MIXING IN STARS, MIXING IN MESA

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Outline

Lecture 1 (today):

- Compositional mixing

Lecture 2 (tomorrow):

- Angular momentum transport

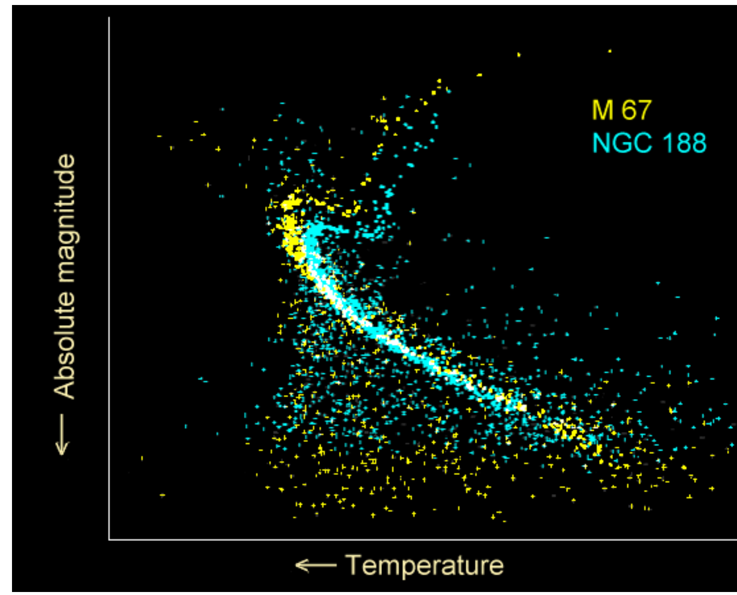
Outline

Lecture 1: Compositional mixing

A. Introduction

The success of standard stellar models

- Stellar evolution theory is incredibly successful at explaining stellar observations, as for instance:
- Properties of HR diagrams



Source: Wikimedia Commons

The success of standard stellar models

- Stellar evolution theory is incredibly successful at explaining stellar observations, as for instance:
- Properties of HR diagrams
- Helioseismic observations
- ….

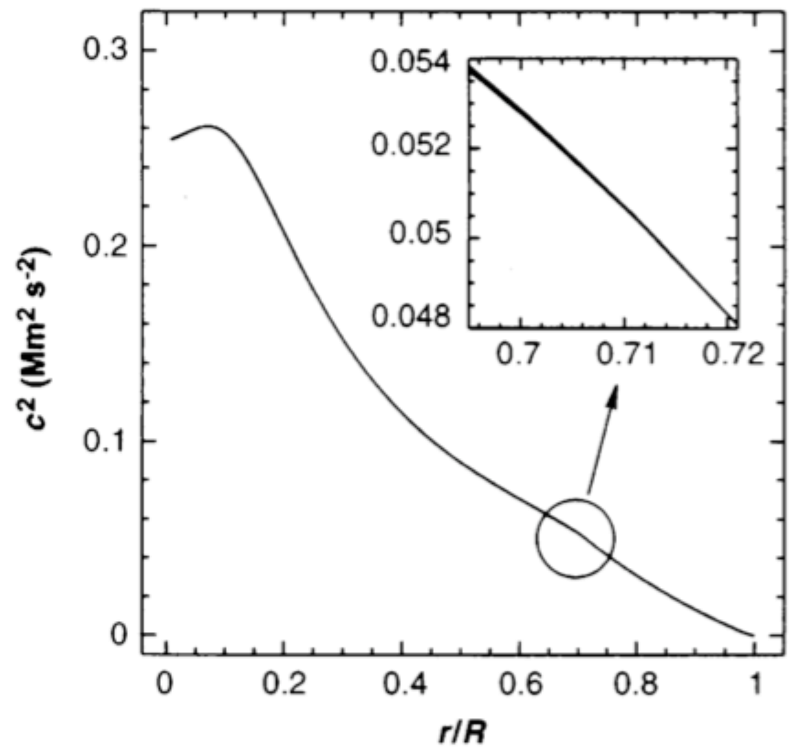


Fig. 3. The dashed curve is the square of the spherically averaged sound speed in the sun. The solid curve corresponds to a standard theoretical model.

Gough et al, 1996

The success of standard stellar models

- Stellar evolution theory is incredibly successful at explaining stellar observations, as for instance:
- Properties of HR diagrams
- Helioseismic observations
- ….

Today, stellar evolution models are sufficiently reliable to be used as tools for other purposes in physics/astrophysics.

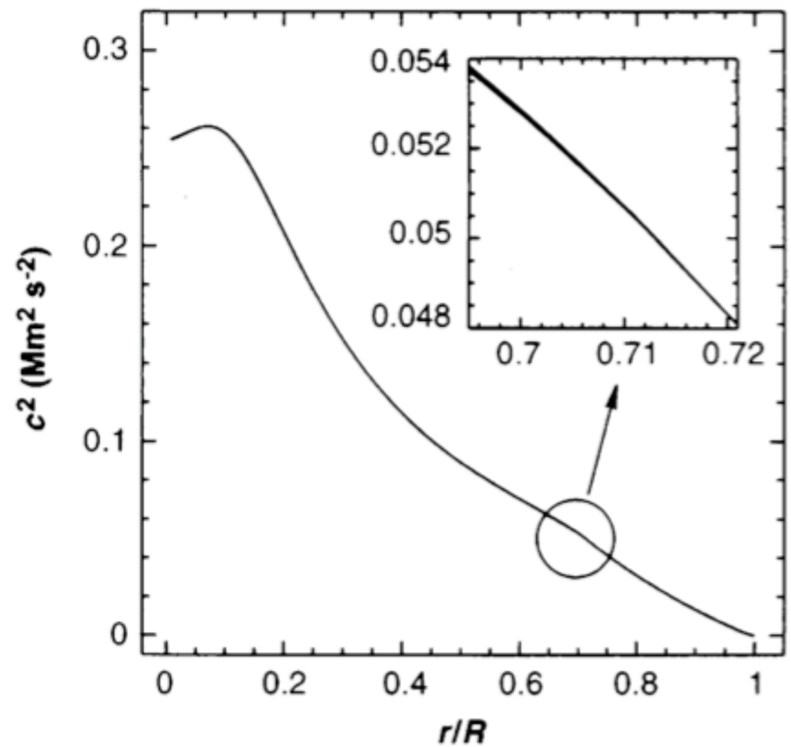


Fig. 3. The dashed curve is the square of the spherically averaged sound speed in the sun. The solid curve corresponds to a standard theoretical model.

Gough et al, 1996

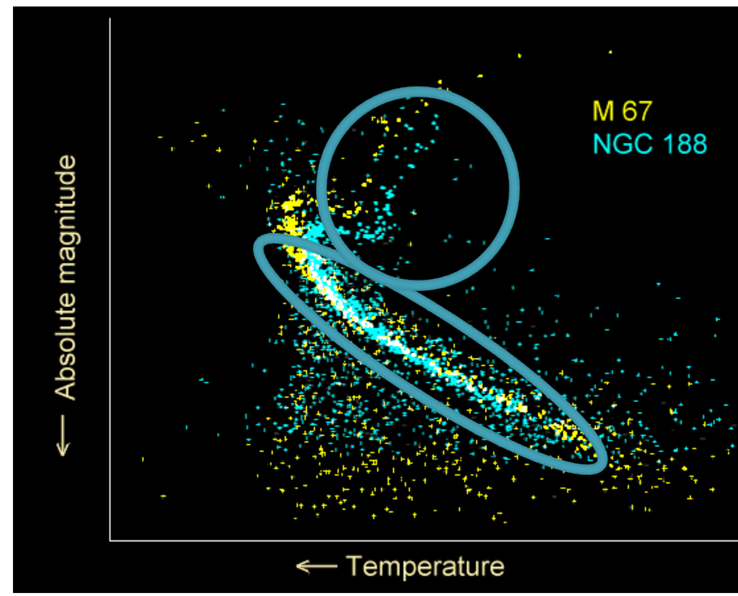
The success of standard stellar models

- This success is perhaps surprising given the "simplicity" of the majority of stellar models:
  - Spherically symmetric, hydrostatic equilibrium
  - Convective zones are chemically homogeneous, energy transport is modeled with mixing-length theory
  - Radiative zones are quiescent, no (or little) chemical mixing, energy transport is radiative.

→ Most salient properties of stellar evolution lie in the microphysics, which are "well-represented" in models:
  - Equation of state
  - Nuclear reaction rates
  - Opacities
  - Surface boundary conditions/atmosphere model

However…

- Currently used models for the *macroscopic* transport of chemical species and angular momentum are very crude.
- Discrepancies between models and observations remain, suggesting need for improvement.
- These manifest themselves both on the Main Sequence and in the Post-MS phase.



Outline

Lecture I: Compositional mixing

B. Missing mixing on the MS

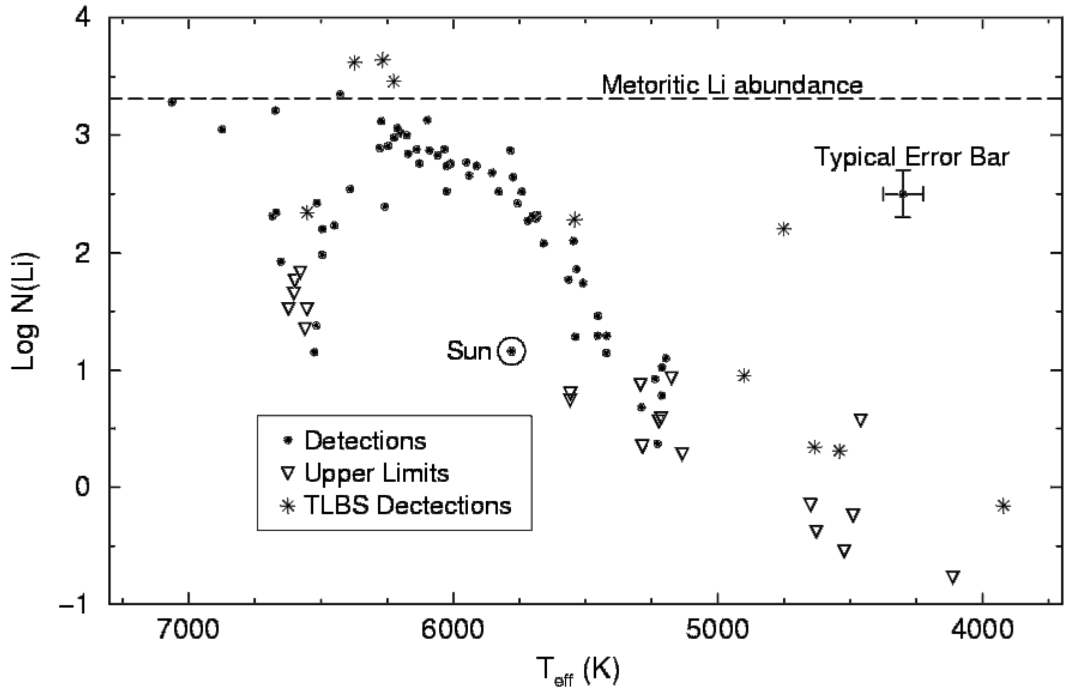
1. The Li abundance problem (part 1)
2. How to model mixing mathematically
3. Convective mixing & its implementation in MESA
4. MESA activity #1: Li depletion in the Hyades
5. The Li abundance problem (part 2)
6. Overshoot & its implementation in MESA
7. MESA activity #2: Li depletion in the Sun
8. Other compositional mixing?

1. The Lithium problem

Example: the Pleiades & Hyades are young clusters in the Taurus constellation.

1. The Lithium problem

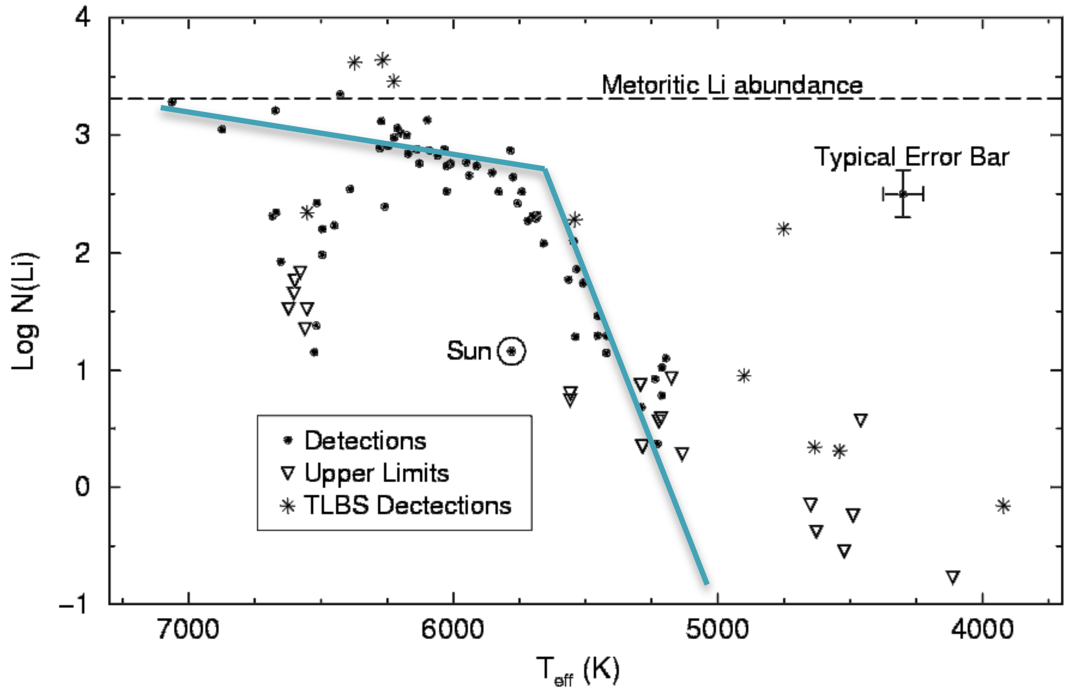
The surface Lithium abundance of Hyades stars as a function of mass (T_{eff}) reveals interesting features.

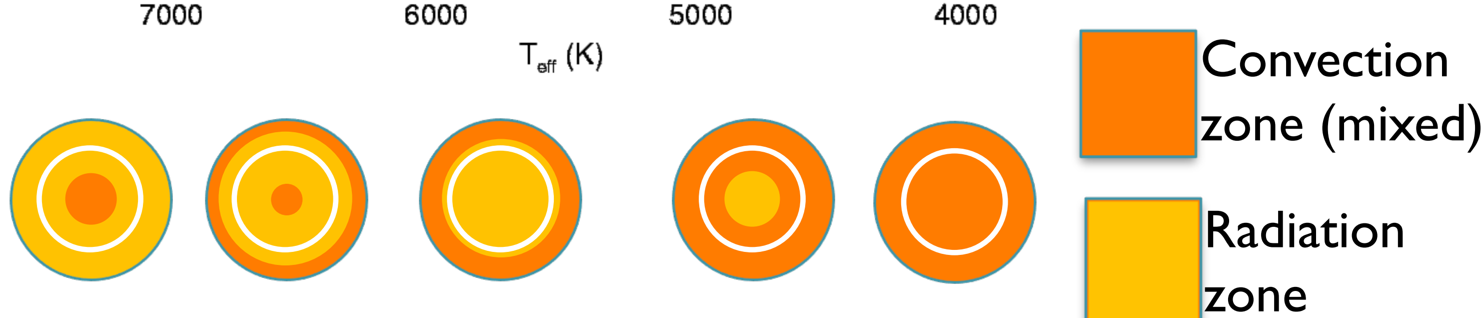


General trend: at same age, high-mass stars (usually) have more Li than low-mass stars.

1. The Lithium problem

This trend can be understood by thinking of convection, and noting that Li is destroyed by nuclear reactions at $T > 2.5\ 10^6$ K.





2. How to model mixing mathematically

- Modeling transport in stellar interiors always starts from the basic conservation equations of fluid mechanics:

$$\underbrace{\frac{D\rho}{Dt}}_{1} = \underbrace{-\rho\nabla\cdot\mathbf{u}}_{2}, \quad \text{and} \quad \underbrace{\frac{D\rho_S}{Dt}}_{1} = \underbrace{-\rho_S\nabla\cdot\mathbf{u}}_{2} \underbrace{-\nabla\cdot\mathbf{F}_s}_{3} + \underbrace{\left(\frac{D\rho_S}{Dt}\right)_{nucl}}_{4}$$

1. Lagrangian change in local density "following the fluid"
2. Effect of compression or expansion of the fluid
3. Additional flux of chemical species in or out of fluid parcel
4. Nuclear reactions

2. How to model mixing mathematically

- Modeling transport in stellar interiors always starts from the basic conservation equations of fluid mechanics:

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\mathbf{u}, \quad \text{and} \quad \frac{D\rho_S}{Dt} = -\rho_S\nabla\cdot\mathbf{u} - \nabla\cdot\mathbf{F}_s + \left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

- Usually we are interested in the mass fraction of a particular species:

$$X_s = \frac{\rho_s}{\rho}$$

$$\rightarrow \frac{DX_s}{Dt} = \frac{1}{\rho}\frac{D\rho_s}{Dt} - \frac{\rho_s}{\rho^2}\frac{D\rho}{Dt}$$

$$= \frac{1}{\rho}\left[-\rho_s\nabla\cdot\mathbf{u} - \nabla\cdot\mathbf{F}_s\right] - \frac{\rho_s}{\rho^2}\left[-\rho\nabla\cdot\mathbf{u}\right] + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

$$\rightarrow \frac{DX_s}{Dt} = -\frac{1}{\rho}\nabla\cdot\mathbf{F}_s + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

2. How to model mixing mathematically

$$\frac{DX_s}{Dt} = -\frac{1}{\rho}\nabla\cdot\mathbf{F}_s + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

- It is common to assume that $\mathbf{F}_s = -D_{mix}\rho\nabla X_s$
  - D is a diffusivity, and has units of cm^2/s (in cgs).

- However, other cases can also arise, where the flux is proportional to the pressure gradient, or to the temperature gradient, etc… These are important for "atomic diffusion", recently implemented in MESA (not the subject of this lecture), however.

2. How to model mixing mathematically

- Combining these equations, we get

$$\frac{DX_s}{Dt} = \frac{1}{\rho}\nabla\cdot(\rho D_{mix}\nabla X_s) + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

$$= \frac{1}{\rho r^2}\frac{\partial}{\partial r}\left(r^2\rho D_{mix}\frac{\partial X_s}{\partial r}\right) + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

$$\boxed{\frac{DX_s}{Dt} = \frac{\partial}{\partial m}\left((4\pi r^2\rho)^2 D_{mix}\frac{\partial X_s}{\partial m}\right) + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}}$$

- This is the mathematical equation actually implemented in MESA. The only question left is:

What is D_{mix}?

2. How to model mixing mathematically

- The mixing coefficient D_{mix} can be due to
  - Basic collisional processes (i.e. microscopic)
  - Turbulent processes (i.e. macroscopic).

- Since D_{mix} has units of length^2/time, or length x velocity, we often (not always) estimate it from

$$D_{mix} \propto \underbrace{v}_{\text{Characteristic velocity}} \underbrace{l}_{\text{Characteristic lengthscale}}$$

- In general, the turbulent processes (if present) lead to much larger values of D_{mix} than microscopic processes.

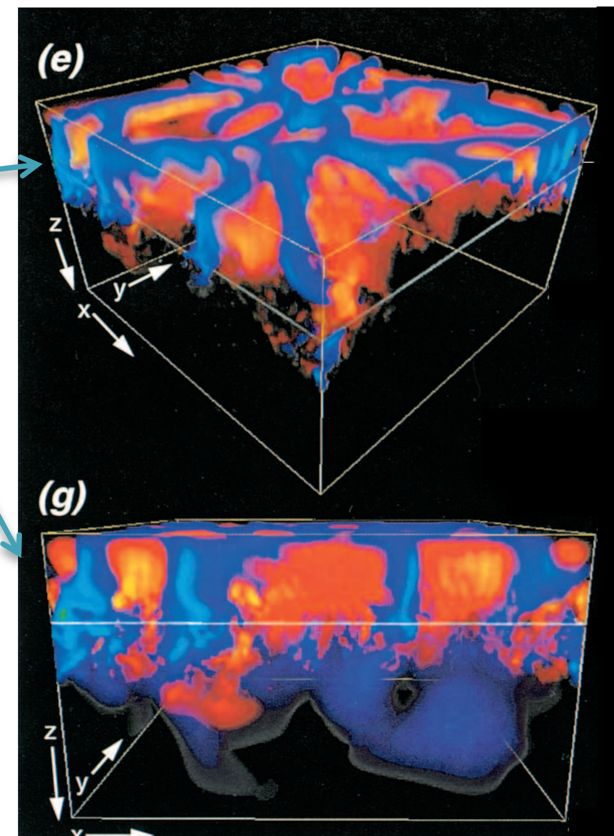
3. Convective mixing in MESA

- There are 2 mixing processes usually taken into account in "canonical models of stellar mixing":
  - Convection
  - Overshoot (see later)

- Convection occurs in regions where

$$\frac{\partial \rho}{\partial p} < \left(\frac{\partial \rho}{\partial p}\right)_{ad}$$

Note: This generalized criterion includes both Schwarzschild criterion (if no compositional gradient present) and Ledoux criterion (if there is one).



Brummell et al. 2002

3. Convective mixing in MESA

- Mixing of chemical species by convection is usually done by assuming that :
  - v is the mean velocity of the convective eddies v_{conv}
  - l is the mean travel distance of the eddies (the "mixing length" l_{conv}).

- In standard mixing-length-theory (cf. Cox & Giuli), we have

$$v_{conv} = l_{conv}\sqrt{\frac{1}{8}\frac{g}{H_p}\delta(\nabla - \nabla_e)}$$

where g is gravity, H_p is the pressure scaleheight, and

$$\delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}, \quad \nabla = \left(\frac{d \ln T}{d \ln P}\right)_{background}, \quad \nabla_e = \left(\frac{d \ln T}{d \ln P}\right)_{element}$$

3. Convective mixing in MESA

The convective mixing coefficient is then calculated as

$$D_{conv} = \frac{1}{3} v_{conv} l_{conv}$$

This is all done in MESA in the `mlt.f` routine in `mlt/private/`, in the subroutines `standard_scheme` and `Get_results`.

Side note: MLT assumes that there is no compositional gradient in a convection zone to calculate the mixing coefficient for composition … slight inconsistency, but all stellar evolution codes do it!

4. Activity I: Li depletion in the Hyades

- Pick a stellar mass randomly in mass range 0.5 – 1.5 M_{sun}
- Starting from a PMS model, integrate MESA model up to Hyades age (625Myr).
- Record Li abundance at Hyades age in Google Sheet (see Agenda).

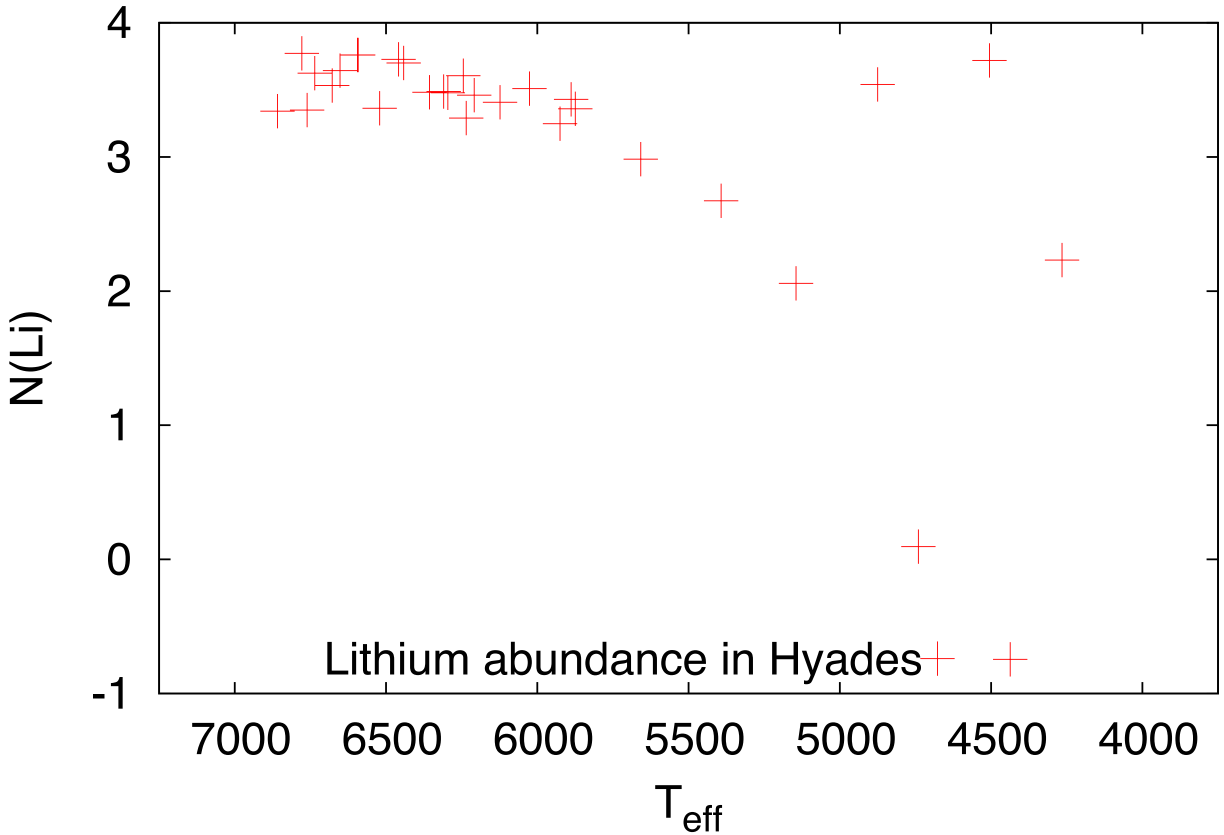
See detailed instructions + downloads for inlist, etc.. in

http://mesastar.org/teaching-materials/2014-mesa-summer-school-working-dir

Download `garaud_day1.tar.gz`, untar and unzip.

Use `ms/` directory.

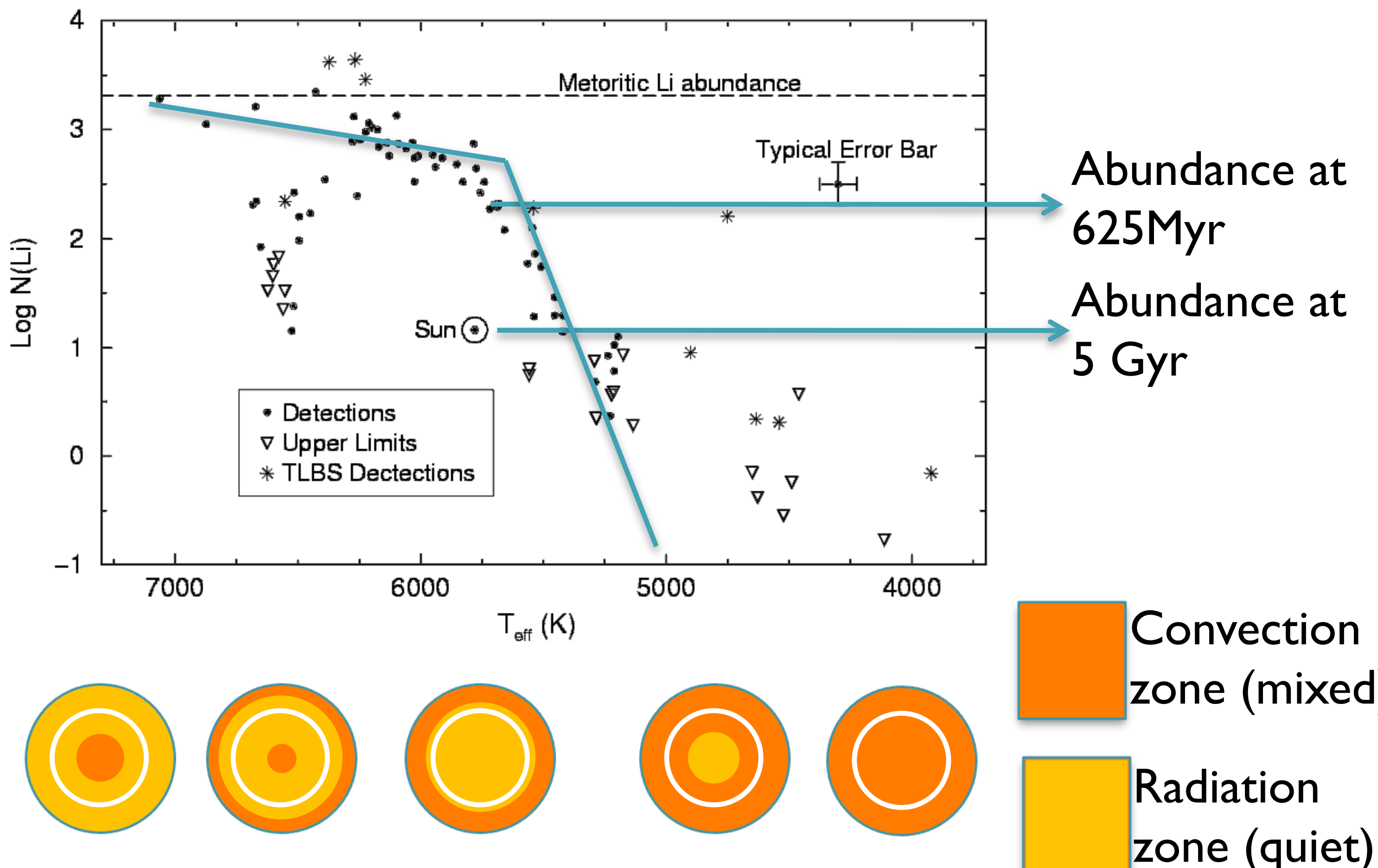
Results



4
3
2
1
0
-1
N(Li)
Lithium abundance in Hyades
7000
6500
6000
5500
5000
4500
4000
T_{eff}

5. The Lithium problem (part 2)

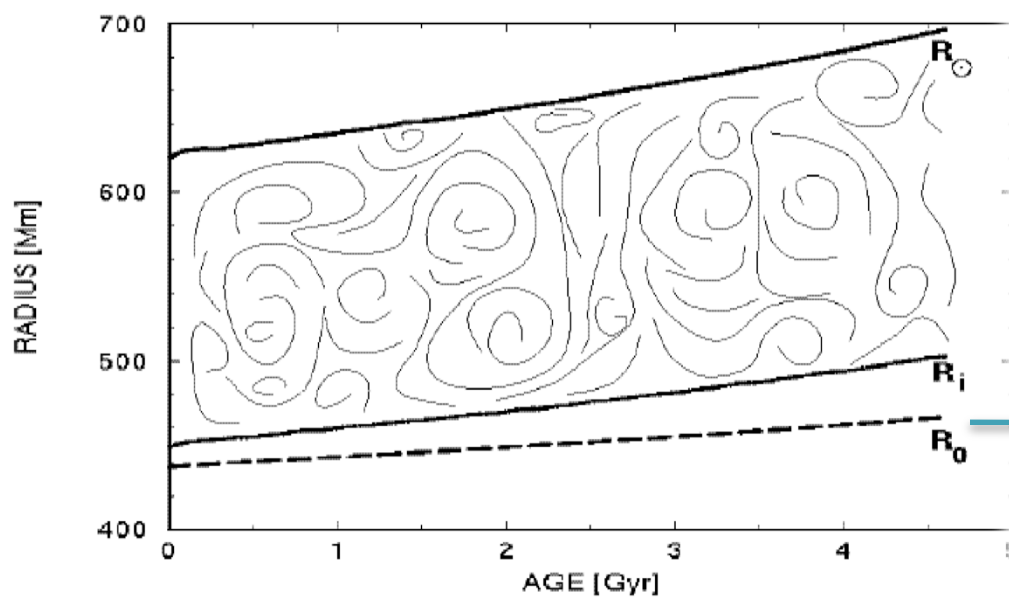
Note the position of the Sun on this diagram:



5. The Lithium problem (part 2)

However, the Li-burning radius never overlaps the convection zone in stars of mass 1Msun or larger:

- Li depletion on the Main-Sequence suggests that there must be additional mixing below the outer convection zone.
- Could this be due to convective overshooting?



Rudiger & Pipin 2000

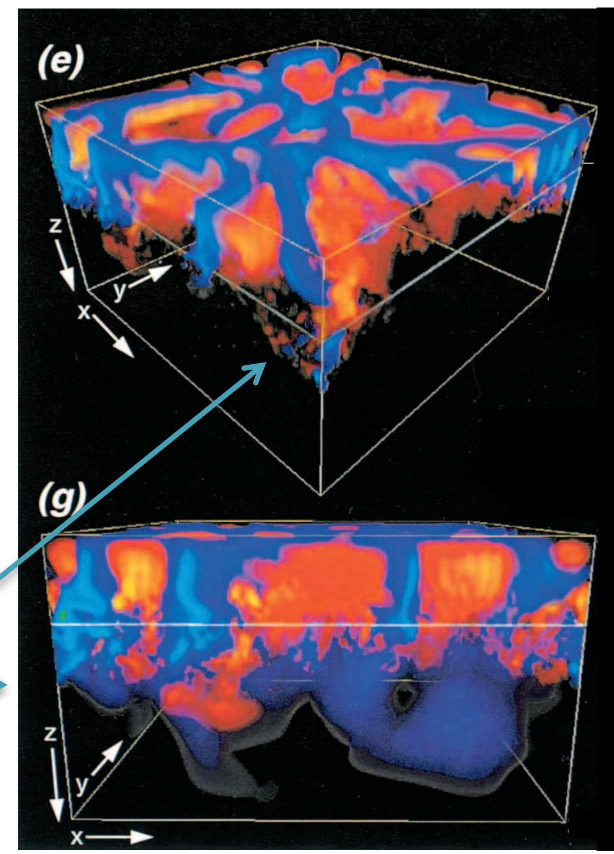
Li destruction radius in the Sun

6. Overshoot in MESA

- There are 2 mixing processes usually taken into account in "canonical models of stellar mixing":
    - Convection
    - Overshoot
- Convection occurs in regions where

$$\frac{\partial \rho}{\partial p} < \left(\frac{\partial \rho}{\partial p} \right)_{ad}$$

- Convective plumes do not "stop" at the edge of the convection zone, but instead, "overshoot" into the nearby radiative region.



Brummell et al. 2002

6. Overshoot in MESA

- There are many possible overshoot models in the literature. Furthermore, there are different models depending on the location/nature of the overshoot layer (above/below a CZ, etc..)

- MESA implements by default mixing by overshoot in the routine `star/private/overshoot.f`. It uses the model of Herwig (2000), in which

$$D_{overssht}(r) = D_{conv,edge} \exp\left(- \frac{2|r - r_{edge}|}{f_{overssht} H_p} \right)$$

where:

  - $D_{conv,edge}$ is the convective diffusion coefficient just inside the convection zone
  - r_{edge} is the position of the edge of the convection zone
  - H_p is the local pressure scaleheight
  - $f_{overssht}$ is user-defined. We can define different values for overshoot above/below a CZ, and use different values for burning CZs and non-burning ones.

7. Activity 2: Li depletion in the Sun

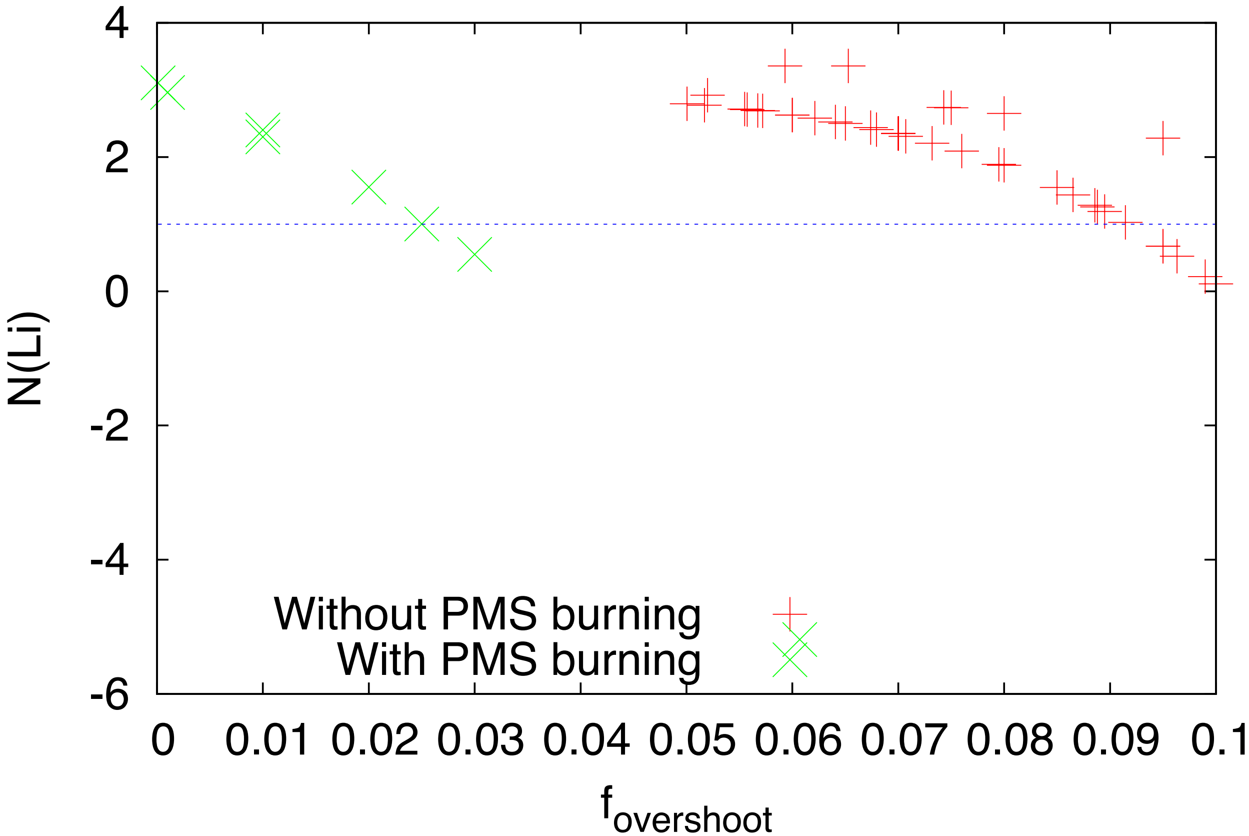
- Using stellar mass $=1M_{sun}$
- Activate overshoot, and pick random number for $f_{overshoot}$
- Starting from a ZAMS model, integrate MESA model up to todays' solar age (4.6Gyr)
- Record $f_{overshoot}$ and corresponding Li abundance at solar age in Google Sheet (see Agenda).

See detailed instructions + downloads for inlist, etc.. in

http://mesastar.org/teaching-materials/2014-mesa-summer-school-working-dir

Continue to use `ms/` directory.

Results



4
2
0
-2
-4
-6
N(Li)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
$f_{overshoot}$
Without PMS burning
With PMS burning

8. Additional mixing processes.

Conclusion:

- $f_{overshoot}$ needed to explain solar Li is really large, much larger than what we believe is reasonable for standard overshoot. The general consensus is that overshoot alone cannot explain Li depletion in the Sun.

- For this reason, a number of other processes have been discussed to explain observed Li depletion in the Sun (and other stars)
  - Mixing by breaking gravity waves (e.g. Charbonnel & Talon 2005)
  - Mixing by large-scale meridional flows (e.g. Gough & McIntyre, 1998)
  - Mixing by fingering convection caused by planetary infall (e.g. Theado & Vauclair 2012)
  - …

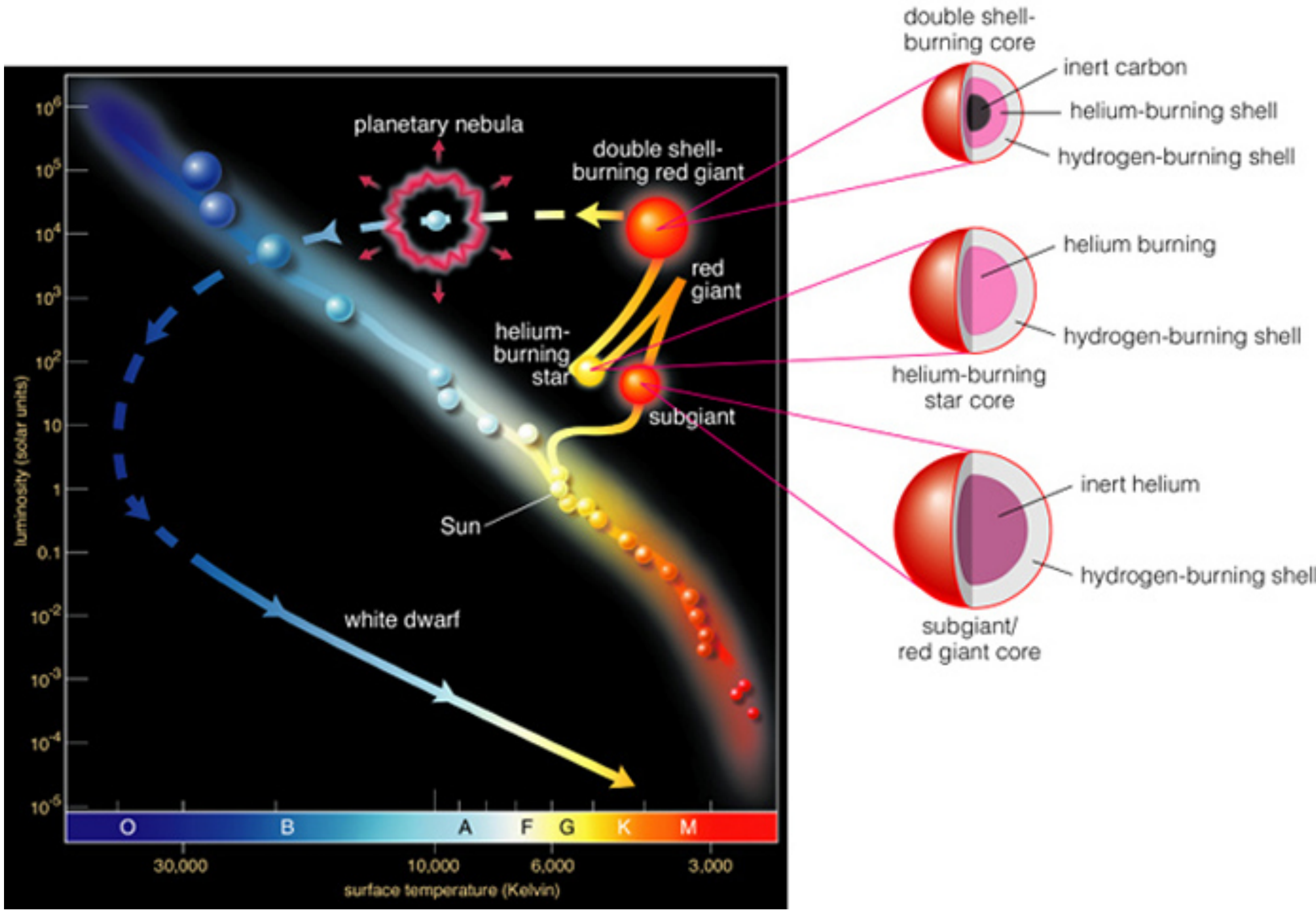
Outline

Lecture I: Compositional mixing

B. Missing mixing on the RGB

1. Introduction to the RGB abundance problem
2. MESA Activity #3: Canonical mixing in RGB stars.
3. Mixing by fingering convection
4. MESA Activity #4: Fingering convection in RGB stars.
5. Other mixing mechanisms.
6. MESA Activity #5: Add-your-own-mixing

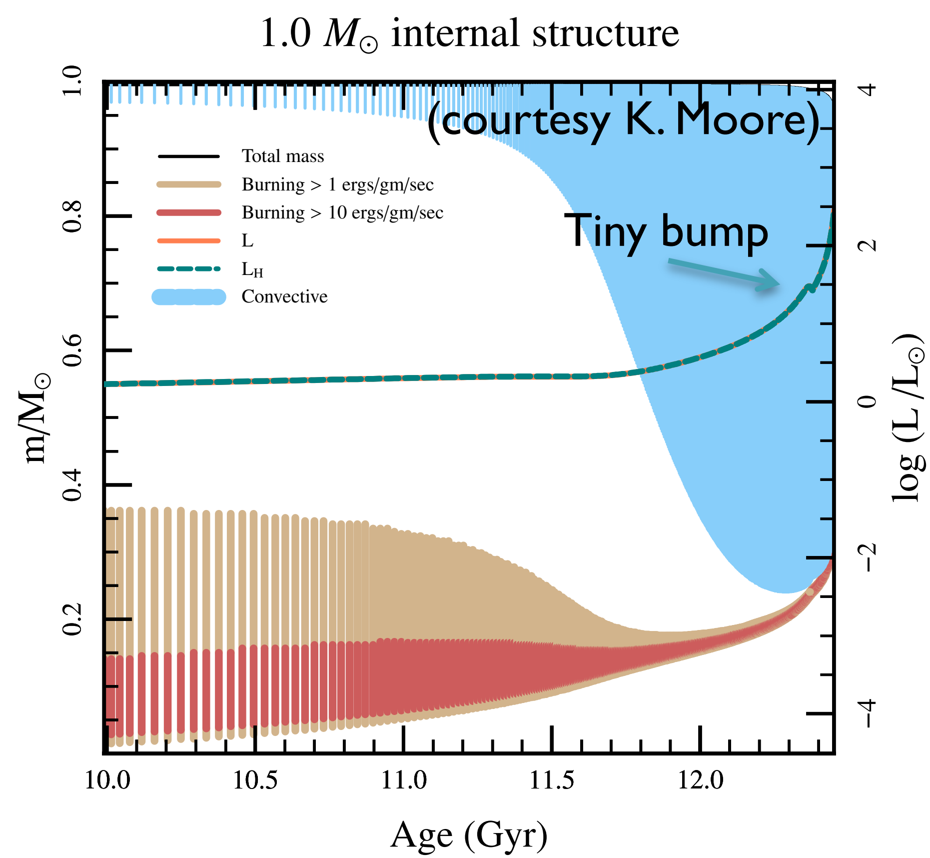
I. Introduction: The Red Giant Branch



1. Introduction: Canonical mixing on RGB

- Upon arrival onto the RGB, the outer convection zone expands and dredges up material from deep within the star: first dredge-up.

- After this event, the base of the convection zone retreats again as the Hydrogen burning shell moves outwards. The two never overlap.

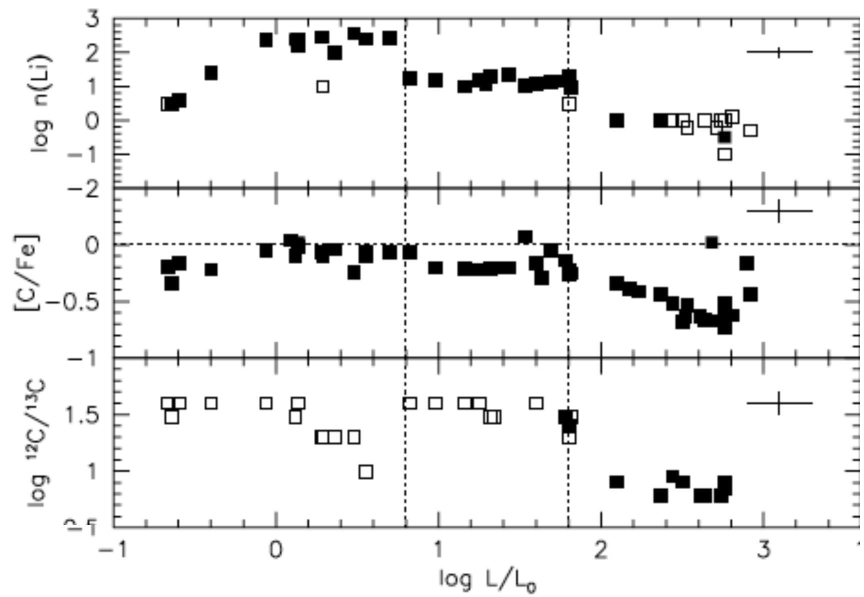


- Because of the prior compositional homogenization, no more changes in surface element abundances are expected on the RGB.

I. Introduction:
Evidence for missing mixing on the RGB

- However, surface abundance data does not support this claim.

- What could cause additional mixing?



Gratton et al. 2000

1st dredge-up: convective mixing

2nd dip in abundances: Overshoot ??

2. Activity 3: Li depletion in RGB (part 1)

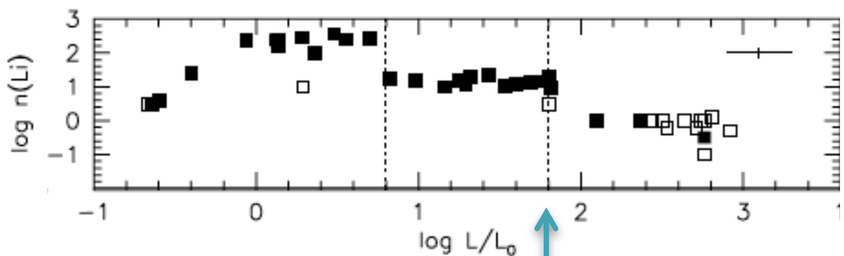
- Starting from a 1Msun, low-metallicity star, with fixed overshoot depth (use inlist provided), study evolution of star on the RGB + its Li surface abundance
  - Note first dredge up, position of Hydrogen burning, etc..
  - Note how deepening of CZ causes dip in Li abundance
  - Note luminosity bump.
- Save model somewhat post-dredge up.

See detailed instructions

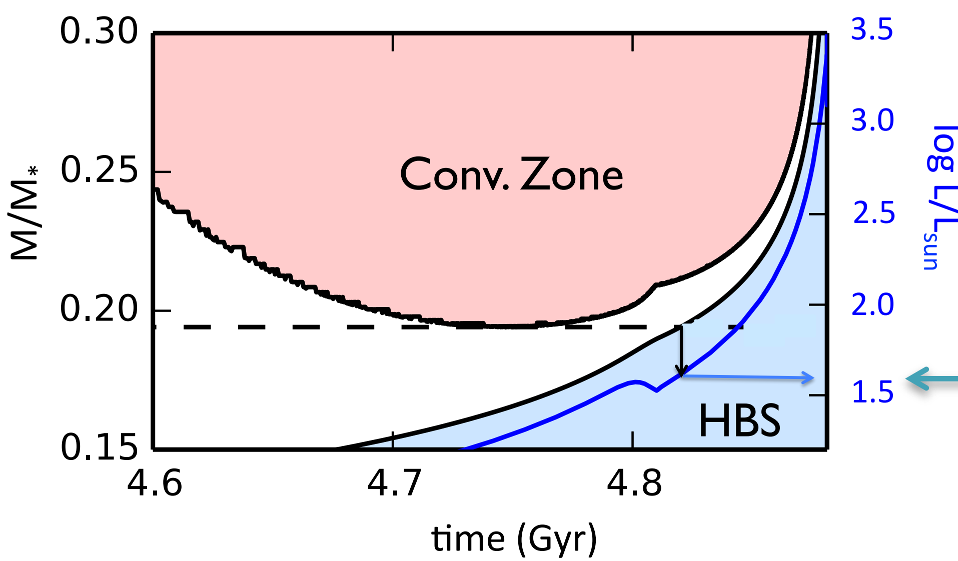
http://mesastar.org/teaching-materials/2014-mesa-summer-school-working-dir

Use `rgb/` directory this time.

3. Mixing by fingering convection



Recall: we want to explain the "second-dip" in abundances.



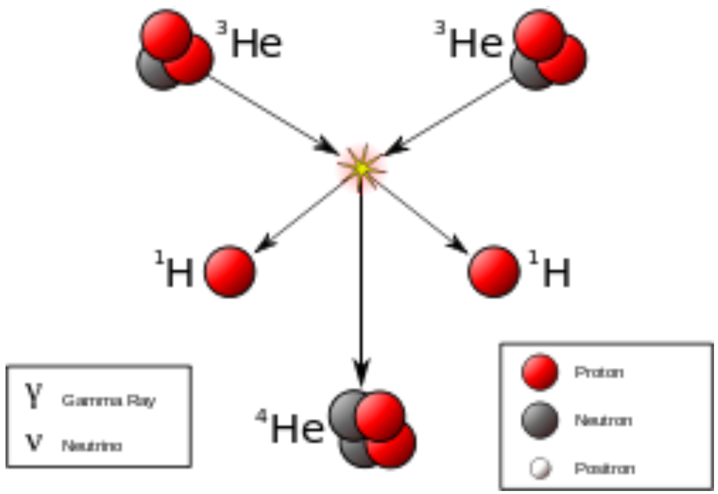
Note how second dip in RGB surface abundances corresponds to luminosity bump in star.

The luminosity bump also happens when the hydrogen-burning shell moves into the region previously mixed by dredge-up.

This is not a coincidence (Eggleton et al., 2006; Charbonnel & Zahn 2007).

3. Mixing by fingering convection

- Near the outer edge of the hydrogen burning shell, the dominant reaction is second part of PP chain.



(Source: Wikipedia)

2 particles of total mass 6

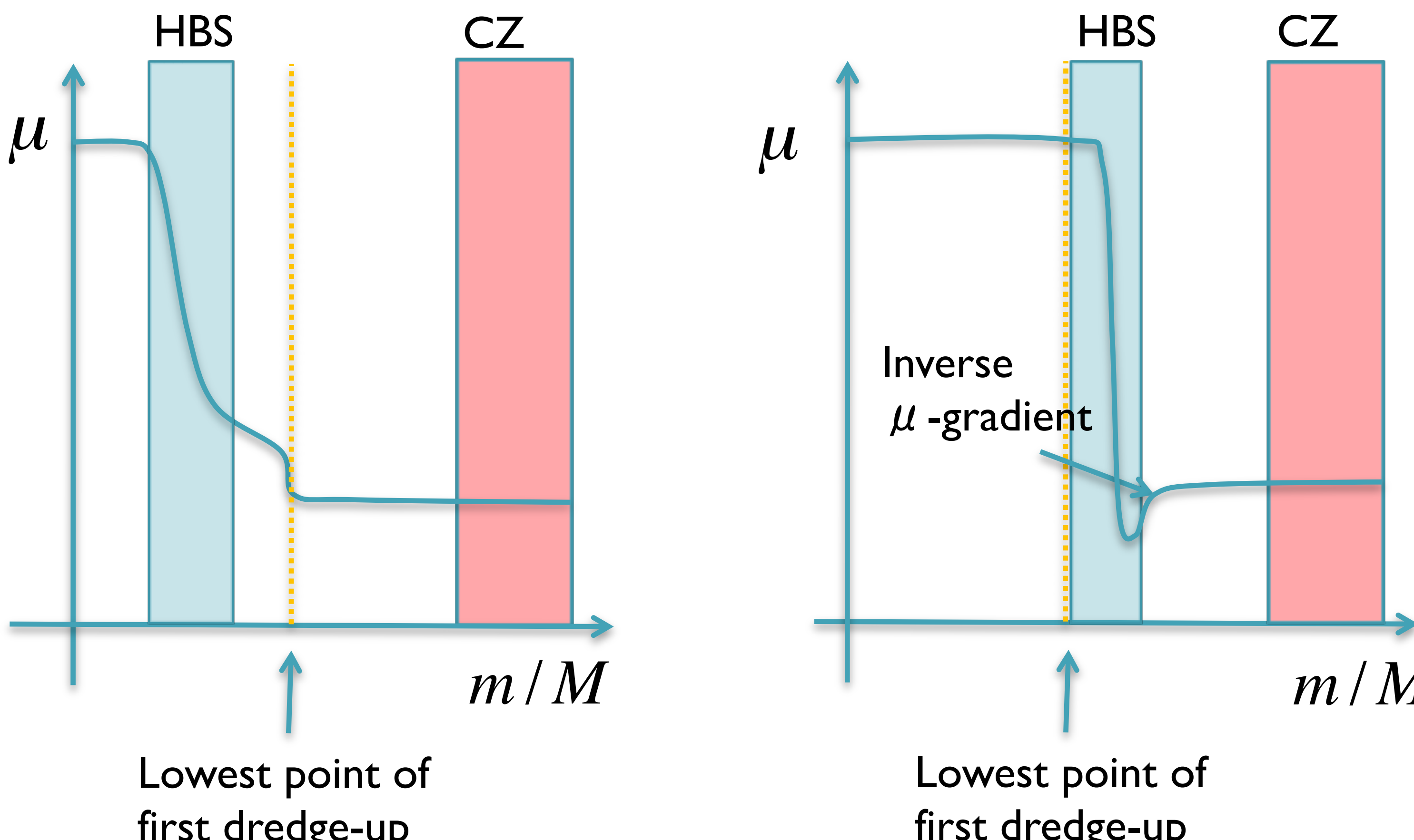
↓

3 particles of total mass 6

This reaction locally destroys ^{3}He and decreases the mean molecular weight.

3. Mixing by fingering convection

- As a result, an inverse μ –gradient can form after luminosity bump, but not before…



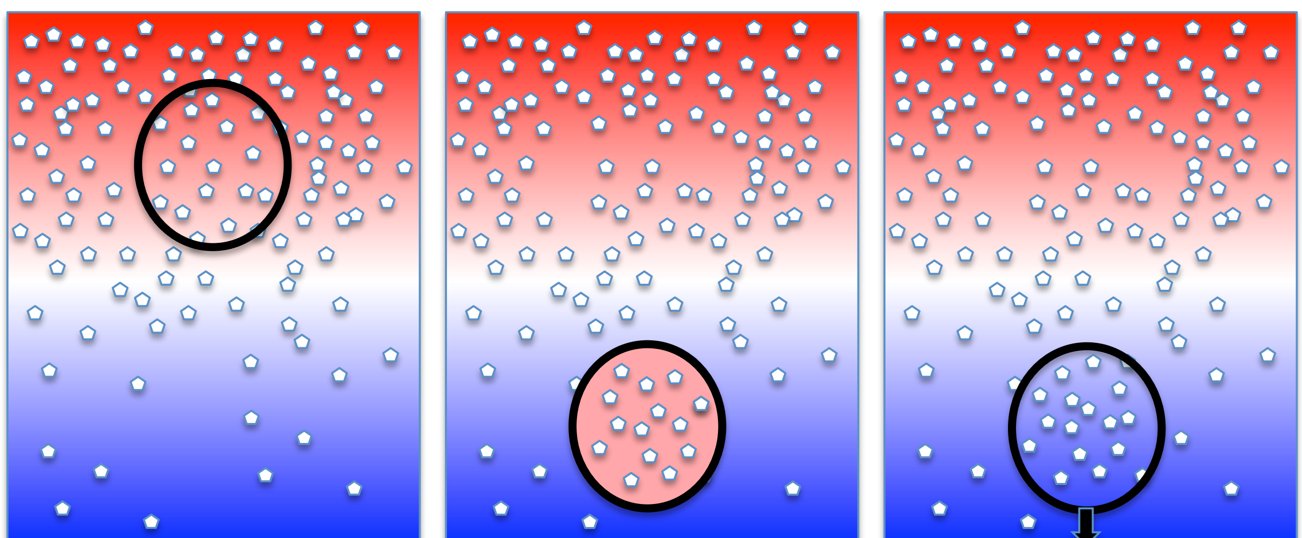
3. Mixing by fingering convection

- Strong enough inverse μ - gradients can trigger convection. This is given by the Ledoux criterion:

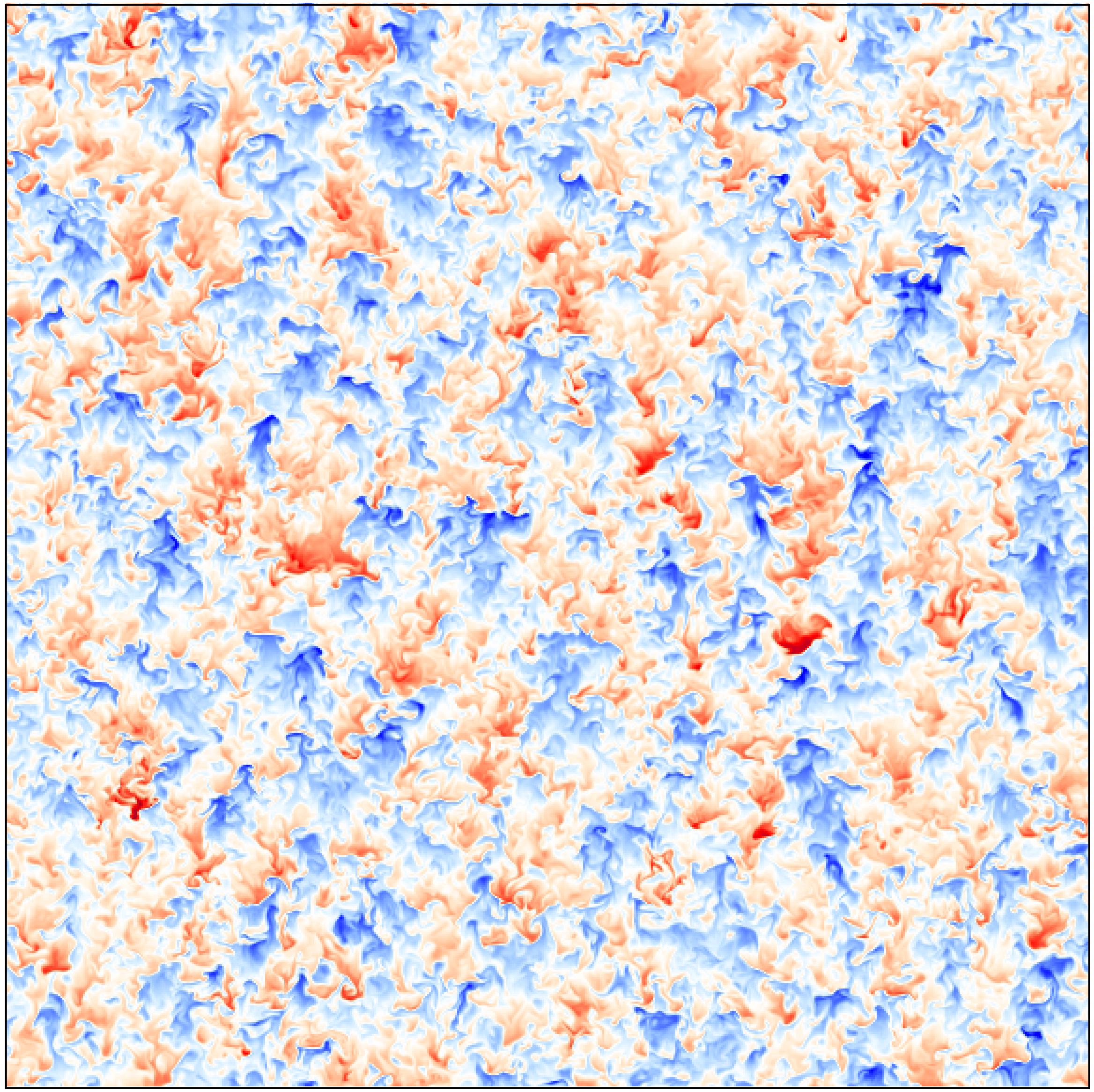
$$\delta\left(\nabla - \nabla_{ad}\right) > \phi\nabla_{\mu}$$

➔ Not the case here, system is always Ledoux-stable.

- *Weak* inverse μ - gradients *stable* to the Ledoux criterion can still trigger **fingering** convection (often called thermohaline convection by analogy with case of similar instability in salt water)

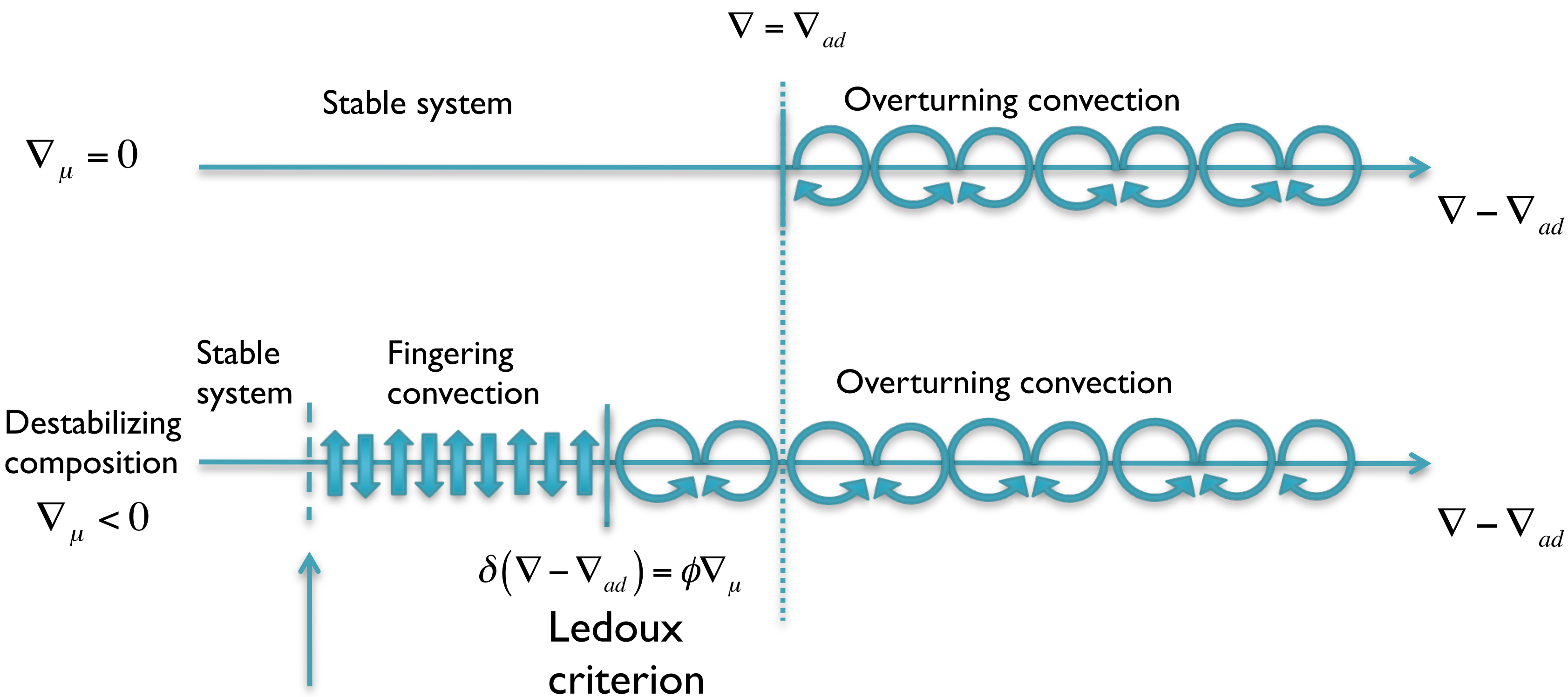


Concentration field



(Gratuitous, self-promoting pretty picture)

3. Mixing by fingering convection



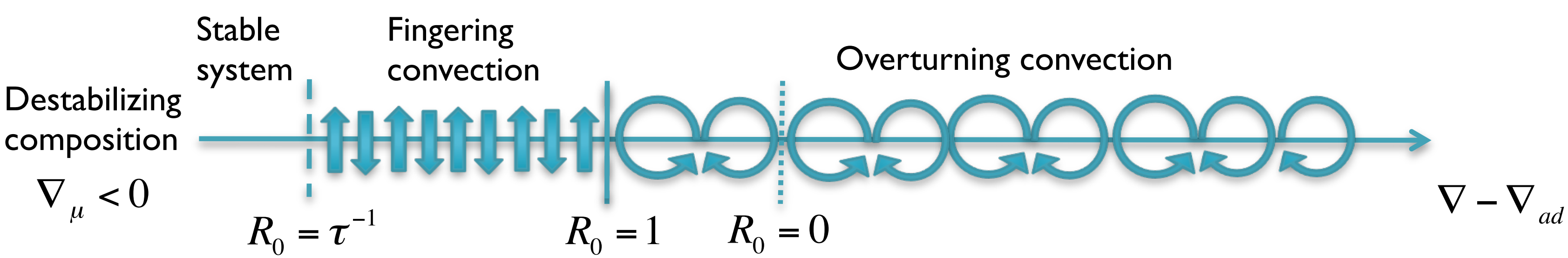
Limit for fingering instability has

$$\frac{\delta(\nabla - \nabla_{ad})}{\phi\nabla_\mu} = \frac{\kappa_T}{\kappa_\mu} = \frac{\text{thermal diff.}}{\text{compositional diff.}} = \frac{1}{\tau} \approx 10^7 \text{ in stars.}$$

3. Mixing by fingering convection

- The efficiency of mixing by fingering instabilities depends on the non-dimensional **density ratio**:

$$R_0 = \frac{\delta\left(\nabla - \nabla_{ad}\right)}{\phi\nabla_\mu}$$



- Ulrich (1972), Kippenhahn et al. (1980) proposed that $D_{finger} = \frac{K}{R_0}\kappa_T$

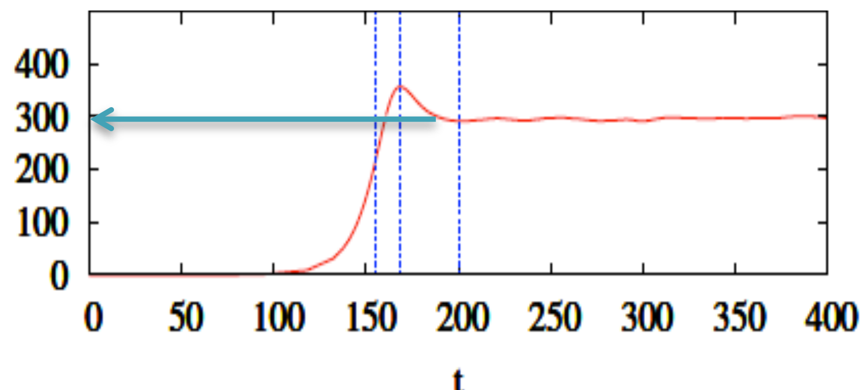
- The value of the constant K was subject of heated debate (estimates range from a few to a few thousands) for decades.

Fingering convection

Numerical simulations help!

$$\frac{D_{finger}}{\kappa_\mu}$$

Nu_μ

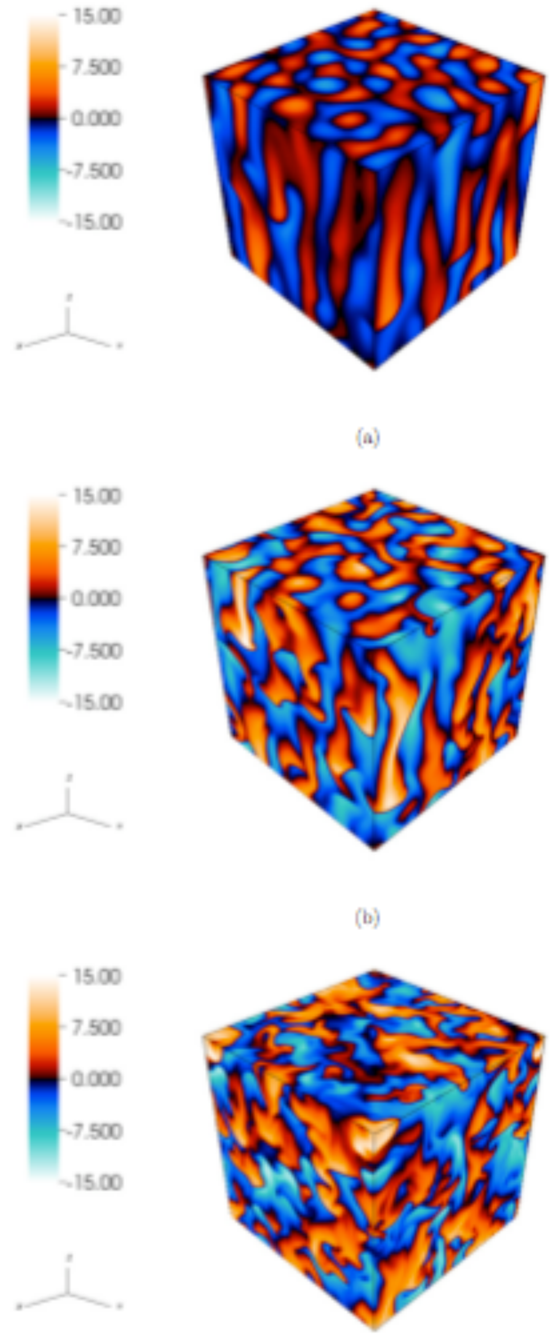


Simulation with:

$$\mathrm{Pr} = \frac{\nu}{\kappa_T} = 0.1,\ \ \tau = \frac{\kappa_\mu}{\kappa_T} = 0.03,\ \ R_0 = 3.0$$

(note: very non-stellar parameters!)

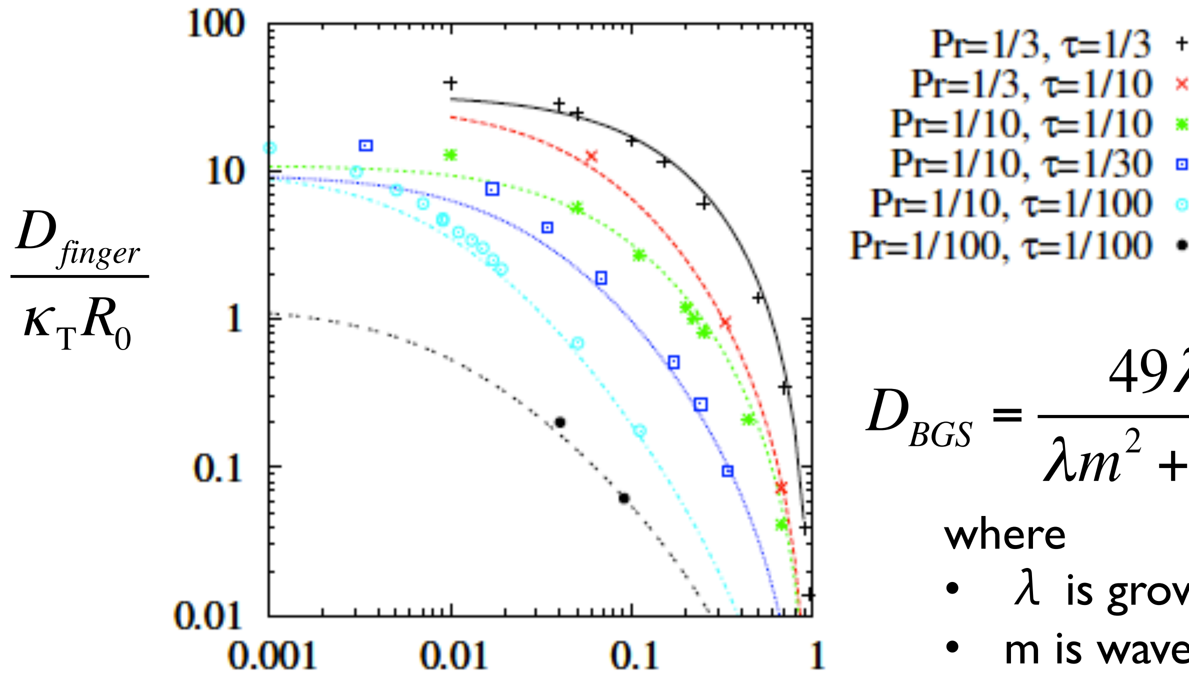
For each given parameter set, we can measure the turbulent diffusivity associated with fingering convection.



Brown, Garaud & Stellmach (2013)

3. Mixing by fingering convection

We can then create models to explain the entire dataset.



$$r = \frac{R_0 - 1}{R_{0,crit} - 1}$$

$$D_{BGS} = \frac{49\lambda^2}{\lambda m^2 + \tau m^4}\kappa_T \equiv Nu_\mu \kappa_\mu$$

where

- λ is growth rate
- m is wavenumber

of most unstable mode from linear instability theory.

These models can now be extrapolated to stellar parameters!

Brown, Garaud & Stellmach (2013)

4. Activity 4: Li depletion in RGB (part 2)

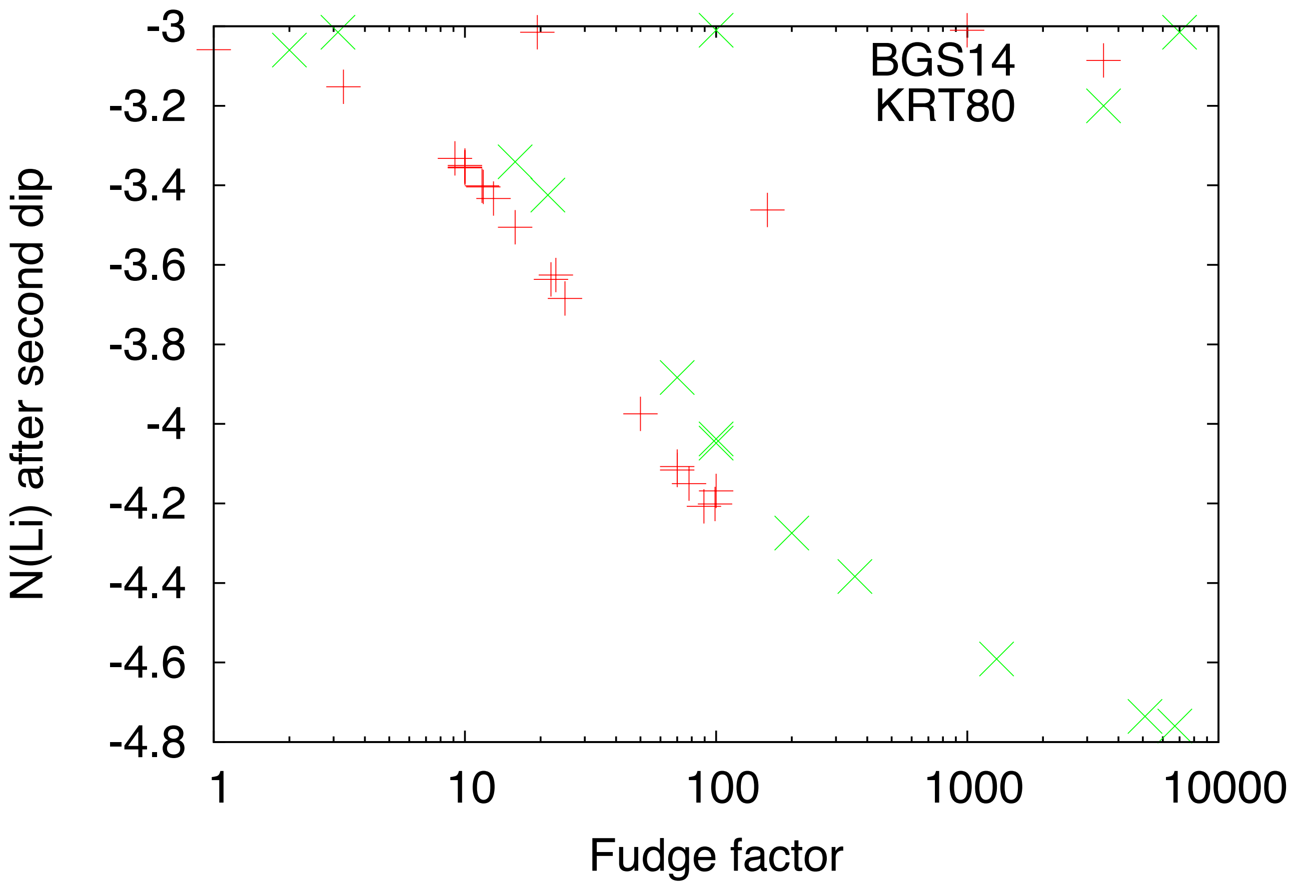
- Have a look at `mlt/mlt.f`, in the routine `set_thermohaline`, and study how mixing by fingering convection is implemented in MESA
- Choose your favorite mixing formalism (Kippenhahn or BGS), and set it up in inlist with your favorite `thermo_haline_coeff`.
- Starting from saved model, run forward and study evolution of Kippenhahn diagram, and Li abundance.
- What models, with what values of `thermo_haline_coeff`, can explain presence of second dip?
- Record result in spreadsheet.

See detailed instructions in

http://mesastar.org/teaching-materials/2014-mesa-summer-school-working-dir

Continue to use `rgb/` directory

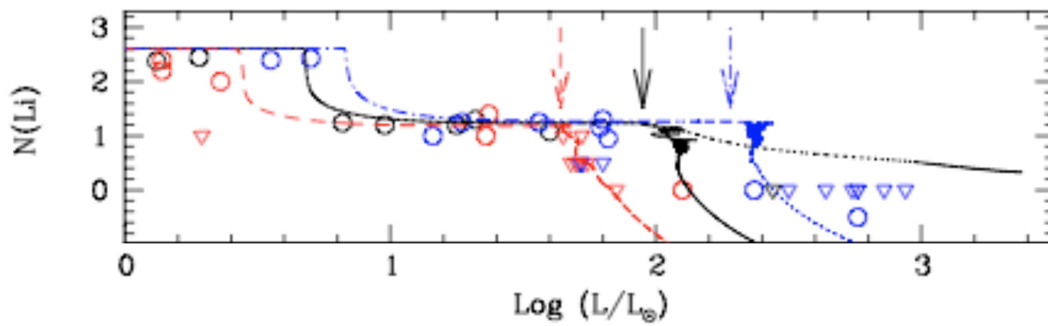
Results



-3
-3.2
-3.4
-3.6
-3.8
-4
-4.2
-4.4
-4.6
-4.8
N(Li) after second dip
1
10
100
1000
10000
Fudge factor
BGS14
KRT80

4. Activity 4: Li depletion in RGB (part 2)

Using Ulrich / Kippenhahn et al. models on 0.9M_{sun} star.

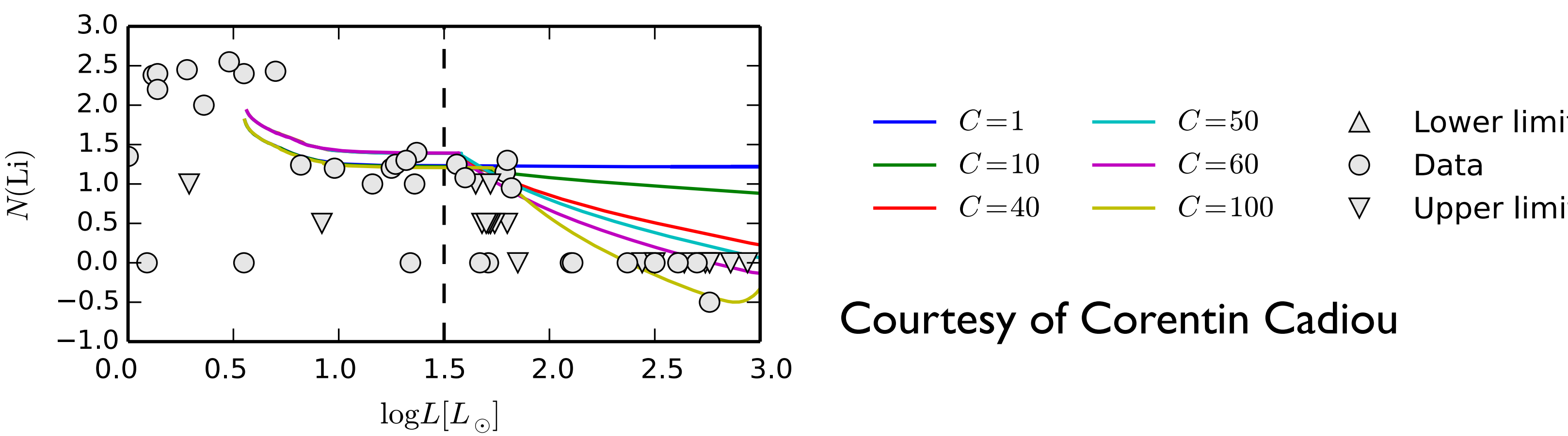


Solid lines: K = 1000

Dotted line: K = 100

Charbonnel & Zahn 2007

Using BGS model on 1.3M_{sun} star.

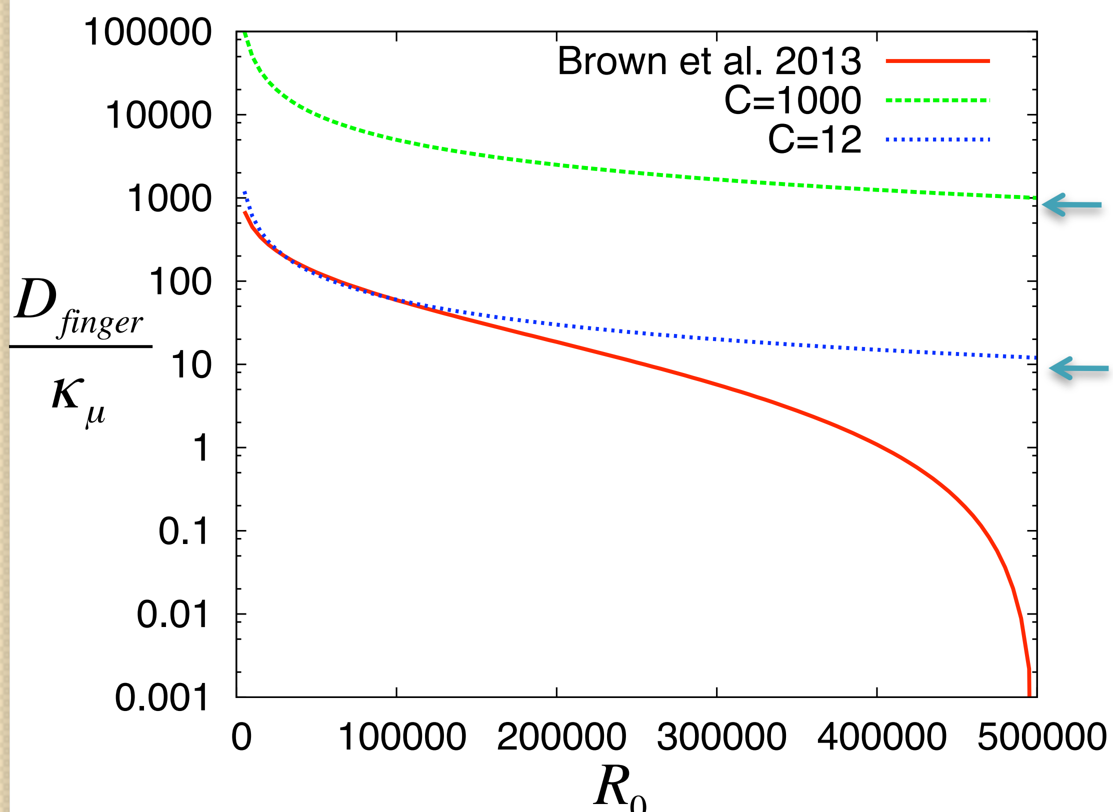


Courtesy of Corentin Cadiou

Mixing by fingering

Consequence:

- Mixing by fingering probably cannot explain RGB abundances



Required by observations (Charbonnel & Zahn 2007)

Original Kippenhahn et al. 1980 proposal (C=12)

$$\frac{D_{finger}}{\kappa_\mu} \propto \frac{K}{\tau R_0}$$

5. Beyond fingering convection?

It looks like fingering convection may not be enough to explain Li abundance. Only by multiplying mixing efficiency by large fudge factor can we make it work.

BUT: This model of fingering convection assumes that there are only horizontal gradients of composition/temperature, while in real stars, there can be both horizontal and vertical gradients.

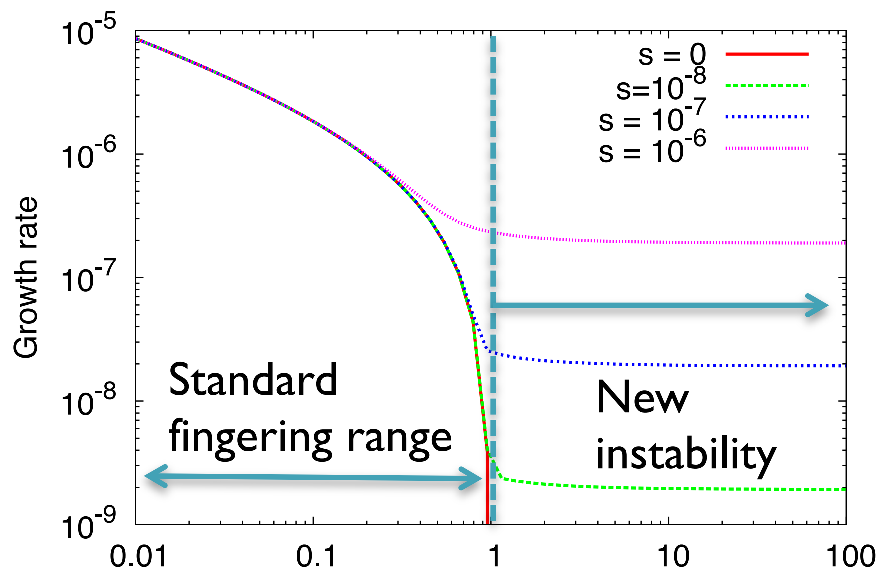
Horizontal gradients can be created by:

- The effect of rotation/differential rotation
- Tides (for stars with companions)
- Large-scale meridional flows …

Interesting new dynamics happen with horizontal gradients…

5. Beyond fingering convection?

- Consider a system which has both vertical and horizontal gradients of temperature and composition.
- We find that there is a now an instability regardless of the background stratification, as long as there is even the tiniest inverse mu-gradient



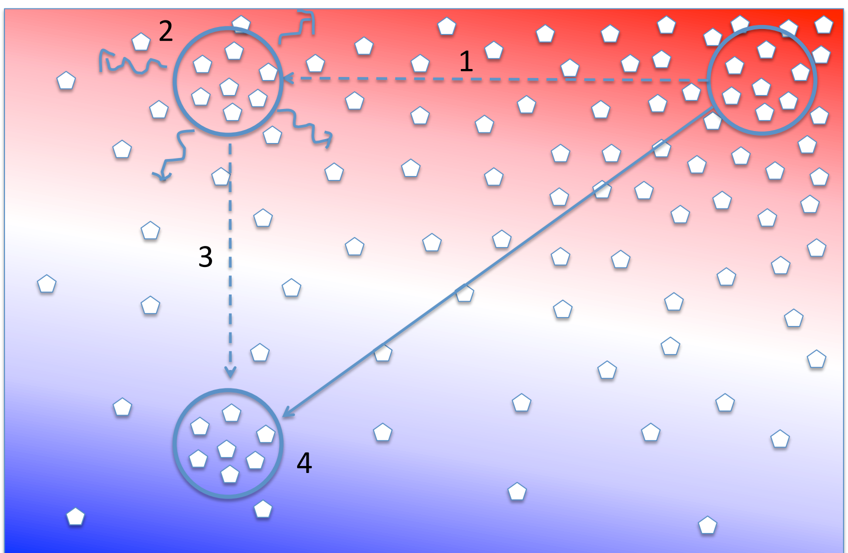
$$r = \frac{R_0 - 1}{R_{0,crit} - 1}$$

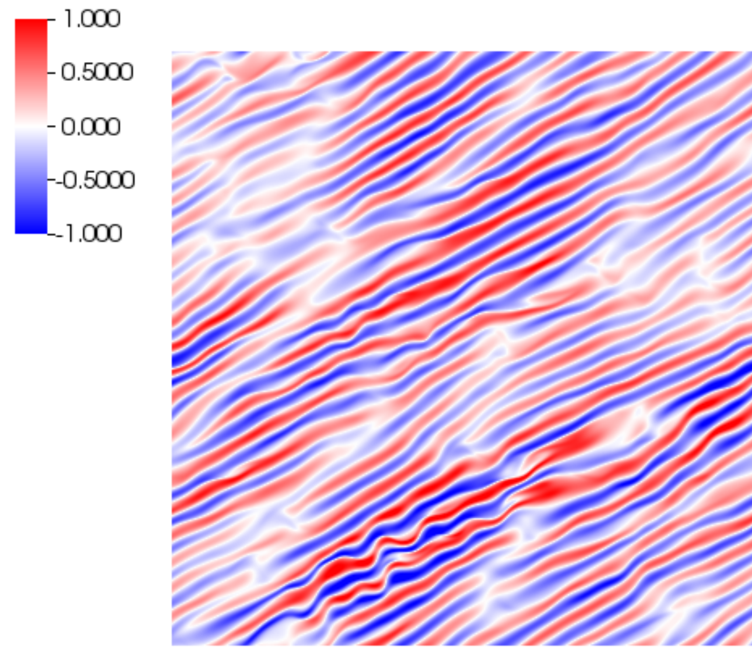
s is the slope of the isotherms.

Medrano, Garaud & Stellmach, 2014

5. Beyond fingering convection?

The instability now takes a slightly different form, however: instead of vertical fingers, we have slanted ones…

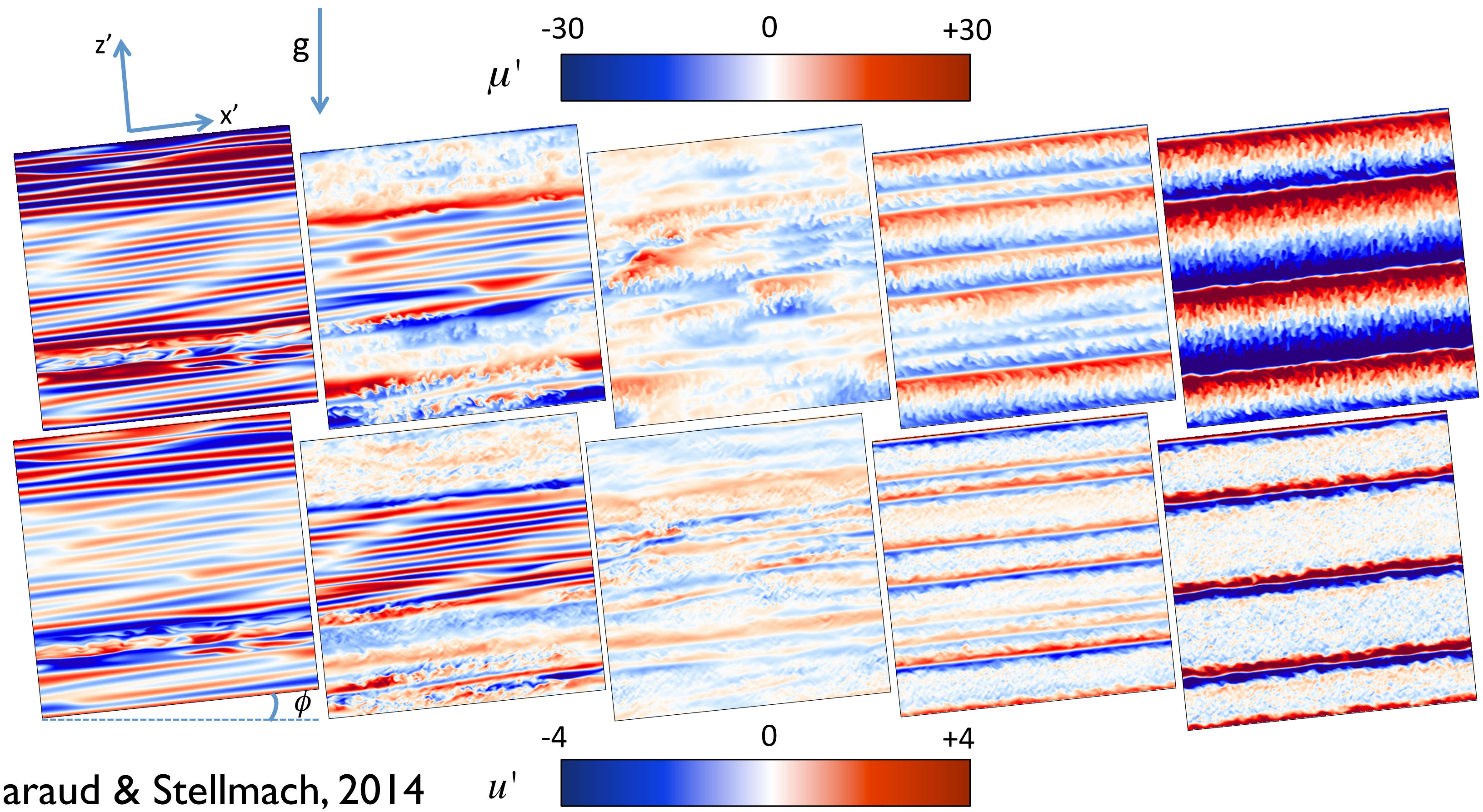




Medrano, Garaud & Stellmach, 2014

5. Beyond fingering convection?

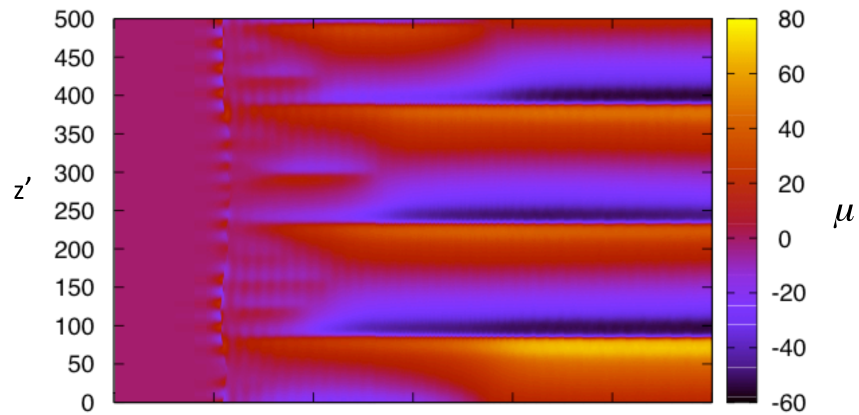
Later on, the slanted modes organize into stacks of (slanted) fingering layers, separated by sheared stable interfaces. These layers progressively merge.



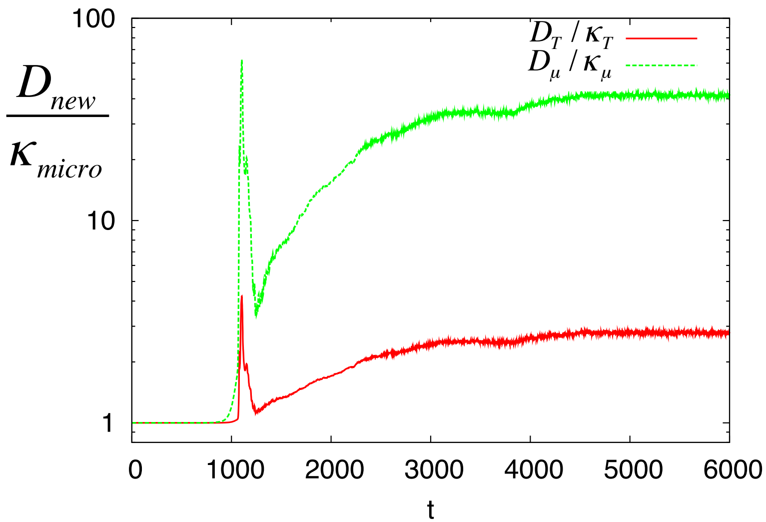
Medrano, Garaud & Stellmach, 2014

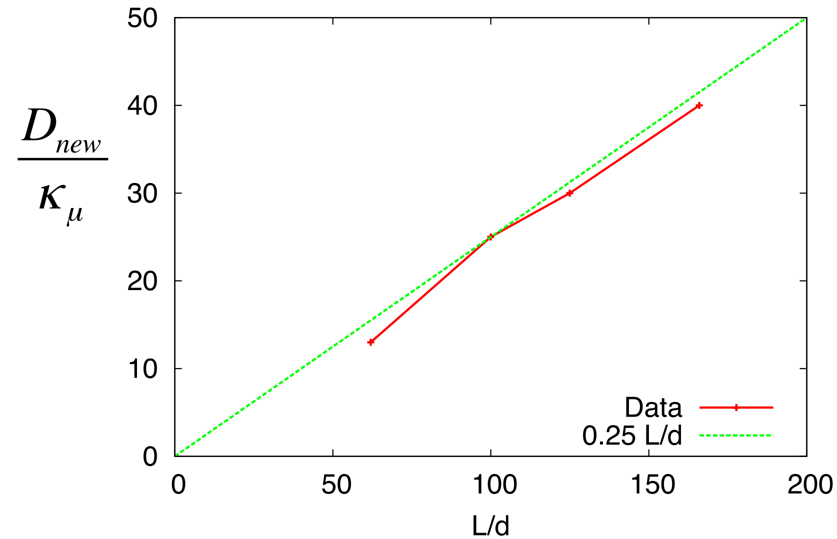
Beyond fingering convection?

With each merger, the total flux of chemical species in the system increases slightly…



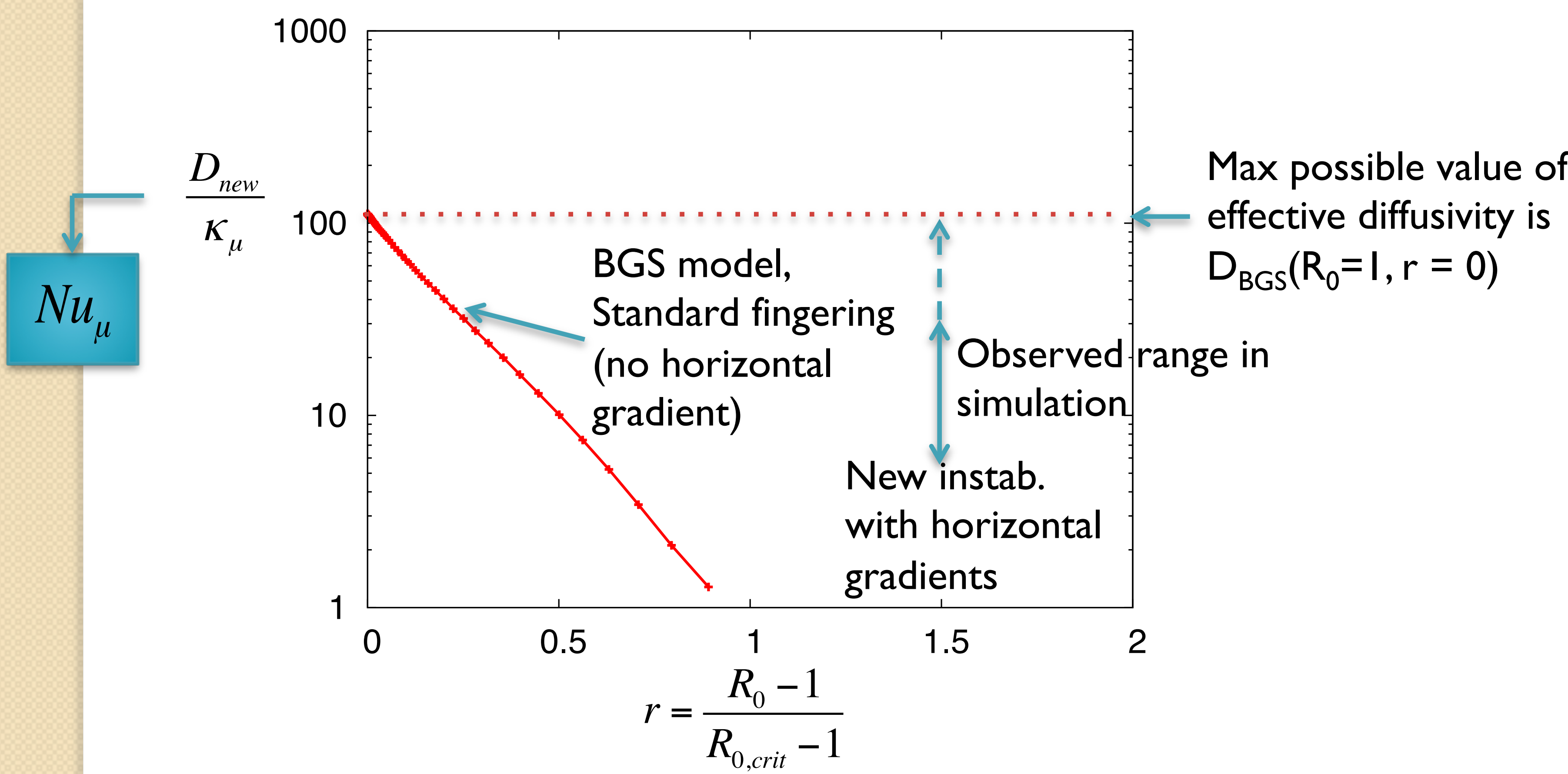
The effective diffusivity D_{new} seems to be linearly related to the layer height L, at least for this data.





5. Beyond fingering convection?

We know, however, that the maximum efficiency can't exceed that of the maximum efficiency of actual fingering convection for the same fluid parameters:



5. Beyond fingering convection?

So, we could try to propose a new effective diffusivity that either:

- **Simple option** (recommended for beginners): Always assumes that the transport occurs at the maximum possible rate (assuming layer mergers occur rapidly)

$$D_{new} = \left(Nu_{\mu,BGS}\right)_{\max} \kappa_\mu \text{ where } \left(Nu_{\mu,BGS}\right)_{\max} = Nu_{\mu,BGS}(R_0 = 1)$$

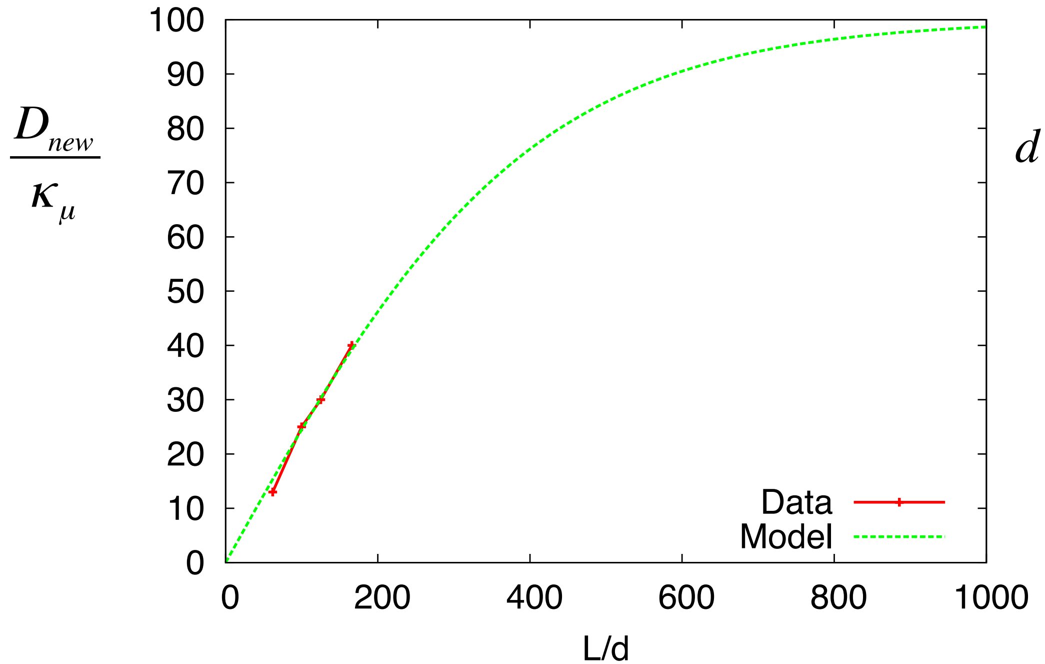
- **Harder option** (recommended for advanced students): Assume that layer mergers stop at some finite height and have a model to describe how the effective diffusivity varies with layer height (bearing in mind that D_{new} can't exceed max possible rate.)

- Note that max possible rate is obtained by calculating D_{new} for R_0 very close (but not equal) to 1. This can be ripped from existing thermohaline routine, by calling `numu` function which returns D_{BGS}/κ_μ

5. Beyond fingering convection?

Possible model for option 2: L/d is a "user-input" parameter.

$$\frac{D_{new}}{\kappa_\mu} = \left(Nu_{\mu,BGS}\right)_{\max} \tanh\left[\frac{L}{4d\left(Nu_{\mu,BGS}\right)_{\max}}\right]$$



$$d = \left(\frac{\kappa_T \nu H_p}{\delta g\left(\nabla_{ad} - \nabla\right)}\right)^{1/4} \quad \text{and } \delta \approx 1$$

6. Activity 5: Li depletion in RGB (part 3)

- Based on your interpretation of these results, create a model for the turbulent diffusivity of the new instability D_{new}
- Use `other_D_mix` hook, and `run_star_extras.f` provided in today's material
- Run your new model. Does it produce a second dip?

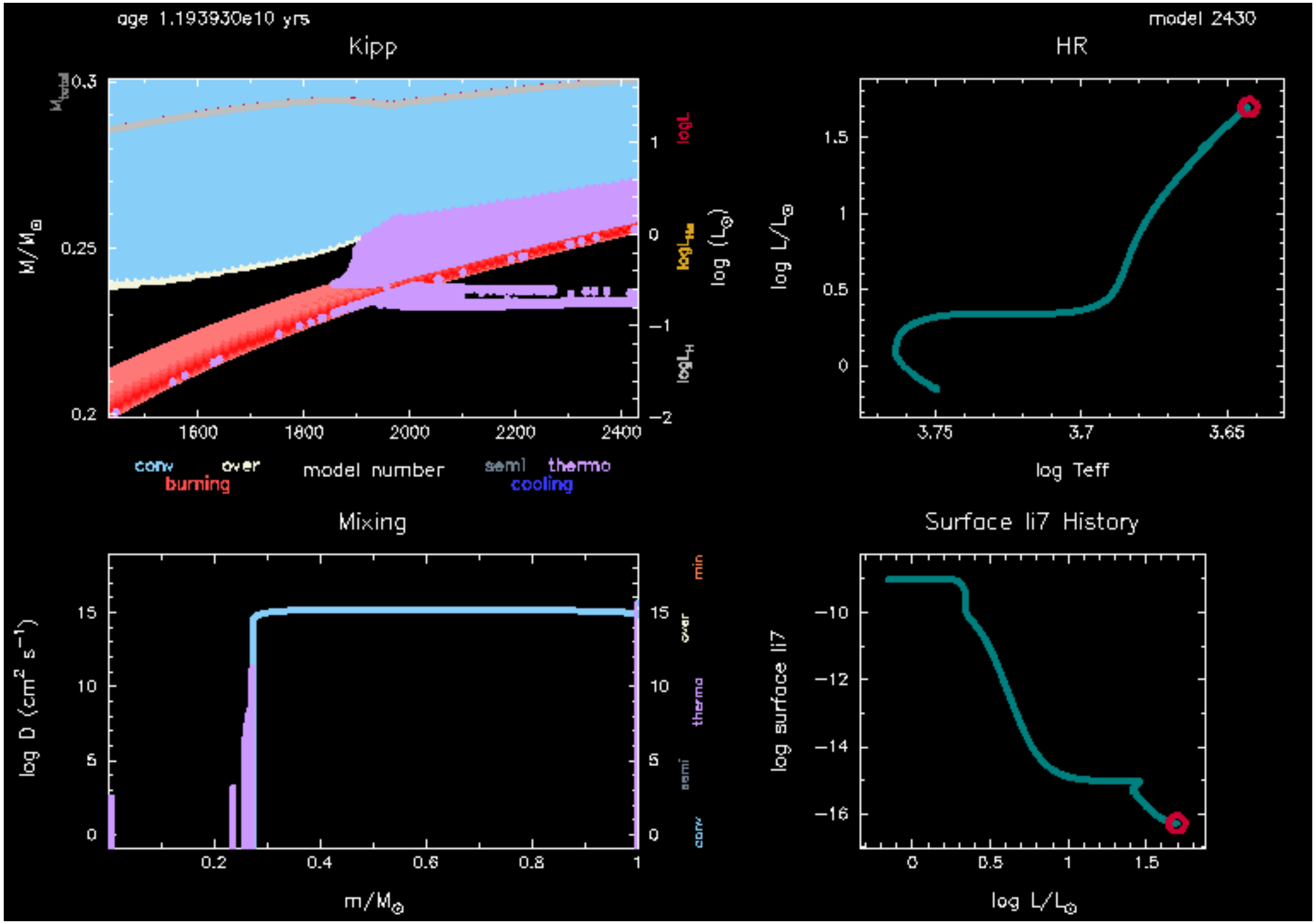
See detailed instructions in

http://mesastar.org/teaching-materials/2014-mesa-summer-school-working-dir

Continue to use `rgb/` directory, this time use `run_star_extra.f` provided as well (set it up as shown by Kevin yesterday).

Result

With basic formulation only: it is possible to get second dip.



Solution to problem (basic case)

Sample code to be added to `run_star_extras.f` can be found in `run_star_extras_solution.f`

Also do not forget to set `use_other_D_mix = .true.` in your inlist.