Role of Average Energy and Plane Waves in the Schrodinger Equation

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In an earlier note (1), it was argued the time-independent Schrodinger equation may be written in a momentum form by using W(x)=Sum over p fp exp(ipx)= wavefunction and V(x)=Sum over p Vp exp(ipx). This lead to an equation for each plane wave of the form: p*p/2m fp + Sum over j+k=p Vj fk = E fp ((1)). In this note, we try to argue that ((1)) does not need to follow from the time-independent Schrodinger equation, but rather seems to be a consequence of fp, which is related to conditional probability P(p/x).

Time-Independent Schrodinger Equation

The time-independent Schrodinger equation may be written as:

(-1/2m) d/dx d/dx W(x) + V(x) W(x) = E W(x) ((2))

If one writes both W(x) and V(x) as a Fourier series and collects coefficients of exp(ipx), one obtains:

p*p/2m fp + Sum over j+k=p Vk fk = E fp ((1)).

It is argued that because each fp uses the same "E", there is a resonance and that equation ((1)) is of the form of a d/dt fp(t) equation leading to exp(iEt) for each p wave.
If one multiplies ((1)) by fp and sums, then:

Average kinetic energy + Sum over p and j fp f(-j+p) Vj = E

so the second term on the left should be the average potential energy Integral dx W(x)W(x) V(x).

Result Not Using the Schrodinger Equation

The focus of this note is to see whether ((1)) may be obtained without using the Schrodinger equation. Consider writing V(x) as a Fourier series and assume each Vj (Fourier transform) actually represents a momentum in terms of its effect on an incoming momentum. Then, by conservation of momentum a collision with Vj could change a particle with momentum k to k+j. Here we are working in one dimension. In such a case, the particle would be represented by exp(ikx) as Vj is associated with exp(i jx). Assume a particle is released into a region where V(x) is present. It will scatter from one momentum state into another, and if a steady state result ensues, there will be a weight fp for each momentum state. Thus, the conditional probability P(p/x) may be written as:

P(p/x) = fp exp(ipx) / W(x)   where W(x) = normalization constant = Sum over p fp exp(ipx) ((3))

If a resonance exists, one may consider different fk's reacting with Vj's to form a particular fp i.e.

Sum j+k = p   Vj fk   ((4))

((3)) is an unusual term as it involves potential energy somehow through Vj and probability through fk. (The overall probability to have a certain p is fp*fp, but fp is a conditional probability related to having a particular p at a point x. It is possible to show that one may go from conditional probabilities related to fp, to non-conditional probability P(p)=fp*fp. This has been argued in (2).)

Now, given a particular fk, it can combine with any Vj to produce different fl's. If fp is related to a conditional probability at x by ((3)), then different fp's represent the relative probabilities for fk to combine with a particular Vl. Thus, one expects:

fk Vj   related to f(k+j) with no other factor depending on momentum ((5))

Thus, there are different probabilities for fk combining with different Fourier transforms Vl.
One must be careful, however, because one is not only dealing with probabilities, but with energy, which is taken to be separate from probability. Thus,

p*p/2m fp + Sum k+j=p   Vj fk = E fp ((6))

Here E and not Ep is used on the right hand side of ((6)) because of ((5)). If one used Ep, which would be a function of p, it would imply there is something more than conditional probability fp governing ((5)). The left hand side of ((6)) contains all energy pieces related to p, so the right hand side should reflect that the conditional probability to have a p at x is proportional to fp with the constant being independent of p. This constant must be average energy due to the nature of the left hand side of ((6)). From ((6)), however, one obtains the time-independent Schrodinger equation.

It should be noted that since one is dealing with a single particle, the fk's are being produced at different points in time. The sum in ((6)) cycles through all fk's for each p. Thus, ((6)) might be interpreted as carrying time information with iE fp = d/dt fp(t). Then, all fp's would carry a factor of exp(iEt) representing cycling in a time of 1/E. Here E is the average energy.

Conclusion

In conclusion, we argue in this brief note that one may obtain the time-independent Schrodinger equation by considering scattering off a potential V(x) written as a Fourier series. It is assumed that the particle scattering has the form exp(ipx) and that the p wave carries a weight of fp related to a conditional probability P(p/x). A particular fk exp(ikx) may scatter with different Fourier transform values Vl with conditional probabilities related to f(k+l). Given that one writes the interaction as Vl fk, energy and not only probability is involved. Thus, one is led to p*p/2m fp + Sum j+k=p Vj fp = E fp where E, the average energy, does not depend on p.

References

1. Ruggeri, Francesco R. 2x2 Matrix Model and Schrodinger Equation (preprint, zenodo, 2018)

2. Ruggeri, Francesco R. Wavefunction Squared as Spatial Probability Density (preprint, zenodo, 2018)