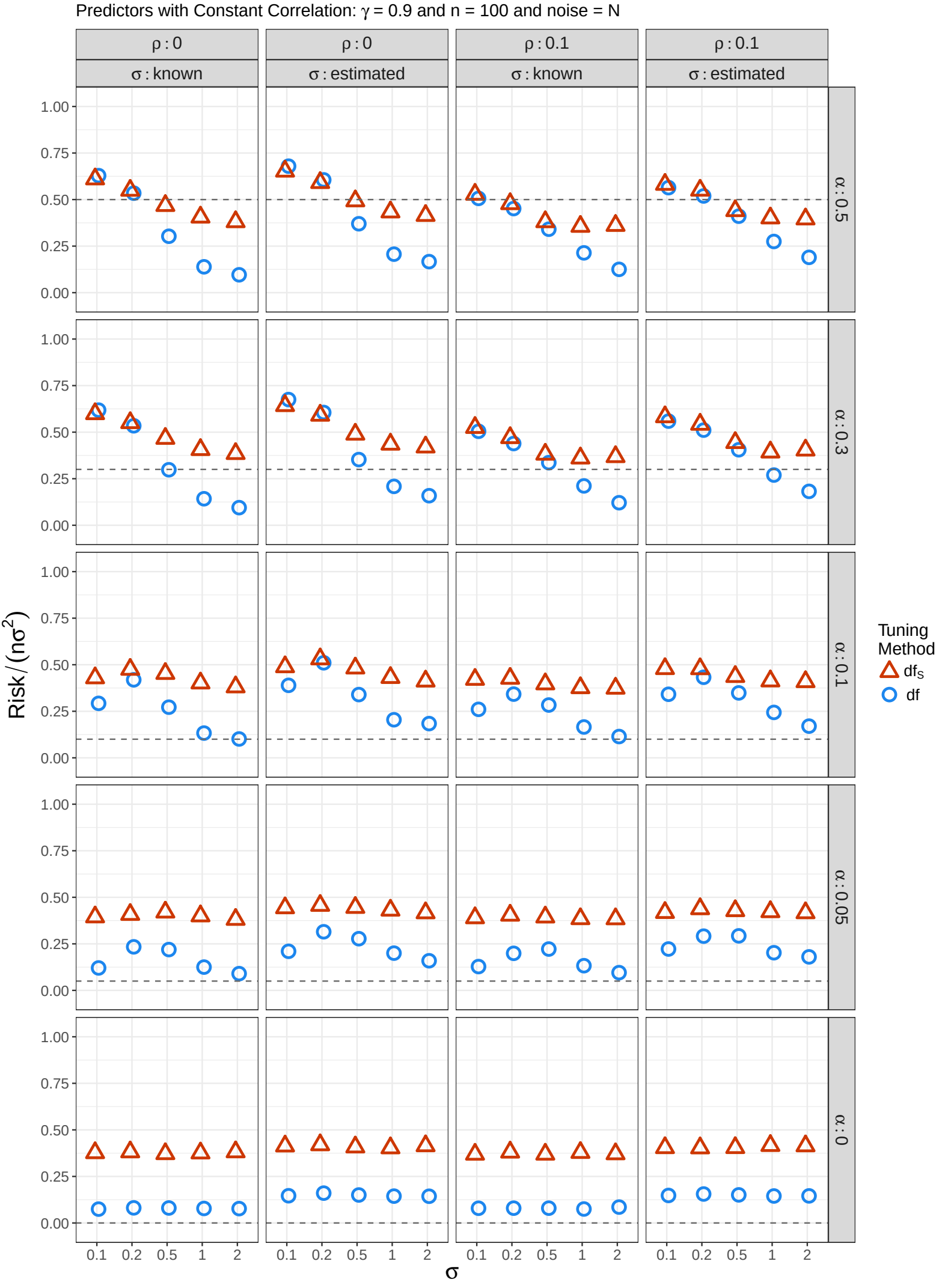
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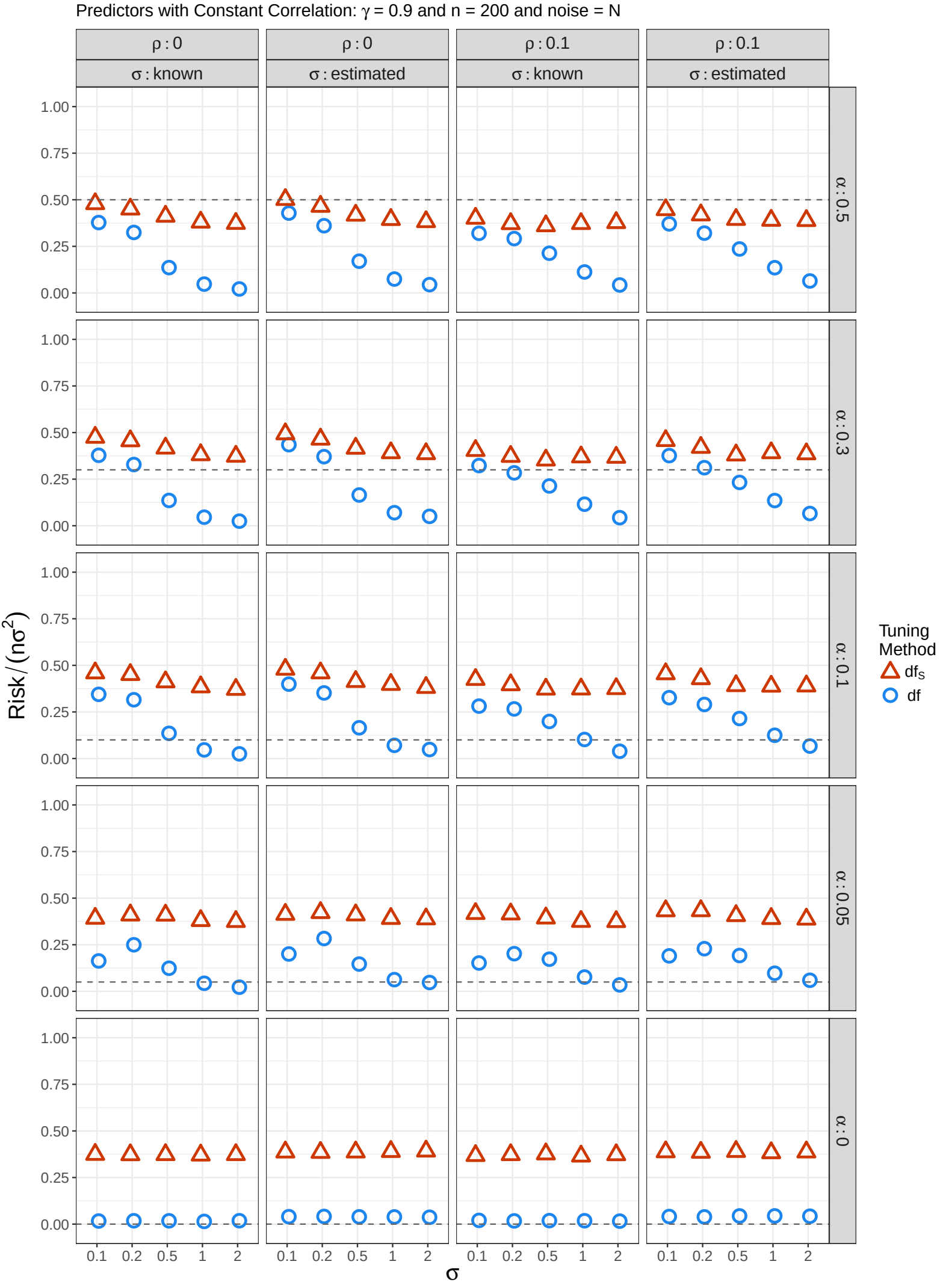
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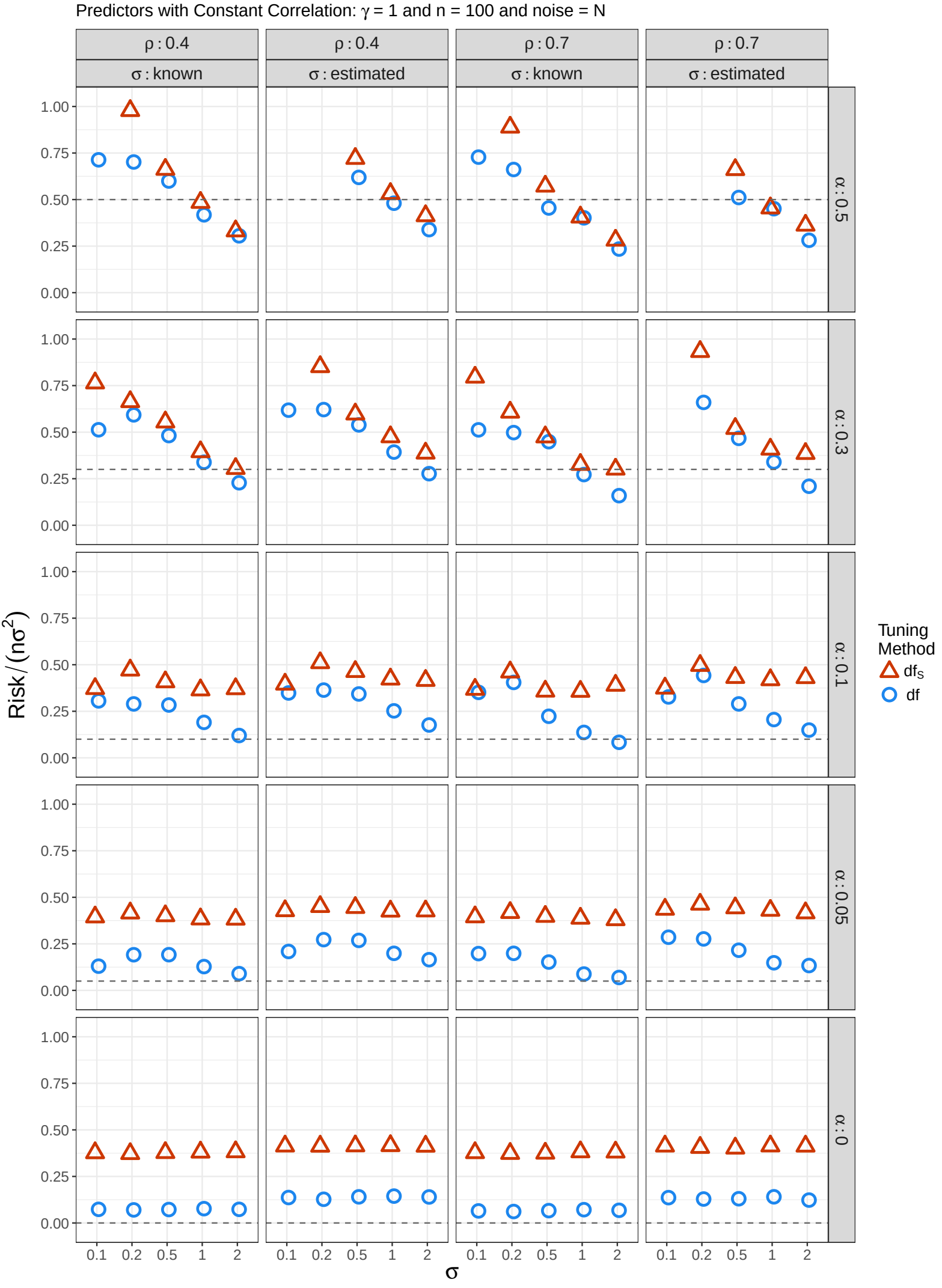
Predictors with Constant Correlation: γ = 1 and n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
dfS
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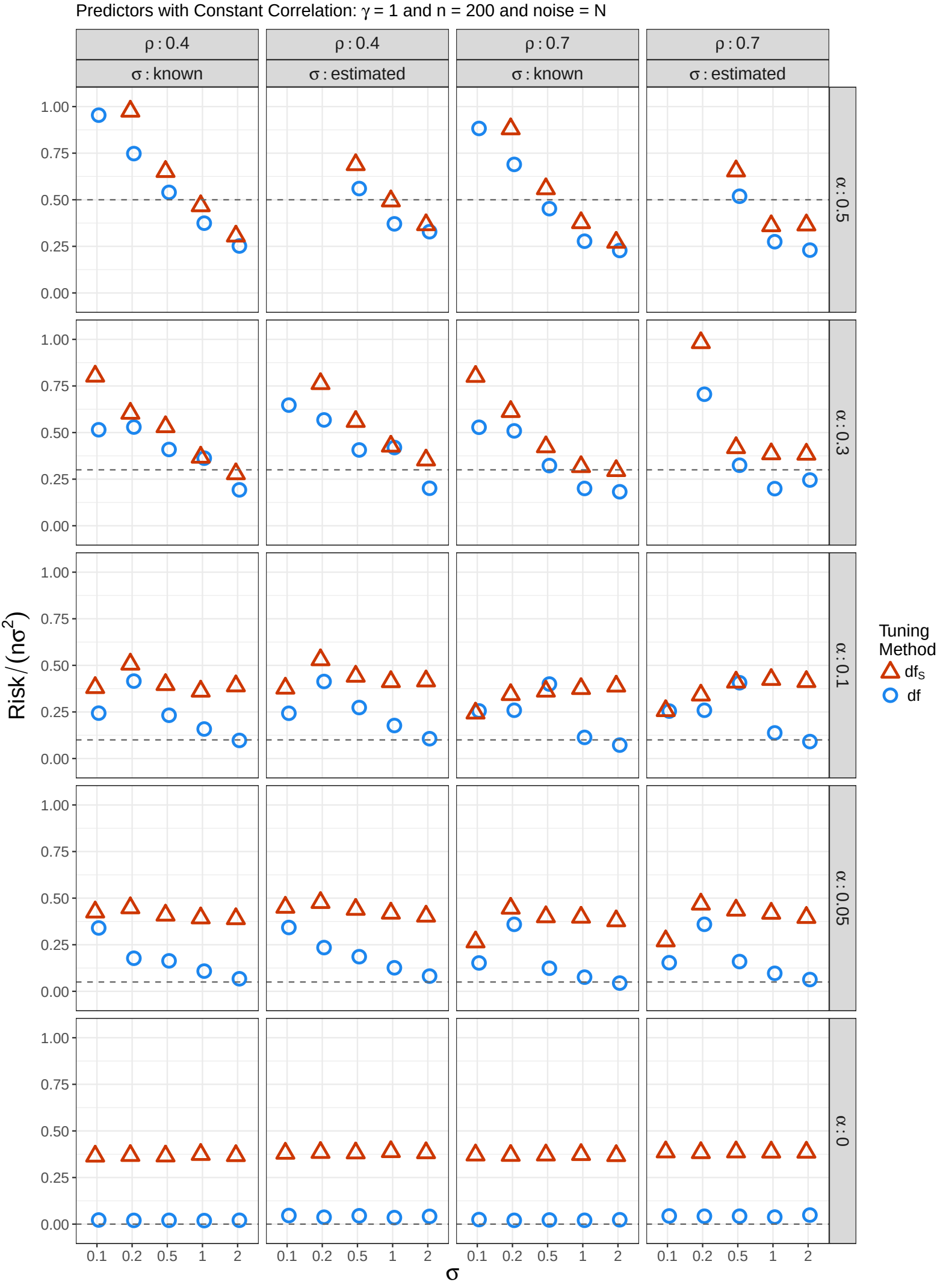
Predictors with Constant Correlation: γ = 1 and n = 200 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk / (nσ²)
1.00
0.75
0.50
0.25
0.00
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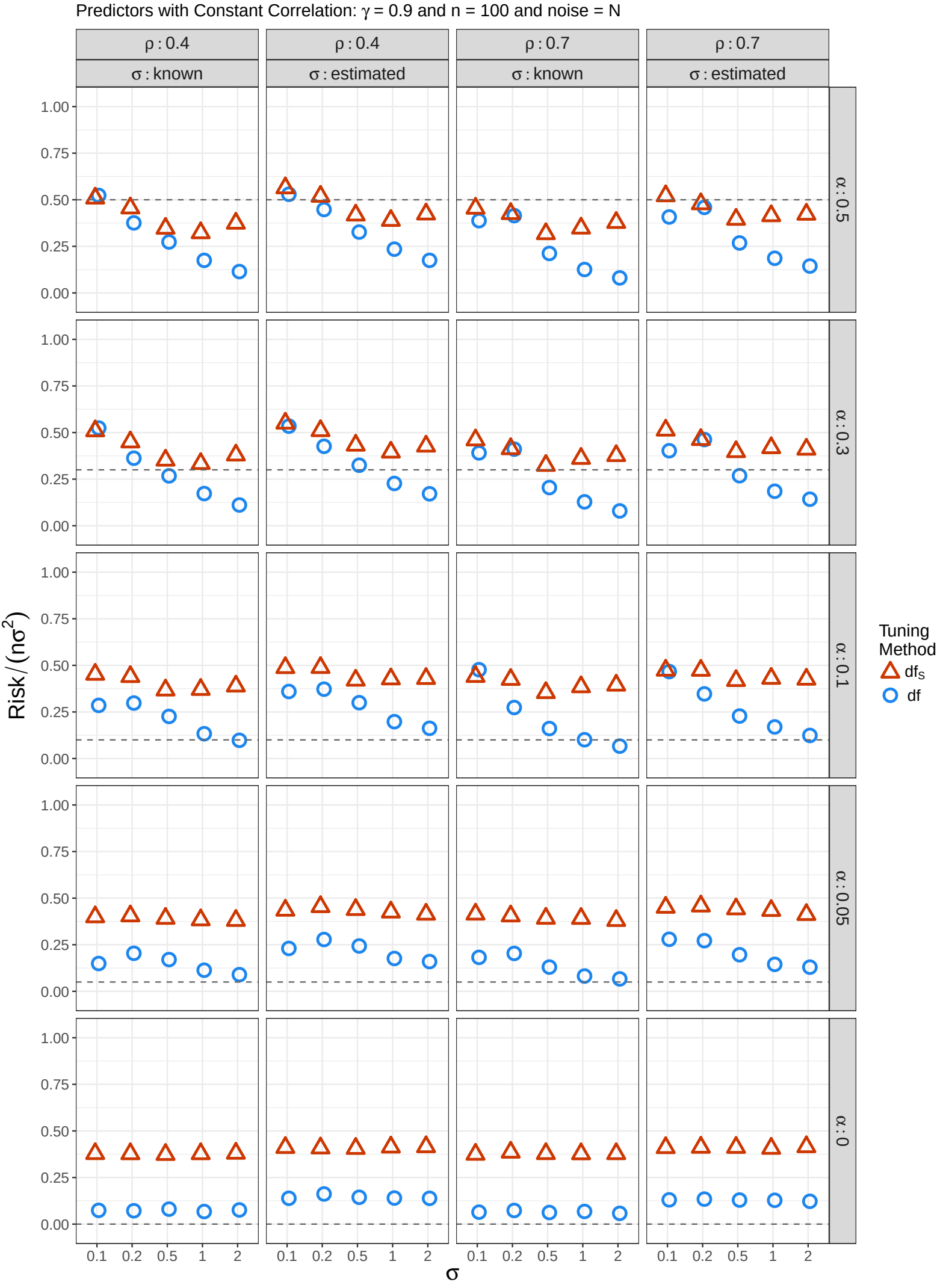
Tuning Method
df_S
df


Predictors with Constant Correlation: γ = 0.9 and n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
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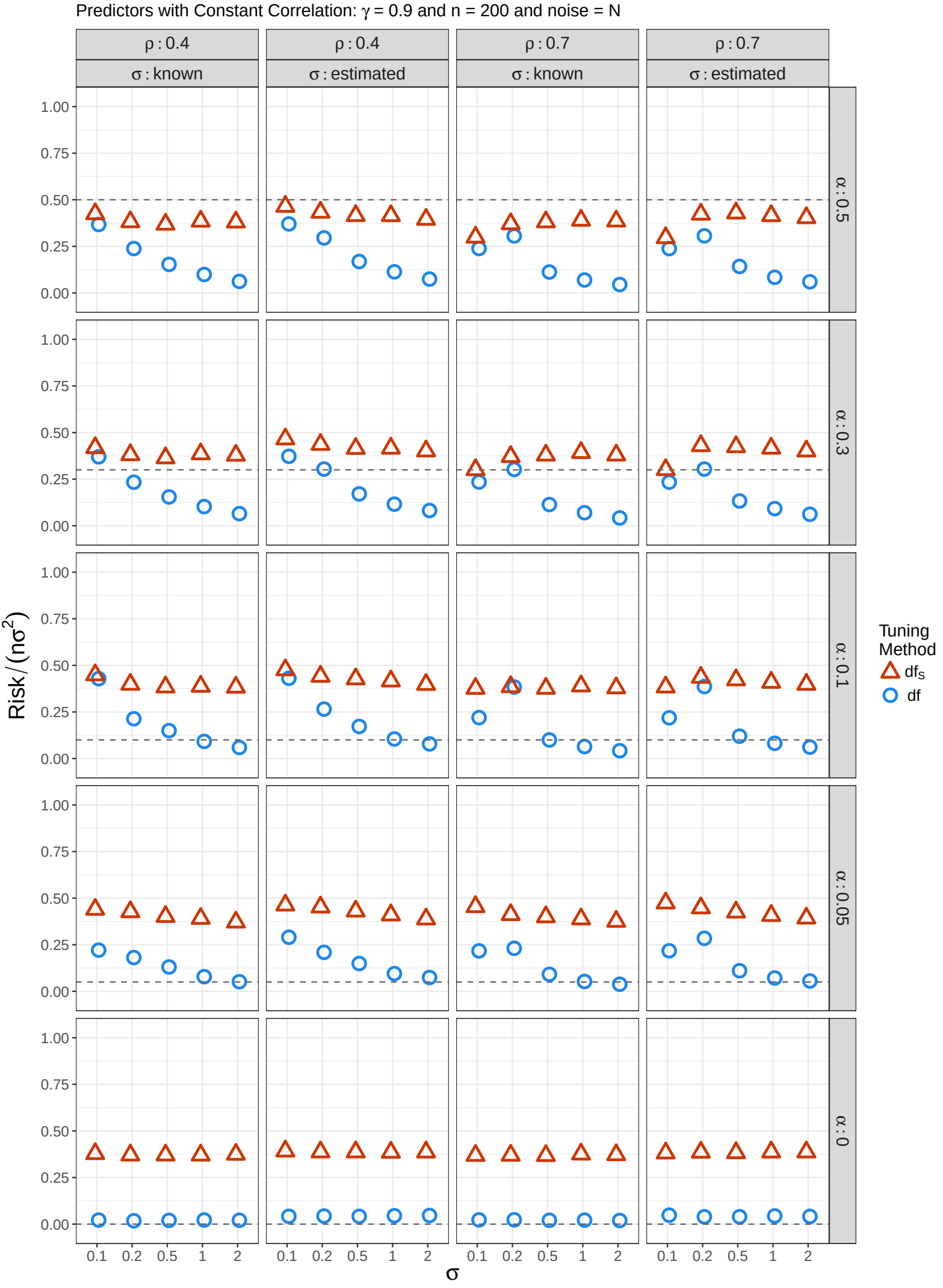
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ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
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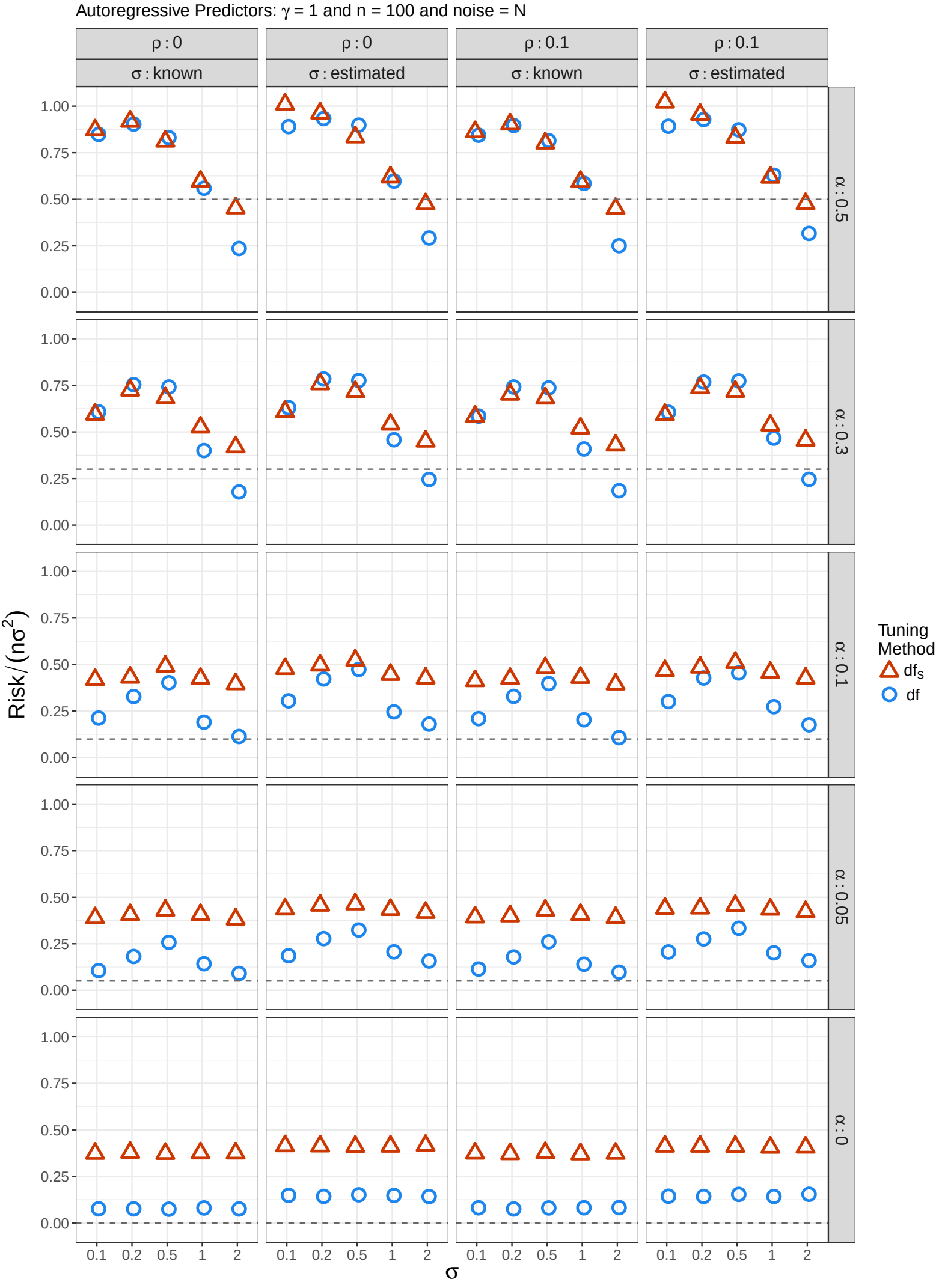
Tuning Method
df_S
df


Autoregressive Predictors: γ = 1 and n = 100 and noise = N
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
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Risk/(nσ²)
1.00
0.75
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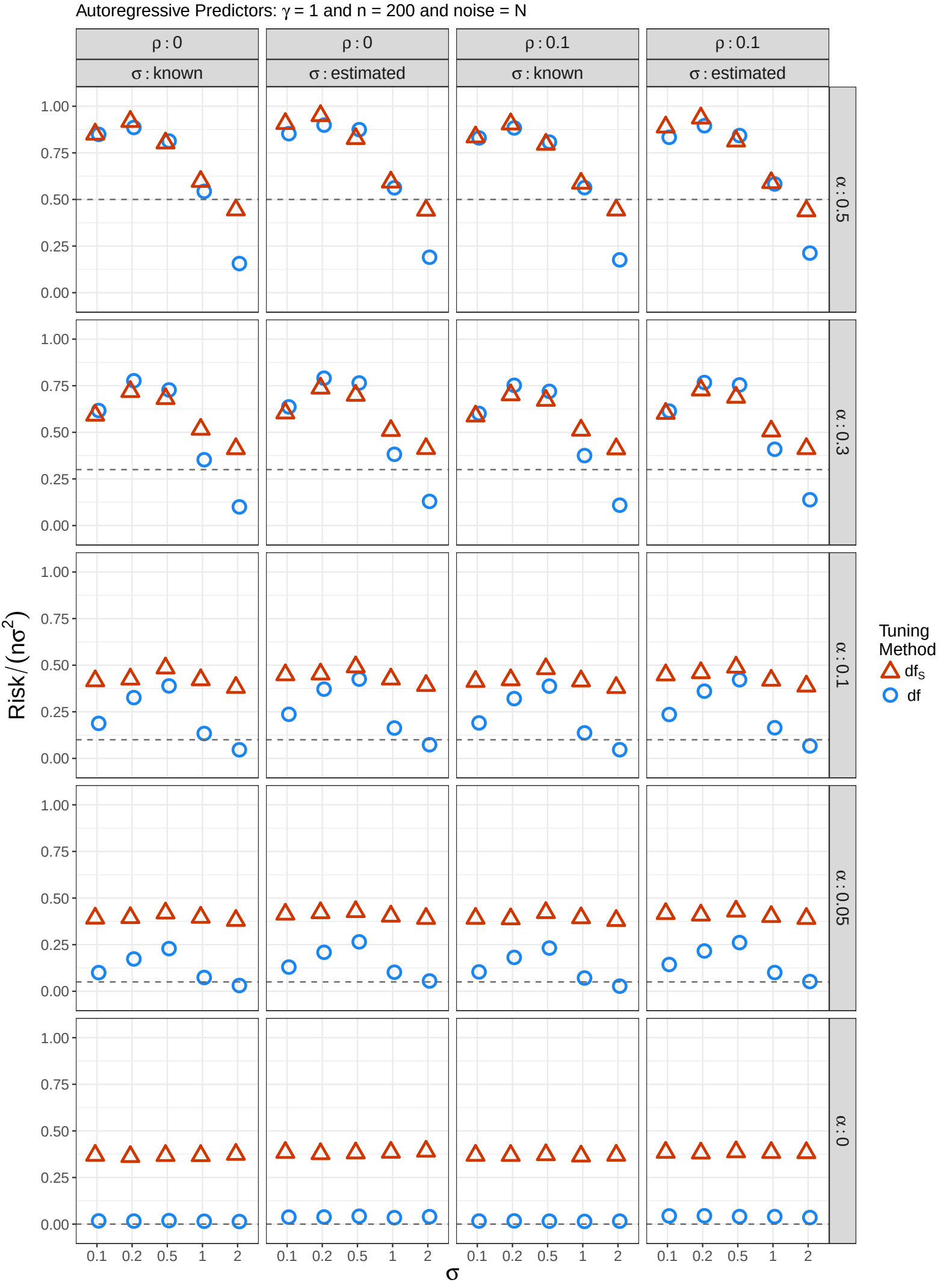
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ρ : 0
ρ : 0
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ρ : 0.1
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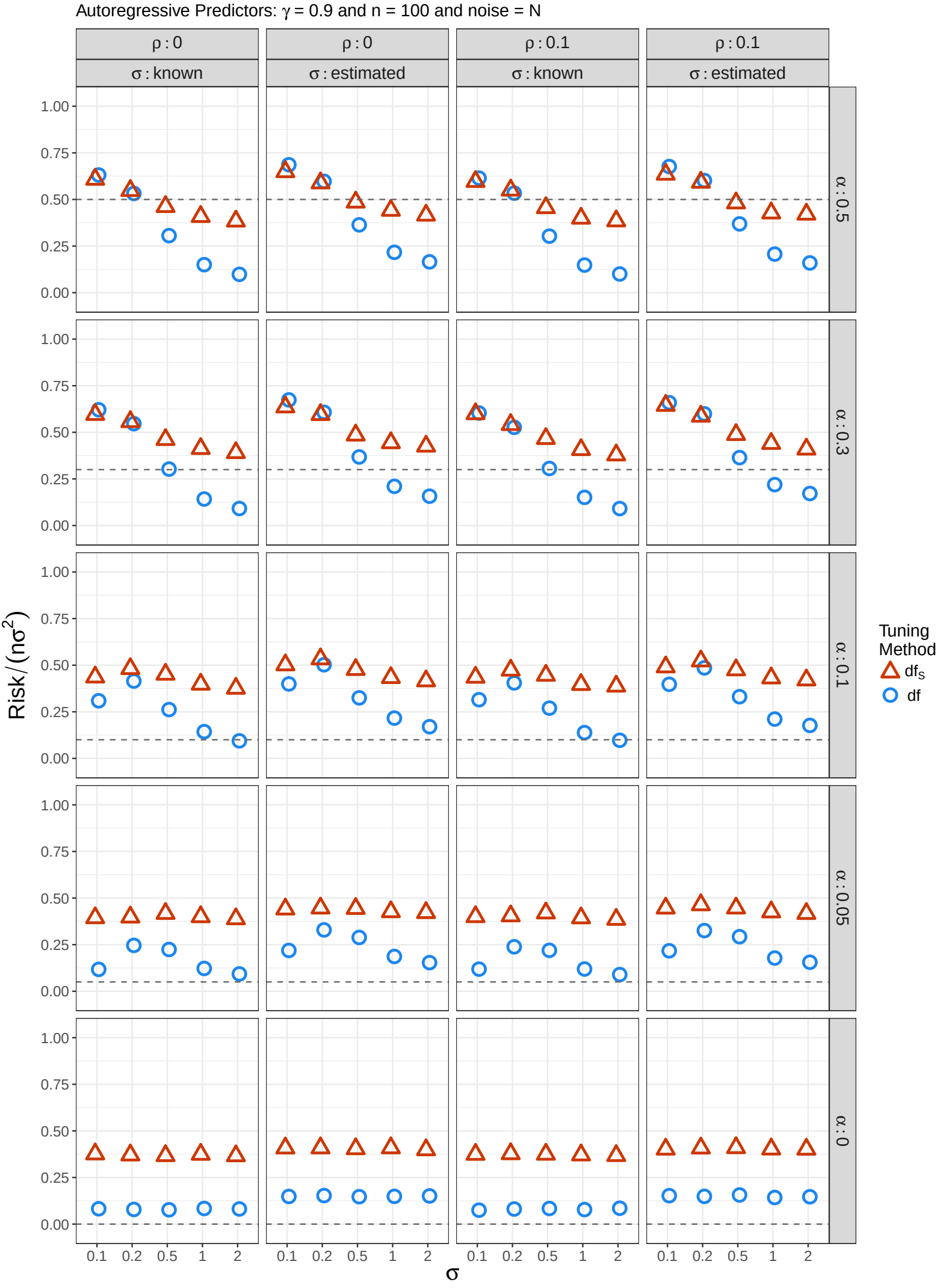
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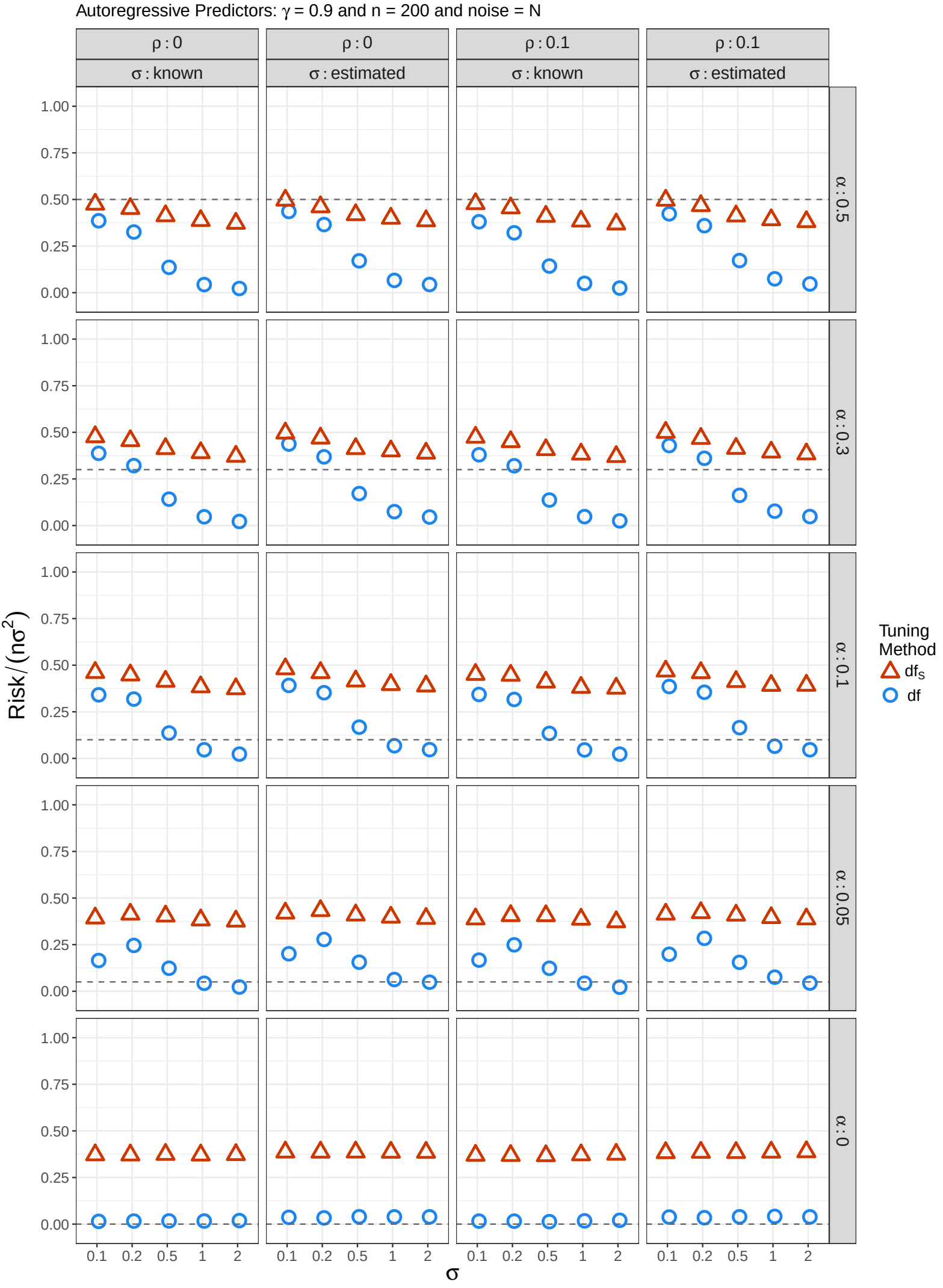
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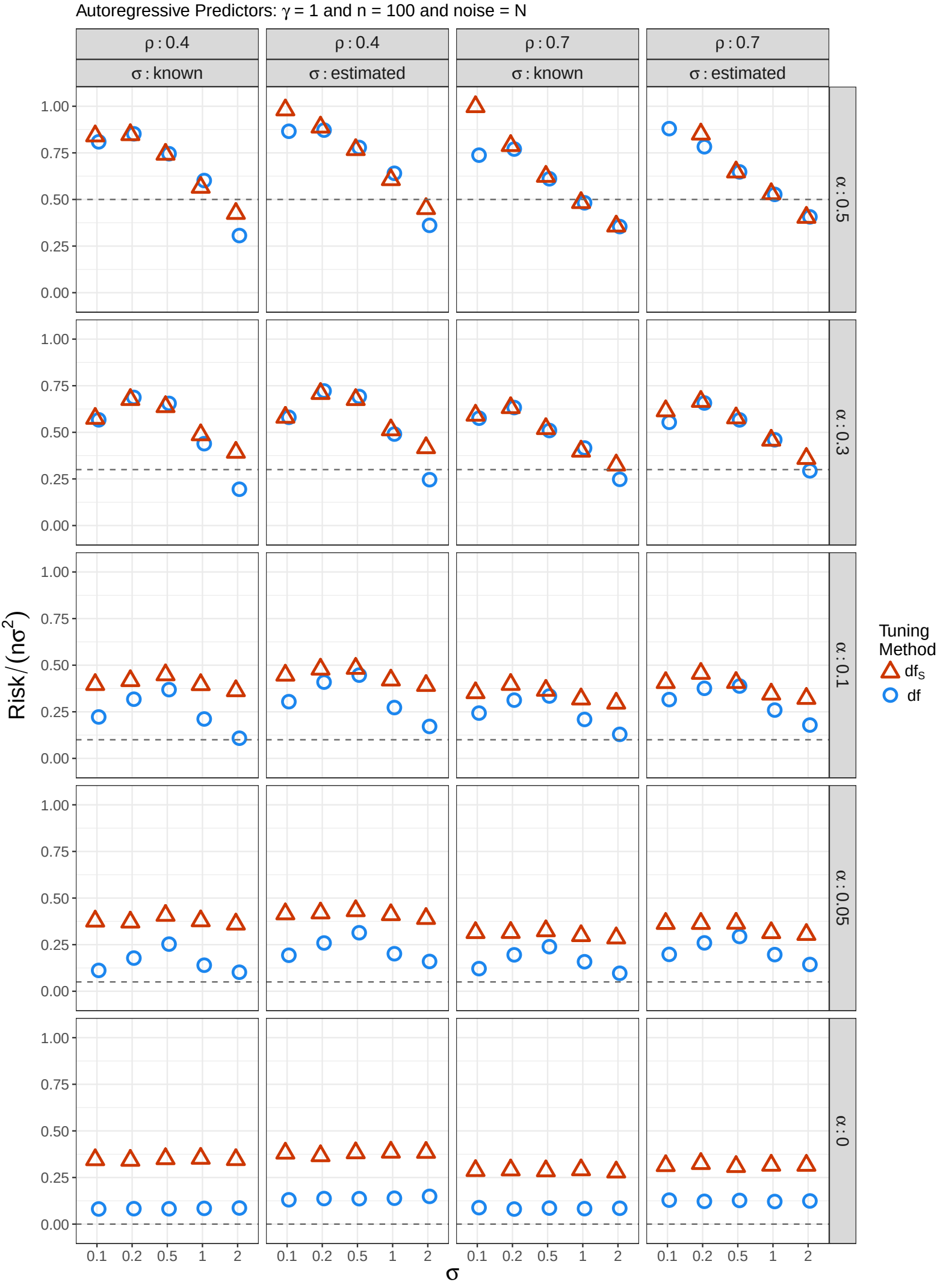
Autoregressive Predictors: γ = 1 and n = 100 and noise = N
ρ : 0.4
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Risk/(nσ²)
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Tuning Method
df_S
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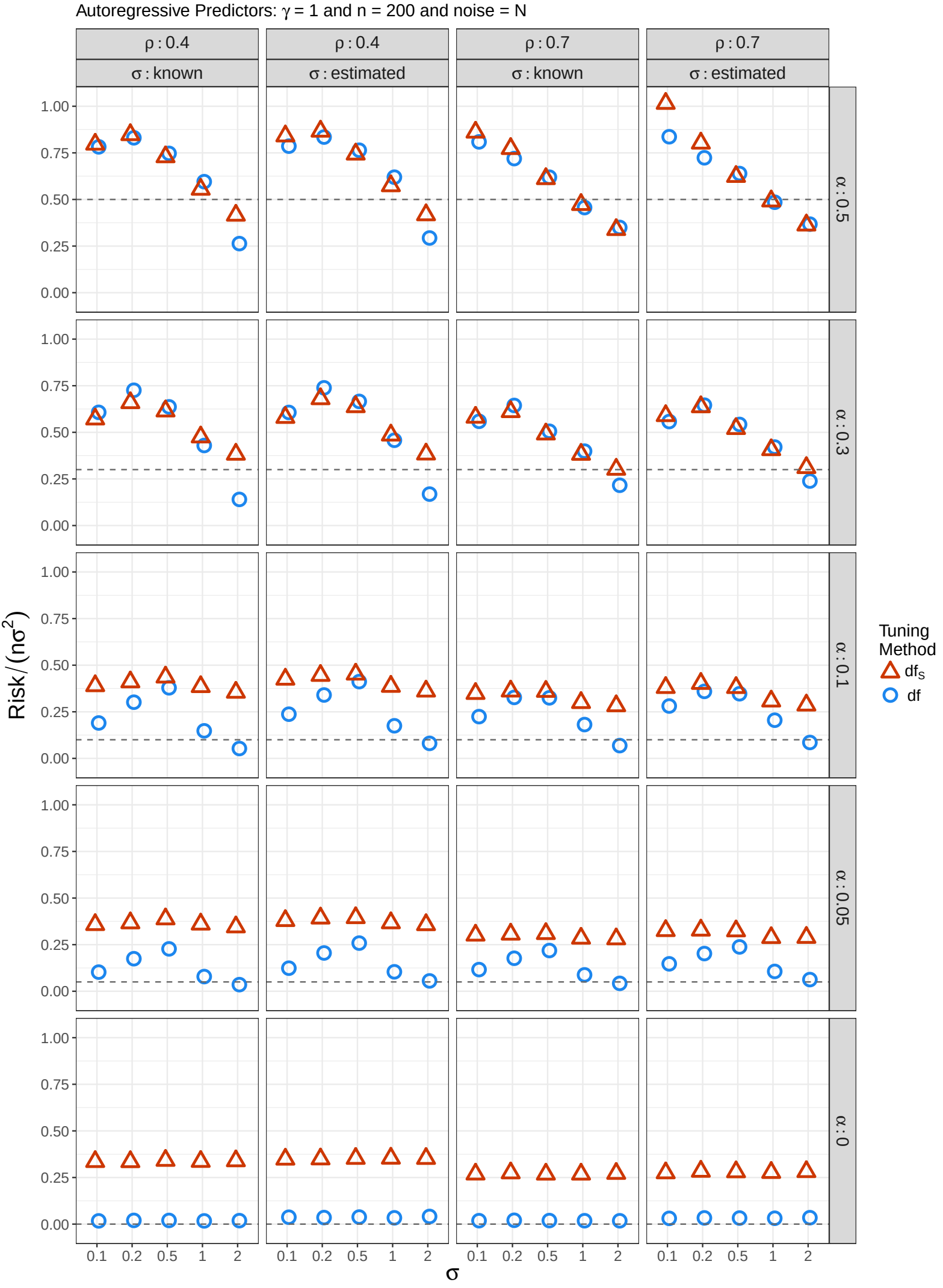
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ρ : 0.4
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Tuning Method
df_S
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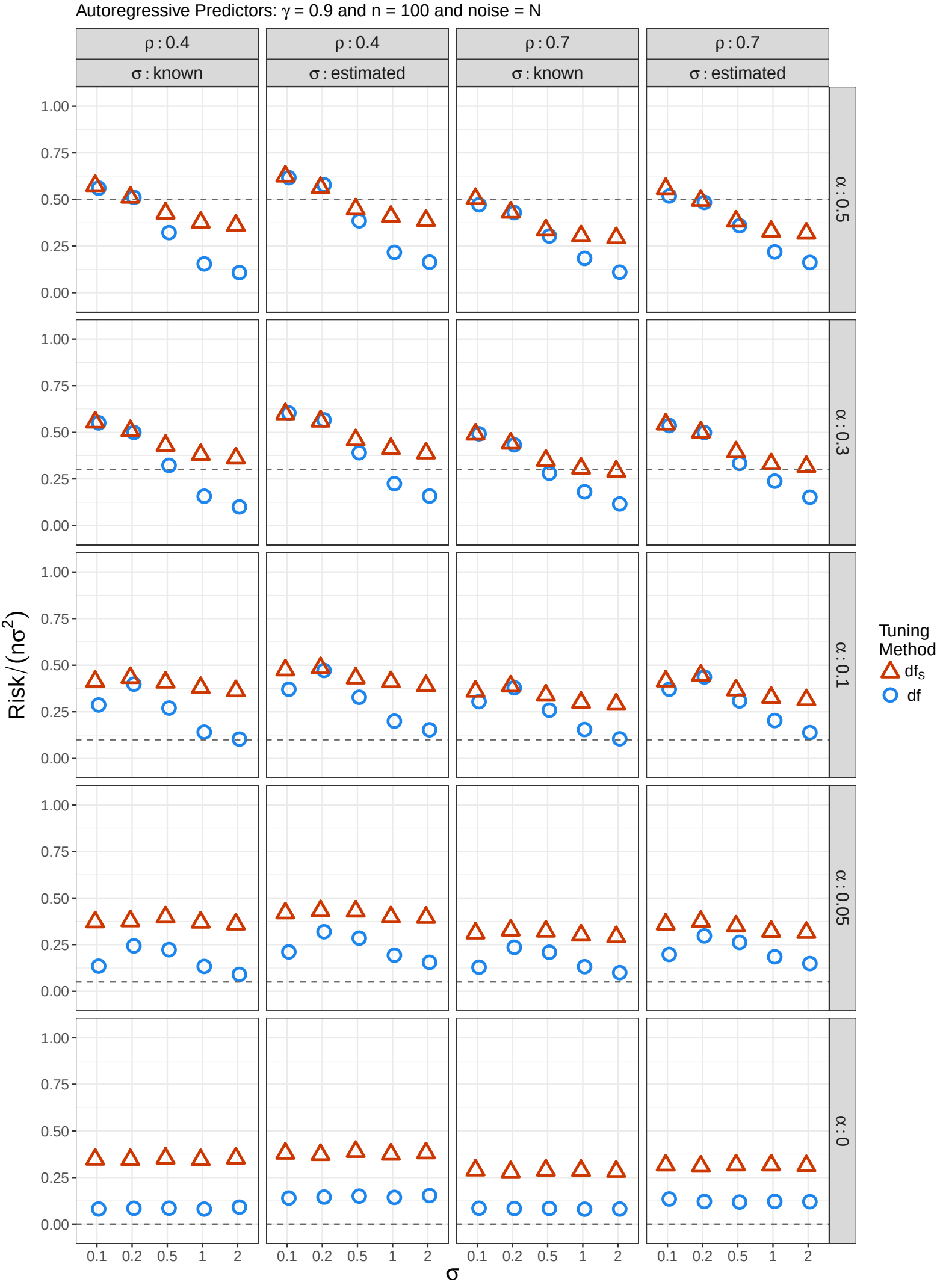
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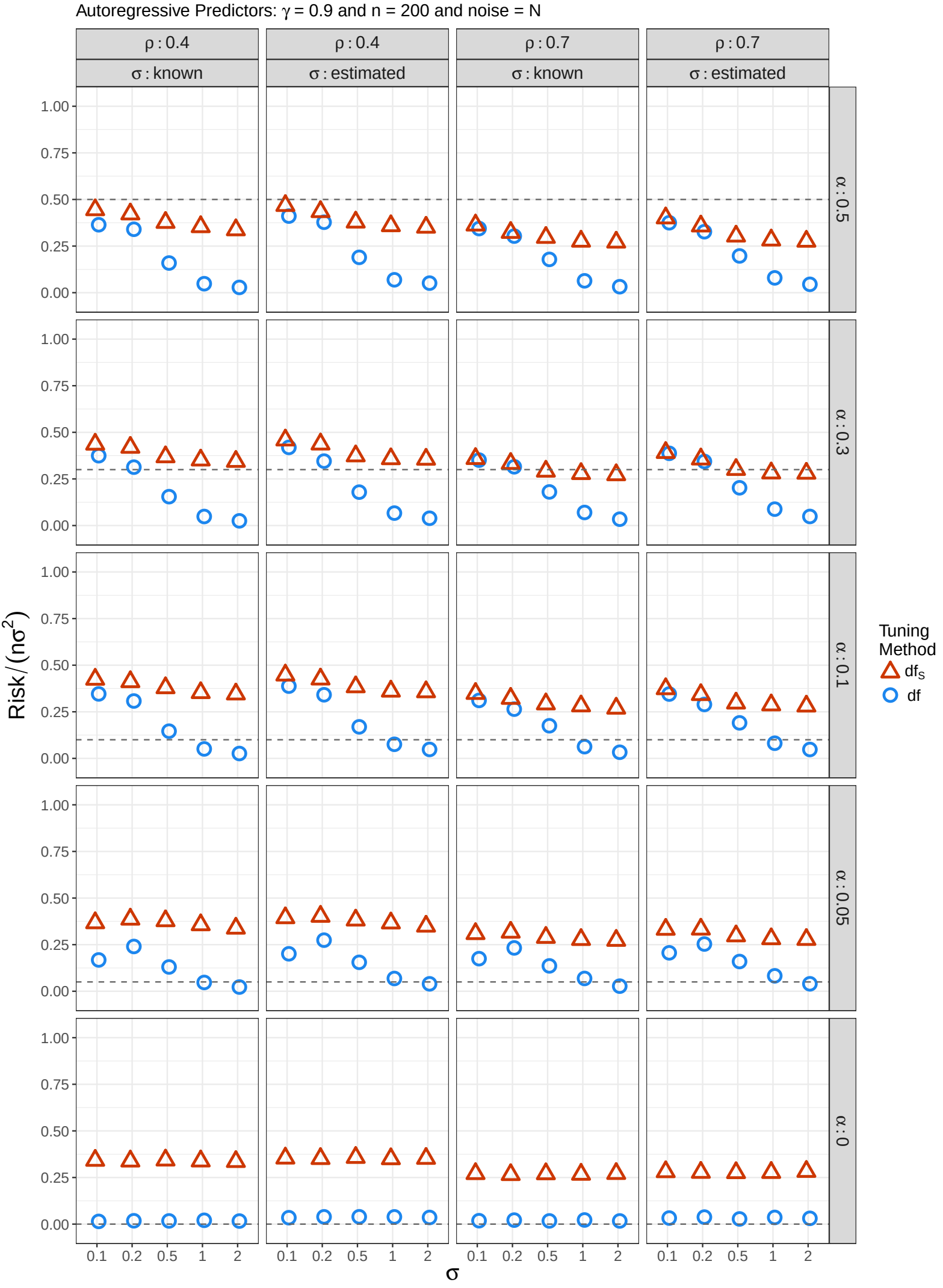
Tuning Method
df_S
df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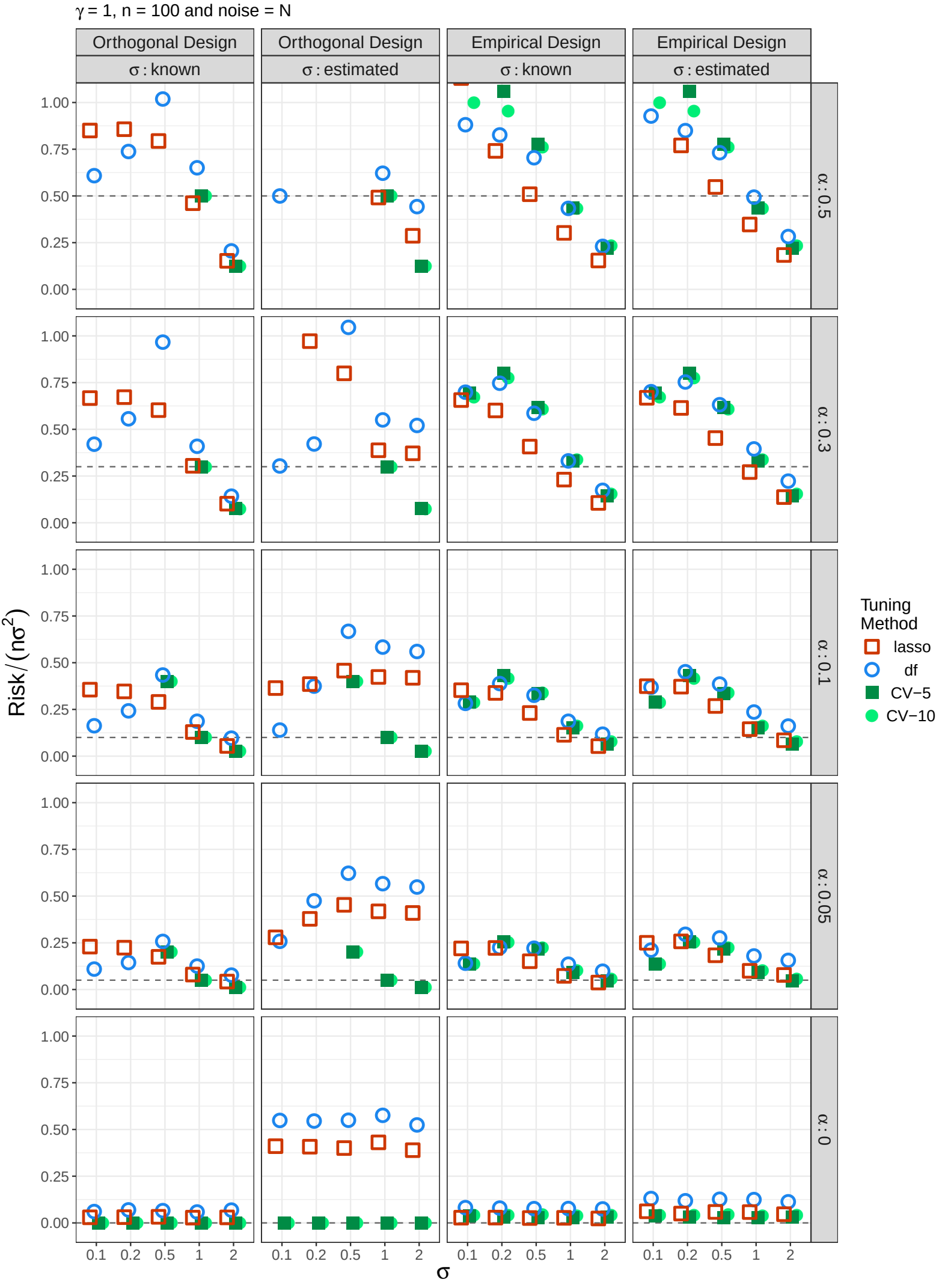
σ
Tuning Method
df_S
df

Plots of the risk estimates relative to $n\sigma^2$ for the estimators $\hat{\mu}_{\text{OLS.l}}^{\hat{\lambda}_{\text{df}}}$, $\hat{\mu}_{\text{OLS.l}}^{\hat{\lambda}_{\text{CV}-5}}$, $\hat{\mu}_{\text{OLS.l}}^{\hat{\lambda}_{\text{CV}-10}}$ and $\hat{\mu}_{\text{lasso}}^{\hat{\lambda}_{\text{df}_S}}$. The dashed lines are the relative risks for the oracle-OLS estimator.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0.1
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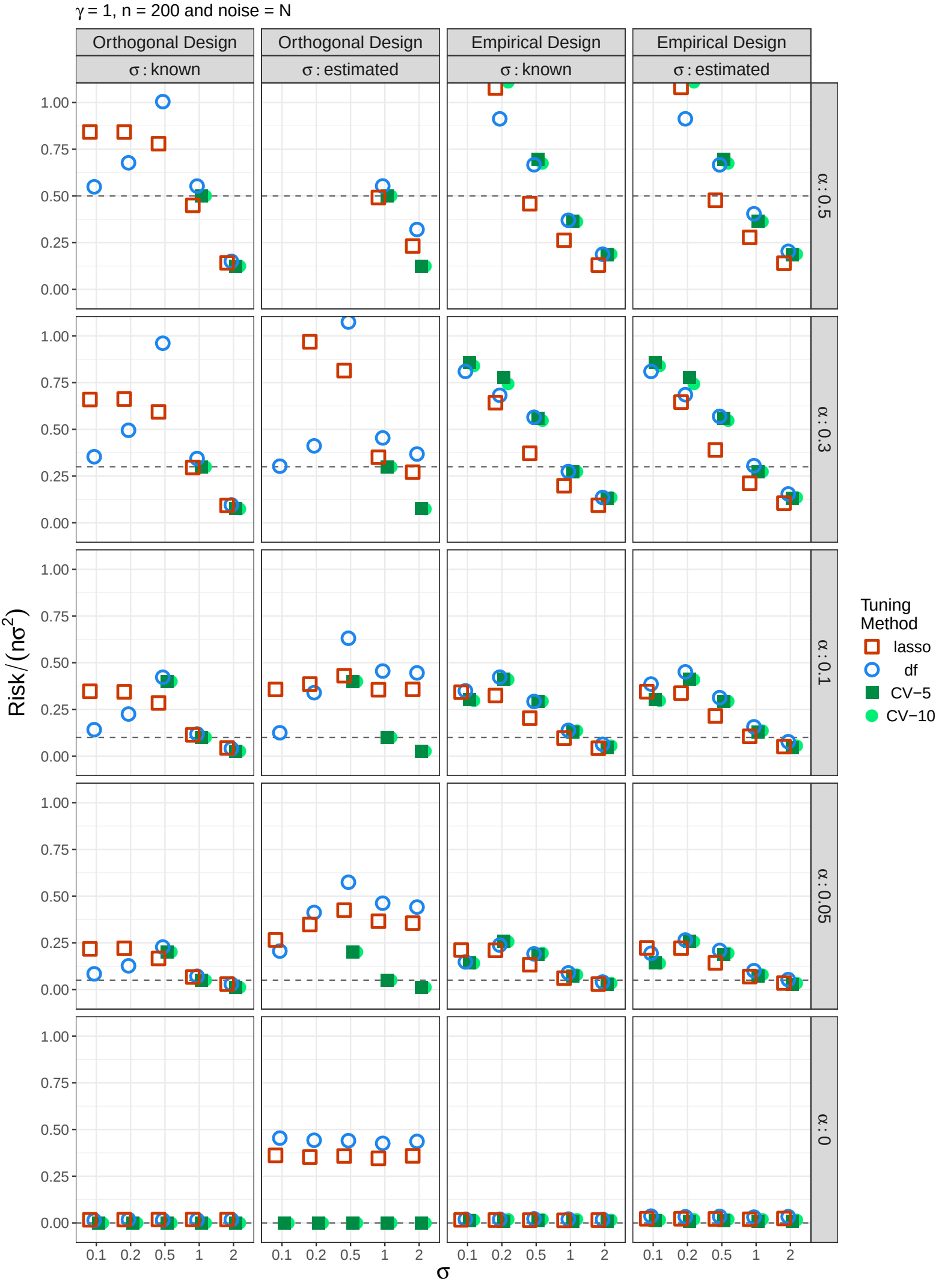
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10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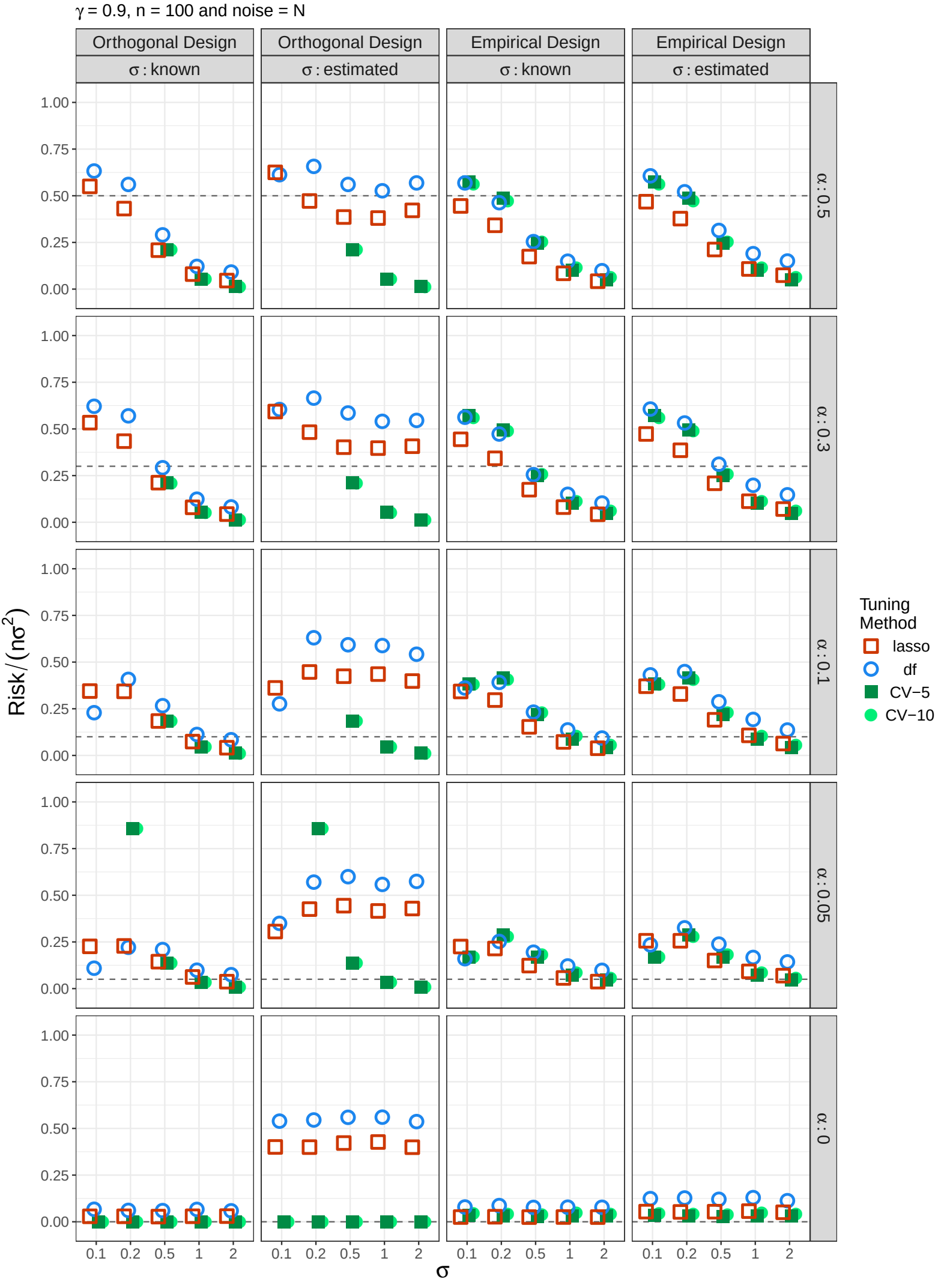
1.00
Tuning Method
lasso
df
CV−5
CV−10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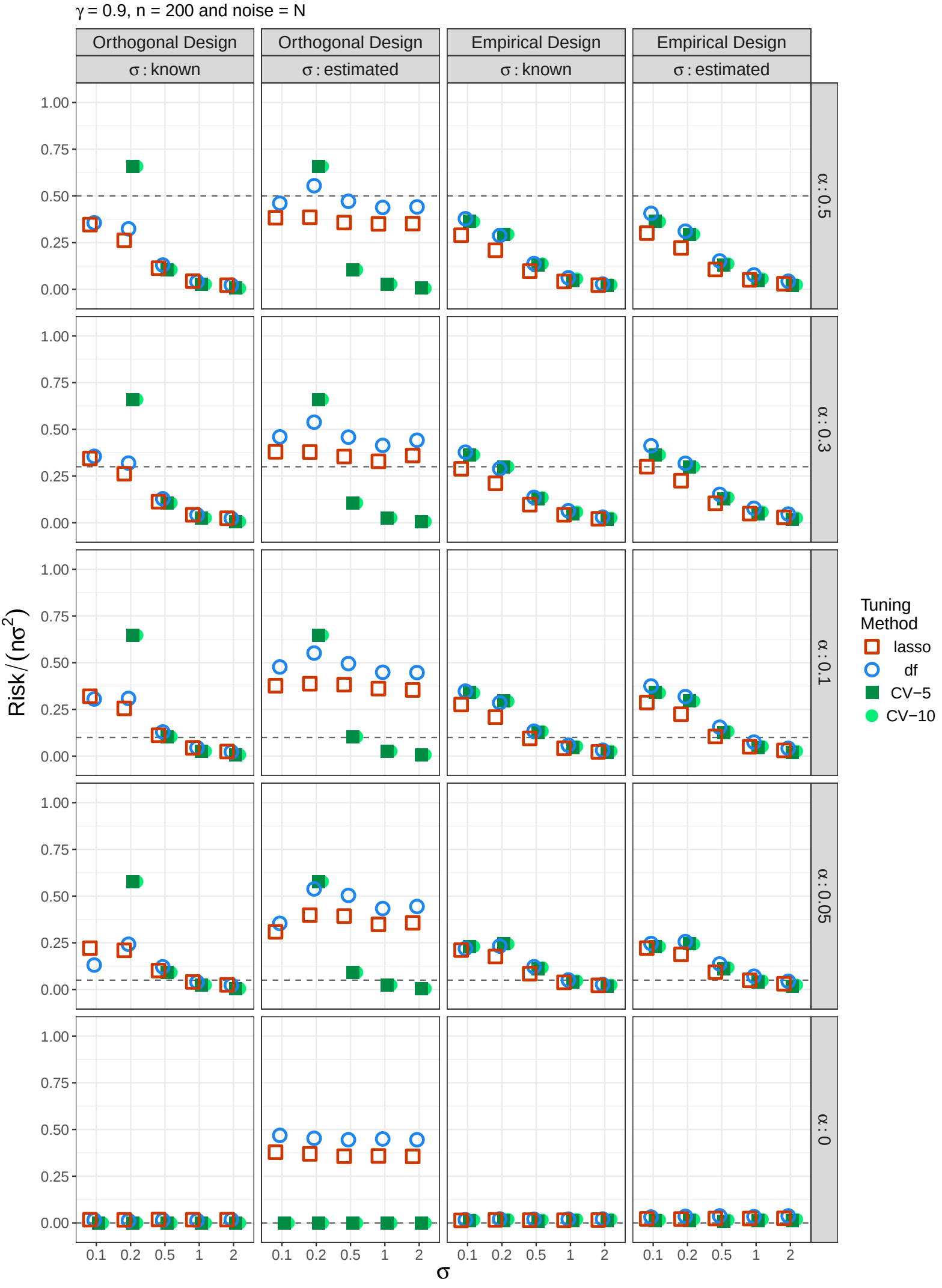
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10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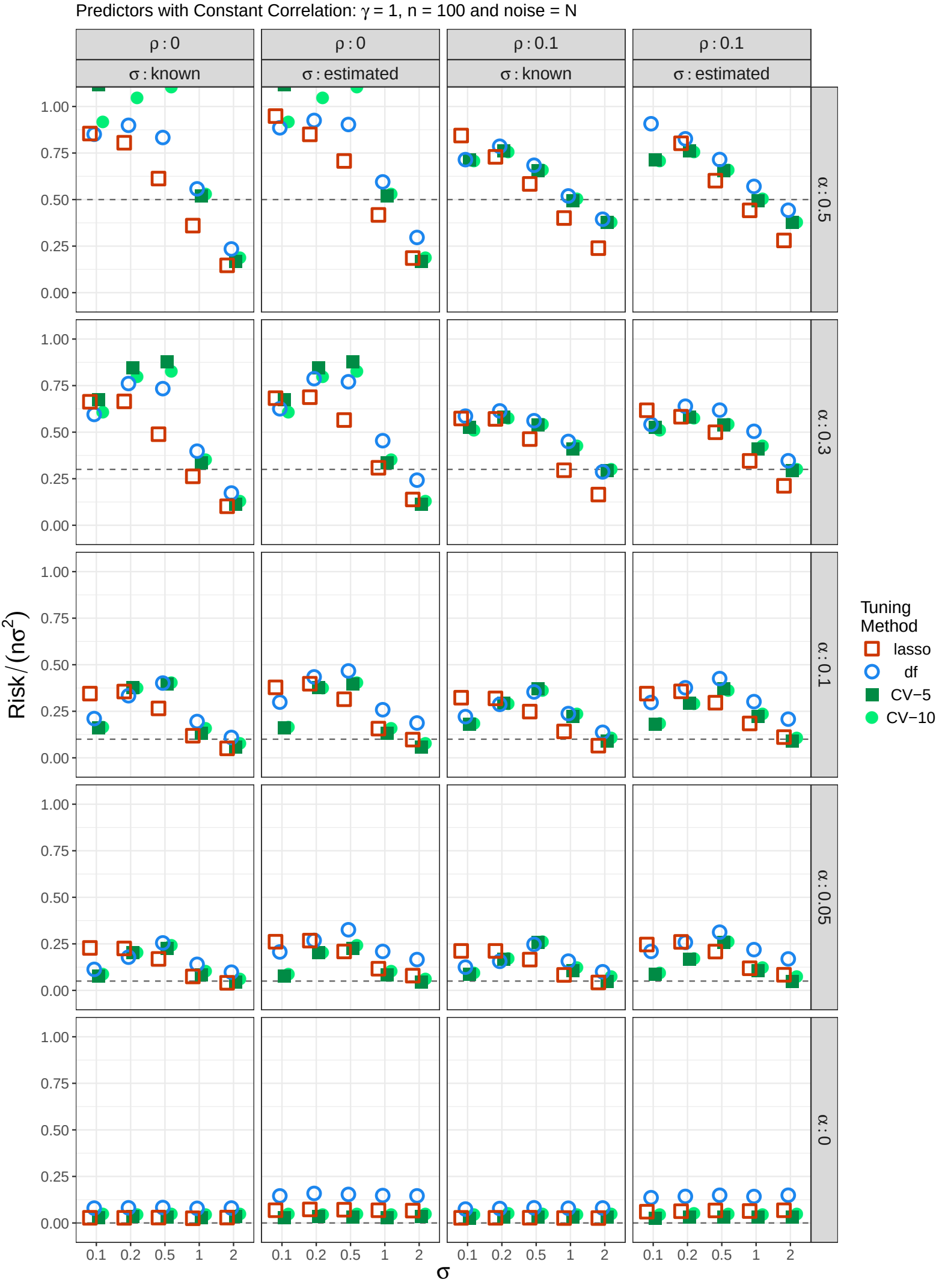
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 100 and noise = N
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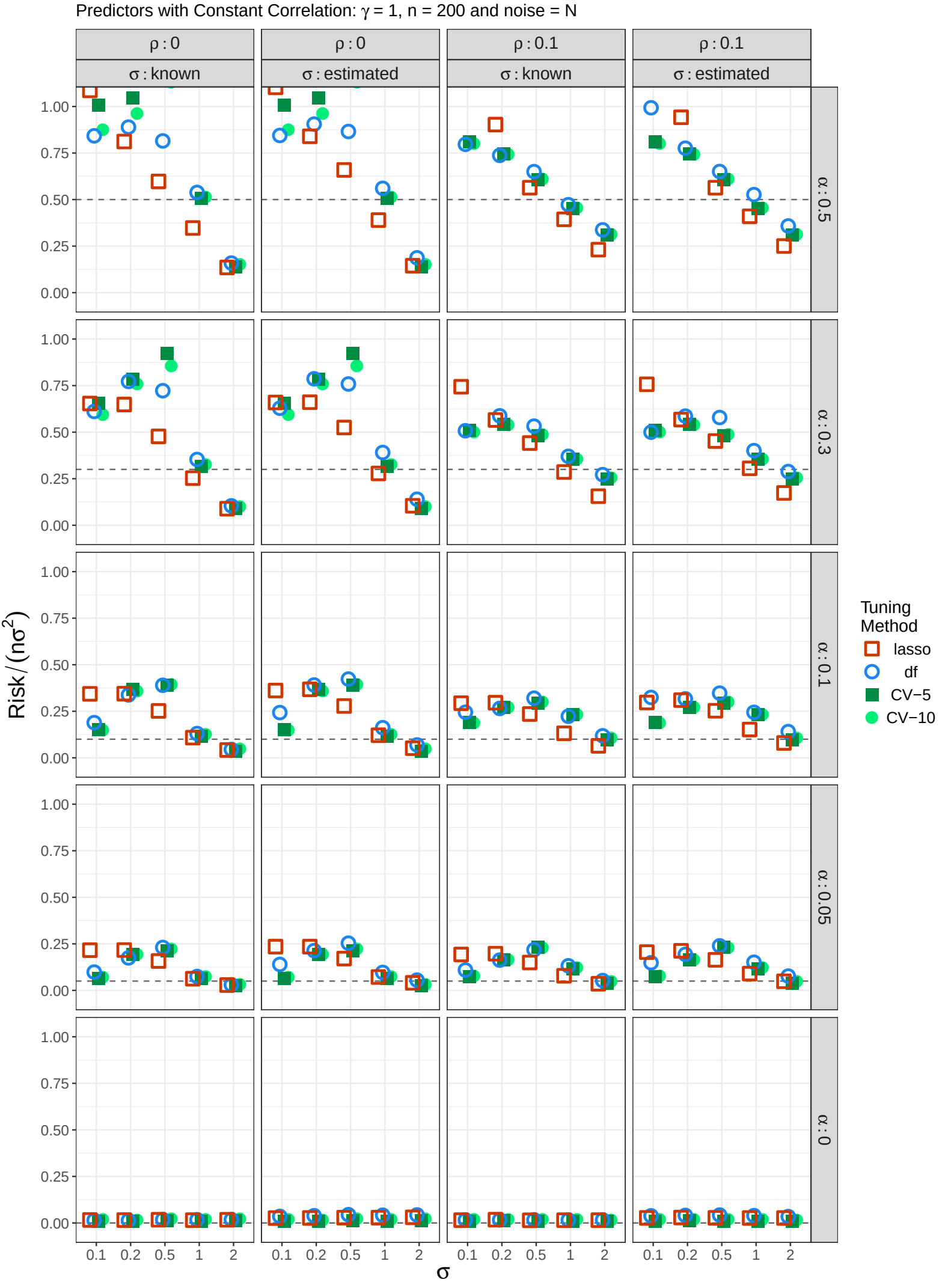
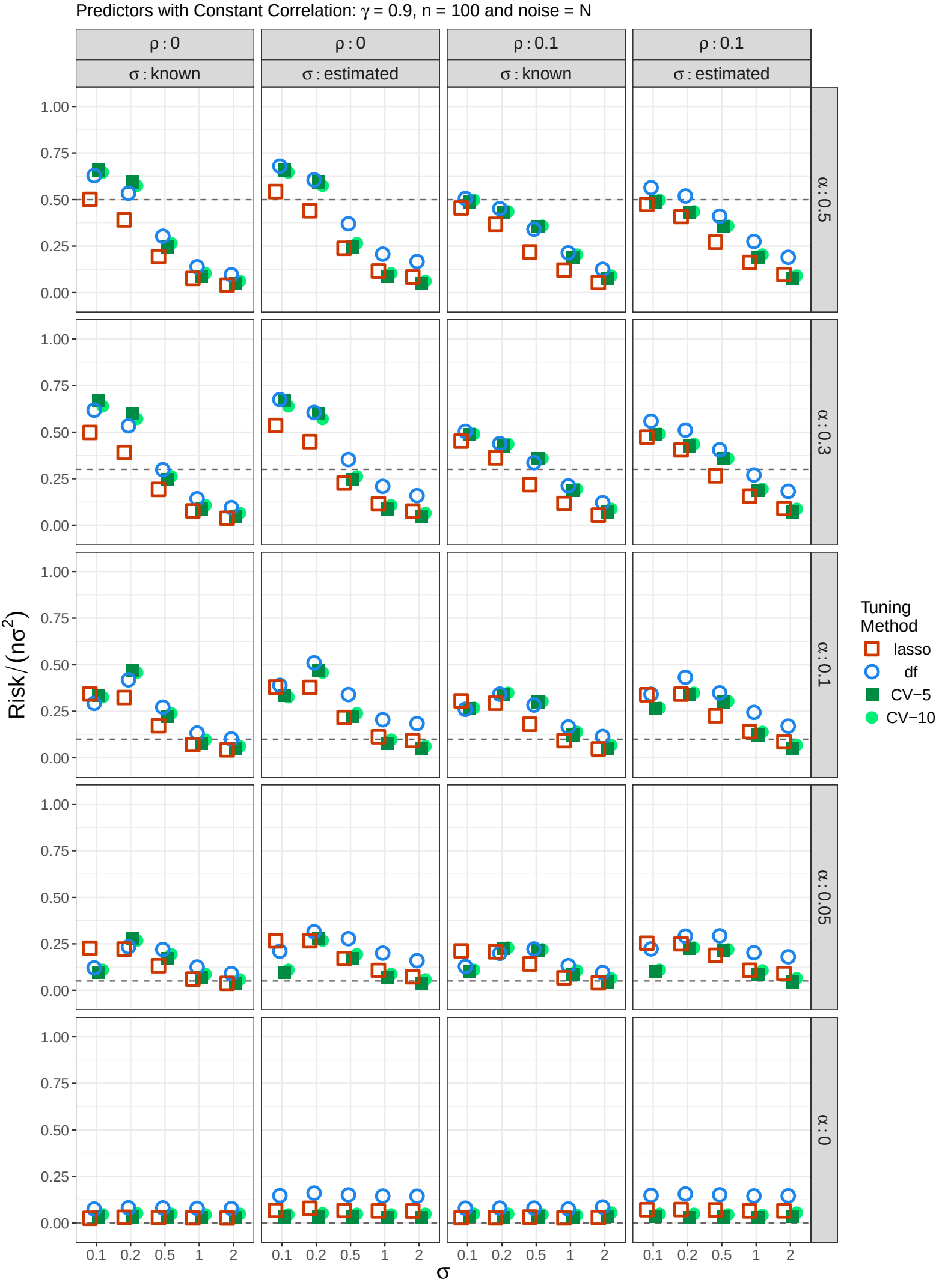
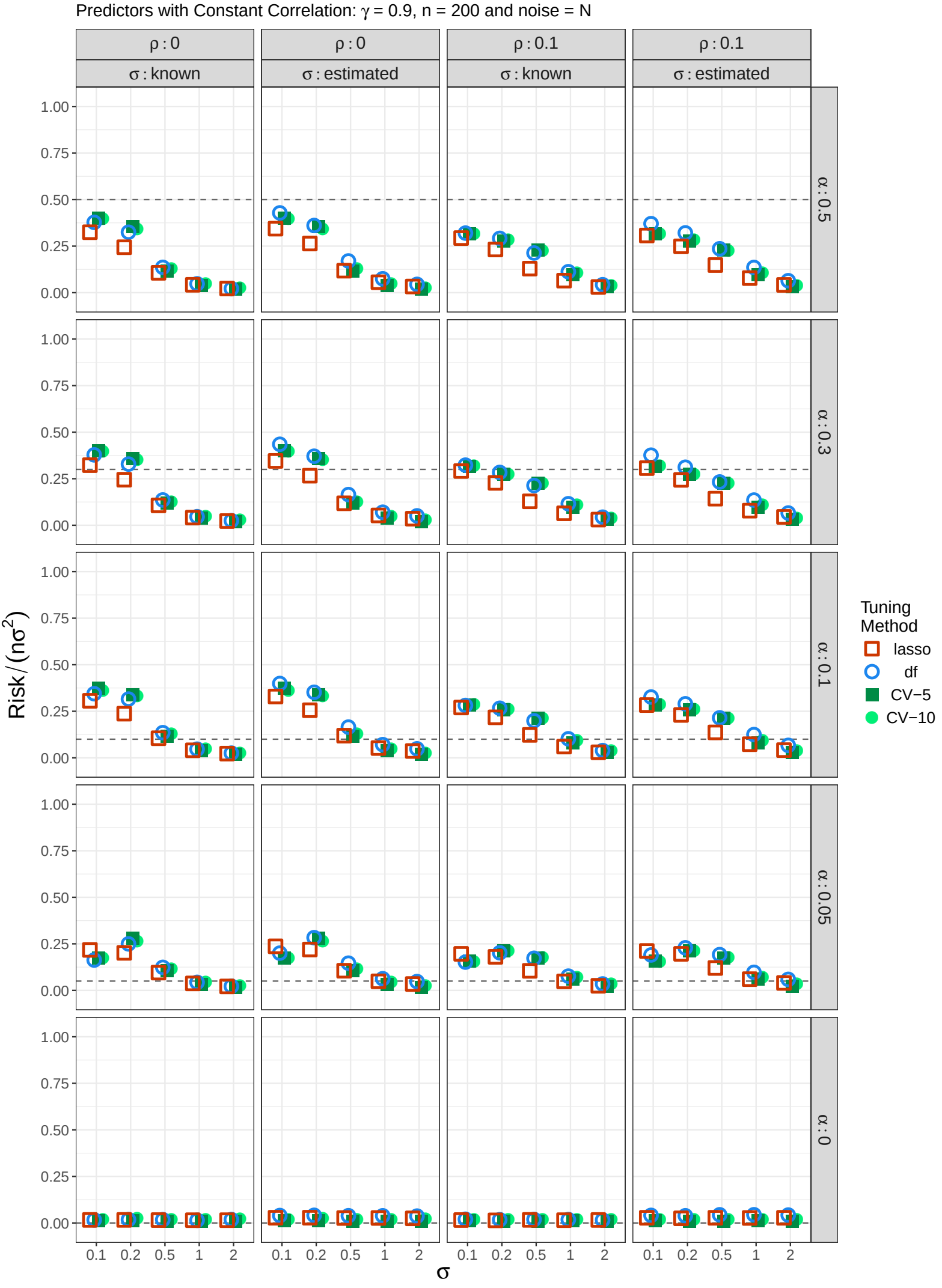
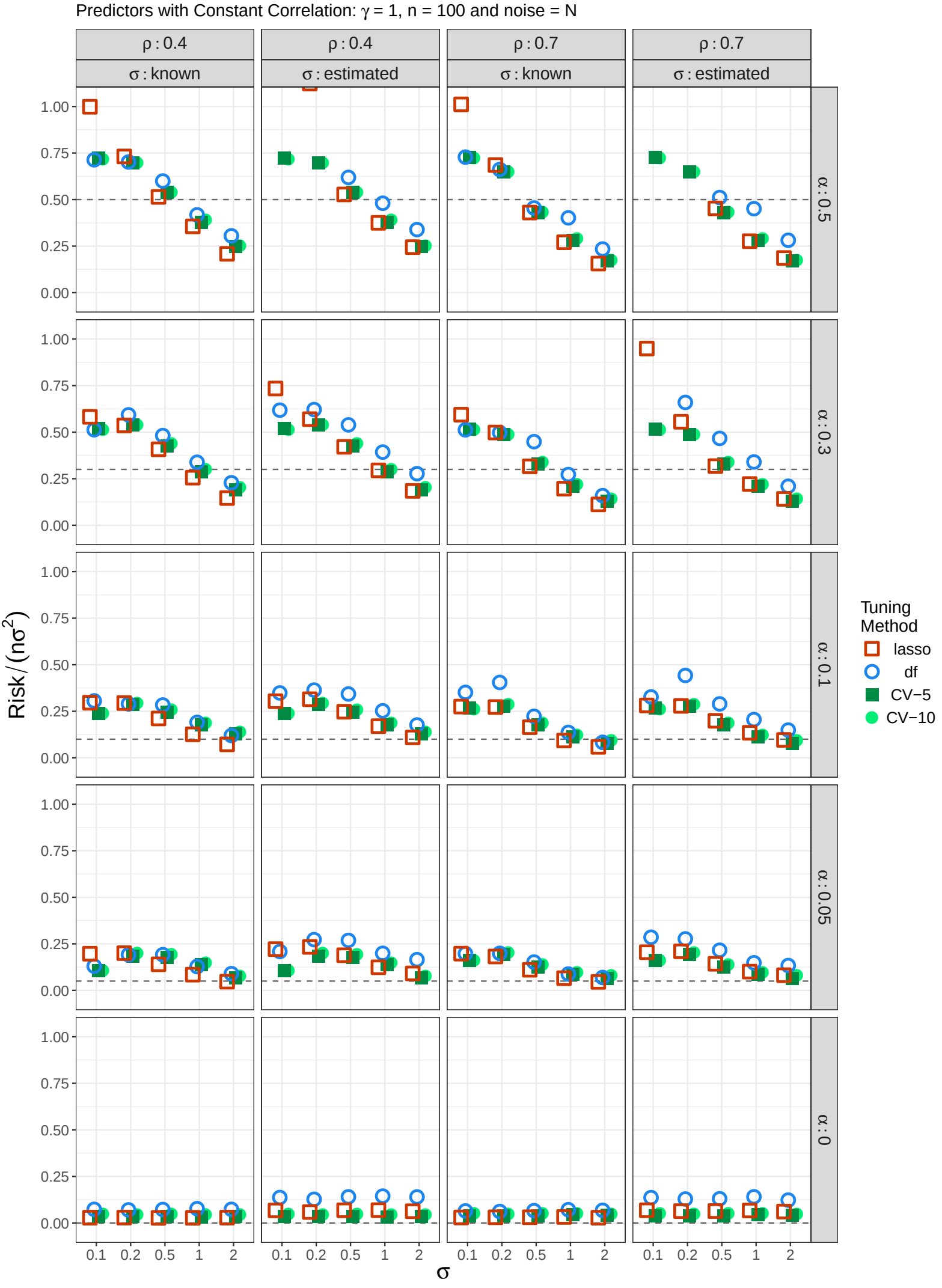
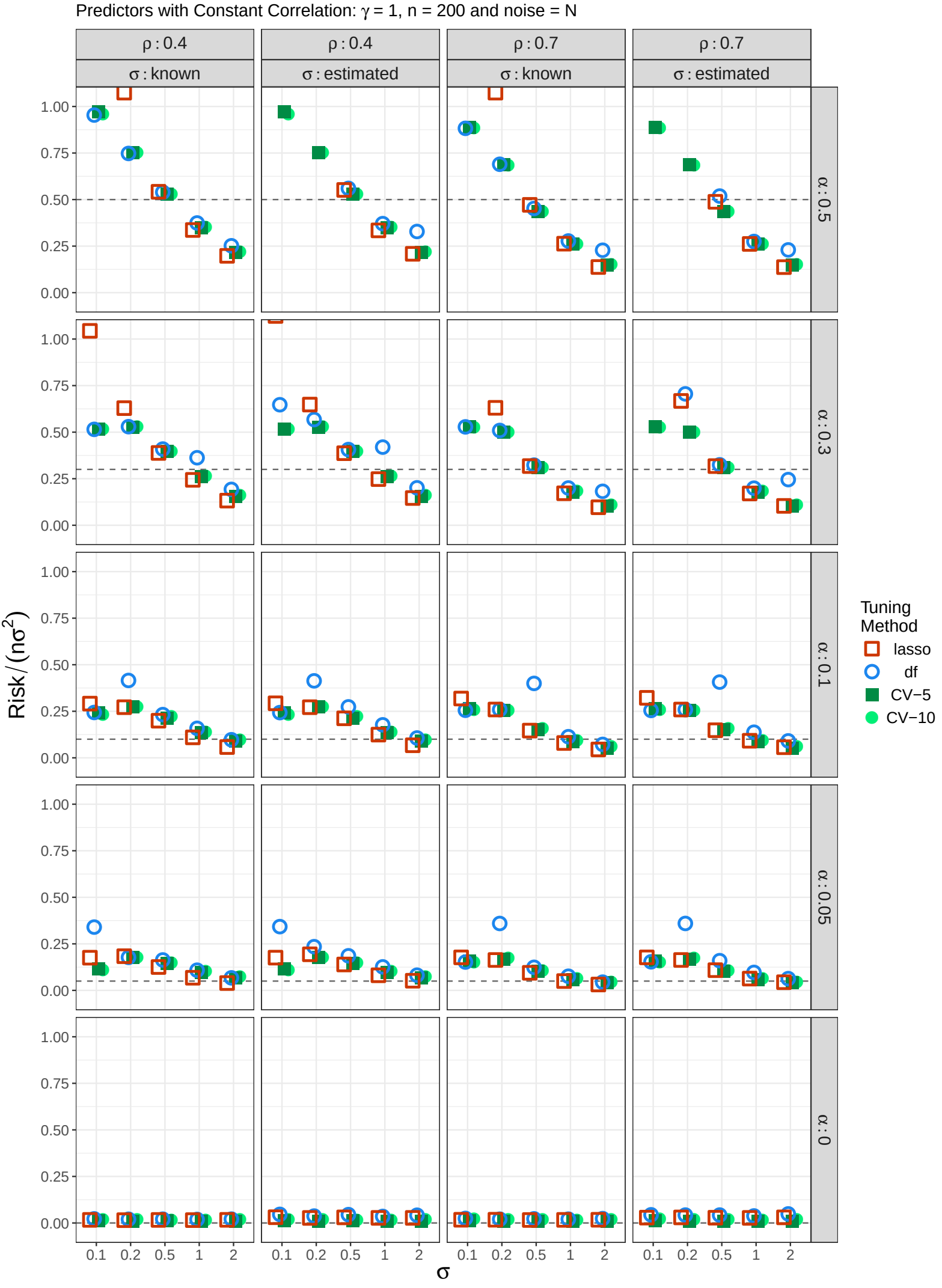
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

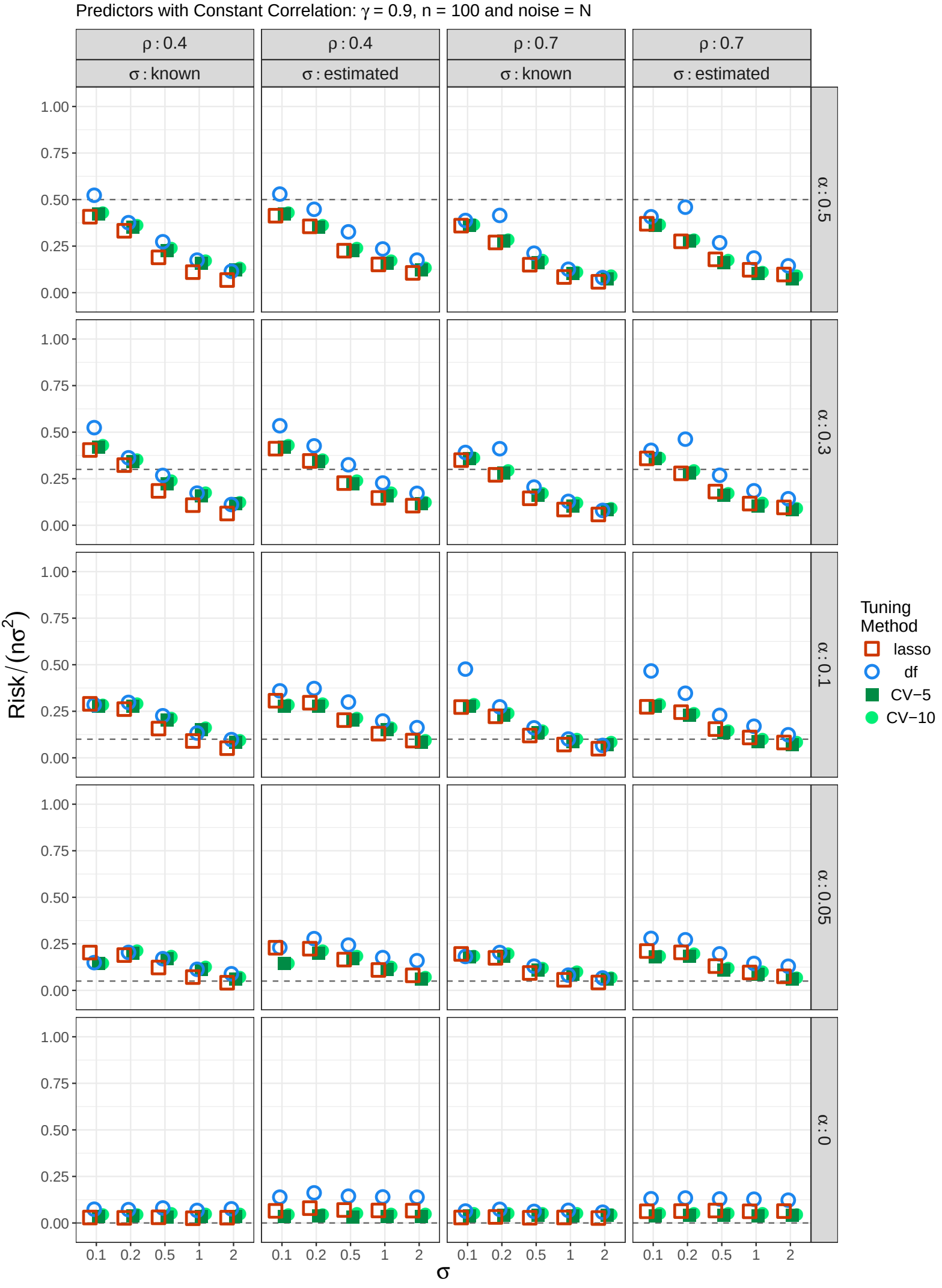

Predictors with Constant Correlation: γ = 1, n = 200 and noise = N
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = N
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = N
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 200 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = N
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
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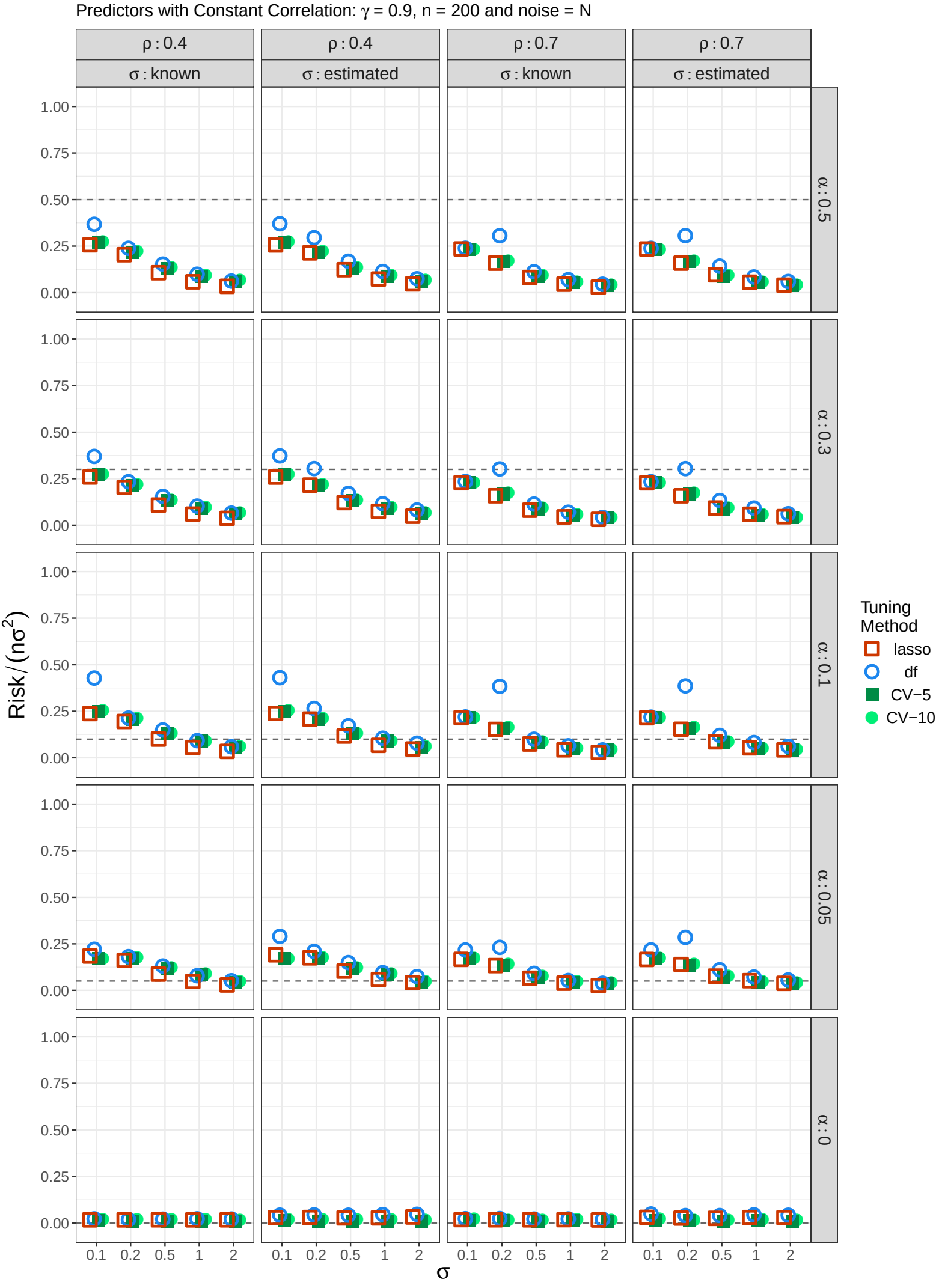
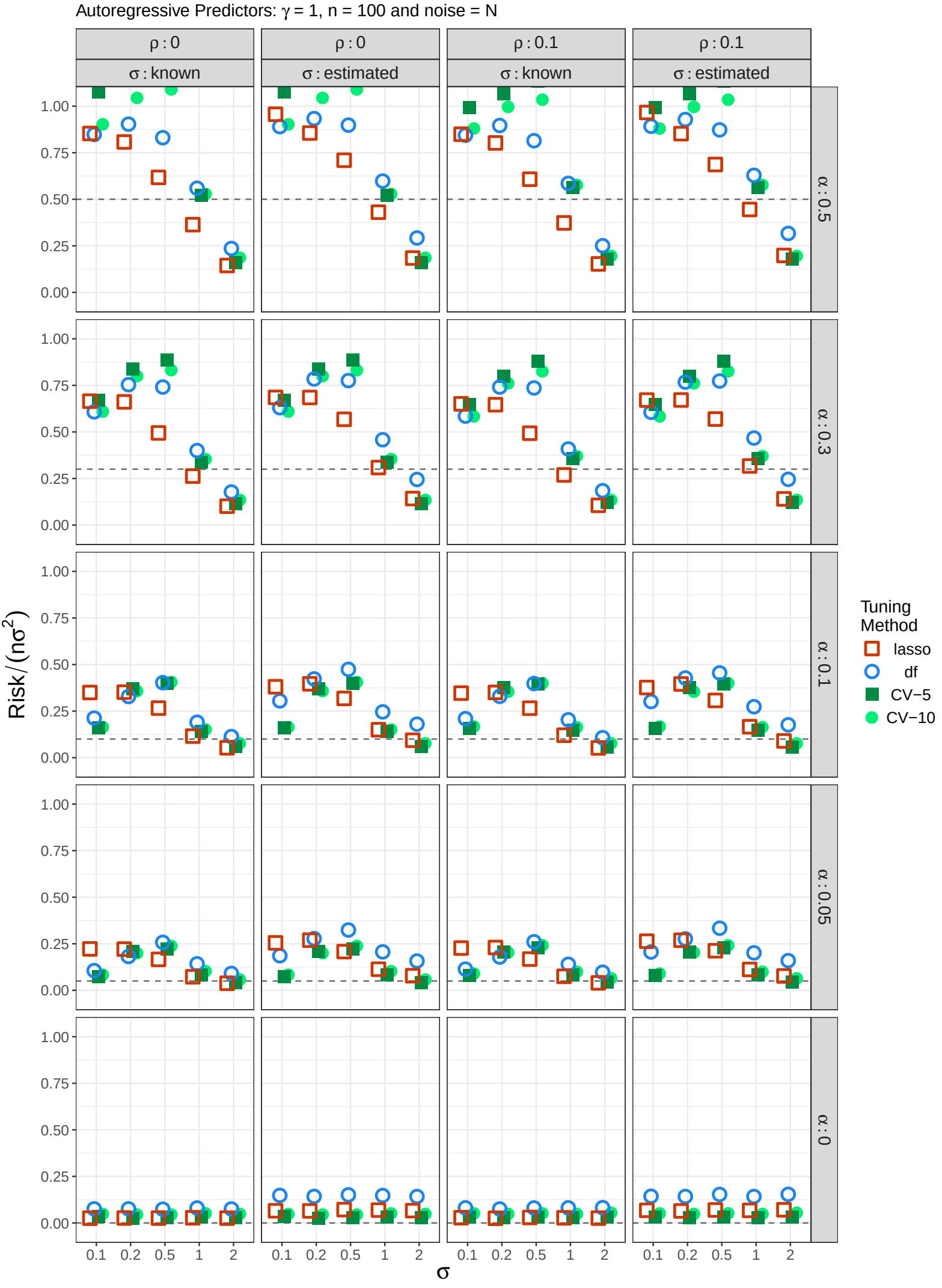
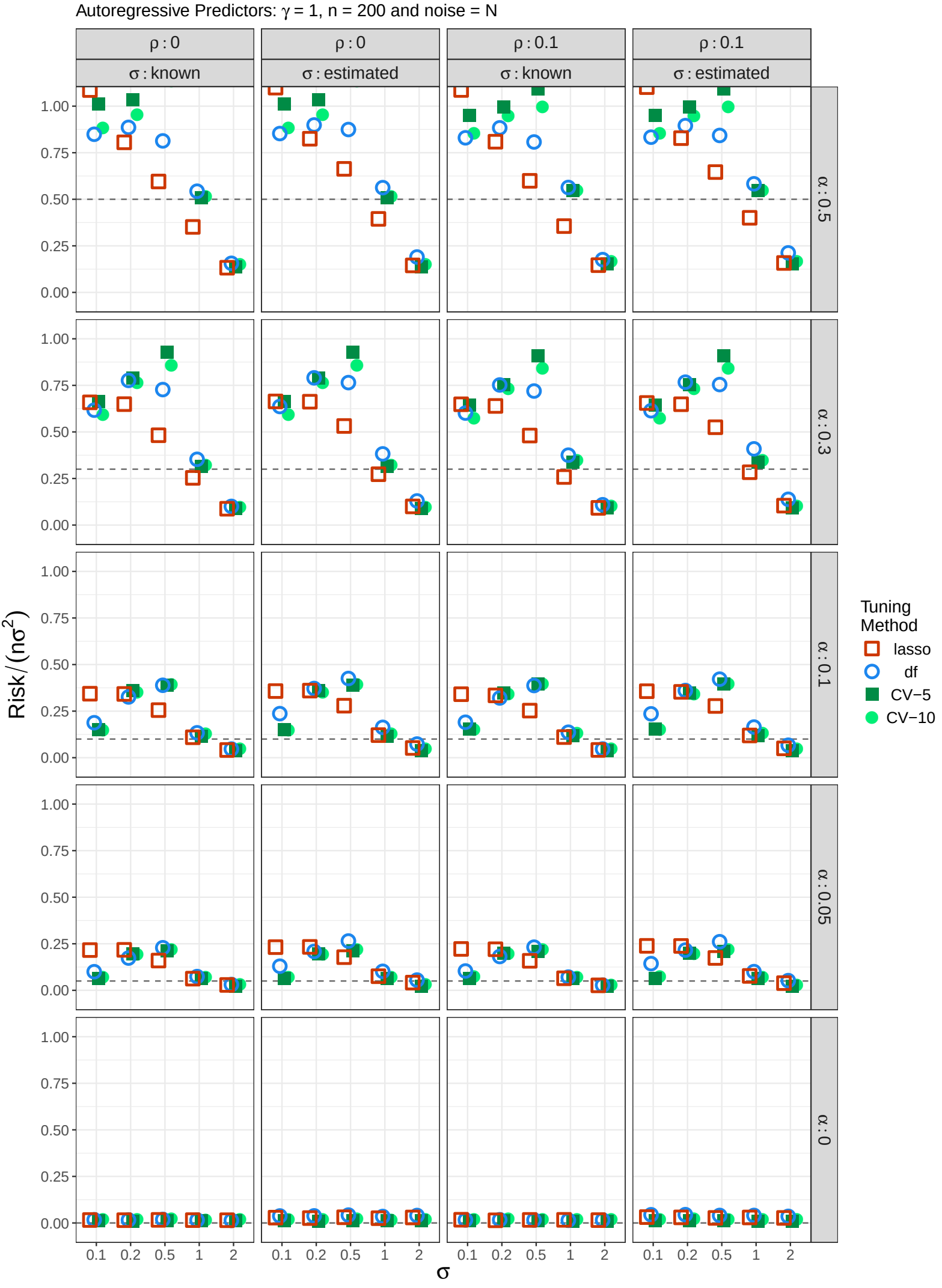
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = N
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 100 and noise = N
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 200 and noise = N
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
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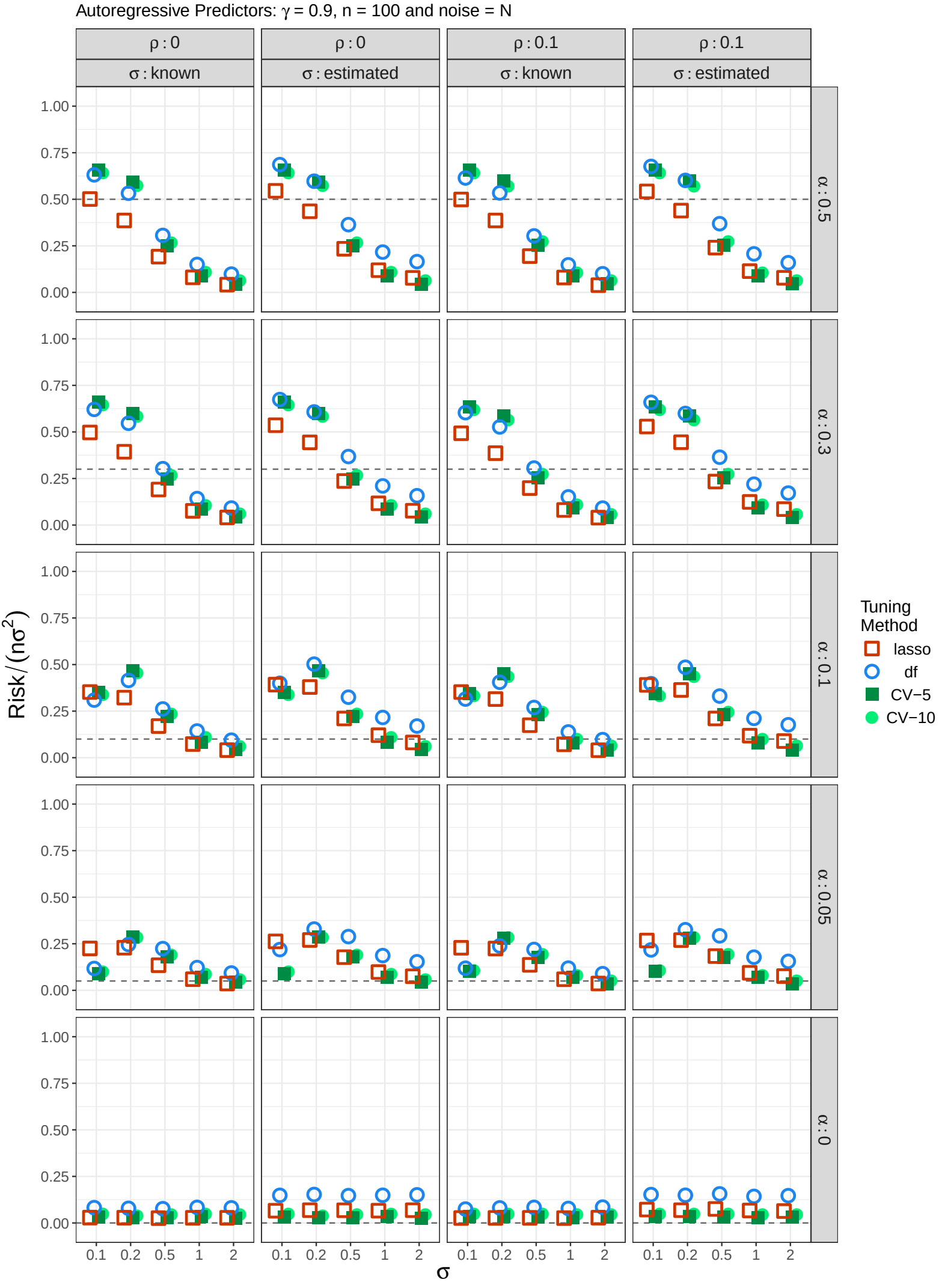
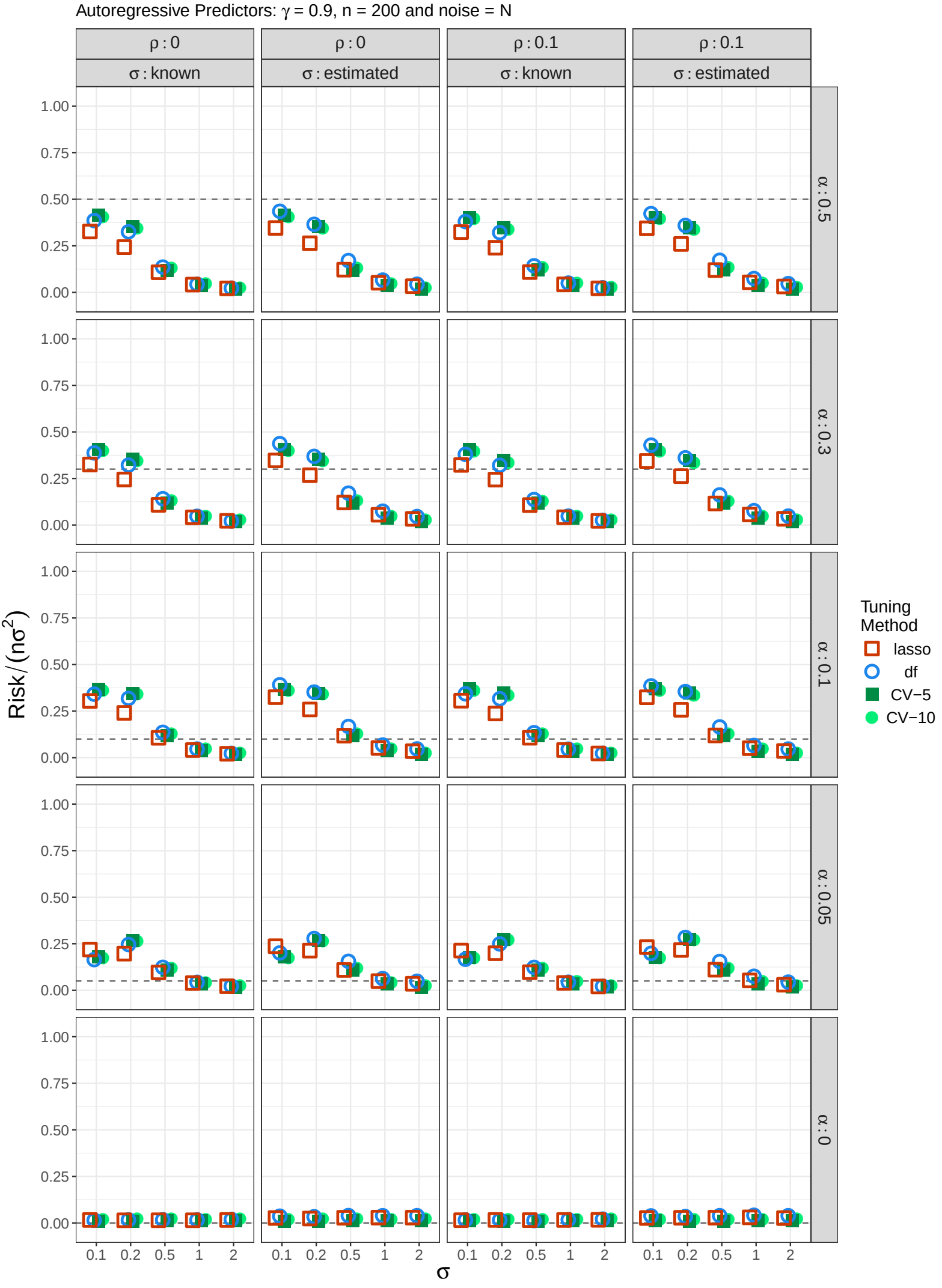
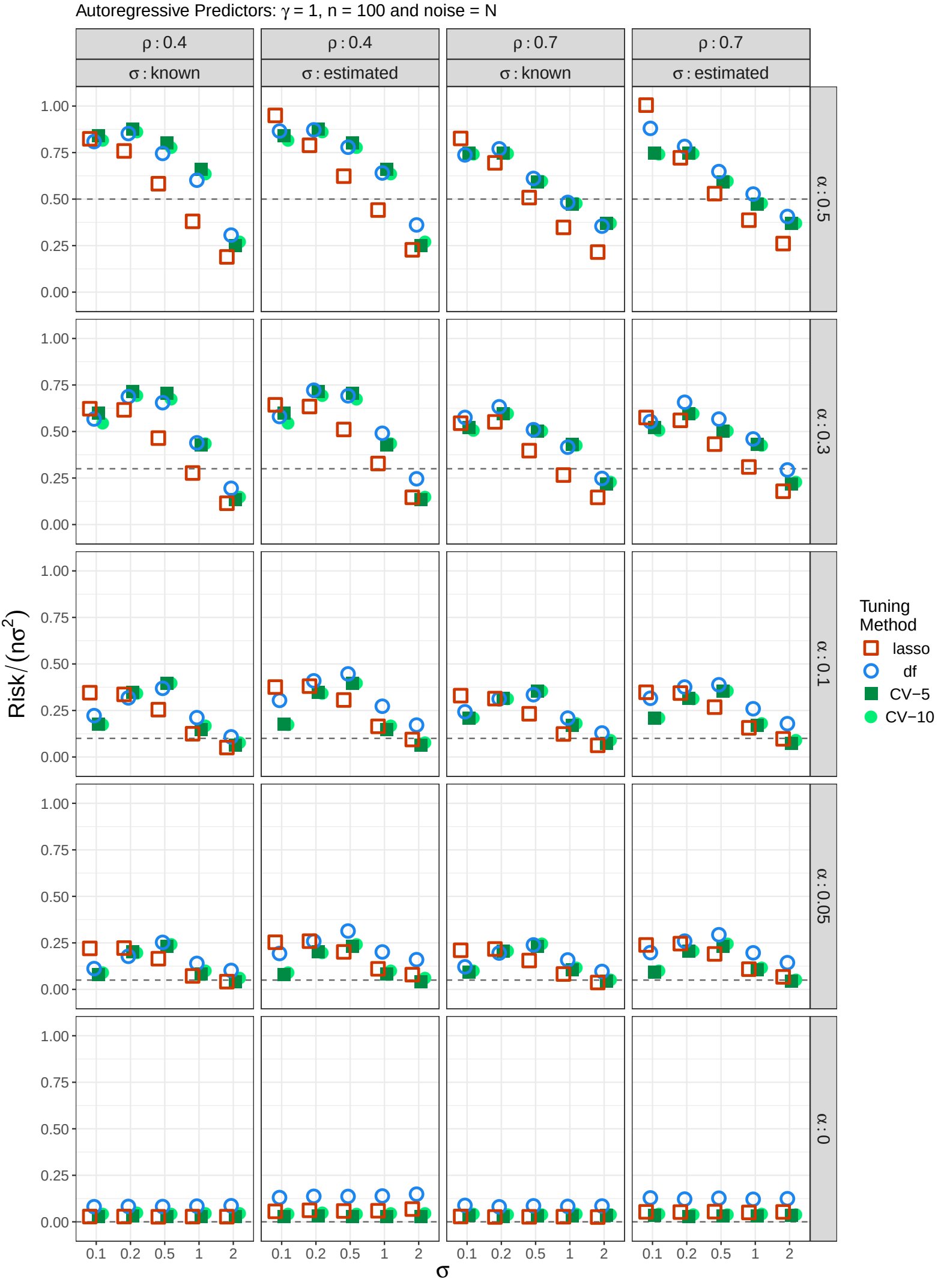
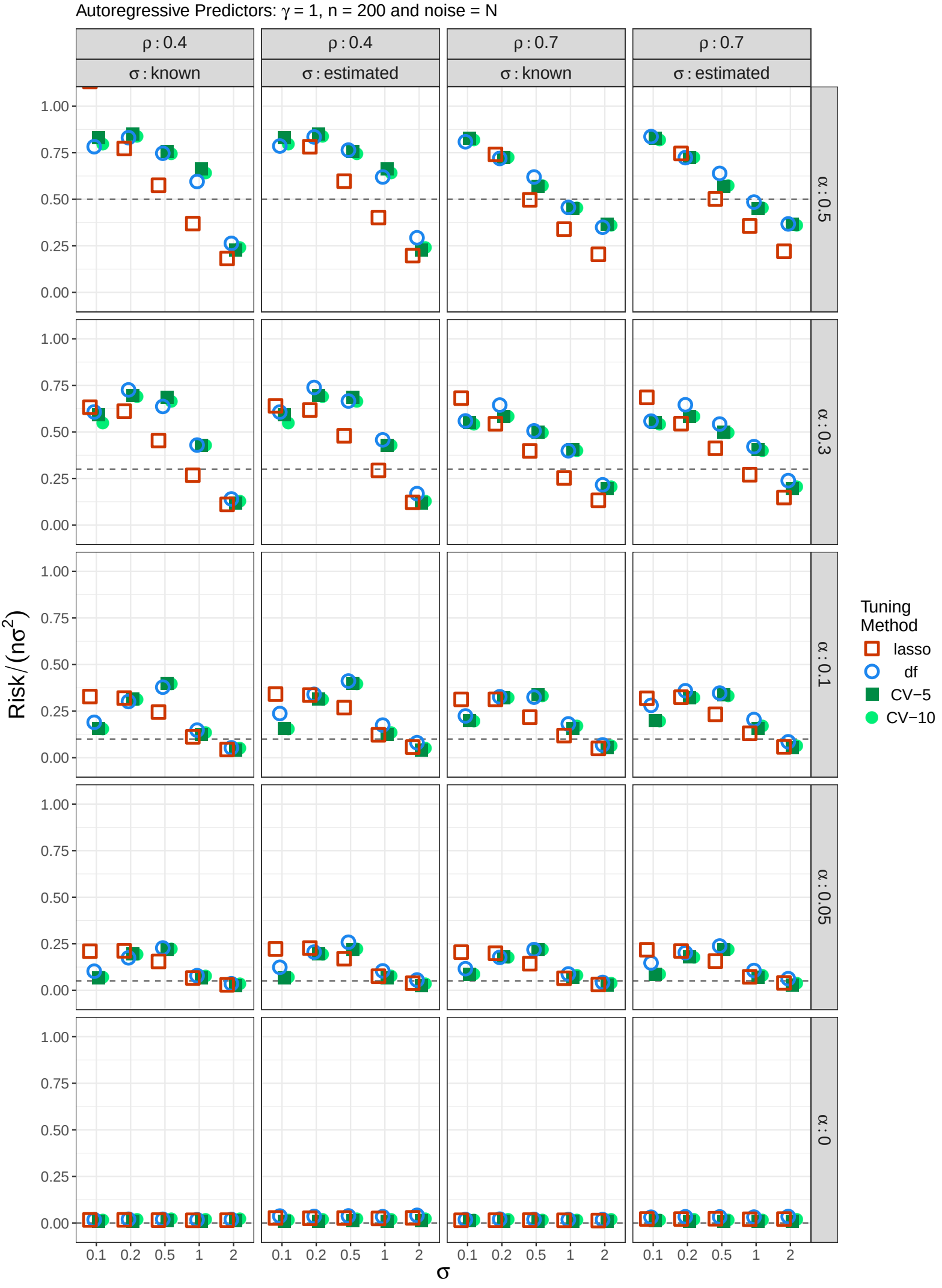
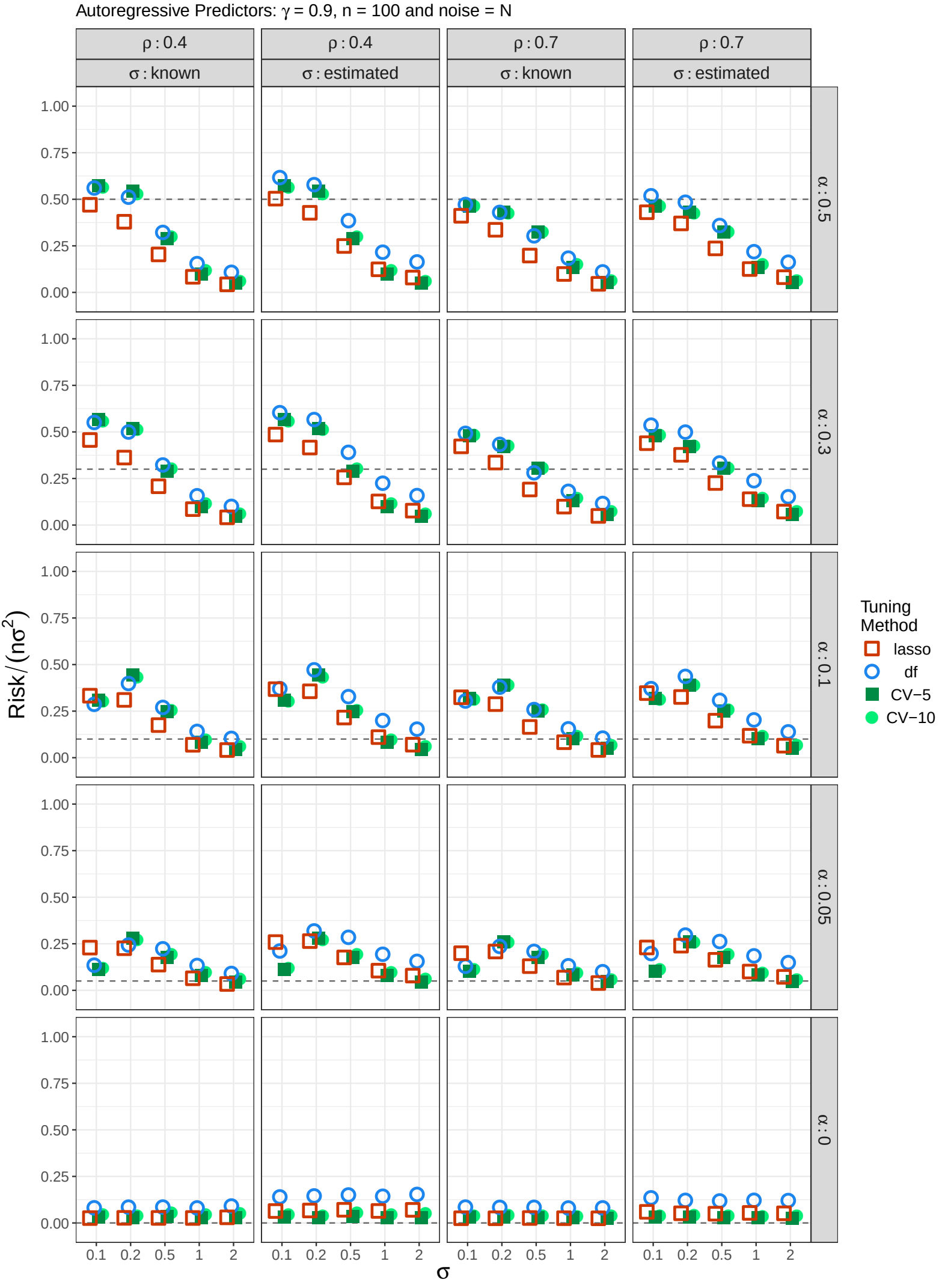
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

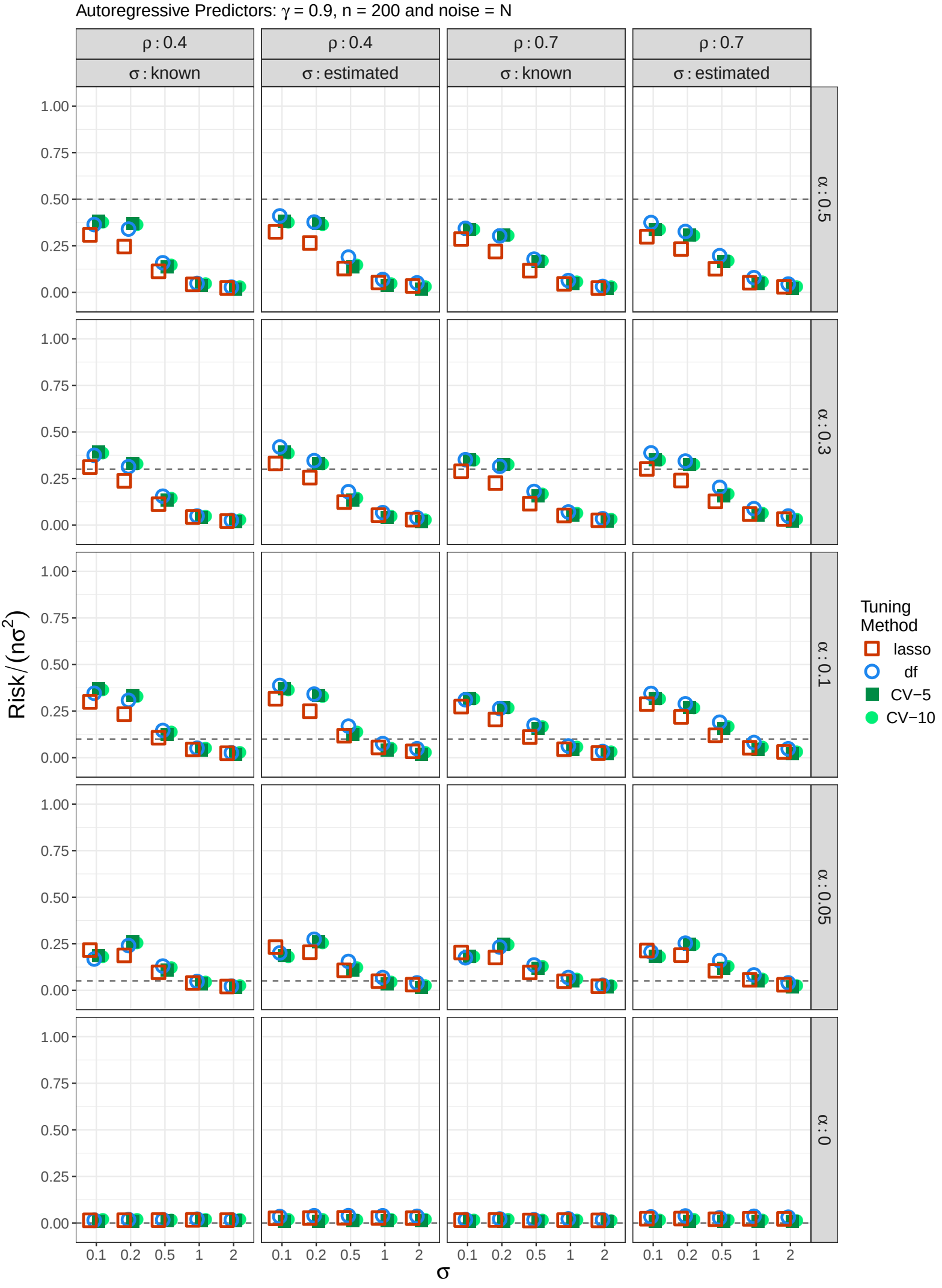

Autoregressive Predictors: γ = 0.9, n = 100 and noise = N
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

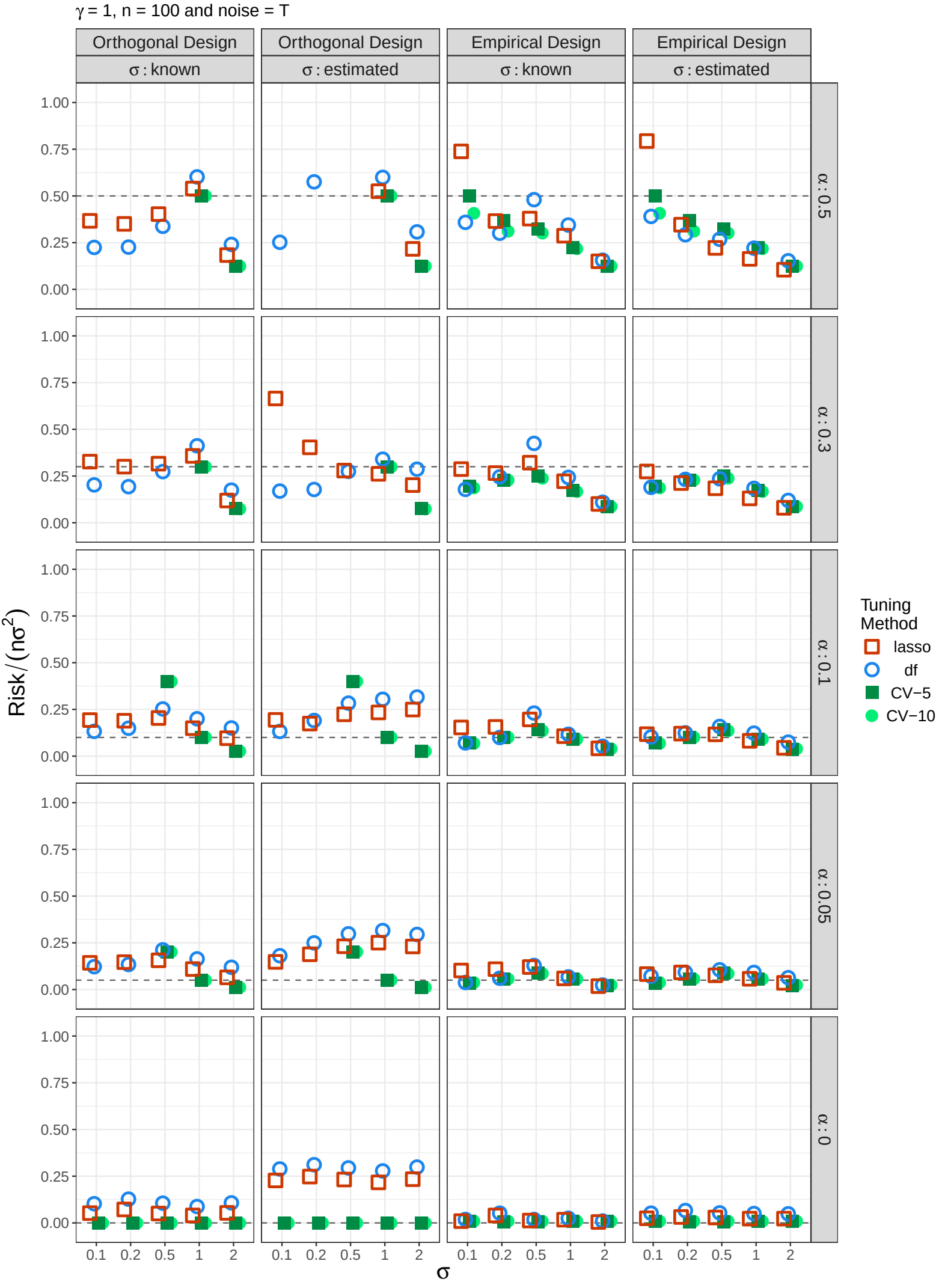

Autoregressive Predictors: γ = 0.9, n = 200 and noise = N
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

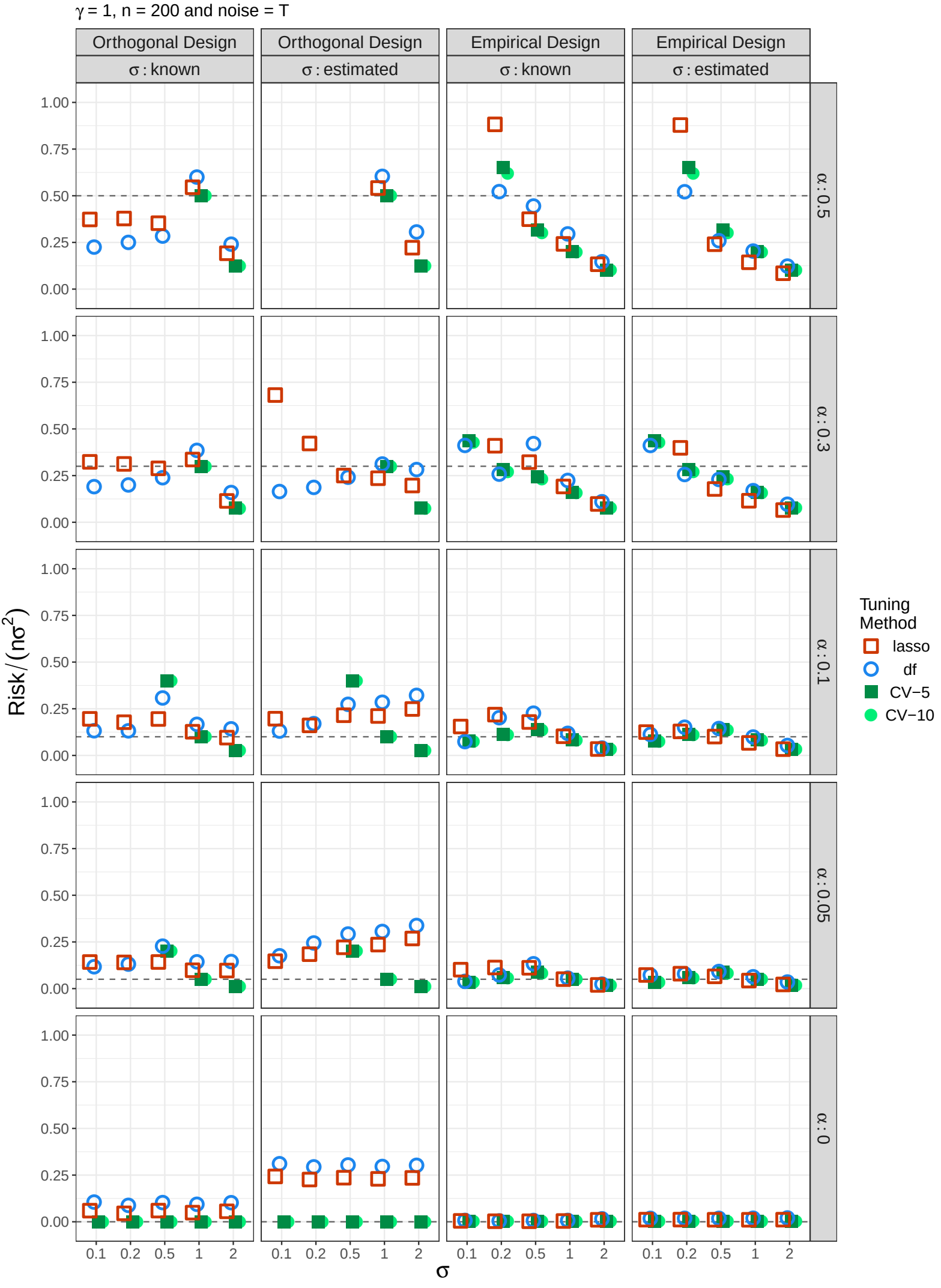

Autoregressive Predictors: γ = 1, n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

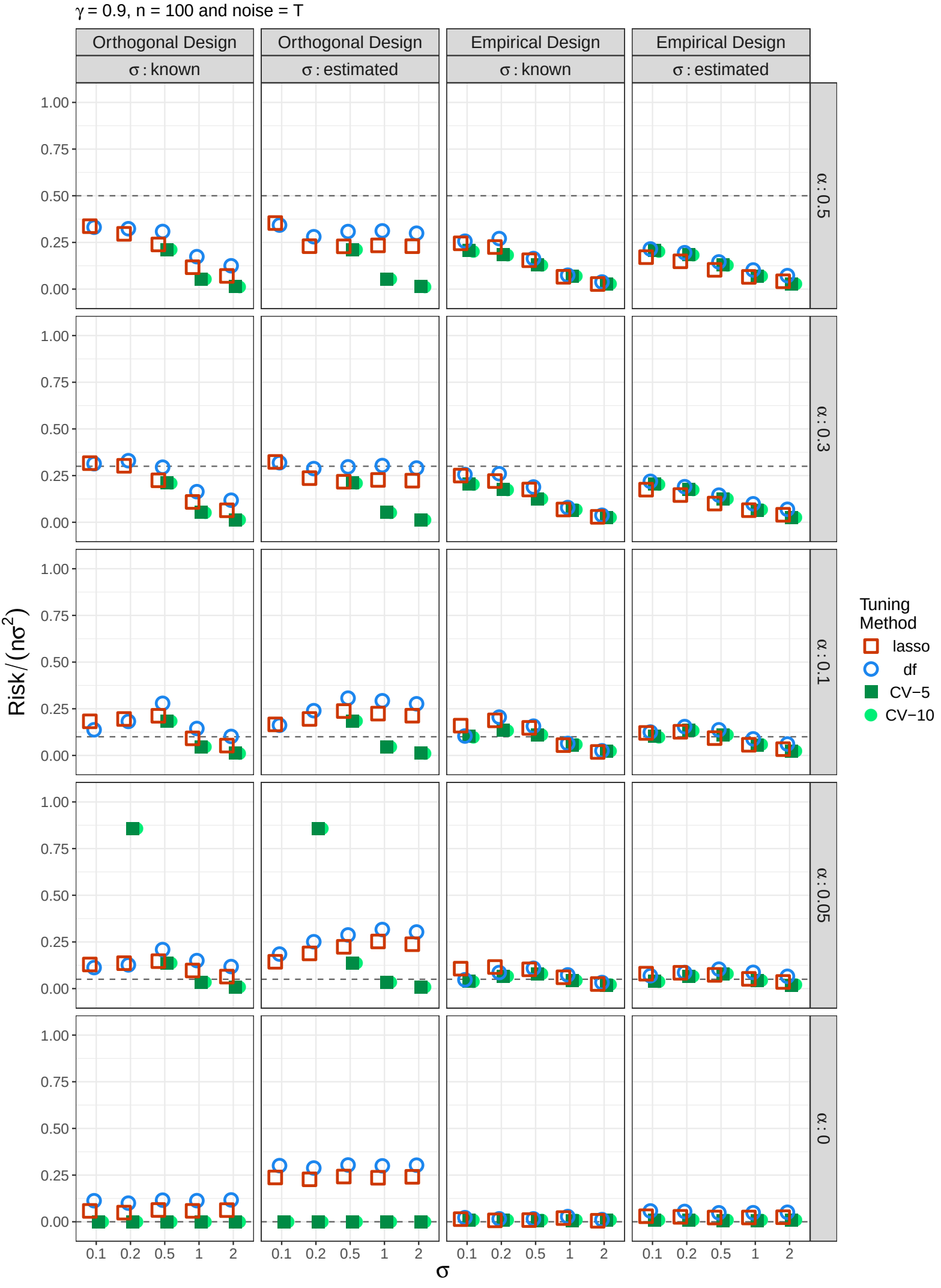

Autoregressive Predictors: γ = 1, n = 200 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

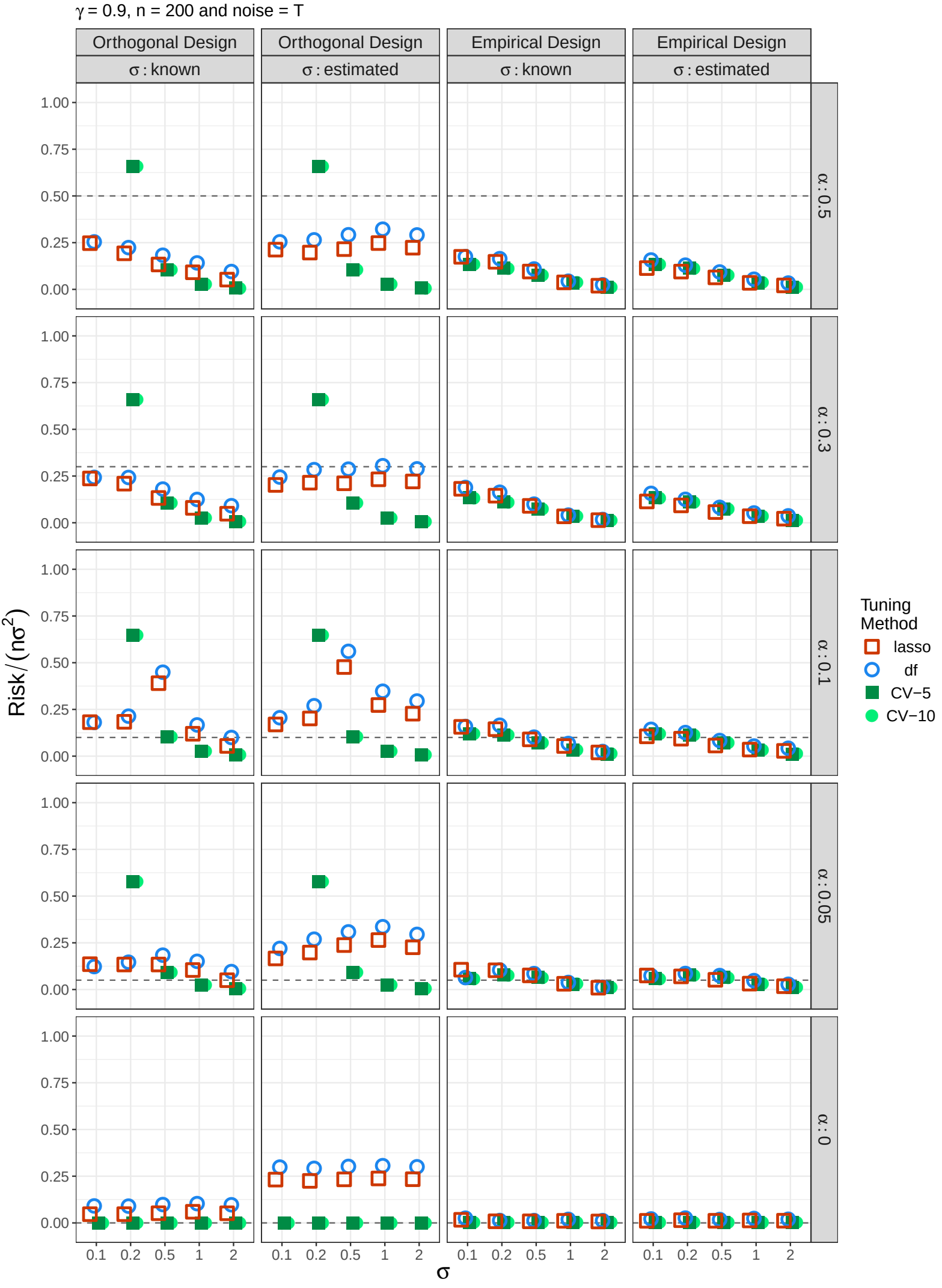

Autoregressive Predictors: γ = 0.9, n = 100 and noise = N
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 200 and noise = N
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
1.00
0.75
0.50
0.25
0.00
Risk/(nσ²)
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


γ = 1, n = 100 and noise = T
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


γ = 1, n = 200 and noise = T
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


γ = 0.9, n = 100 and noise = T
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


γ = 0.9, n = 200 and noise = T
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
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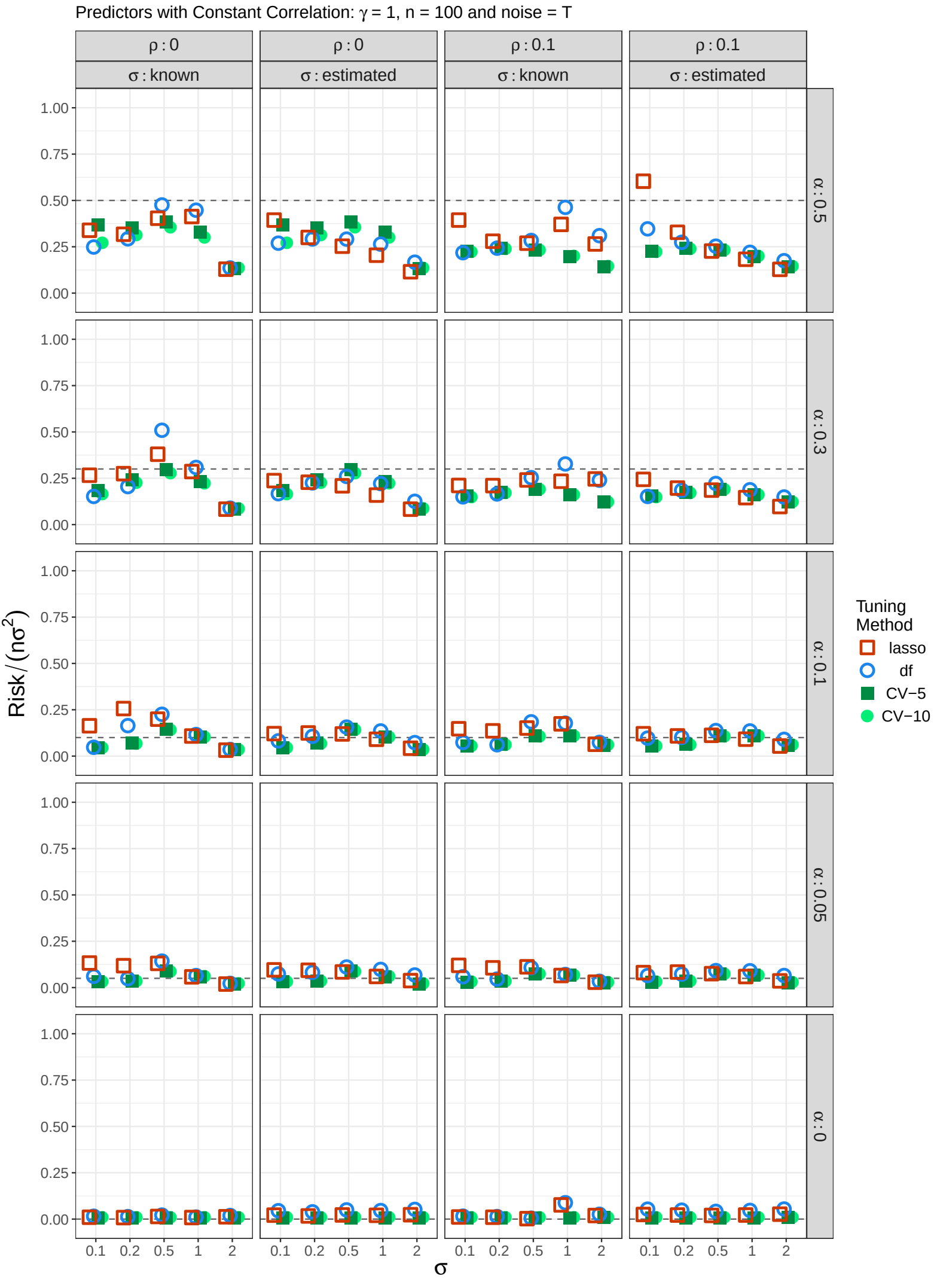
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 100 and noise = T
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
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0.1
0.2
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2
0.00
0.25
0.50
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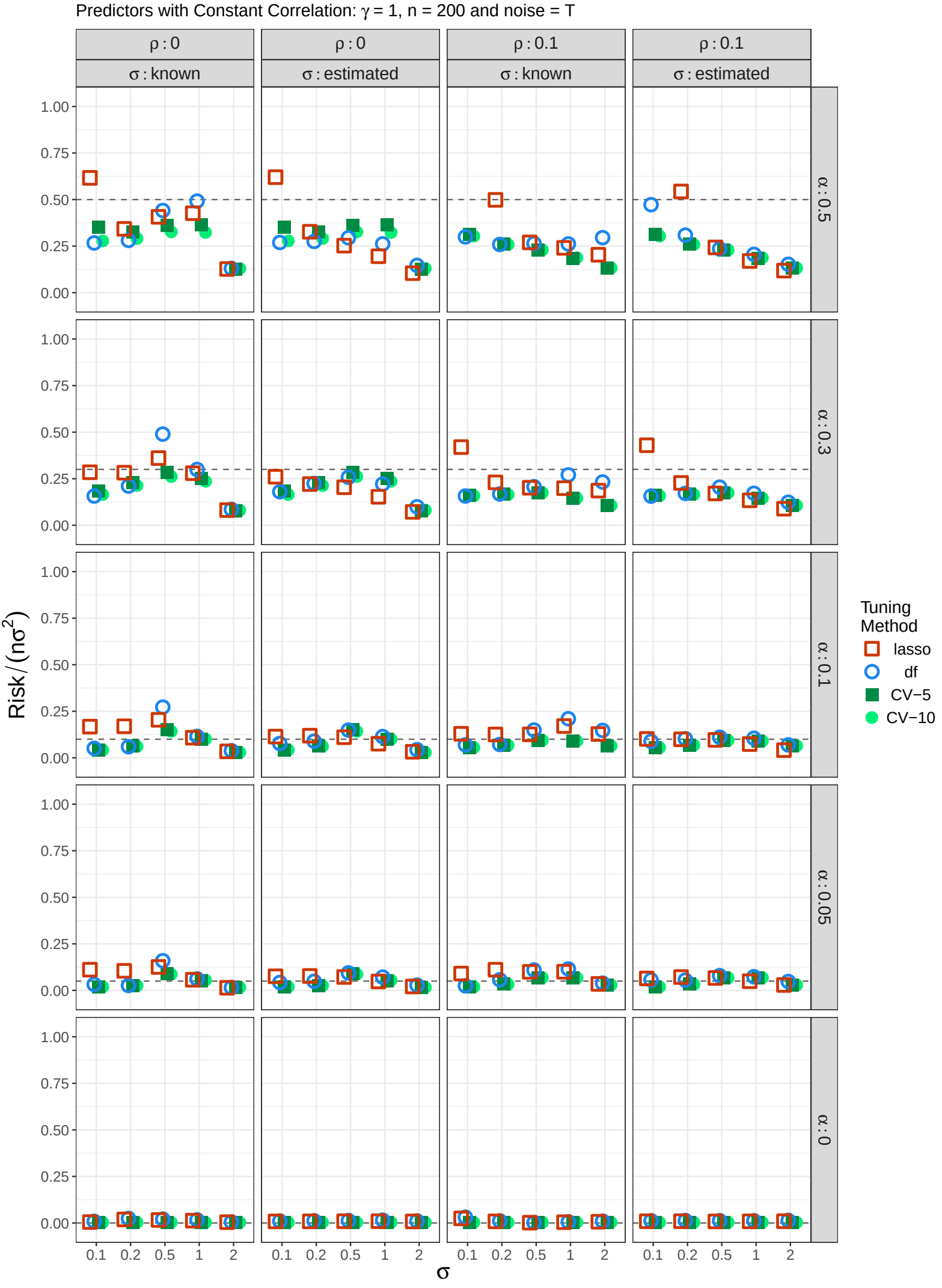
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 200 and noise = T
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
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Risk/(nσ²)
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0.5
1
2
σ
Tuning Method
lasso
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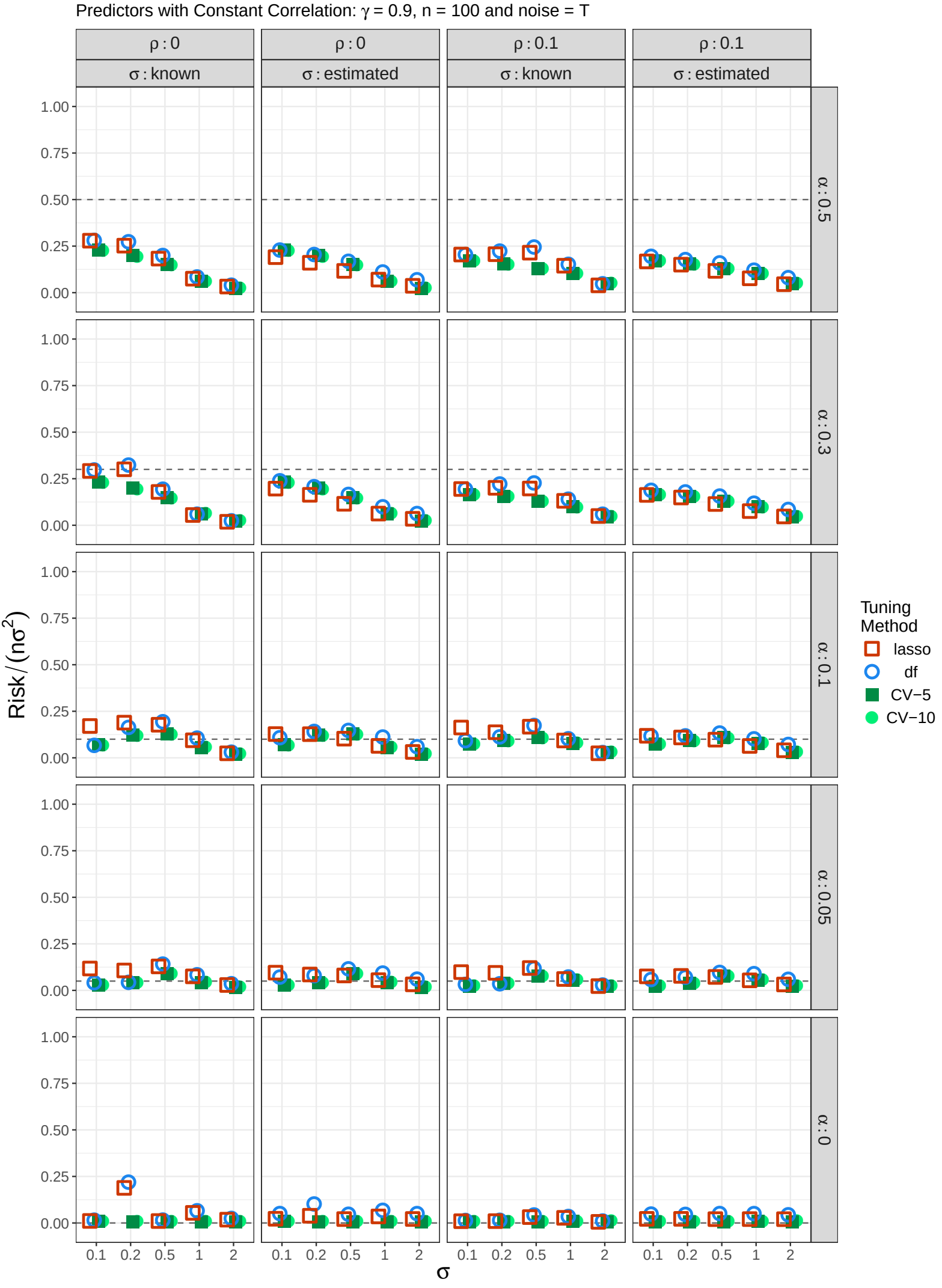
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = T
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
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0.5
1
2
σ
Tuning Method
lasso
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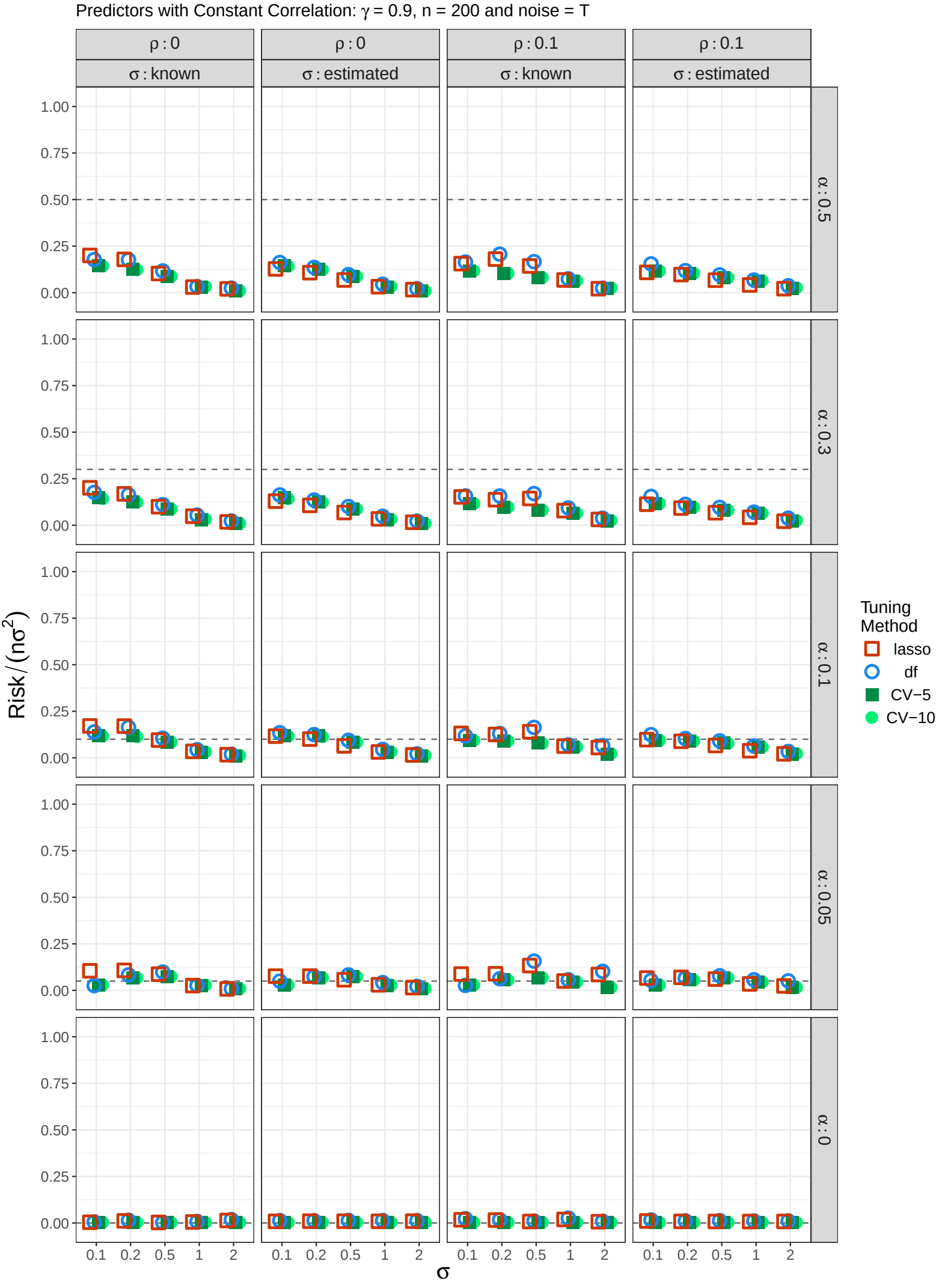
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = T
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
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Risk/(nσ²)
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0.00
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0.2
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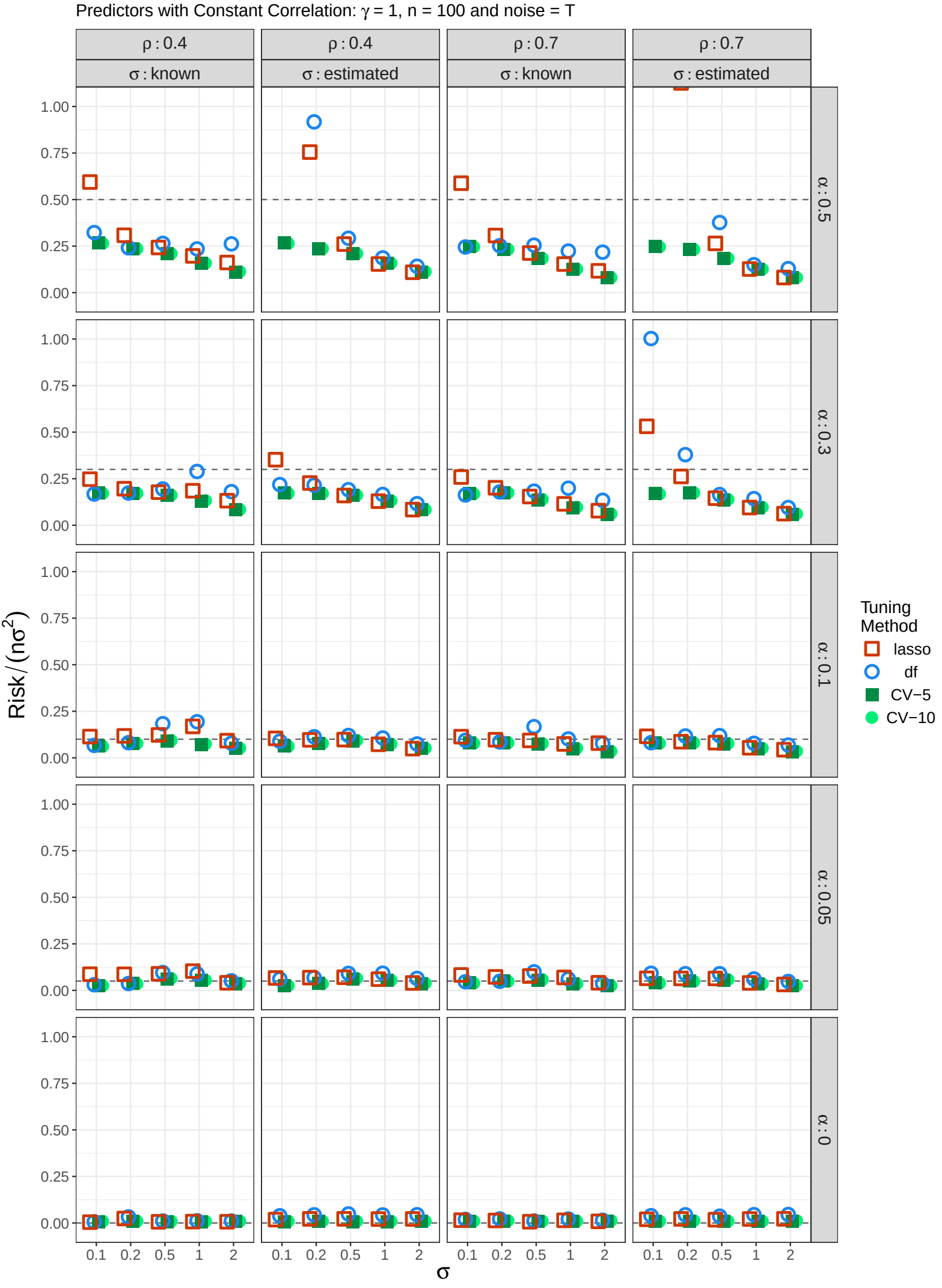
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 100 and noise = T
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
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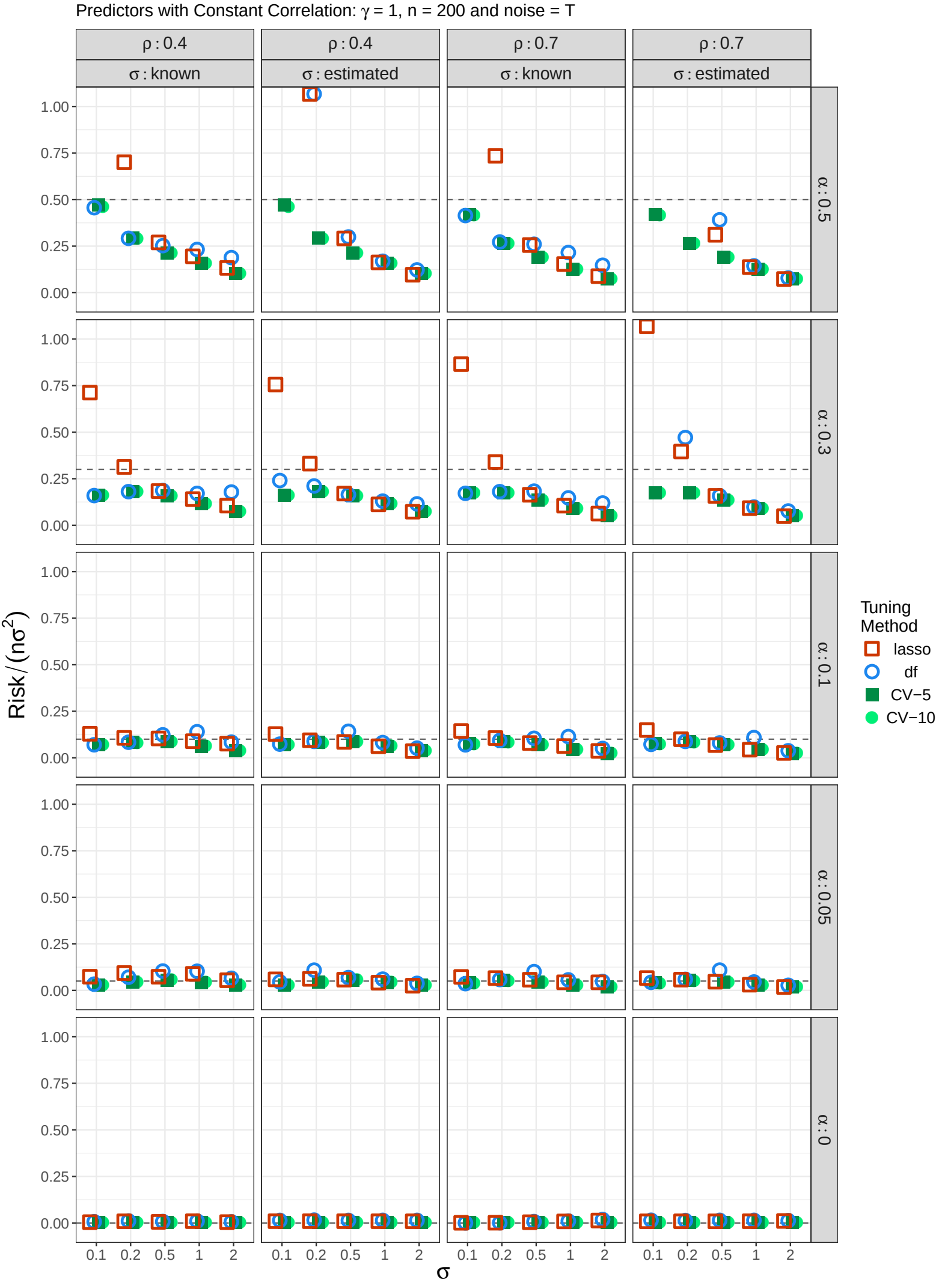
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 200 and noise = T
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
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0.00
0.25
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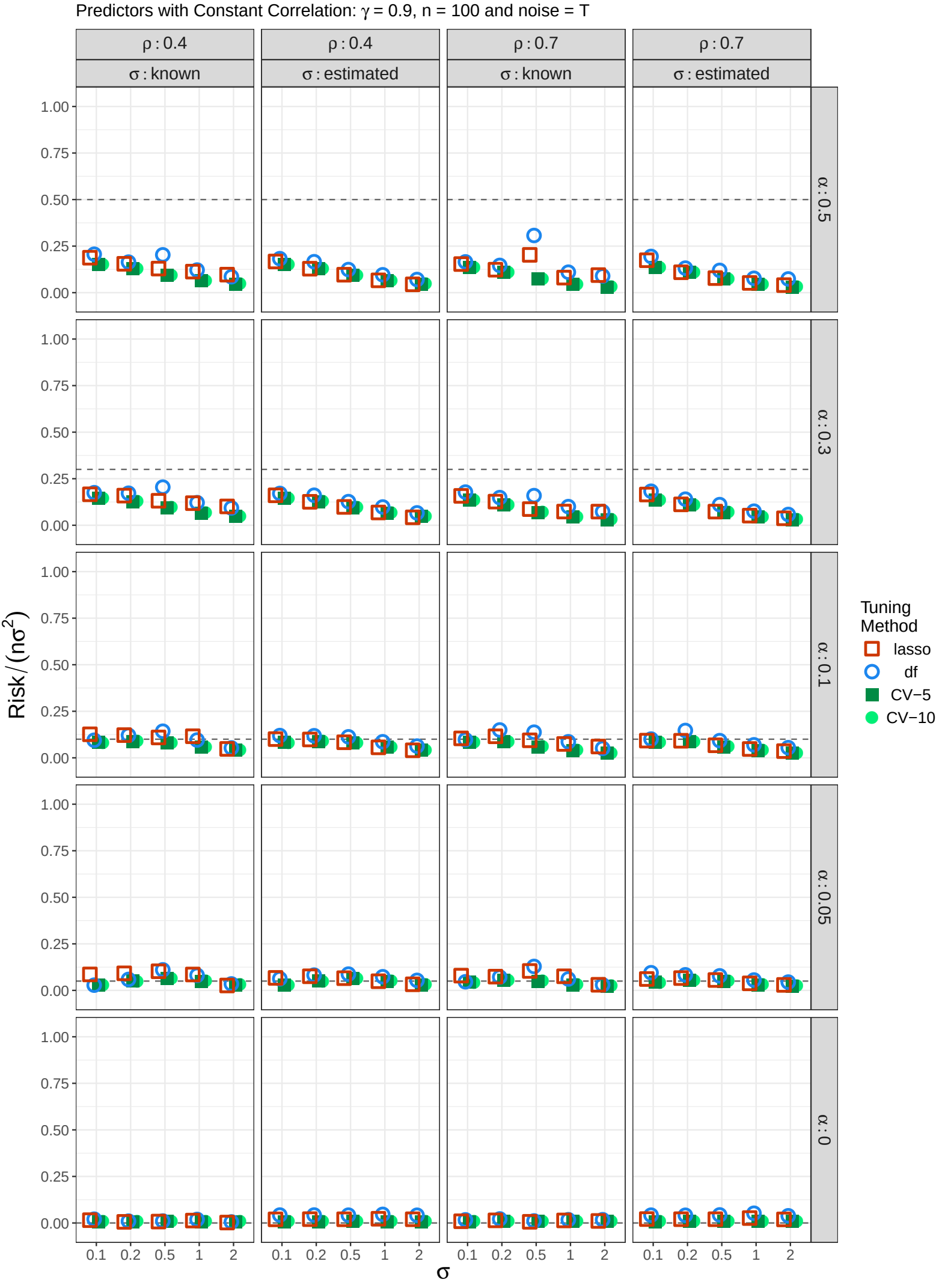
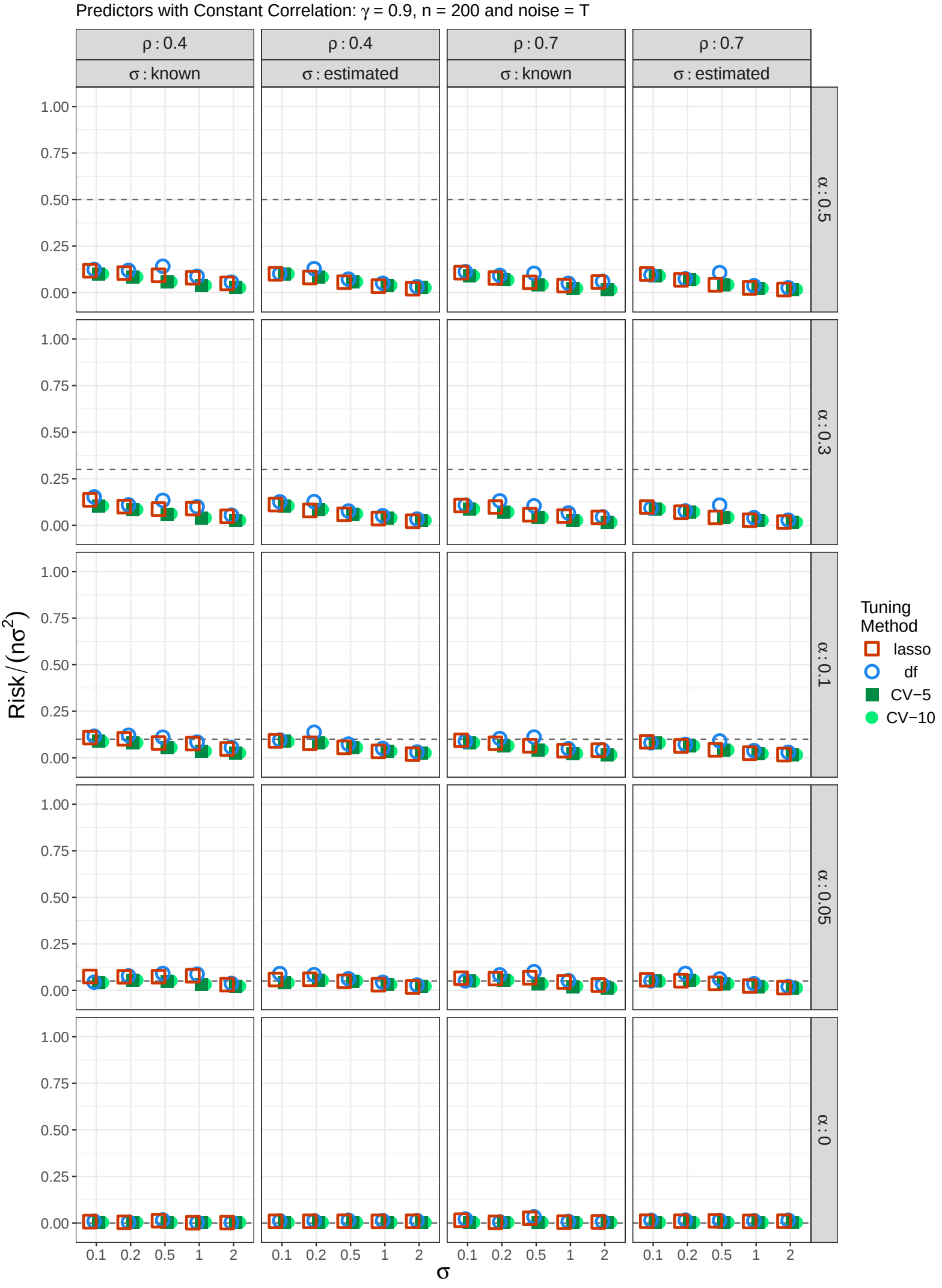
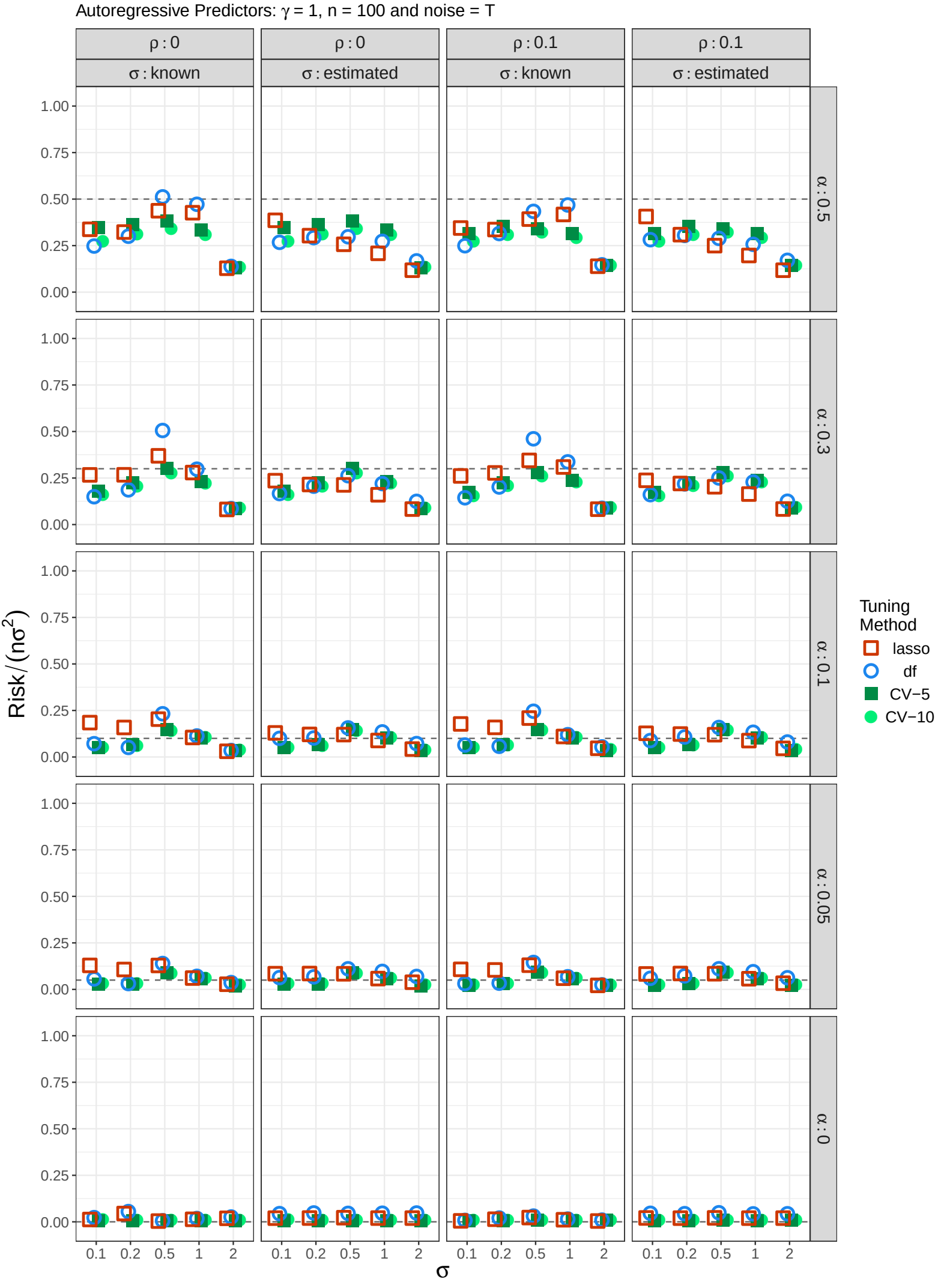
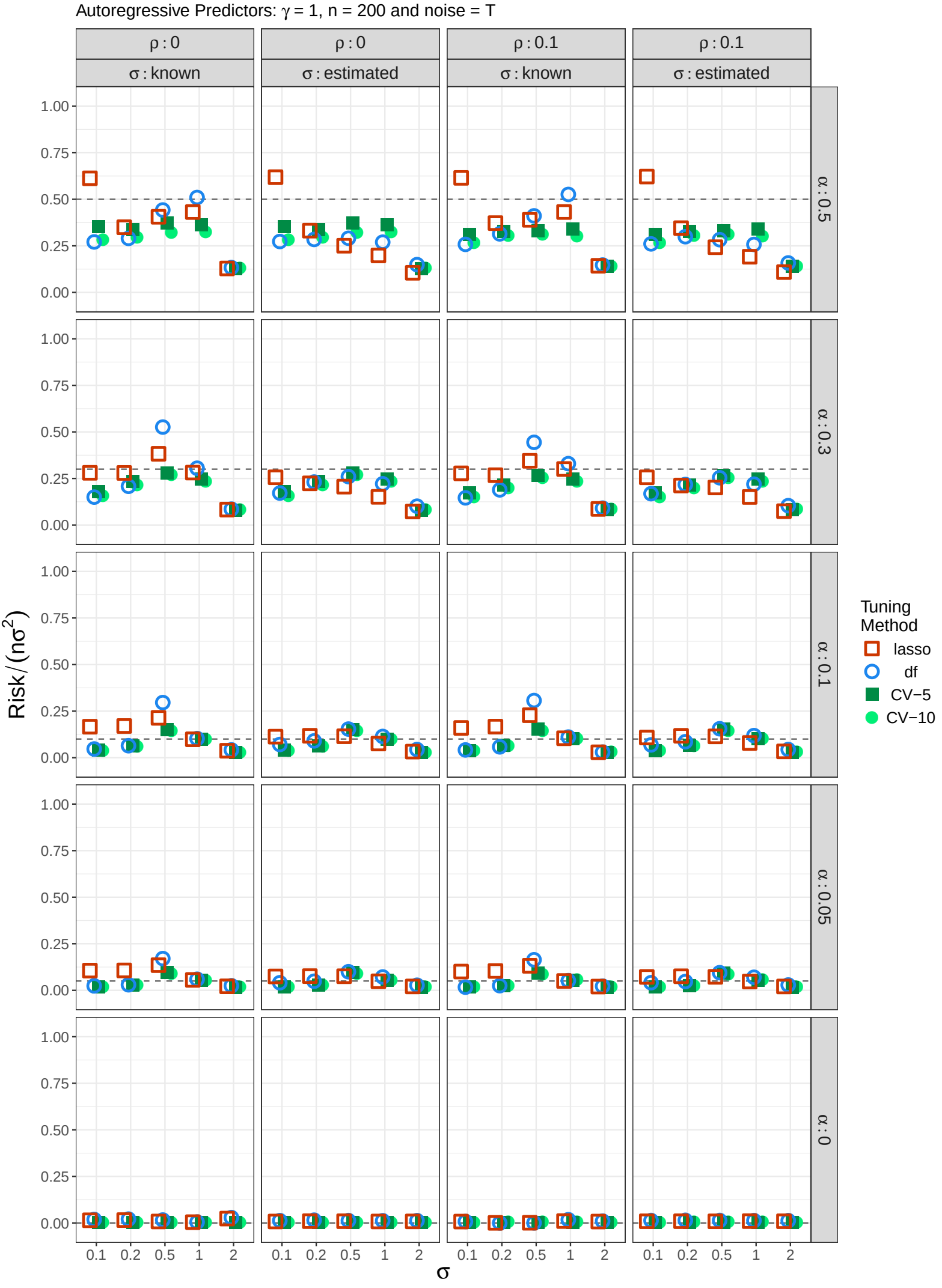
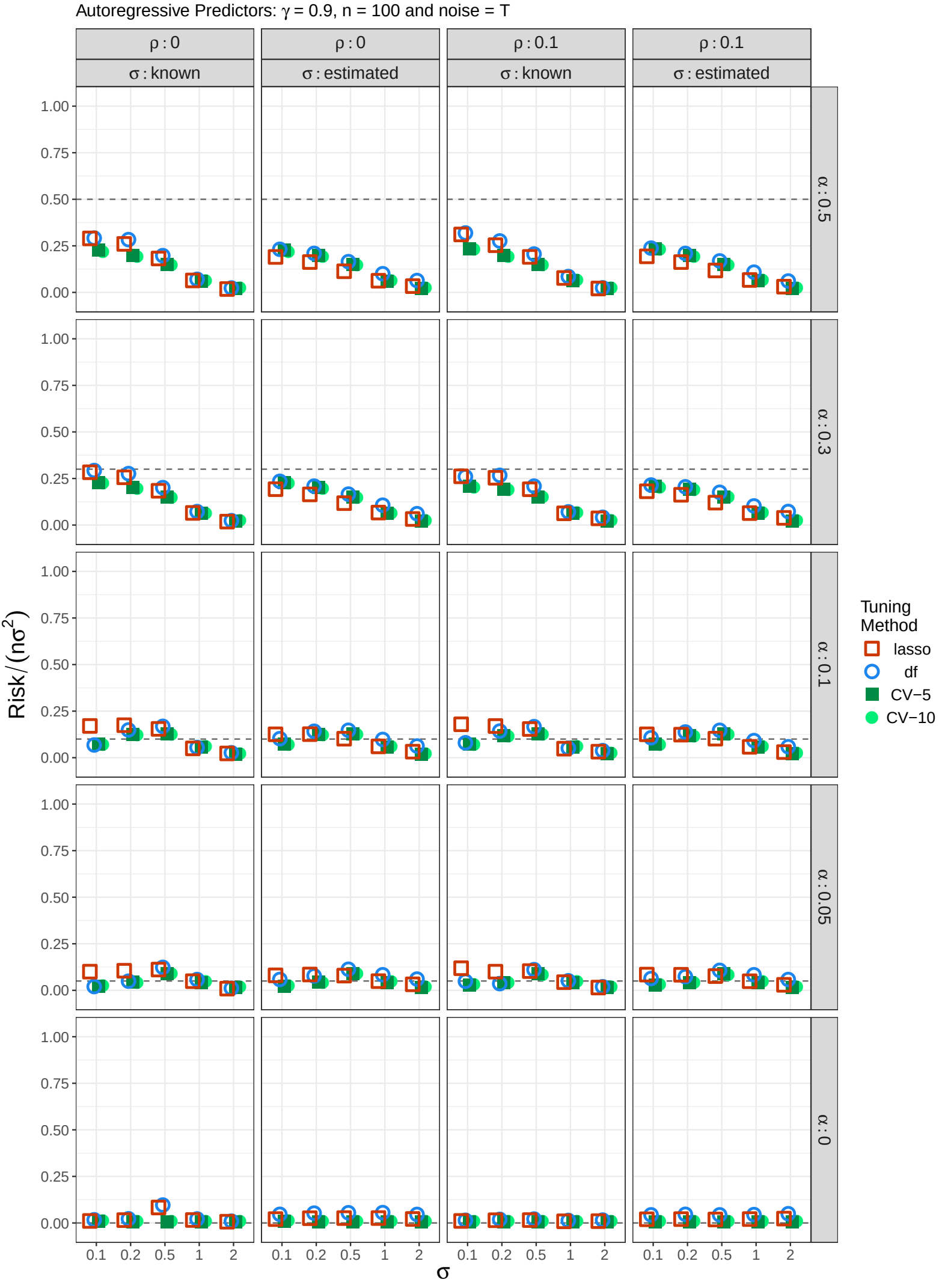
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

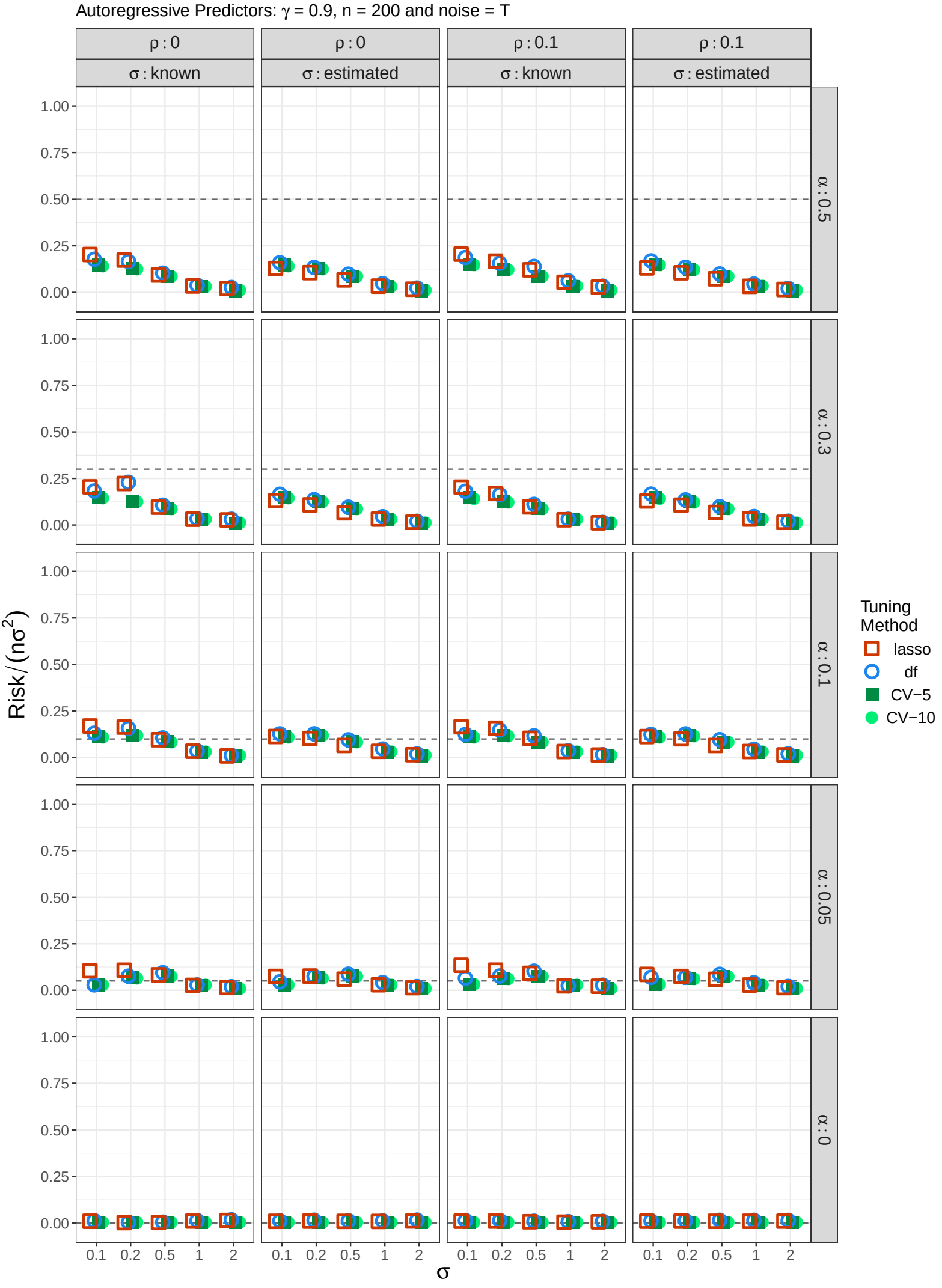

Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = T
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = T
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
1.00
0.75
0.50
0.25
0.00
Risk/(nσ²)
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 100 and noise = T
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 200 and noise = T
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 100 and noise = T
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 200 and noise = T
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
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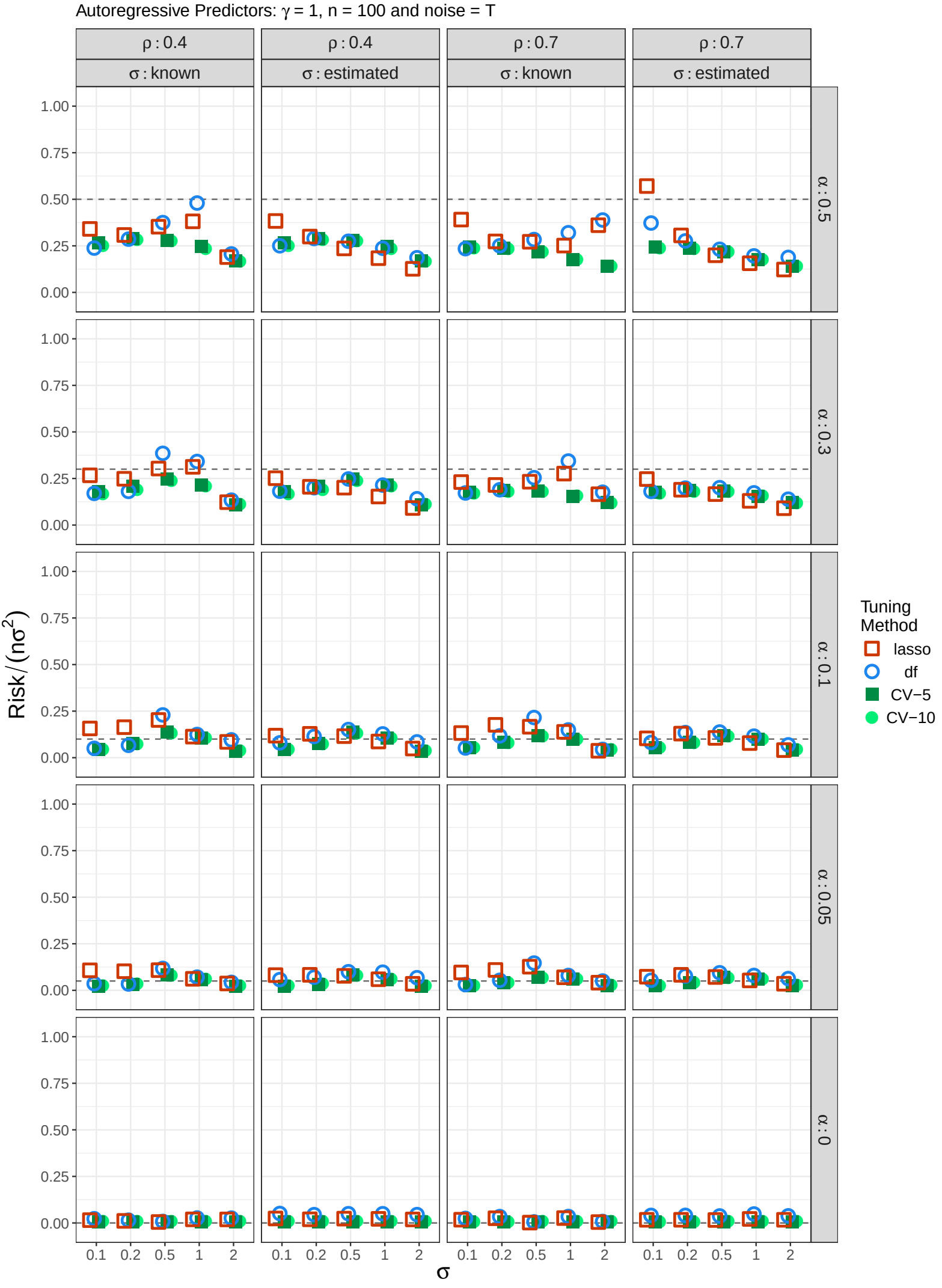
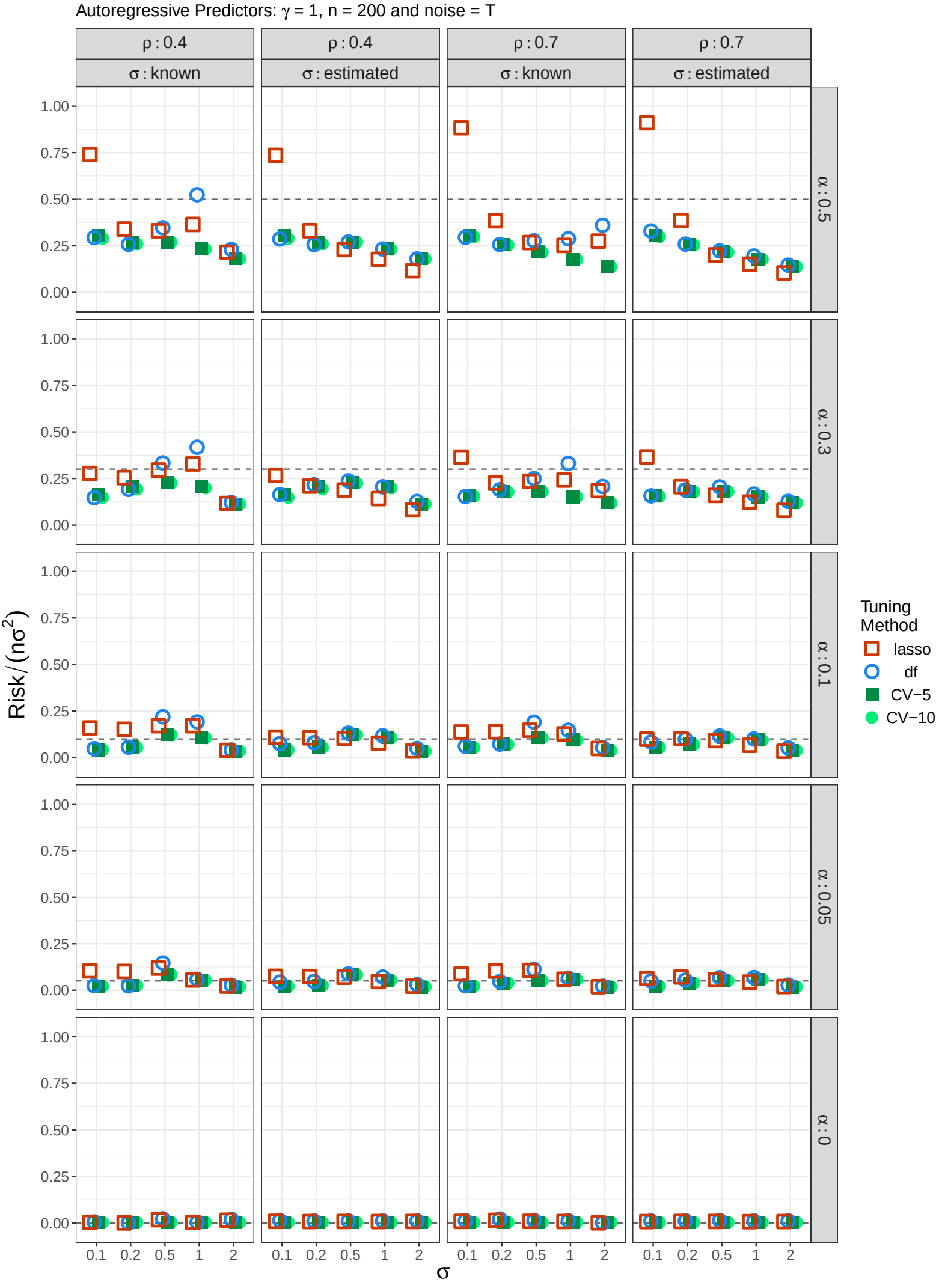
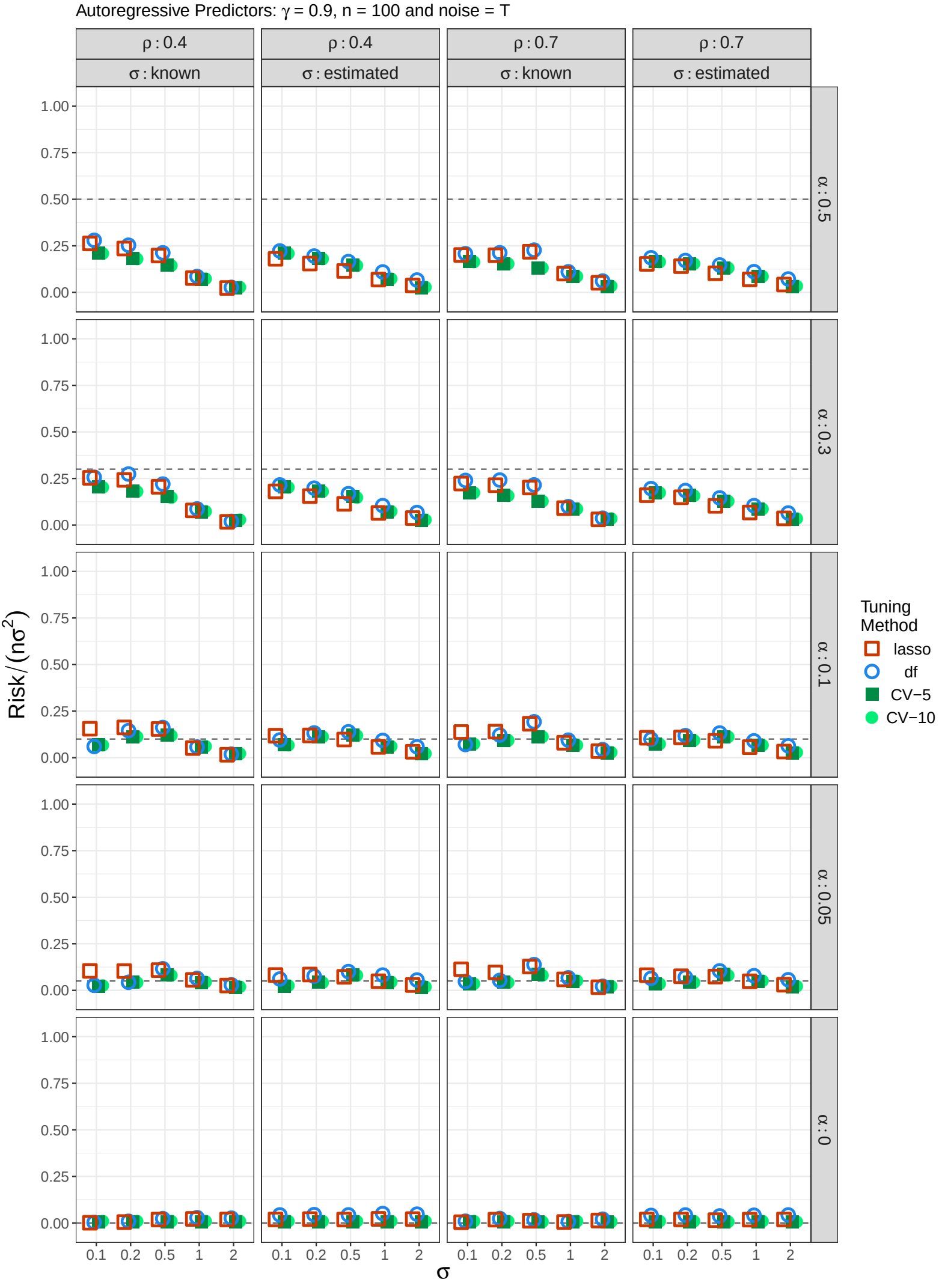
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 100 and noise = T
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 200 and noise = T
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 100 and noise = T
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
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0.25
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0.1
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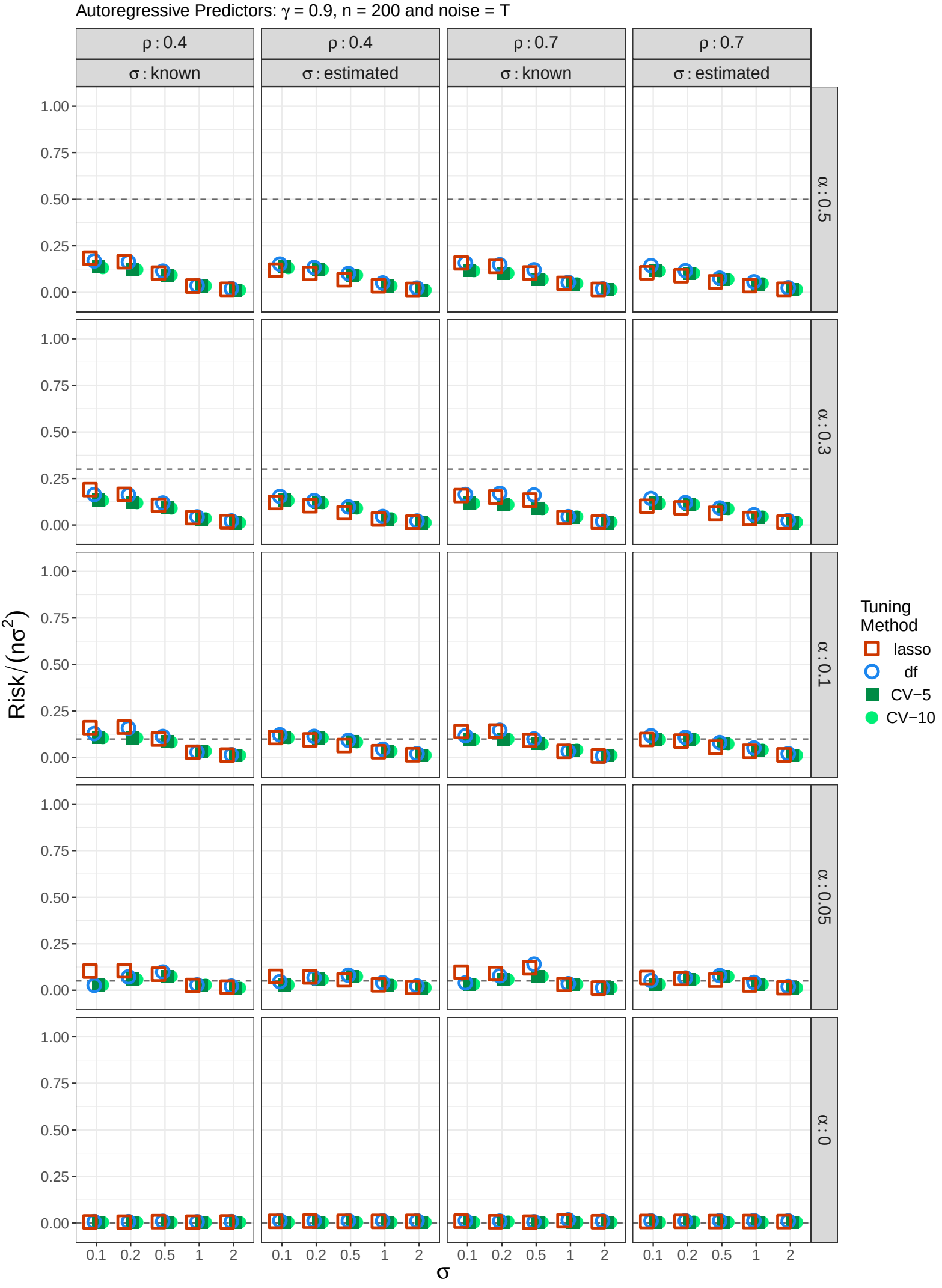
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 200 and noise = T
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
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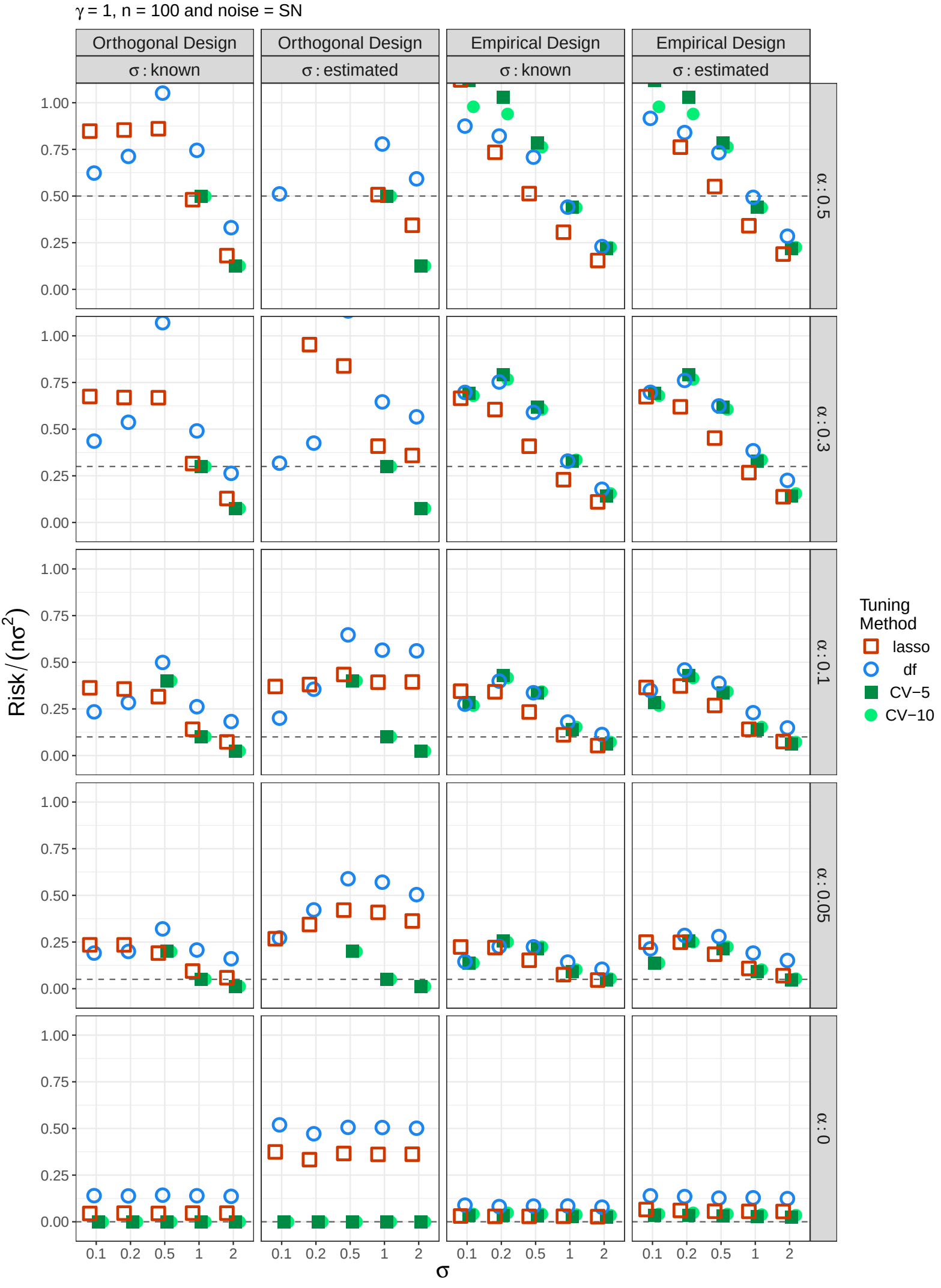
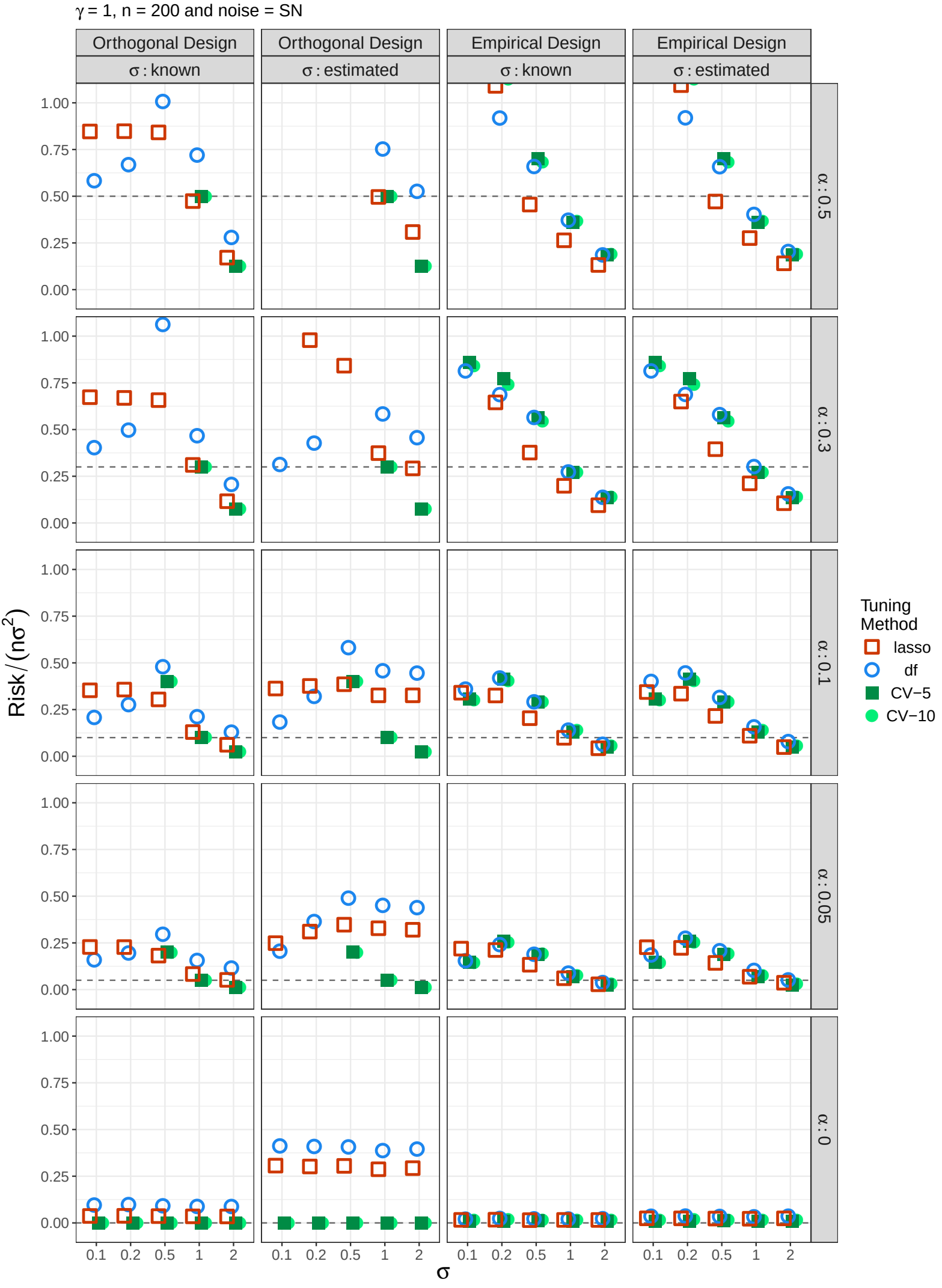
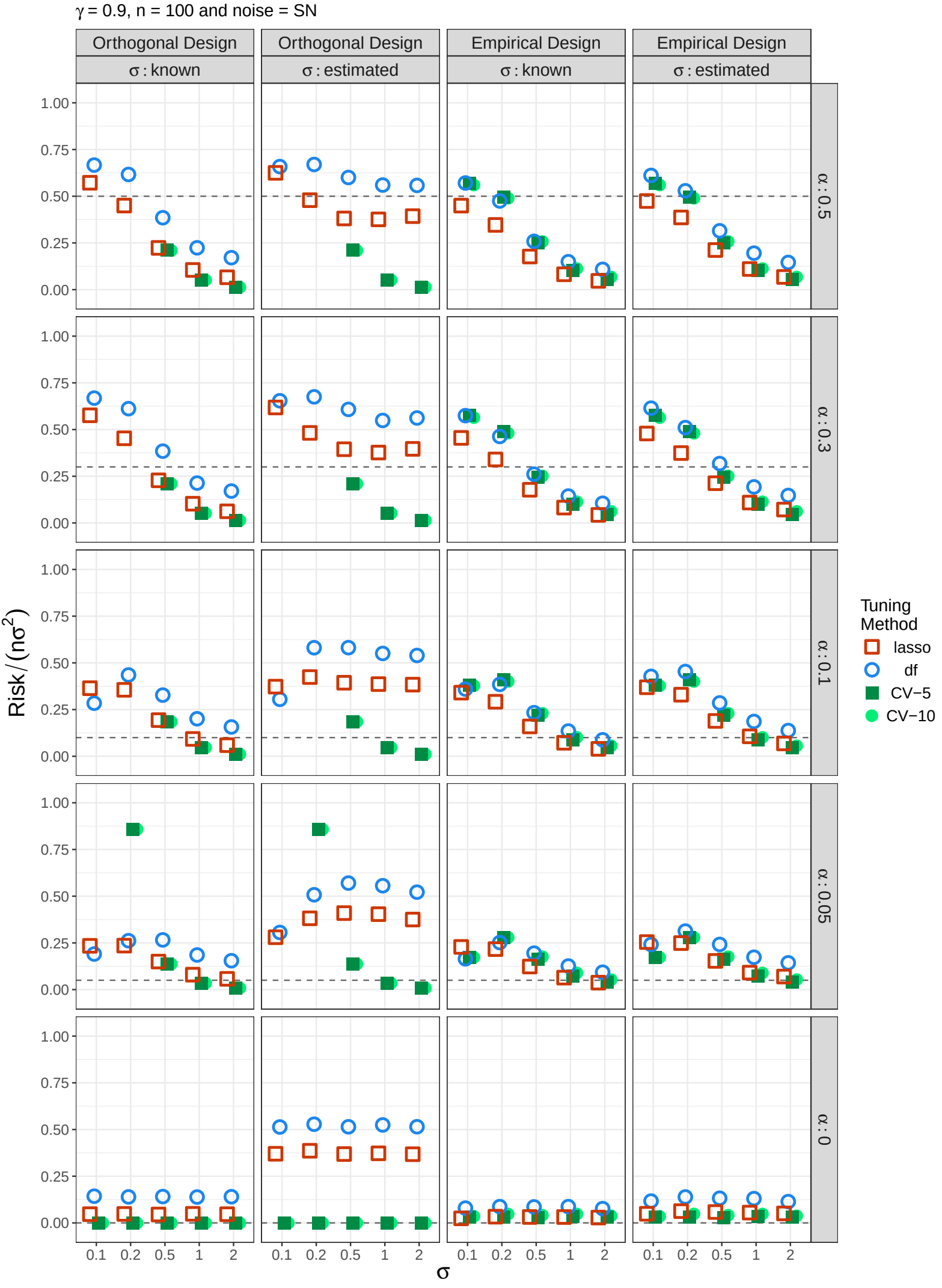
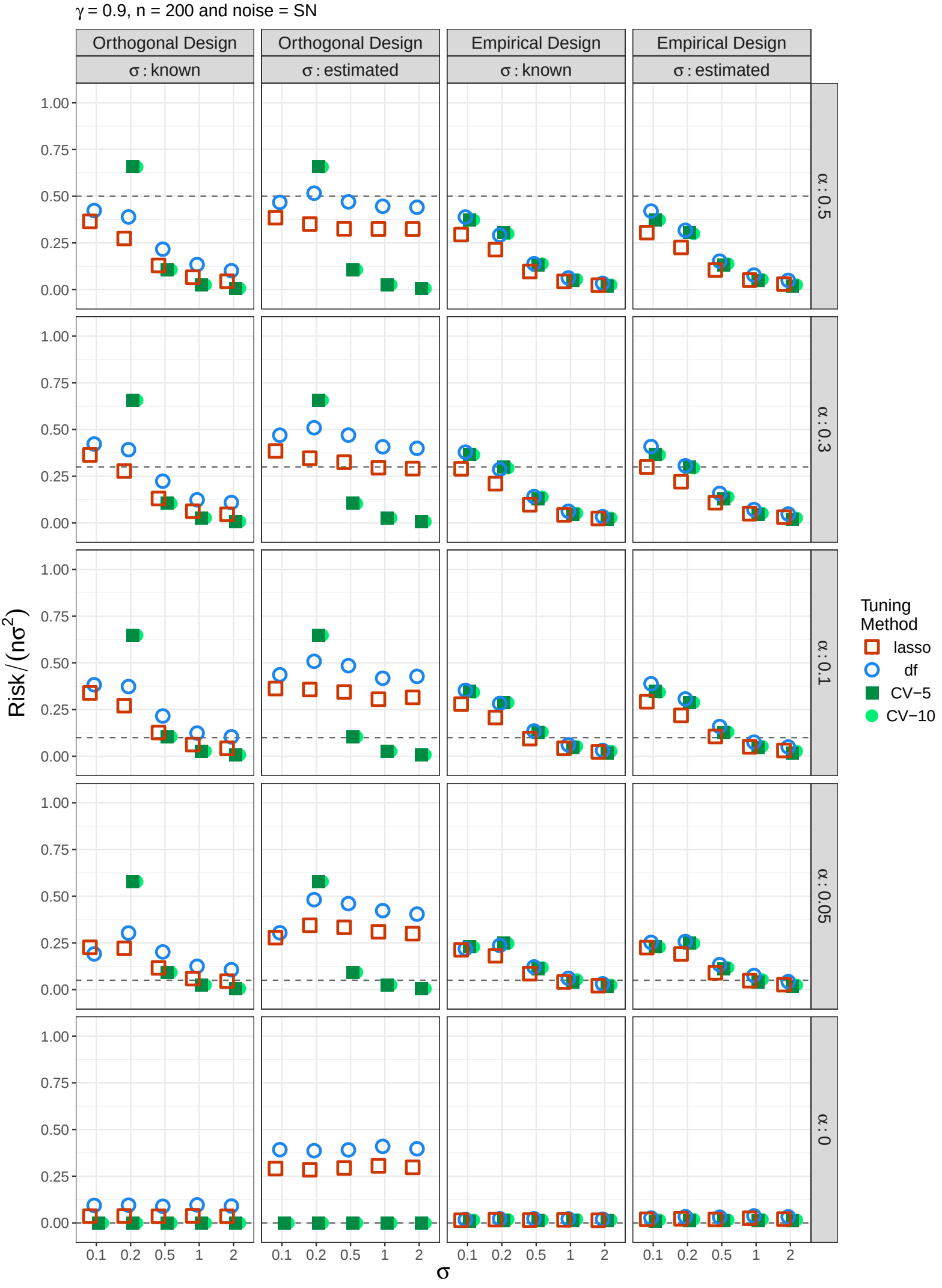
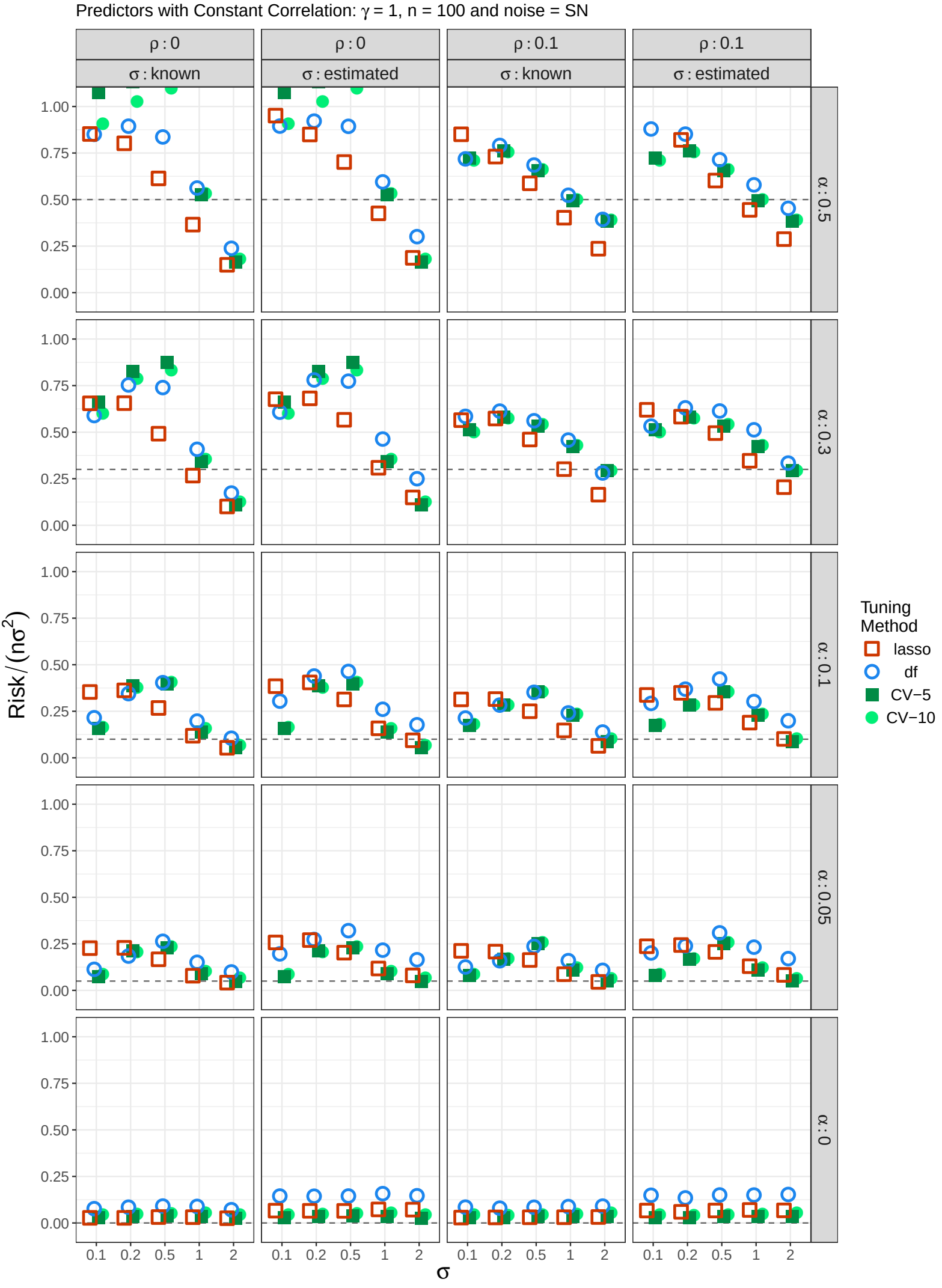
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

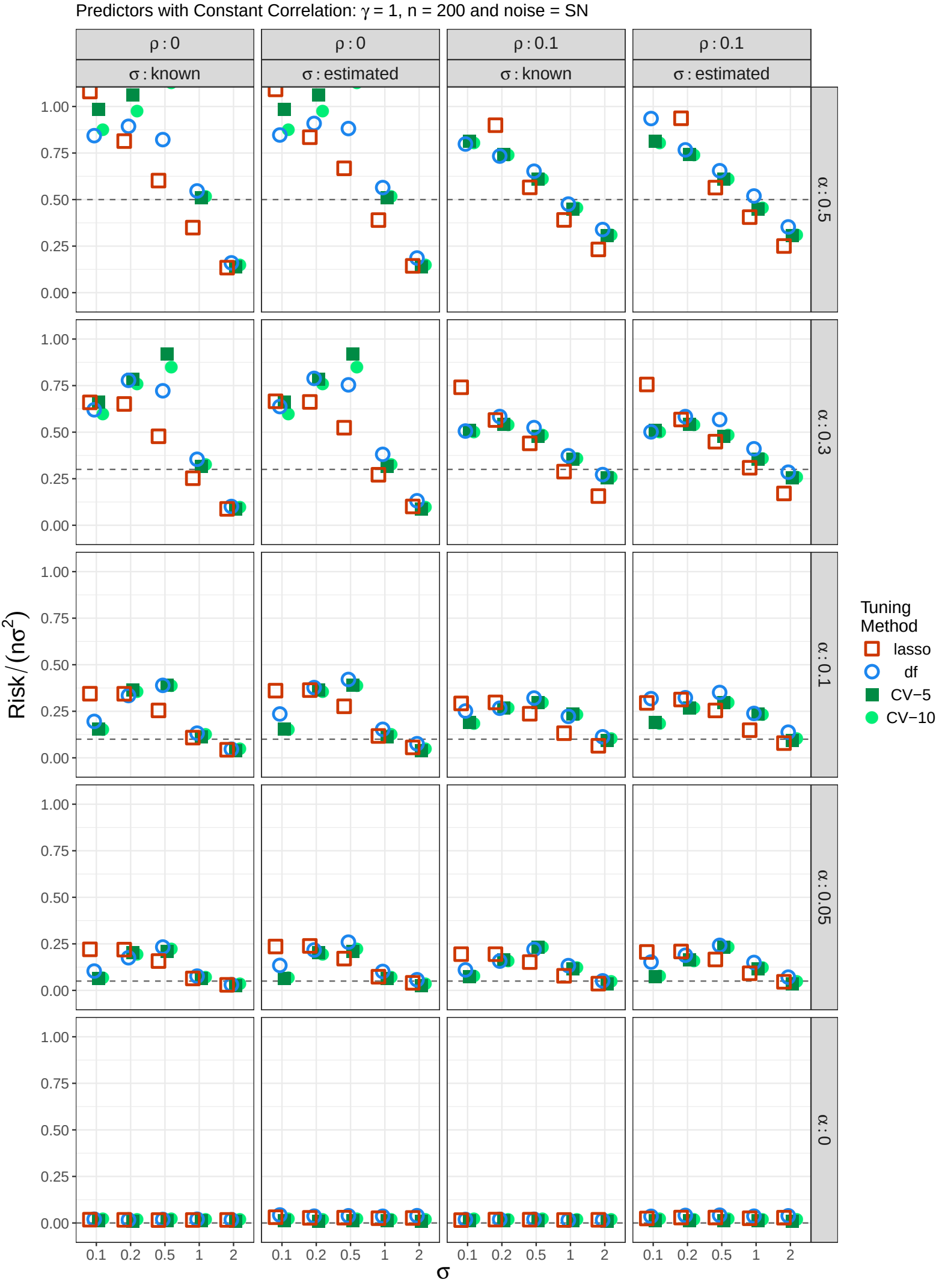

γ = 1, n = 100 and noise = SN
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

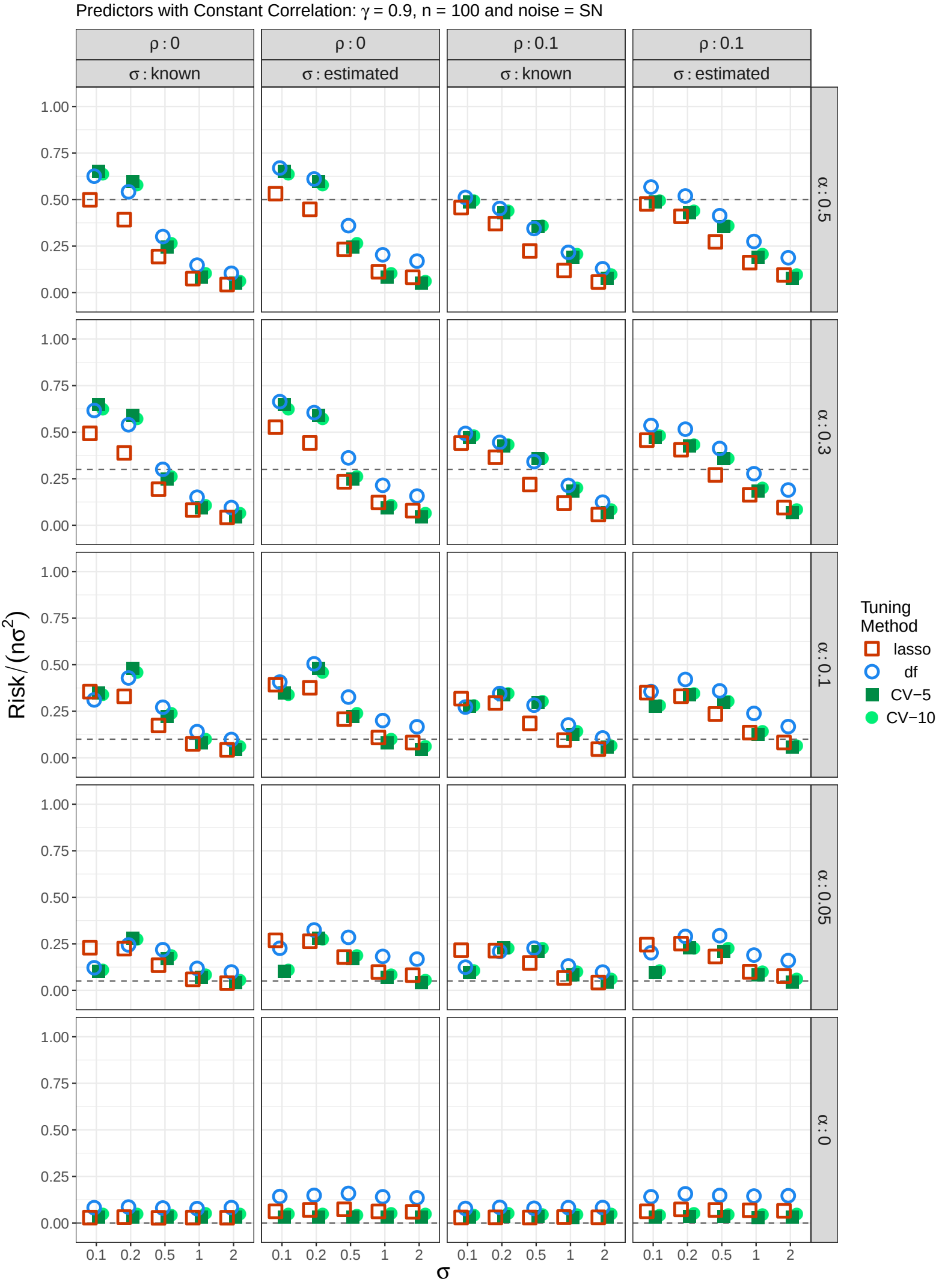

γ = 1, n = 200 and noise = SN
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10

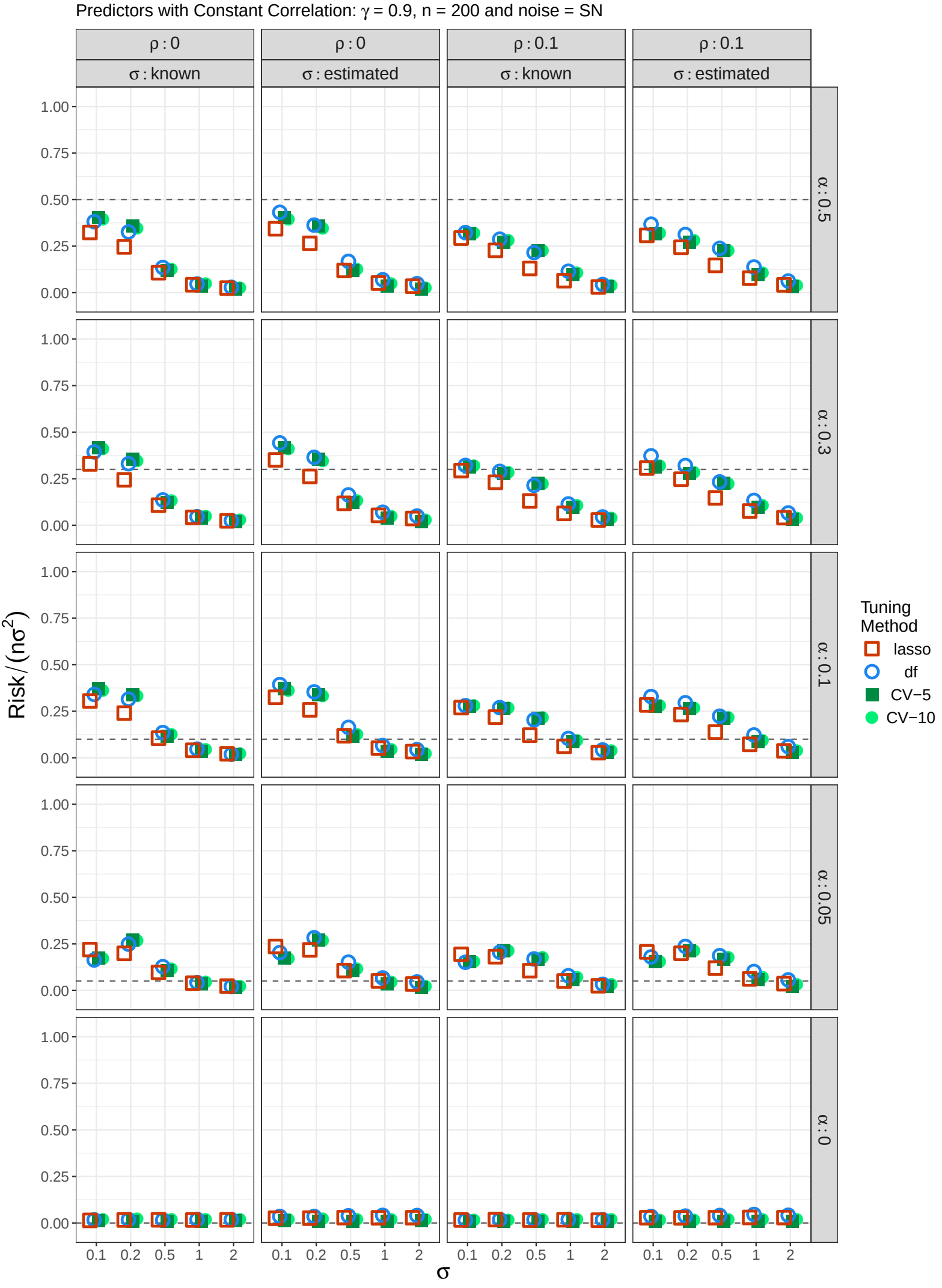

γ = 0.9, n = 100 and noise = SN
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


γ = 0.9, n = 200 and noise = SN
Orthogonal Design
σ : known
Orthogonal Design
σ : estimated
Empirical Design
σ : known
Empirical Design
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 100 and noise = SN
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1 0.2 0.5 1 2
0.00 0.25 0.50 0.75 1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 200 and noise = SN
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1 0.2 0.5 1 2
0.00 0.25 0.50 0.75 1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = SN
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = SN
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
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α : 0.1
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α : 0
Risk/(nσ²)
1.00
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0.50
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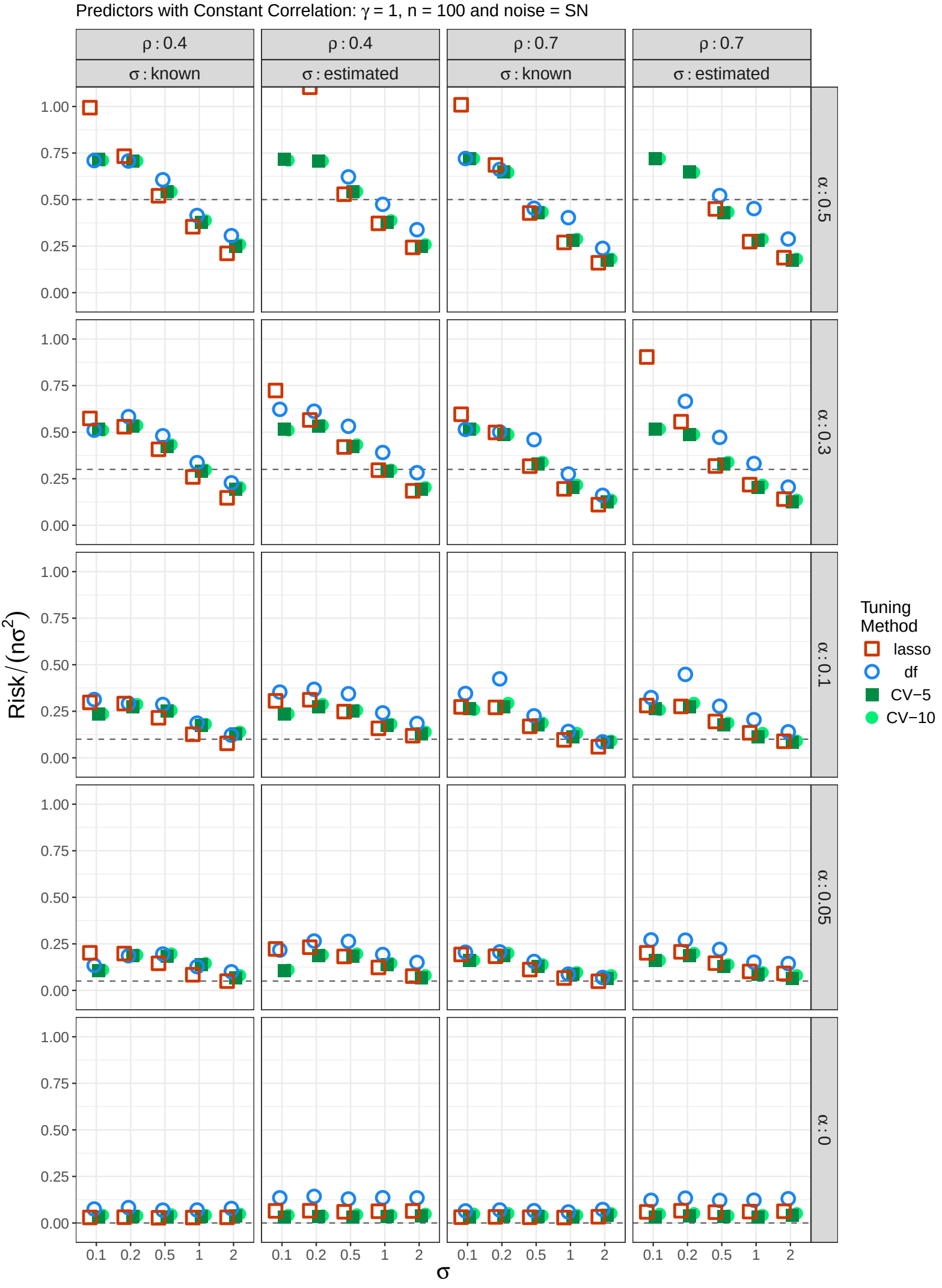
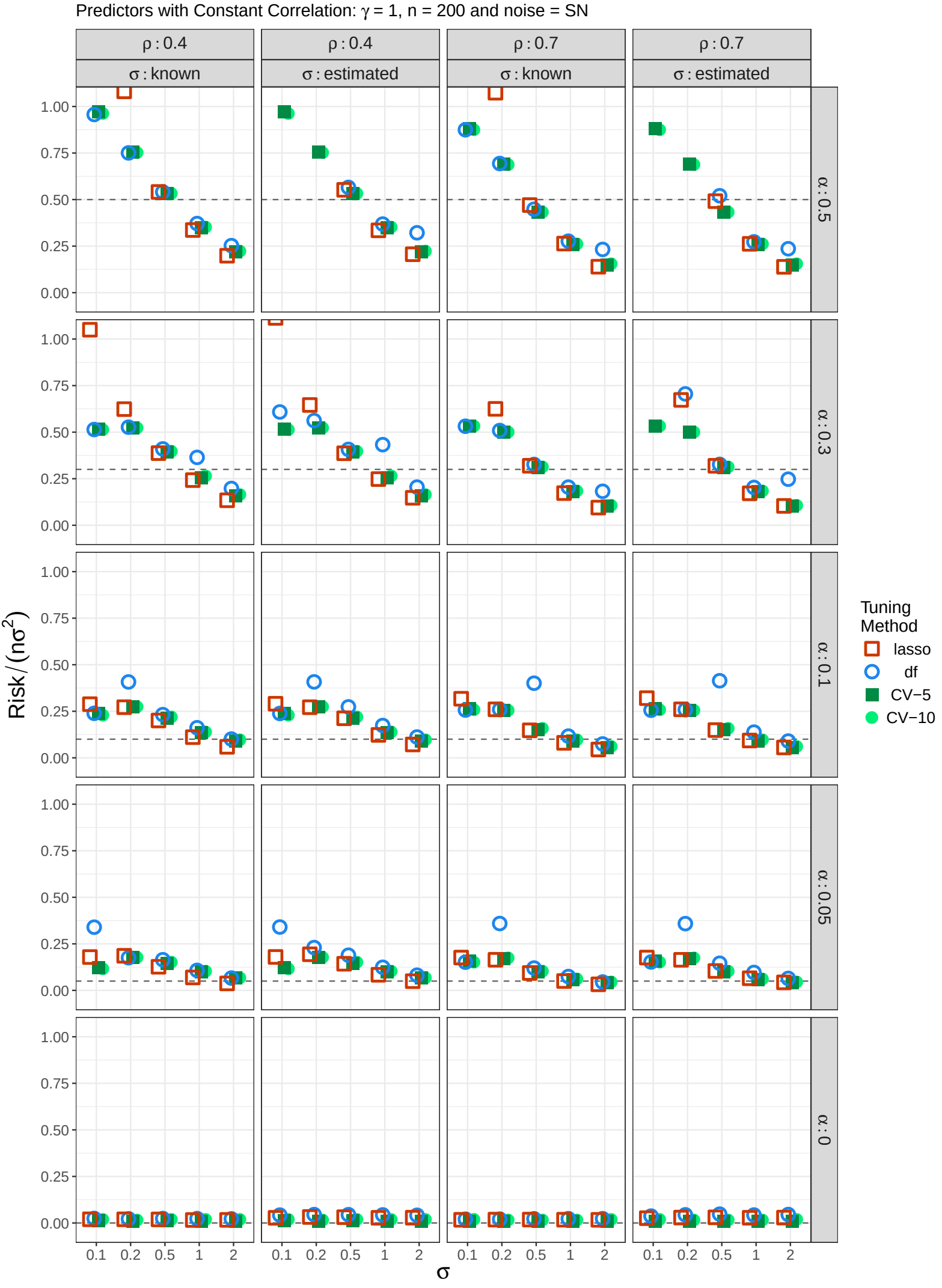
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 100 and noise = SN
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 1, n = 200 and noise = SN
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
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0.2
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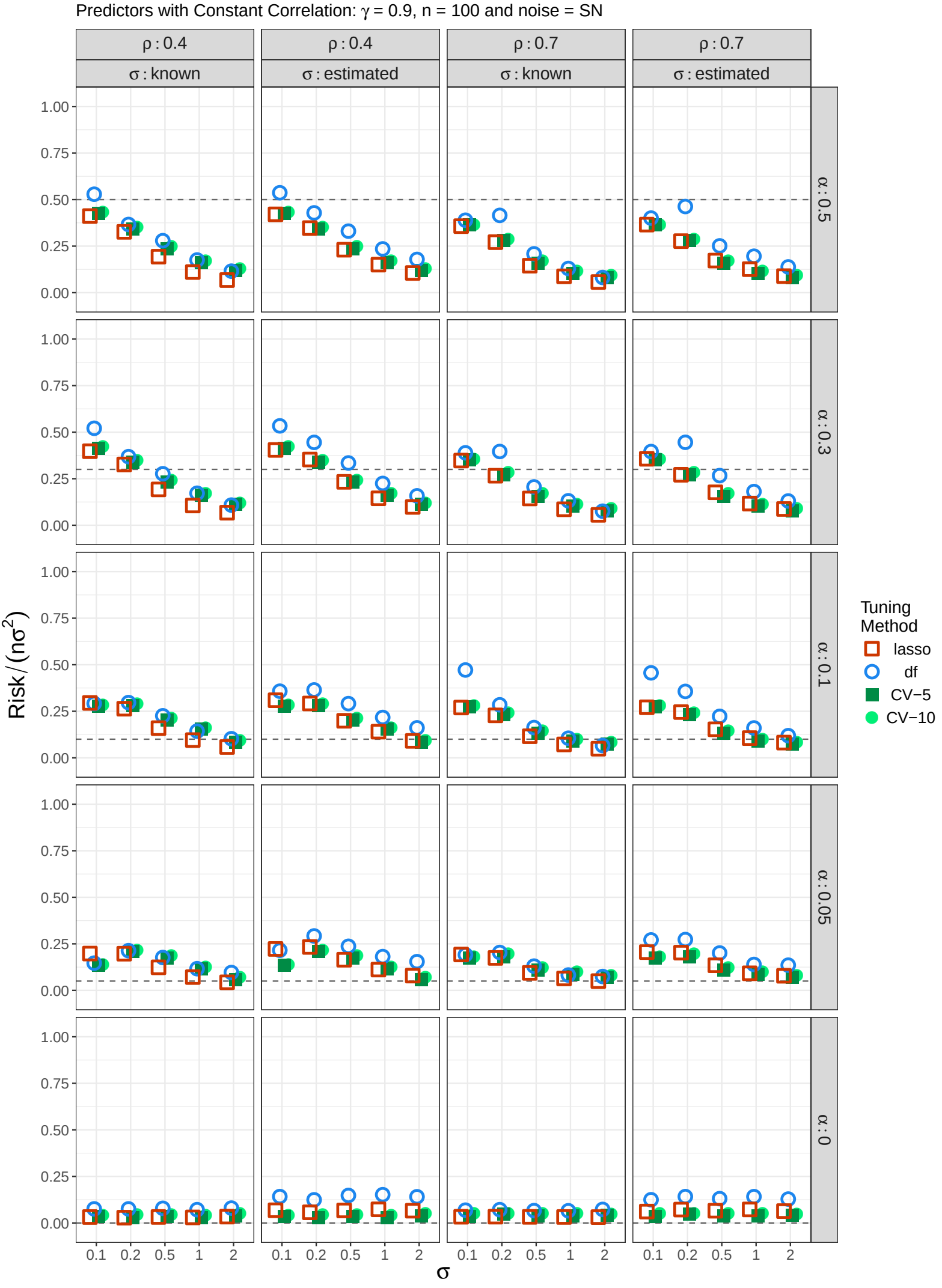
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 100 and noise = SN
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
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σ : estimated
α : 0.5
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α : 0
Risk/(nσ²)
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0.5
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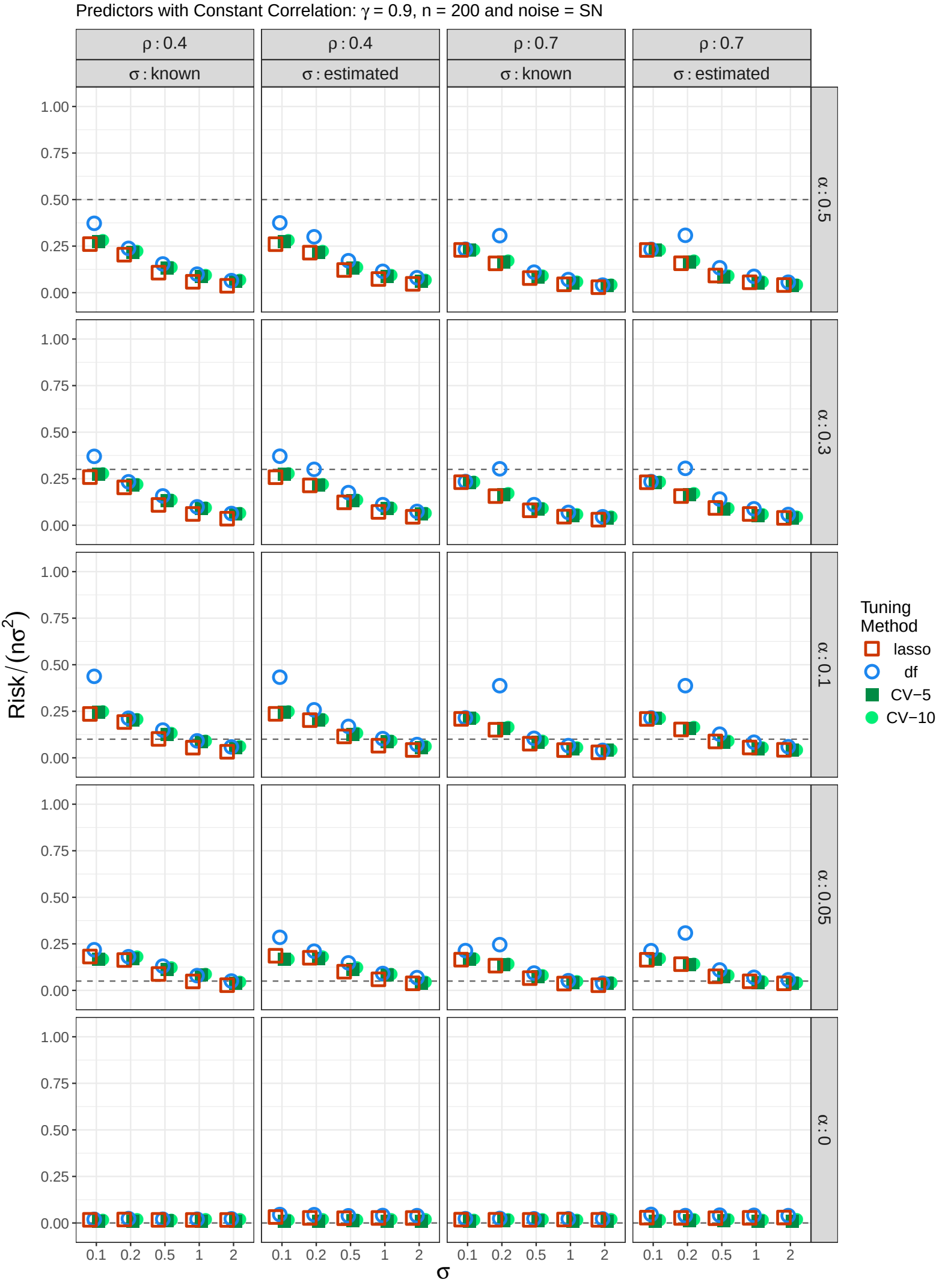
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Predictors with Constant Correlation: γ = 0.9, n = 200 and noise = SN
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
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Risk/(nσ²)
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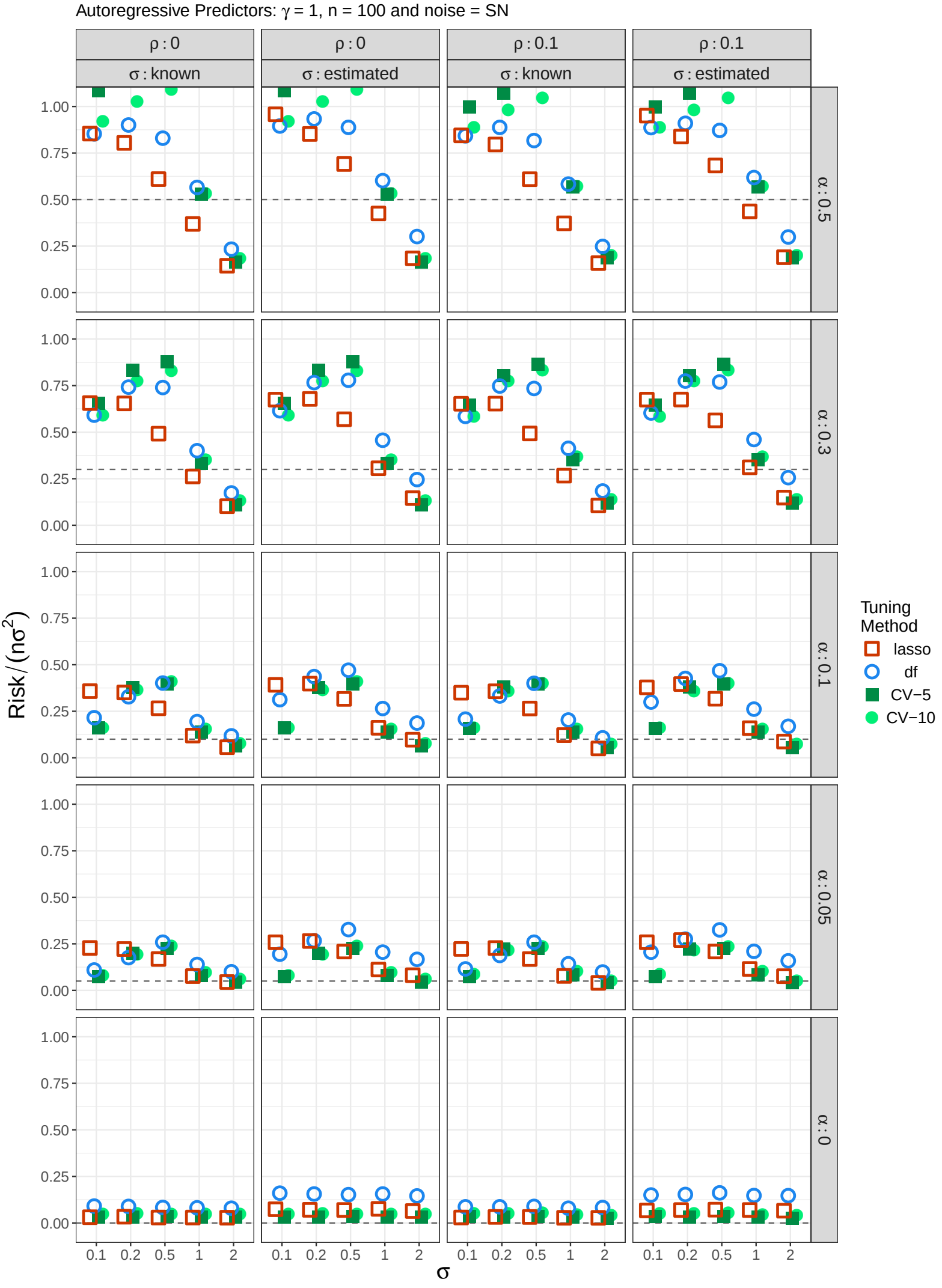
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 100 and noise = SN
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
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Tuning Method
lasso
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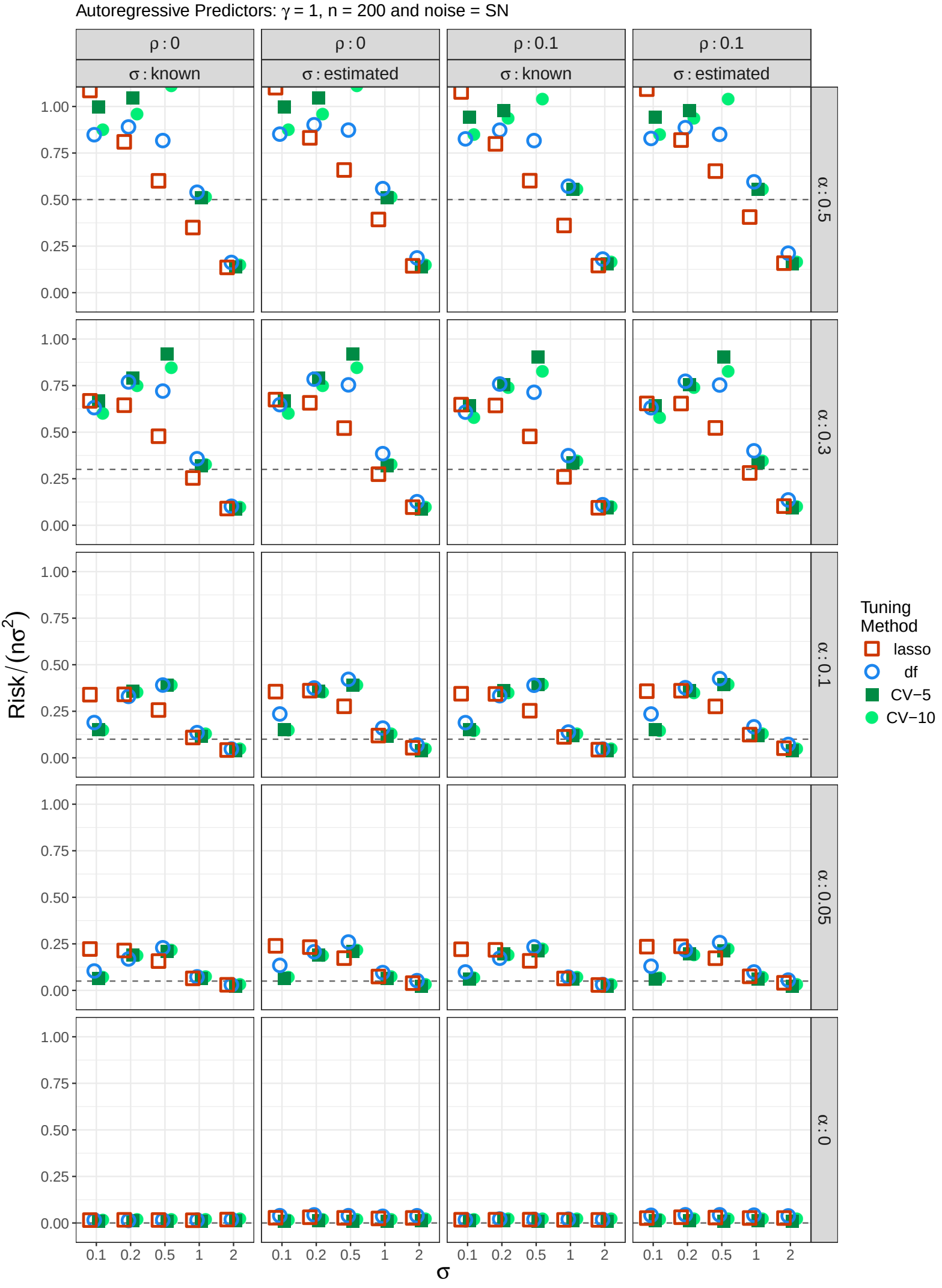
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 200 and noise = SN
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
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σ : estimated
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Tuning Method
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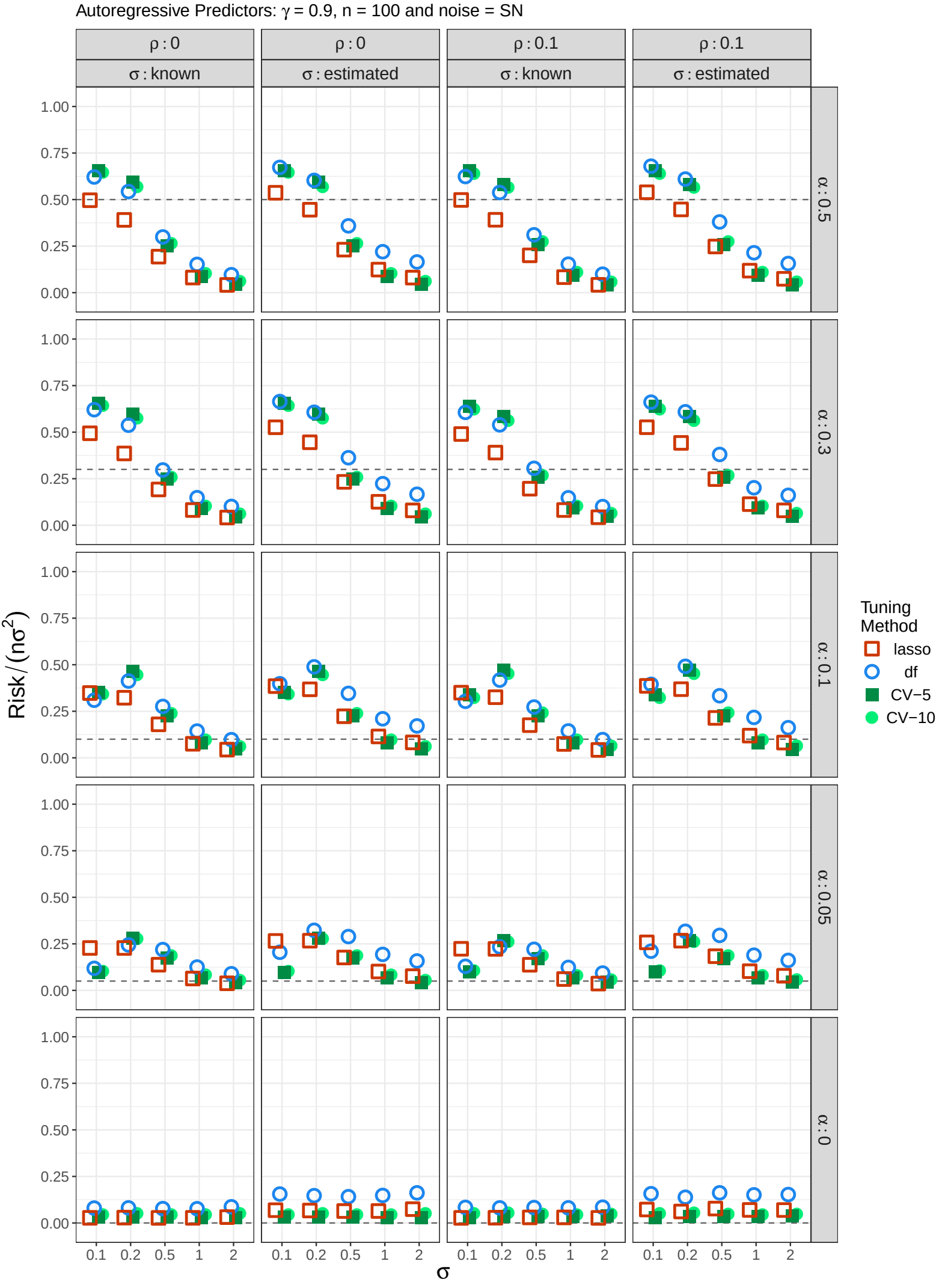
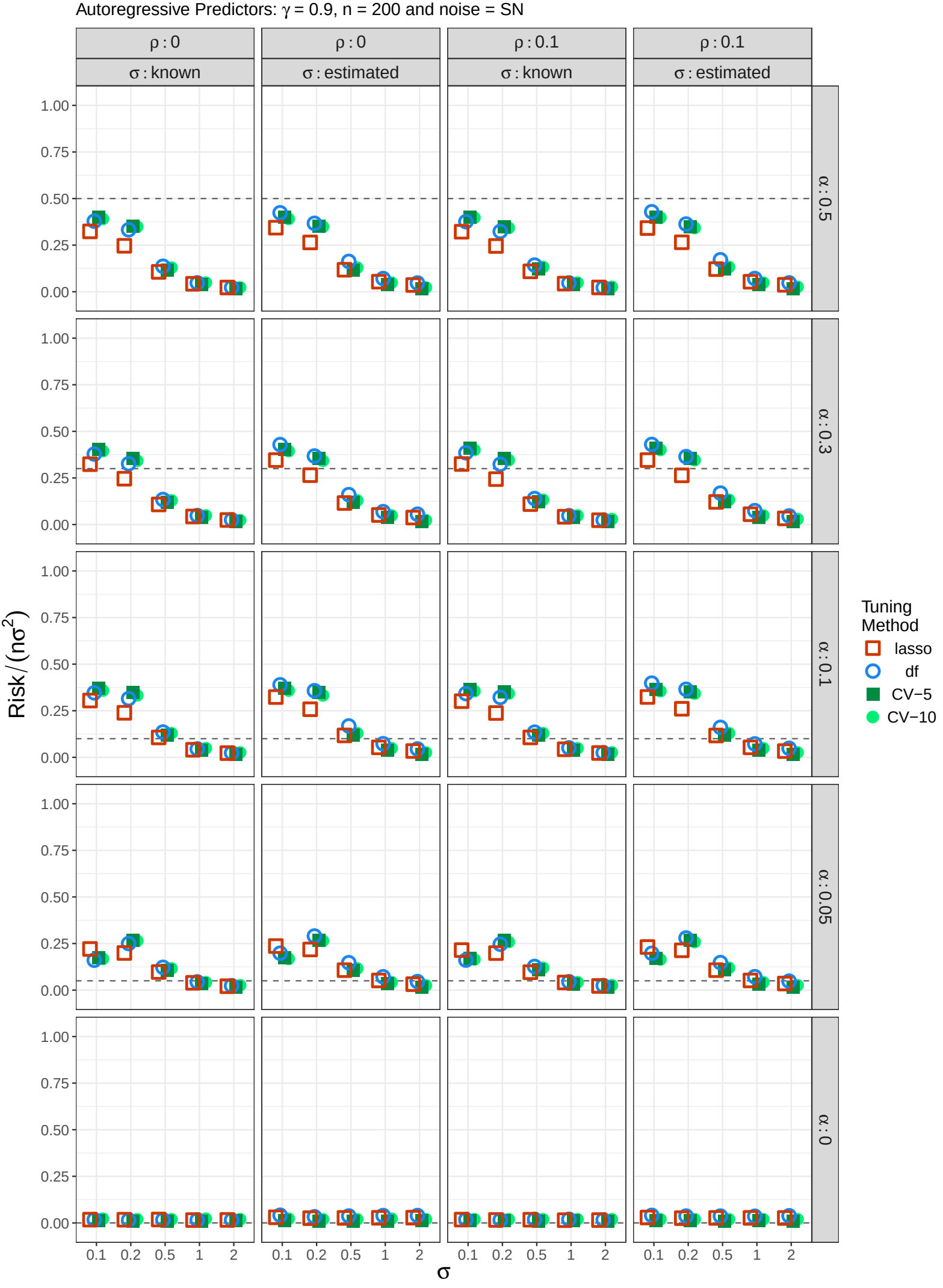
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 100 and noise = SN
ρ : 0
ρ : 0
ρ : 0.1
ρ : 0.1
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 200 and noise = SN
ρ : 0
σ : known
ρ : 0
σ : estimated
ρ : 0.1
σ : known
ρ : 0.1
σ : estimated
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2
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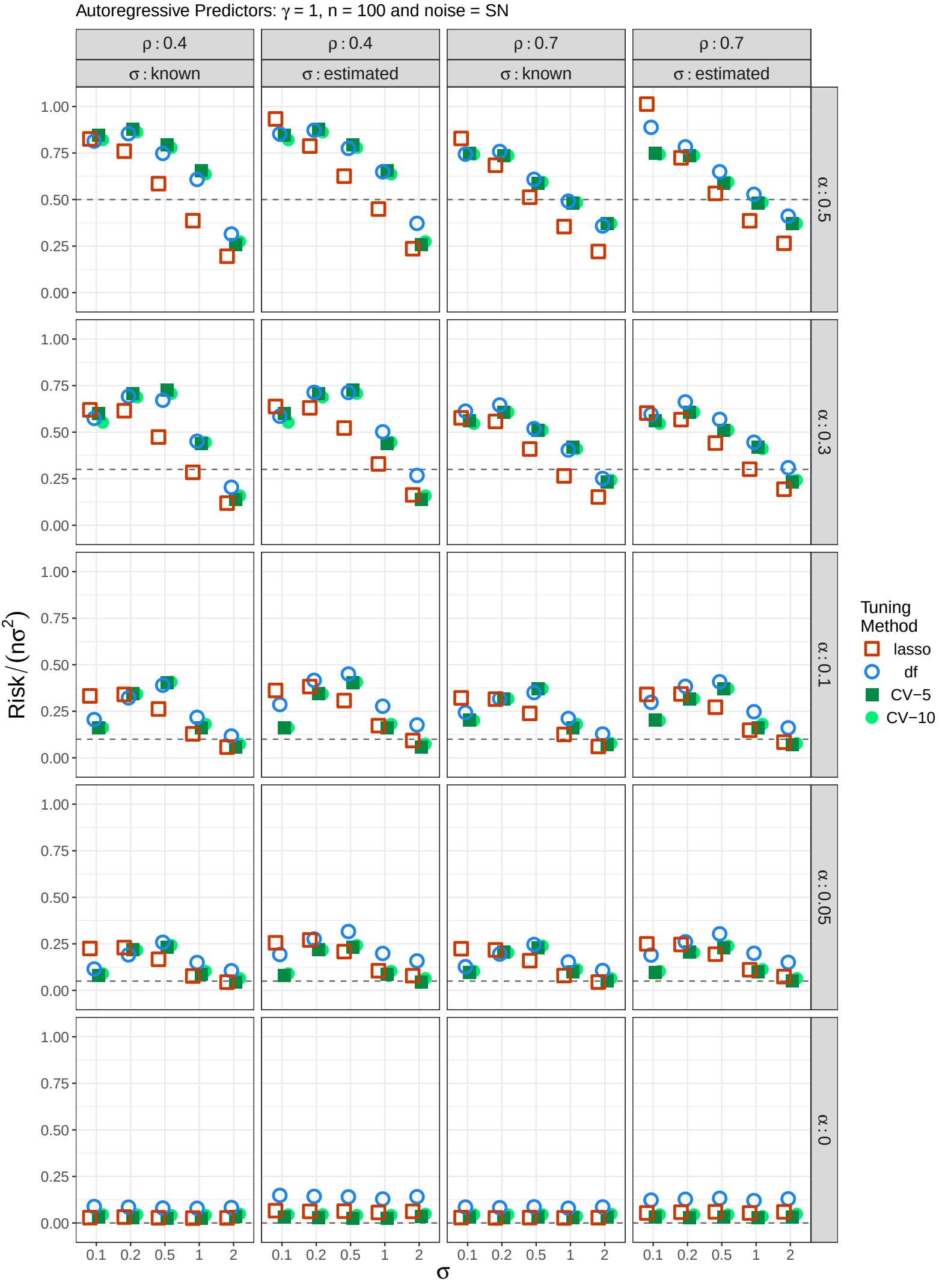
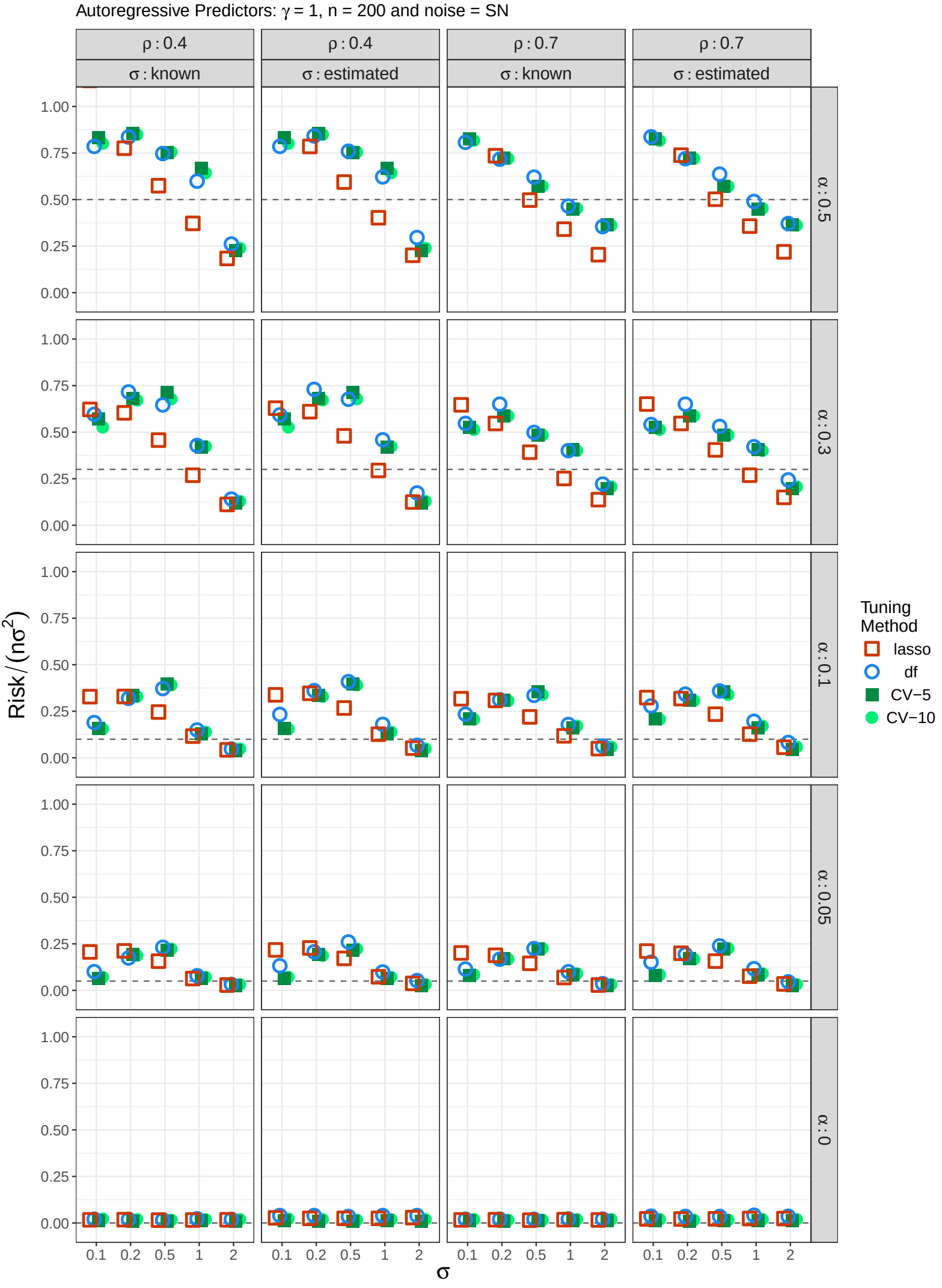
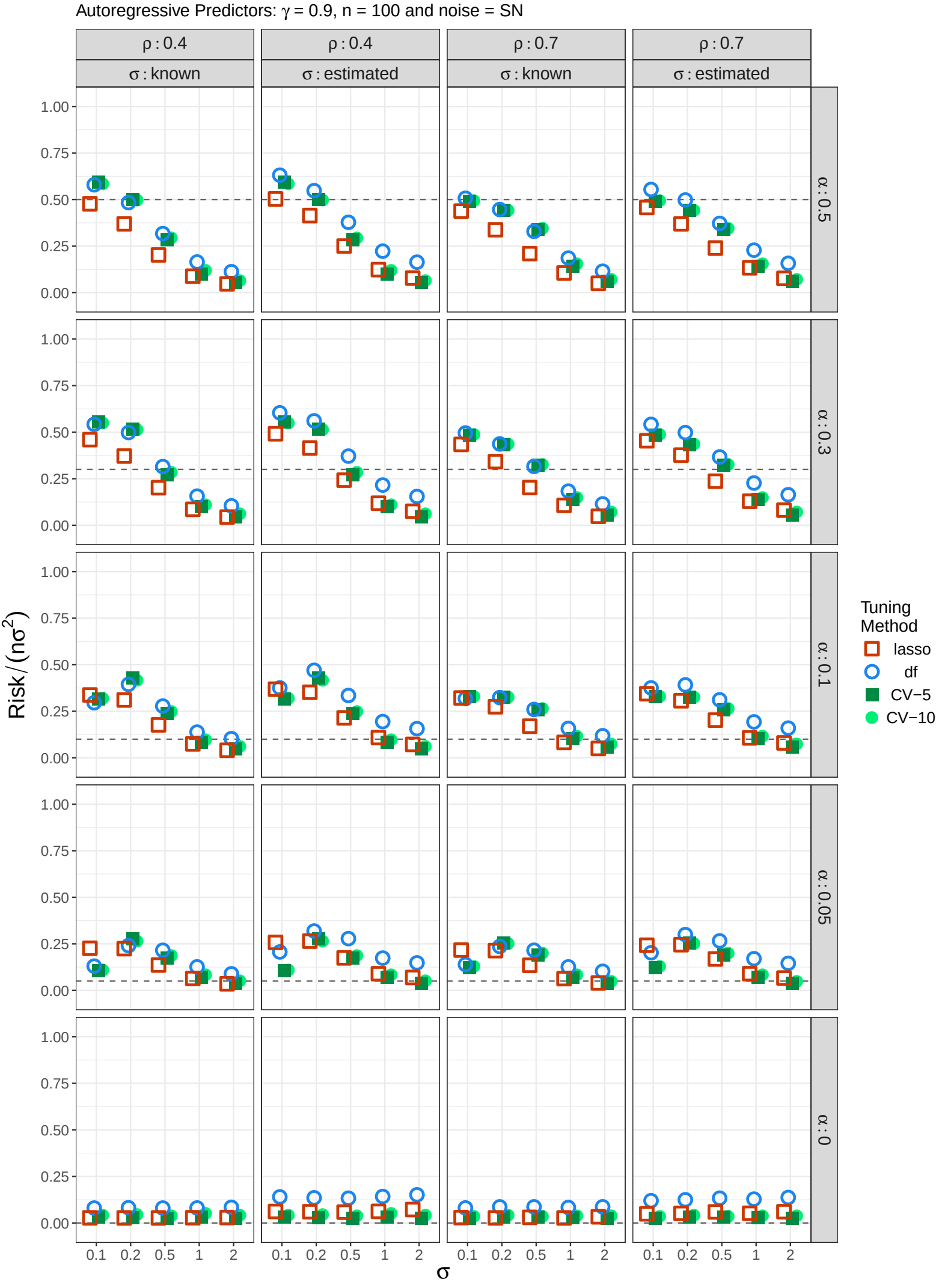
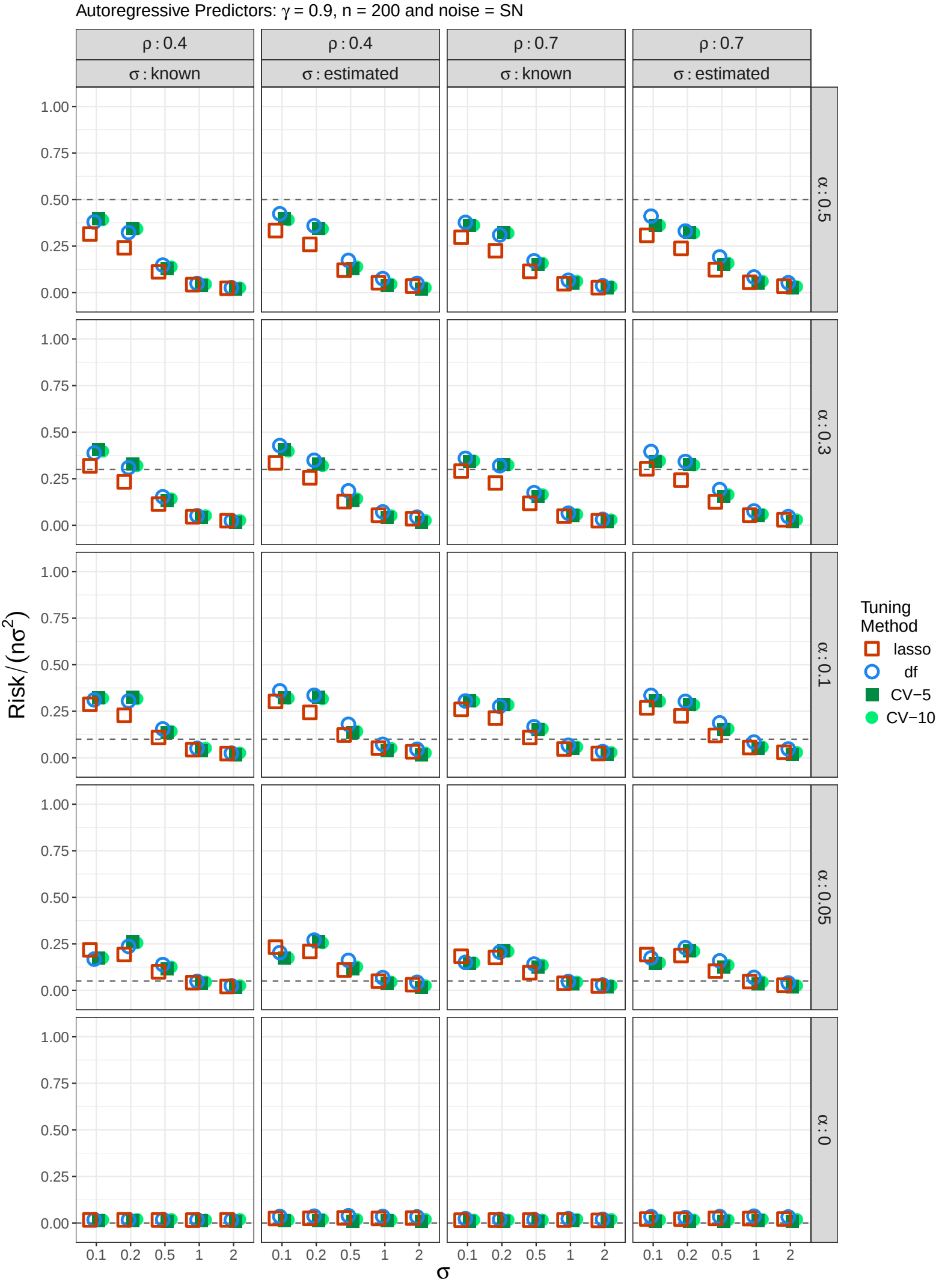
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 100 and noise = SN
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 1, n = 200 and noise = SN
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
σ
0.1
0.2
0.5
1
2
0.00
0.25
0.50
0.75
1.00
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 100 and noise = SN
ρ : 0.4
σ : known
ρ : 0.4
σ : estimated
ρ : 0.7
σ : known
ρ : 0.7
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10


Autoregressive Predictors: γ = 0.9, n = 200 and noise = SN
ρ : 0.4
ρ : 0.4
ρ : 0.7
ρ : 0.7
σ : known
σ : estimated
σ : known
σ : estimated
α : 0.5
α : 0.3
α : 0.1
α : 0.05
α : 0
Risk/(nσ²)
1.00
0.75
0.50
0.25
0.00
0.1
0.2
0.5
1
2
σ
Tuning Method
lasso
df
CV−5
CV−10

SUPPL. MAT. DF FOR PIECEWISE LIPSCHITZ ESTIMATORS

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