MIXING IN STARS, MIXING IN MESA

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Outline

Lecture 1 (yesterday):

- Compositional mixing

Lecture 2 (today):

- Angular momentum transport

Outline

Lecture 2: Angular momentum transport

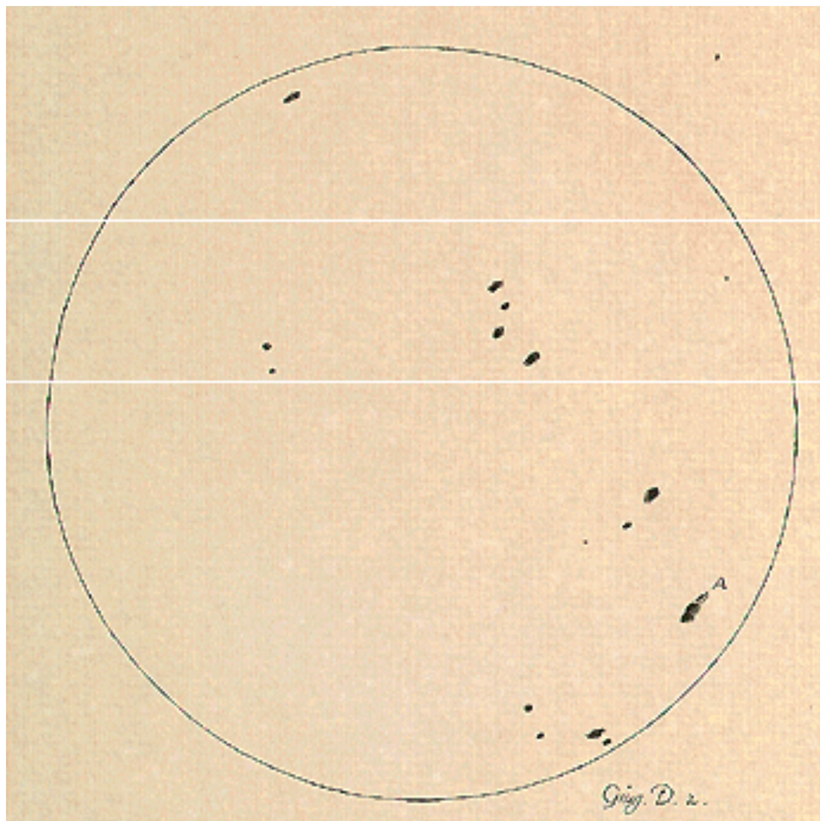
A. Introduction to stellar rotation and angular momentum transport.

1. Evidence for stellar rotation (surface and internal)
2. Effect of rotation & differential rotation on stellar evolution
3. Angular momentum transport: mathematical modeling and MESA implementation.
4. Activity #1: Modeling the rotational evolution of RGB stars using MESA (basic mixing)

1. Evidence for stellar rotation

All stars rotate – but some rotate a lot faster than others. We know this from

- Looking at the Sun (Gallileo, 1612). Rotation period ~ 27 days

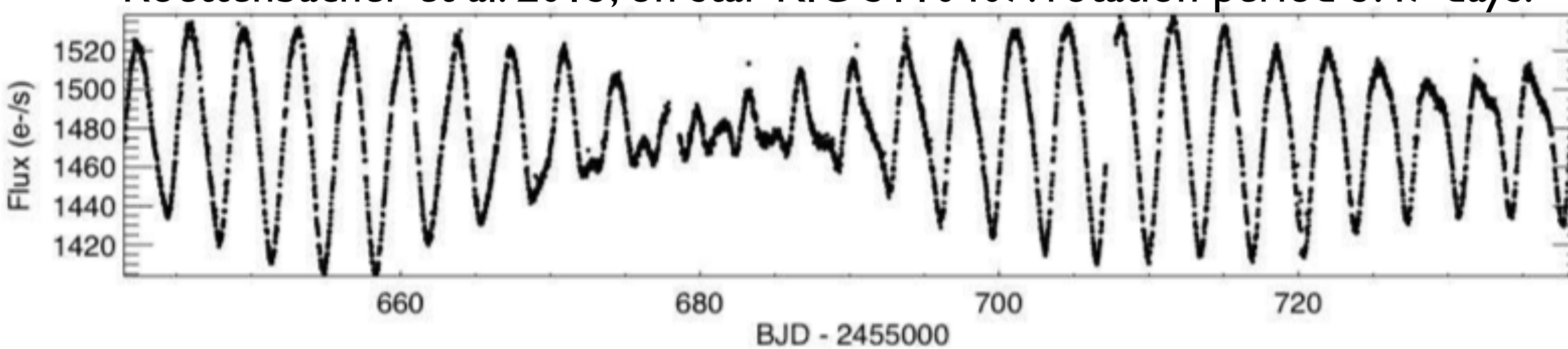


1. Evidence for stellar rotation

All stars rotate – but some rotate a lot faster than others. We know this from

- Looking at the Sun (Gallileo, 1612)
- Basic photometry (starspots modulate lightcurve)

Roettenbacher et al. 2013, on star KIC 5110407: rotation period 3.47 days.

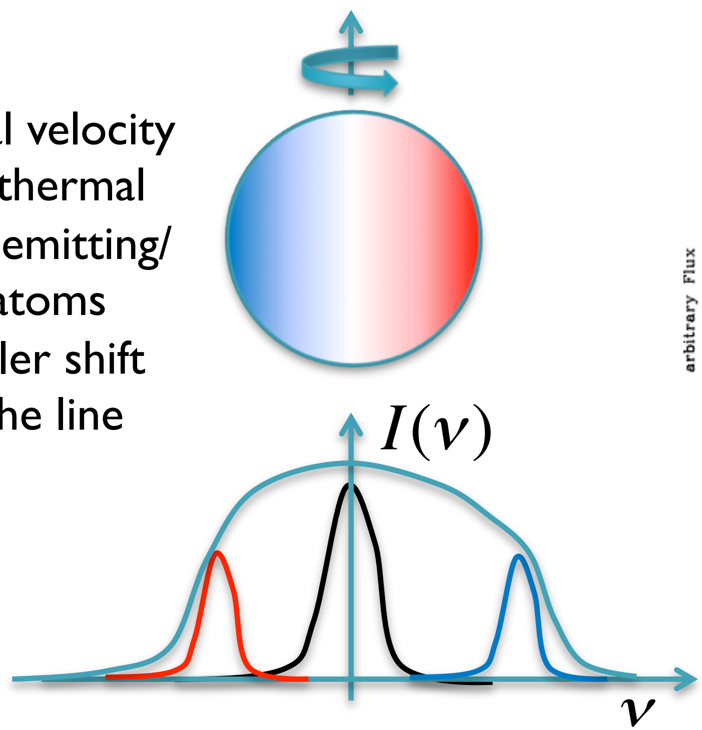


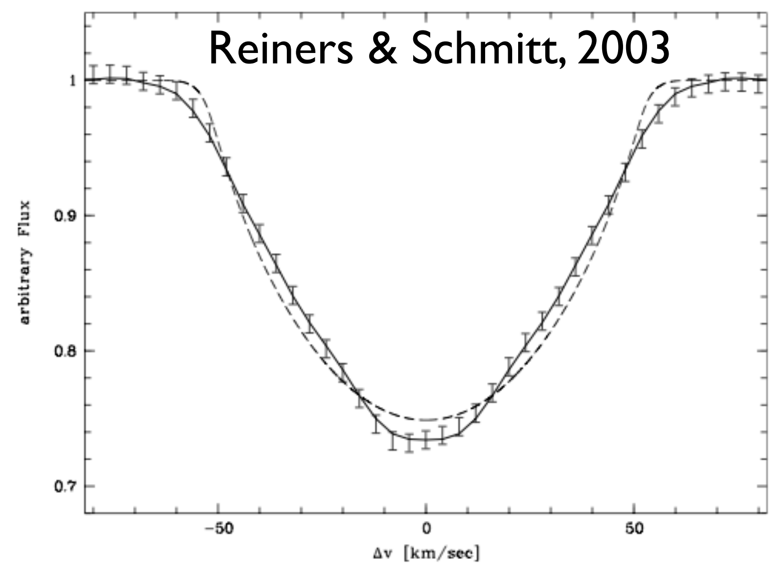
1. Evidence for stellar rotation

All stars rotate – but some rotate a lot faster than others. We know this from

- Looking at the Sun (Gallileo, 1612)
- Basic photometry (starspots modulate lightcurve)
- Spectroscopy (rotation broadens spectral lines).

If rotational velocity of star >> thermal velocity of emitting/ absorbing atoms then Doppler shift broadens the line





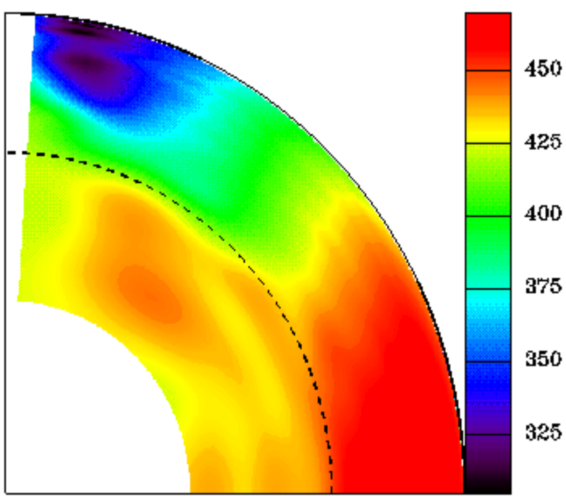
Deviation from dashed line model = evidence for differential rotation!

1. Evidence for stellar rotation

All stars rotate – but some rotate a lot faster than others. We know this from

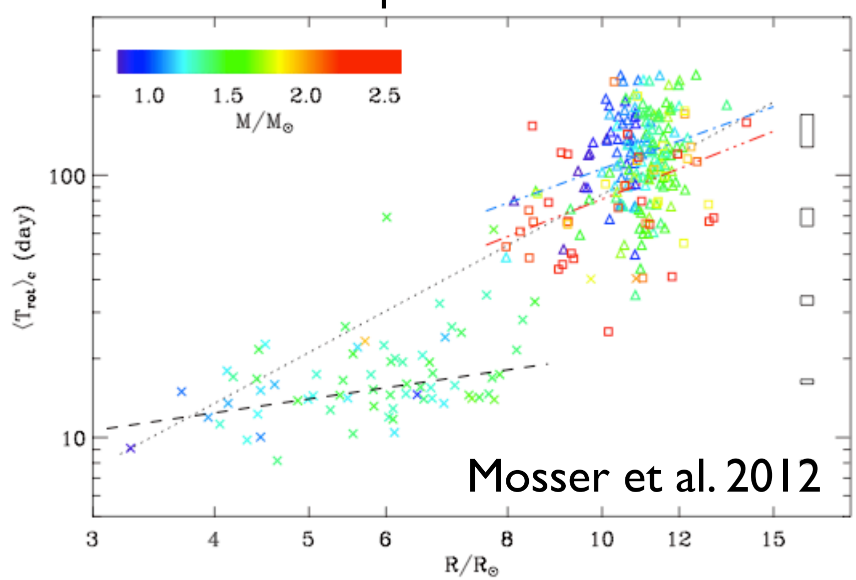
- Looking at the Sun (Gallileo, 1612)
- Basic photometry (starspots modulate lightcurve)
- Spectroscopy (rotation broadens spectral lines)
- Helio/asteroseismology (rotation causes "rotational splittings").

Differential rotation in the Sun



SOI/MDI team.

Core rotation period in RBG stars

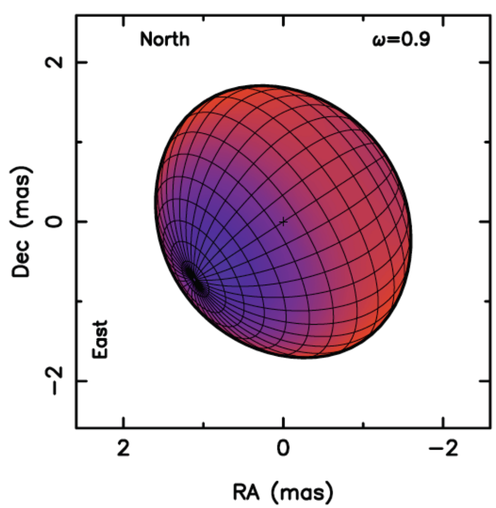


Mosser et al. 2012

1. Evidence for stellar rotation

All stars rotate – but some rotate a lot faster than others. We know this from

- Looking at the Sun (Gallileo, 1612)
- Basic photometry (starspots modulate lightcurve)
- Spectroscopy (rotation broadens spectral lines)
- Helio/asteroseismology (rotation causes "rotational splittings").
- For very rapid rotators only: rotation deforms the star, causing pole-to-equator luminosity variations, detectable via interferometry.



Reconstruction of interferometric data (surface temperature and shape) for Altair by Petersen et al. 2006
Star is rotating near breakup.

2. Effect of rotation on stellar evolution

Three types of effects need to be distinguished:

- The effect of rotation (i.e. solid-body rotation)
- The effect of temporal variations in the rotation rate (spin-up/ spin-down)
- The effect of spatial variations in the rotation rate (differential rotation)

Although they are not entirely independent of one-another, it is sometimes useful to think about them as "different terms in the momentum equation", which helps clarify how they affect the star.

2. Effect of rotation on stellar evolution

Suppose $\Omega(r,\theta,t) = \Omega_*(t) + \tilde{\Omega}(r,\theta,t)$ then momentum equation is:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \underbrace{\underbrace{\mathbf{u}\cdot\nabla\mathbf{u}}_{\text{Inertial forces}} + \underbrace{2\boldsymbol{\Omega}_* \times \mathbf{u}}_{\text{Coriolis force}}}_{\text{Differential rotation, since } \mathbf{u} = (u_r, u_\theta, r\tilde{\Omega})} + \underbrace{\underbrace{\boldsymbol{\Omega}_* \times \boldsymbol{\Omega}_* \times \mathbf{r}}_{\text{Centrifugal force}}}_{\text{Solid-body rotation}} + \underbrace{\underbrace{\dot{\boldsymbol{\Omega}}_* \times \mathbf{r}}_{\text{Euler's force}}}_{\text{Spin-up/ Spin-down}}\right) = -\nabla p + \rho\mathbf{g} + \ldots$$

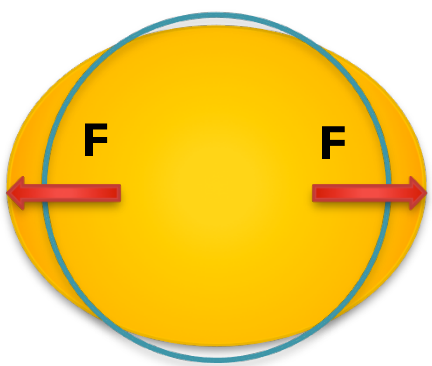
2. Effect of rotation on stellar evolution

Suppose $\Omega(r,\theta,t) = \Omega_*(t) + \tilde{\Omega}(r,\theta,t)$ then momentum equation is:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + 2\boldsymbol{\Omega}_* \times \mathbf{u} + \underbrace{\boldsymbol{\Omega}_* \times \boldsymbol{\Omega}_* \times \mathbf{r}}_{\text{Centrifugal force}} + \dot{\boldsymbol{\Omega}}_* \times \mathbf{r}\right) = -\nabla p + \rho\mathbf{g} + \ldots$$

Each of these "forces" acts in a somewhat different way.

- **Centrifugal force**: usually small (except for very rapid rotators), acts in direction perpendicular to rotation axis.



It deforms the star, and causes discrepancies between isotherms (which remains nearly spherical) and constant pressure/density surfaces (which are more oblate).

This drives global Eddington-Sweet flows, which are a form of mixing (cf. later)

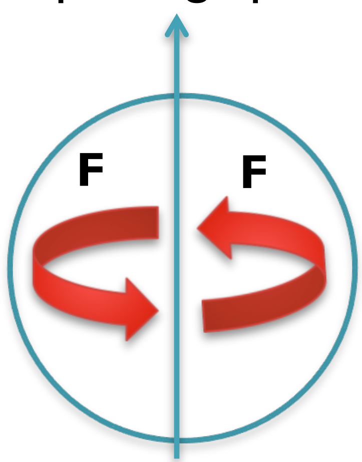
2. Effect of rotation on stellar evolution

Suppose $\Omega(r,\theta,t) = \Omega_*(t) + \tilde{\Omega}(r,\theta,t)$ then momentum equation is:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + 2\mathbf{\Omega}_* \times \mathbf{u} + \mathbf{\Omega}_* \times \mathbf{\Omega}_* \times \mathbf{r} + \underbrace{\dot{\mathbf{\Omega}}_* \times \mathbf{r}}_{\text{Euler's force}}\right) = -\nabla p + \rho\mathbf{g} + \ldots$$

Each of these "forces" acts in a somewhat different way.

- **Euler's force**: only appears when the *frame of reference* is spinning up or down.



It acts in the azimuthal direction.

No direct consequence on stellar evolution, but it ensures conservation of angular momentum in an inertial frame.

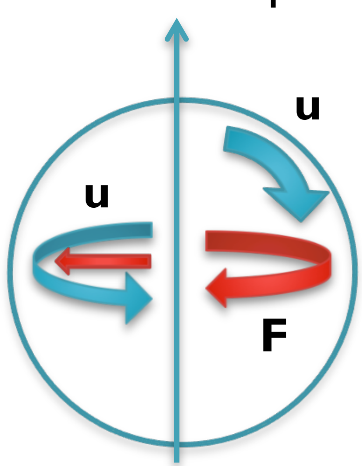
2. Effect of rotation on stellar evolution

Suppose $\Omega(r,\theta,t) = \Omega_*(t) + \tilde{\Omega}(r,\theta,t)$ then momentum equation is:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + \underbrace{2\mathbf{\Omega}_* \times \mathbf{u}}_{\text{Coriolis force}} + \mathbf{\Omega}_* \times \mathbf{\Omega}_* \times \mathbf{r} + \dot{\mathbf{\Omega}}_* \times \mathbf{r}\right) = -\nabla p + \rho \mathbf{g} + \ldots$$

Each of these "forces" acts in a somewhat different way.

- **Coriolis force**: apparent deflection of fluid motion in rotating frame compared with inertial frame.



It acts in direction perpendicular to fluid motion and also ensures conservation of angular momentum in an inertial frame.

Can turn meridional flows into azimuthal ones, or vice-versa, which are then sources of mixing.

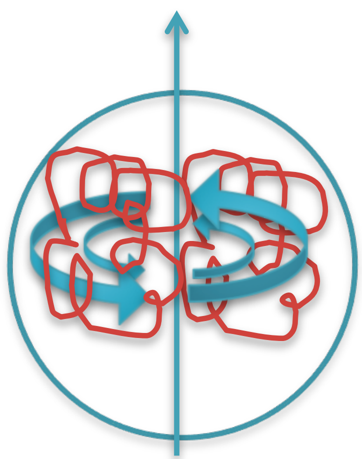
2. Effect of rotation on stellar evolution

Suppose $\Omega(r,\theta,t) = \Omega_*(t) + \tilde{\Omega}(r,\theta,t)$ then momentum equation is:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \underbrace{\mathbf{u}\cdot\nabla\mathbf{u}}_{\text{Inertial forces}} + 2\boldsymbol{\Omega}_* \times \mathbf{u} + \boldsymbol{\Omega}_* \times \boldsymbol{\Omega}_* \times \mathbf{r} + \dot{\boldsymbol{\Omega}}_* \times \mathbf{r}\right) = -\nabla p + \rho \mathbf{g} + \ldots$$

Each of these "forces" acts in a somewhat different way.

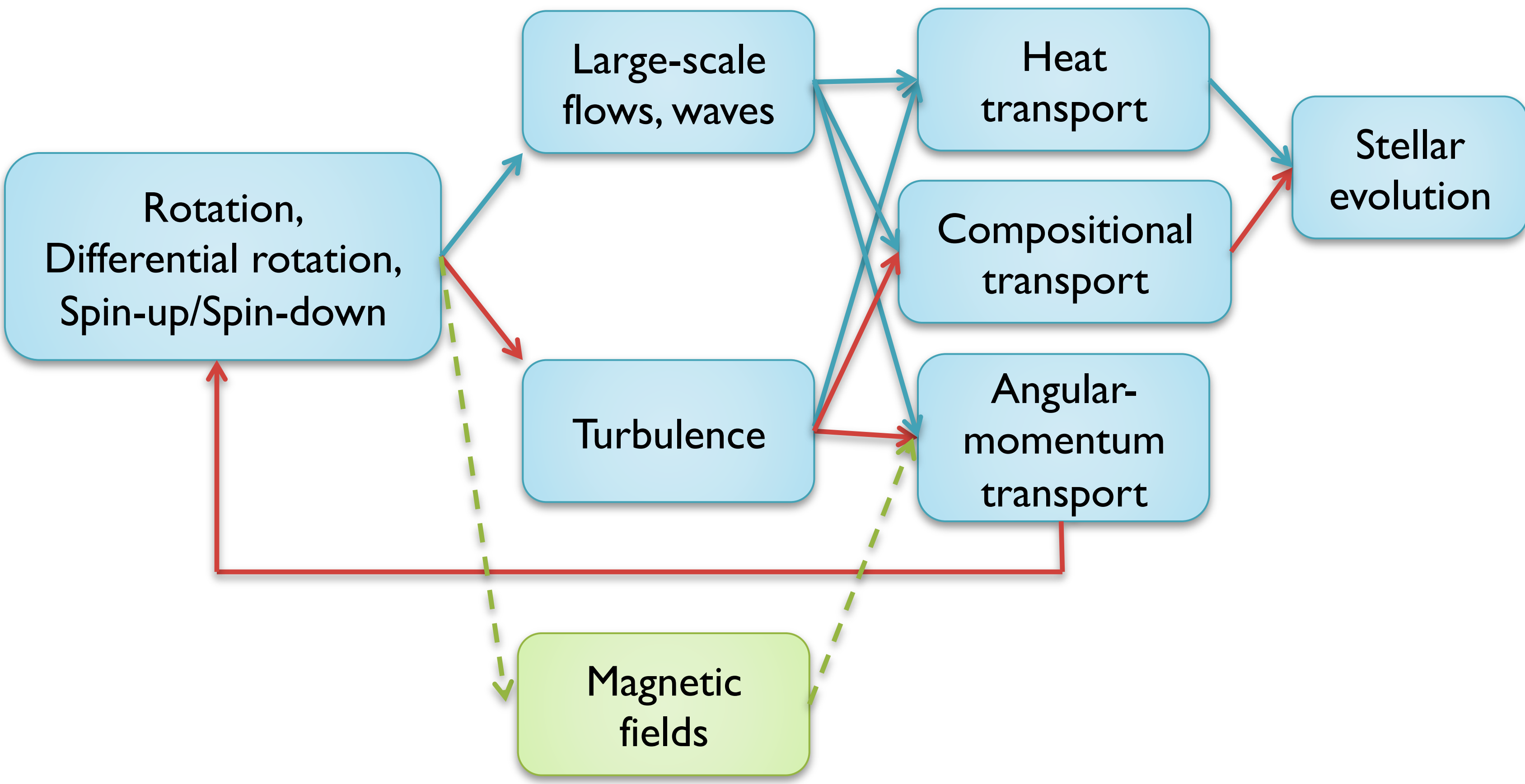
- **Inertial terms**: nonlinear terms, cause development and saturation of waves and instabilities from differential rotation.



Shear instabilities drive turbulence, and therefore turbulent mixing. Waves can also cause mixing, both turbulent and non-turbulent.

2. Effect of rotation on stellar evolution

In short:



3. Modeling angular momentum transport

- As in the case of chemical transport, modeling AM transport starts from the basic equations for fluid dynamics:
  - Mass conservation equation.
  - Momentum conservation equation (in the azimuthal direction).

- Assuming a spherically symmetric star (with a spherically symmetric angular velocity profile $\Omega(m,t)$ then, if only viscous stresses (with viscosity ν) are taken into account, we have

$$\frac{D}{Dt}(r^2\Omega) = \frac{\partial}{\partial m}\left((4\pi r^2\rho)^2 \nu r^2 \frac{\partial \Omega}{\partial m}\right)$$

- This looks quite similar (but not identical) to the equation for the evolution of the compositional field studied yesterday:

$$\frac{DX_s}{Dt} = \frac{\partial}{\partial m}\left((4\pi r^2\rho)^2 D_{mix} \frac{\partial X_s}{\partial m}\right)$$

3. Modeling angular momentum transport

- Viscosity is, however, usually negligible in stellar interiors.

- If angular momentum transport processes other than viscosity are invoked (such as turbulence, or meridional flows), then the expression for the evolution of $\Omega(m,t)$ is more complicated.

- *By analogy*, it is often assumed that these processes act as an effective turbulent viscosity, so that the expression for the evolution of $\Omega(m,t)$ simplifies to:

$$\frac{D}{Dt}(r^2\Omega) = \frac{\partial}{\partial m}\left((4\pi r^2 \rho)^2 \nu_{AM} r^2 \frac{\partial \Omega}{\partial m} \right)$$

Note: this is generally not a good model for angular momentum transport, especially when the latter is mediated by waves, meridional flows or magnetic fields. However, this is what MESA and most other codes do.

3. Modeling angular momentum transport

- The only remaining question is:

  What is ν_{AM}?

- As for D_{mix}, it has the dimensions of a length x velocity, or length^2 / time, and can be estimated from the typical timescale / lengthscale / velocity scale of turbulent eddies (for angular momentum transport by turbulence).

➔ the turbulent components of ν_{AM} should be similar to those of D_{mix}, and the same processes that transport angular momentum can transport chemical elements, and vice-versa.

3. Modeling angular momentum transport

The way in which MESA is organized is as follows:

- Calculate "basic turbulent diffusivities" for each process separately (in `mlt.f` and `rotation_mix_info.f`)
- Multiply them by one prefactor if they are to be used to mix chemical species, and by another prefactor if they are to be used to transport angular momentum. Default prefactor values can be viewed in `star/defaults/control.defaults`.
- Add the desired ones together to create D_{mix} and ν_{AM}

Note: simply adding mixing coefficients together is generally completely wrong, but this is what MESA and most stellar evolution codes do.

3. Modeling angular momentum transport

- In MESA, we have:

```
D_mix = D_mix_non_rotation + f*am_D_mix_factor*
       (D_DSI_factor*D_DSI + D_SH_factor*D_SH
       + D_SSI_factor*D_SSI + D_ES_factor*D_ES
       + D_GSF_factor*D_GSF + D_ST_factor*D_ST)
```

where

- `D_mix_non_rotation` includes mixing from convection, overshoot + all additional compositional mixing coefficients that were calculated in `mlt.f`
- `D_DSI, D_SH,D_SSI`, etc… are basic mixing coefficients associated with instabilities that arise from rotation (see later), and `D_DSI_factor`, etc… are multiplicative coefficients (set to 0 by default)
- `am_D_mix_factor` is a general multiplicative coefficient for the whole thing (set to 0 by default).

3. Modeling angular momentum transport

In MESA, we have:

```
am_nu = am_nu_factor*(am_nu_non_rot + am_nu_rot)
am_nu_non_rot = am_nu_non_rotation_factor*D_mix_non_rotation
am_nu_rot = am_nu_visc_factor*D_visc + am_nu_DSI_factor*D_DSI
          + am_nu_SH_factor*D_SH + am_nu_SSI_factor*D_SSI
          + am_nu_ES_factor*D_ES + am_nu_GSF_factor*D_GSF
          + am_nu_ST_factor*D_ST)
```

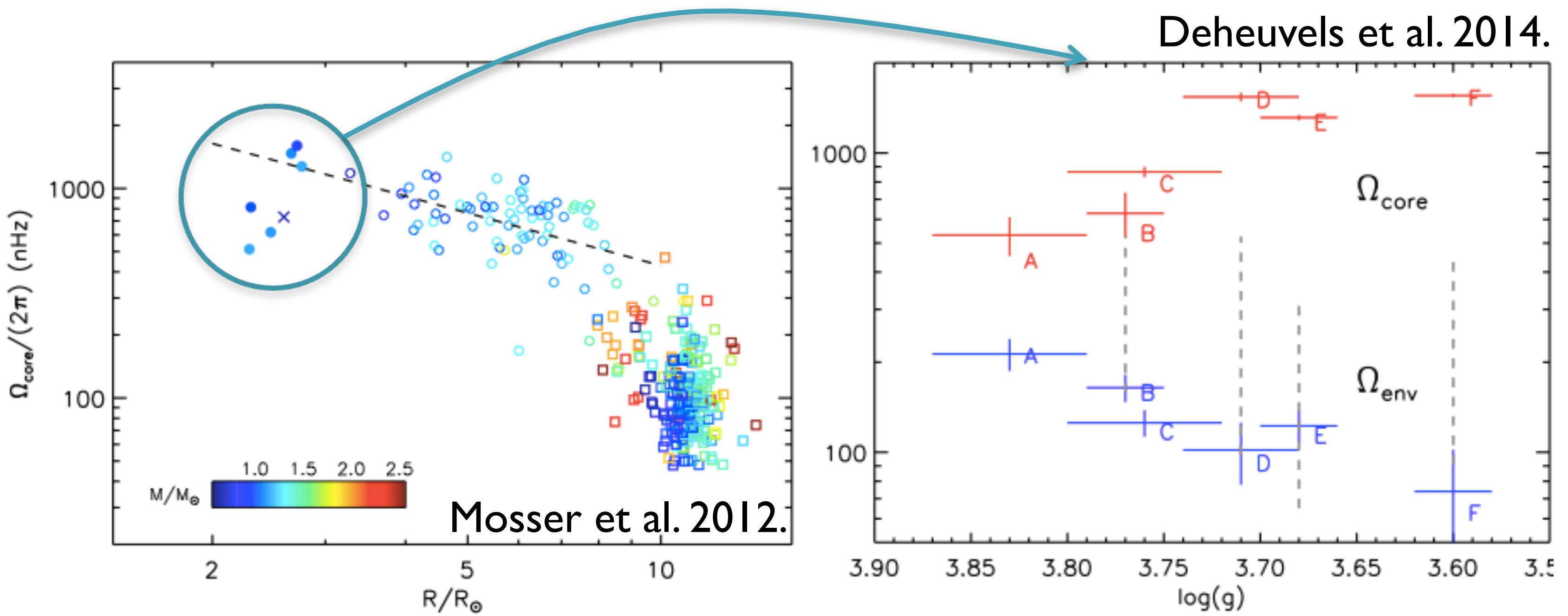
where

- `D_mix_non_rotation` is as before, this time multiplied by `am_nu_non_rotation_factor` multiplicative coefficient (set to 1 by default)
- `D_visc` is the basic viscosity, and `am_nu_visc_factor` is a multiplicative coefficient (set to 1 by default).
- `D_DSI, D_SH,D_SSI,` etc… are as before, this time multiplied by `am_nu_DSI_factor`, etc… multiplicative coefficients (set to 0 by default)

4. Activity #1: Background material

- Recently, observations of the core rotation rates of RGB stars have become available.

Deheuvels et al. 2014.



Mosser et al. 2012.

- Let's see what standard angular momentum mixing models predict for the angular velocity in RGB stars.

4. Activity #1: Modeling rotational evolution of RGB stars with MESA.

- Start from $1M_{sun}$ ZAMS model in solid-body rotation.
- Turn on mass loss (basic prescription)

Part 1

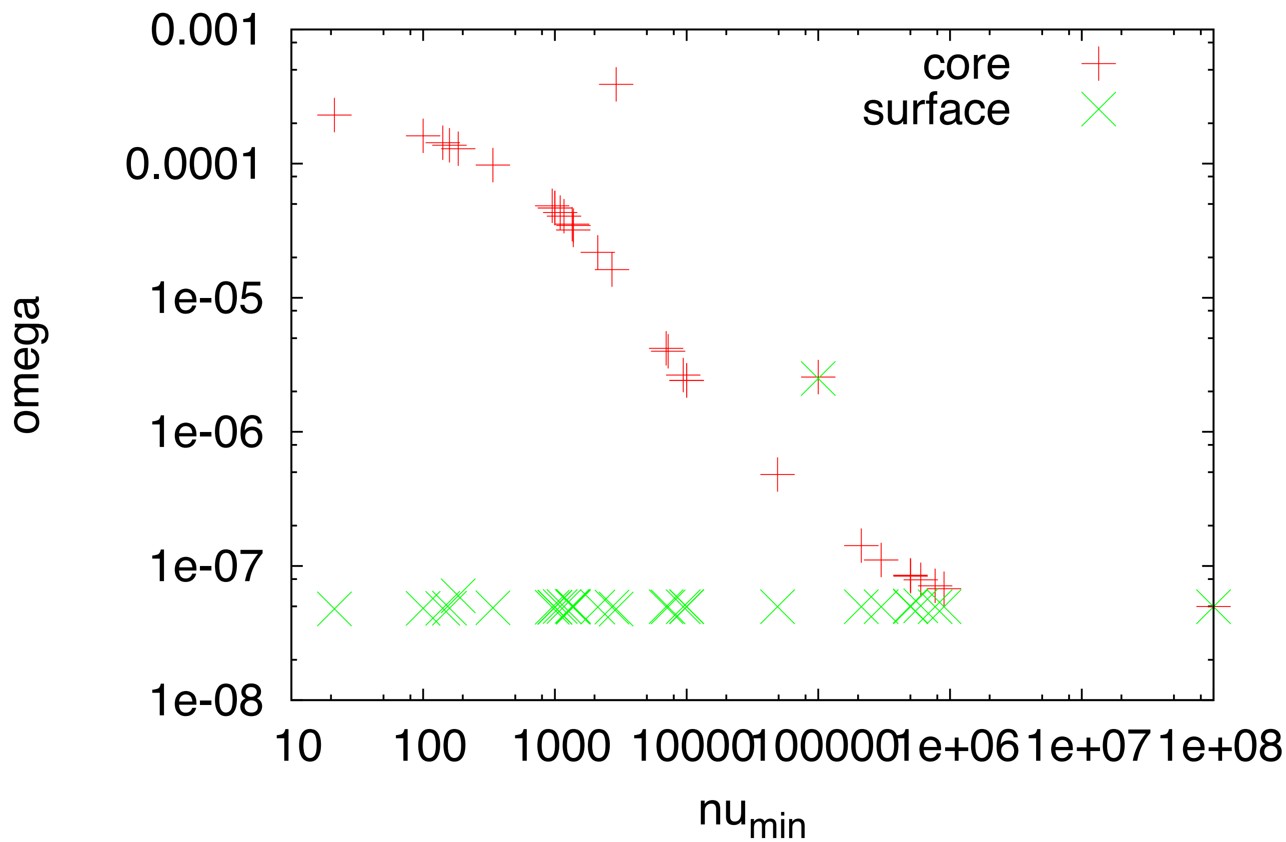
- Turn off all angular momentum mixing except viscosity
- Evolve rotation profile and star all way up RGB (up to $6R_{sun}$)
- Study results.

Part 2

- Turn on minimum value of am_nu_min.
- Study results.

See detailed instructions on mesastar.org, Day 2.

Results



core
surface
omega
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
10
100
1000
10000
100000
1e+06
1e+07
1e+08
nu_{min}

Outline

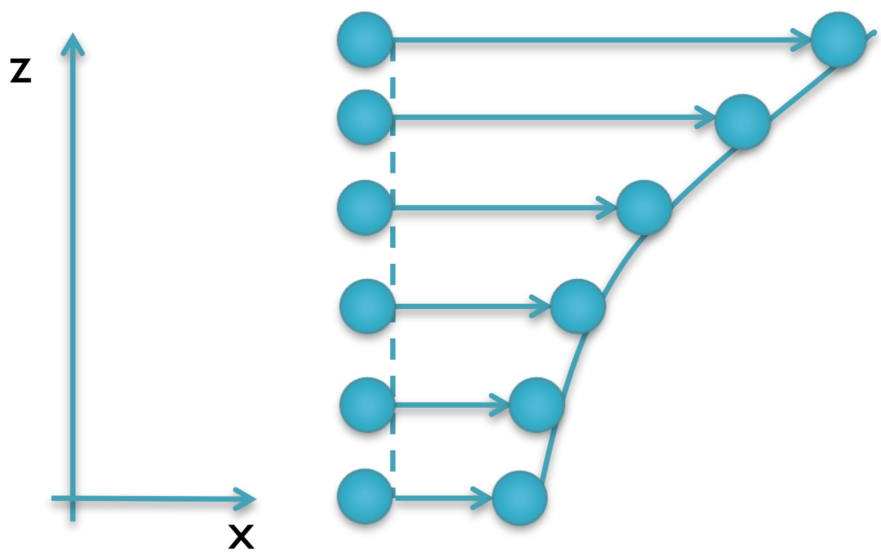
Lecture 2: Angular momentum transport

B. Improving models for angular momentum transport.

1. Case study: shear instabilities.
2. Activity #2: Using existing shear instability routines in MESA.
3. Other shear-induced instabilities.
4. Activity #3: Improving the shear instability routines in MESA.
5. Discussion of the results.

1. Mathematical models of shear instabilities

- Shear occurs when adjacent parcels of fluid are moving at different velocities.



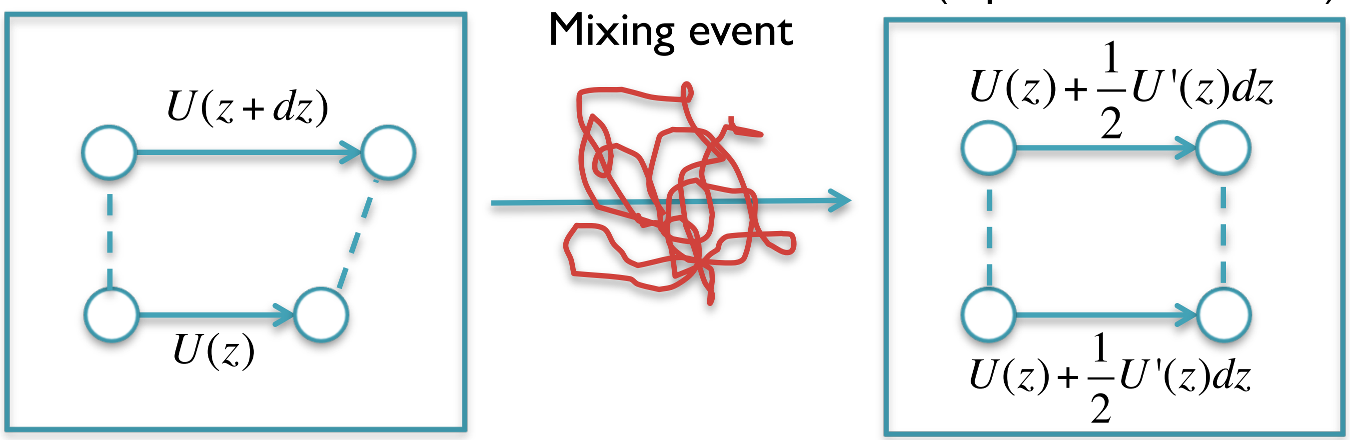
- Interchanging 2 parcels of fluid (while equalizing their momenta, and densities, and conserving the total momentum) can be energetically favorable, thus causing the shear instability.

Mathematical models of shear instabilities

- If the background fluid is neutrally stratified (no change in the density with z), then the interchange is always energetically favorable.

Before interchange

After interchange (equalized momenta)



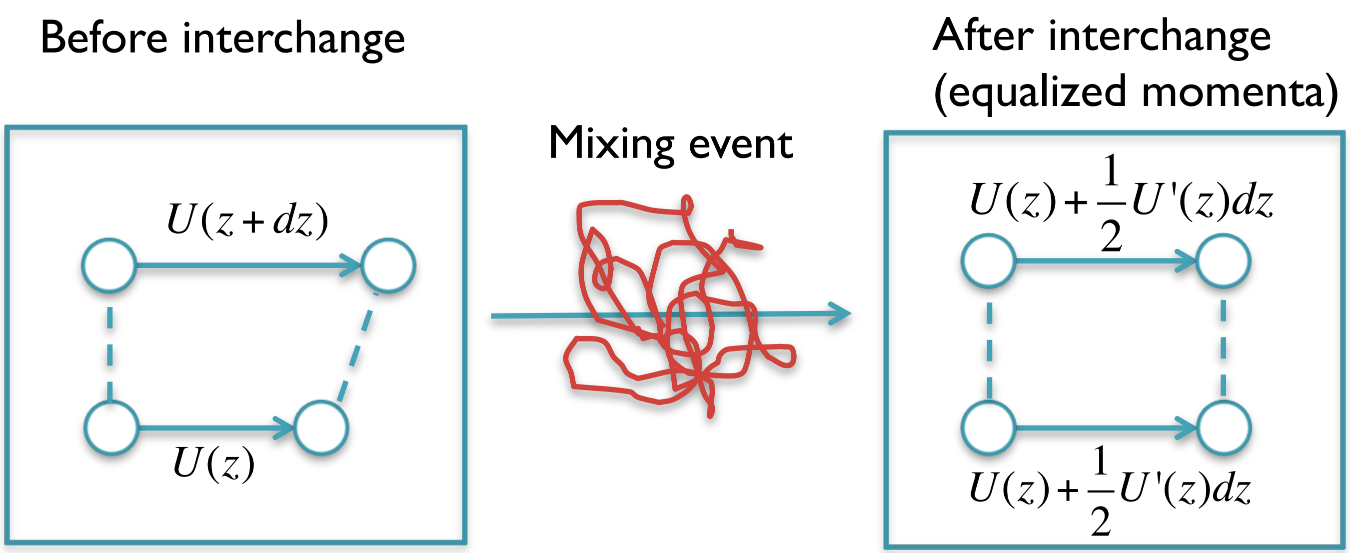
$$E_{tot} = \frac{\rho}{2}\left[U^2(z)+\left(U(z)+U'(z)dz\right)^2\right]$$
$$= \frac{\rho}{2}\left[2U^2(z)+2U(z)U'(z)dz+\left[U'(z)\right]^2 dz^2\right]$$

$$>$$

$$E_{tot} = \frac{\rho}{2}\cdot 2\left[U(z)+\frac{1}{2}U'(z)dz\right]^2$$
$$= \frac{\rho}{2}\cdot\left[2U^2(z)+2U(z)U'(z)dz+\frac{1}{2}\left[U'(z)\right]^2 dz^2\right]$$

Mathematical models of shear instabilities

- If the background fluid is neutrally stratified (no change in the density with z), then the interchange is always energetically favorable.



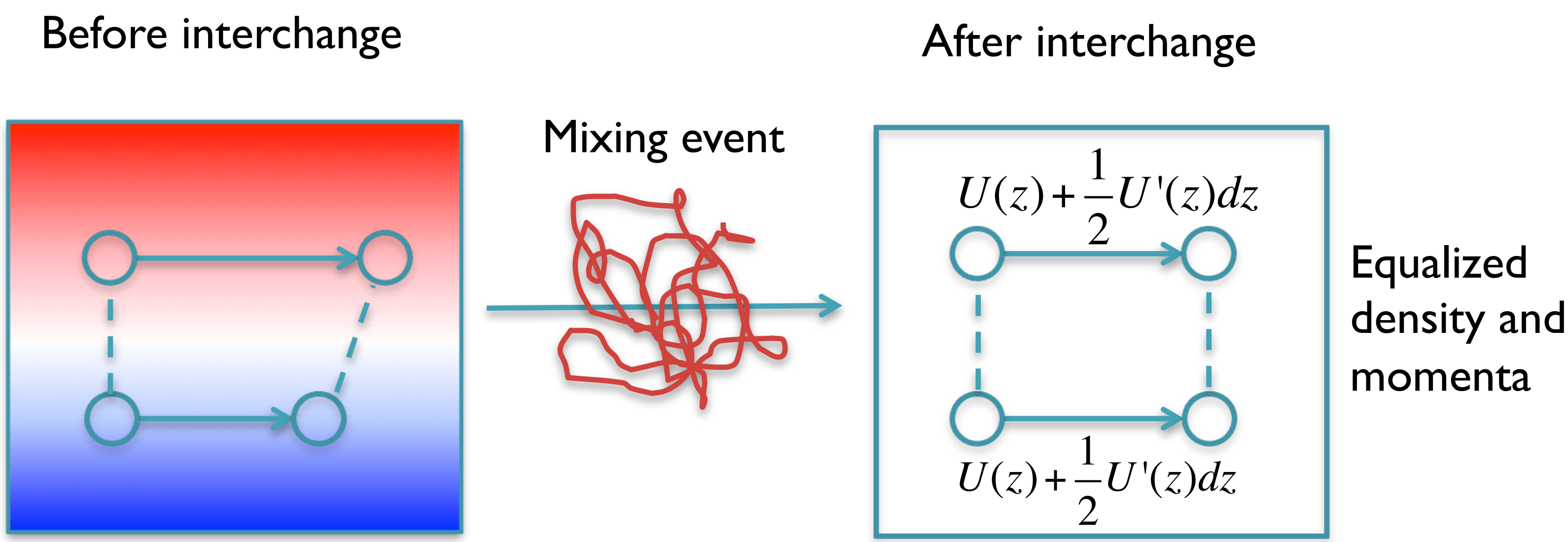
- The (kinetic) energy gained in the exchange:

$$\Delta E = \frac{\rho}{4}\left[U'(z)dz\right]^2$$

can serve to amplify the perturbations ➔ positive feedback loop.

Mathematical models of shear instabilities

- In stellar radiative zones, however, the background density is stably stratified (density decreases with z). Interchange is only favorable if gain in kinetic energy is larger than loss in potential energy.



- To see whether exchange is favorable, consider ratio R:

$$R = \frac{-\Delta E_{pot}}{\Delta E_{kin}} = 2\frac{N^2(z)}{\left[U'(z)\right]^2} \quad \text{where} \quad N^2(z) = -g\left(\frac{d\ln\rho}{dz} - \frac{d\ln\rho_{ad}}{dz}\right)$$

Mathematical models of shear instabilities

- It is common to define the Richardson number Ri as

$$Ri(z) = \frac{N^2(z)}{\left[U'(z)\right]^2} = \frac{N^2(z)}{\left[r\frac{d\Omega}{dr}\right]^2}$$

If $Ri > Ri_c$, no instability

If $Ri < Ri_c$, instability

- This criterion is based on energetics, and is only an estimate. It seems to hold reasonably well in both linear stability analyses and in fully nonlinear calculations, with $Ri_c = 1/4$ to $Ri_c = 1$.

T perturbations

u_x

Gratuitous self-promoting pretty movie.

Mathematical models of shear instabilities

- It is common to define the Richardson number Ri as

$$Ri(z) = \frac{N^2(z)}{\left[U'(z)\right]^2} = \frac{N^2(z)}{\left[r\frac{d\Omega}{dr}\right]^2} \quad \begin{cases} \text{If } Ri>Ri_c\text{, no instability} \\ \text{If } Ri<Ri_c\text{, instability} \end{cases}$$

- This criterion is based on energetics, and is only an estimate. It seems to hold reasonably well in both linear stability analyses and in fully nonlinear calculations, with $Ri_c=1/4$ to $Ri_c=1$.

- However, this only tells us when an instability is expected, but not what the induced turbulent diffusivity may be.

- Using $D_{shear} \propto \nu_{AM,shear} \propto \upsilon l \propto l^2/t$

what could a typical velocity & lengthscale be for these eddies?

Mathematical models of shear instabilities

- In MESA, this process is supposed to be described by the "dynamical shear instability" coefficient, D_DSI
- The calculation of D_DSI is based on the model described in Heger, Langer & Woosley (2000), in which:

$$l = \min\{L, H_p\}\left[1 - \max\left(\frac{Ri}{Ri_c}, 0\right)\right]$$

$$t = t_{dyn} = \sqrt{\frac{r^3}{Gm}}$$

Note: I do not condone the use of this formula for mixing, but this won't change the result for RGB stars (see later).

where L is the height of the region unstable to shear, Ri_c=1/4, and

$$Ri = \frac{N_T^2 + N_\mu^2}{\left(r\frac{d\Omega}{dr}\right)^2} = \frac{\rho g^2}{P}(\delta(\nabla_{ad} - \nabla) + \phi\nabla_\mu)\left(r\frac{d\Omega}{dr}\right)^{-2}$$

5. Activity #2: Using existing angular-momentum mixing routines in MESA.

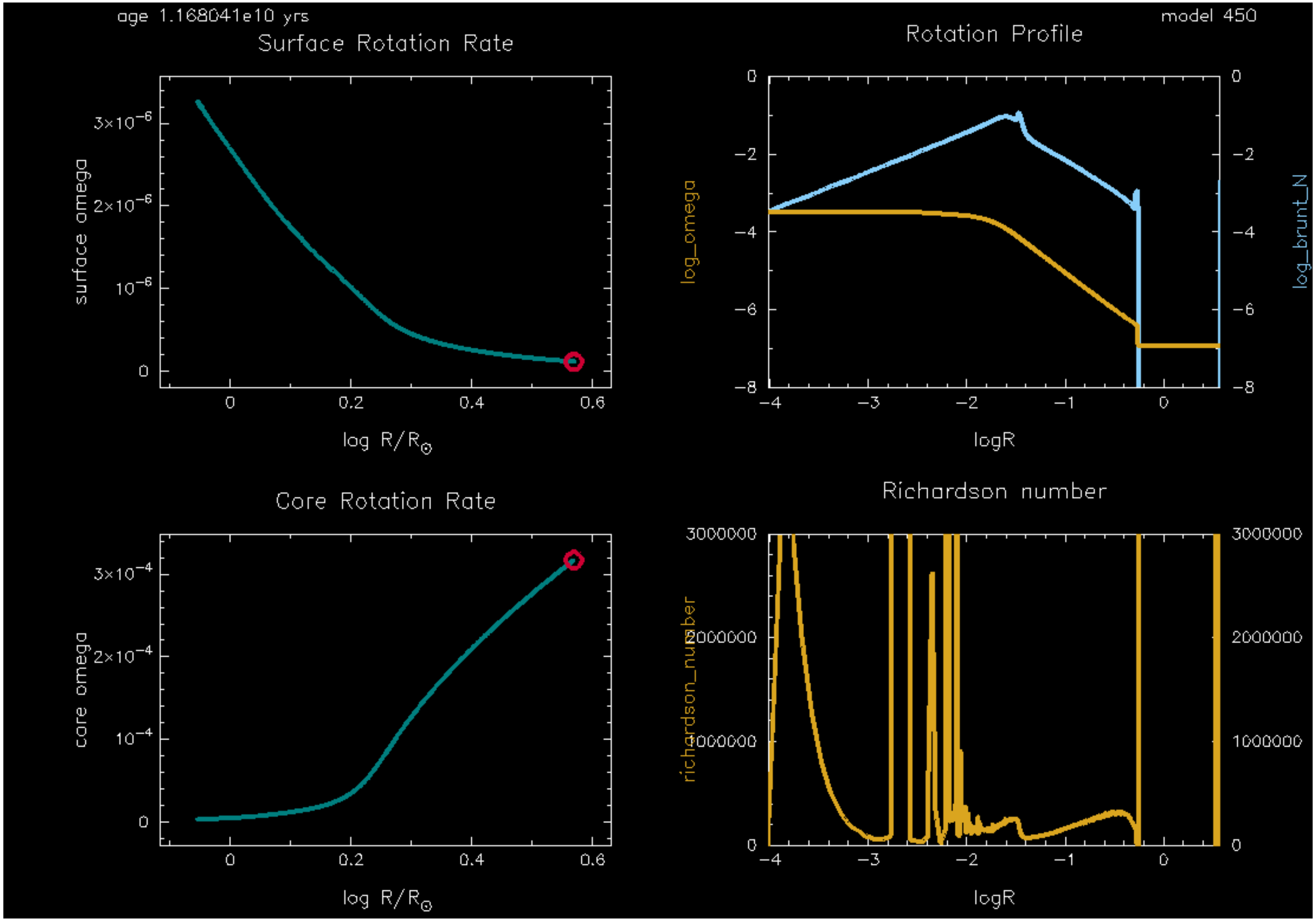
Part 1

- Look at `rotation_mix_info.f` to see where `D_DSI` is calculated.
- Run RGB evolution as before, but this time add dynamical shear instability, with different `am_nu_DSI_factor`
- Study results: what do you notice?

Part 2

- Plot the Richardson number in the star. What do you notice? Does it explain your results?

Results



age 1.168041e10 yrs
model 450
Surface Rotation Rate
surface omega
log R/R$_\odot$
Rotation Profile
log_omega
log_brunt_N
logR
Core Rotation Rate
core omega
log R/R$_\odot$
Richardson number
richardson_number
logR

6. Other shear-induced instabilities.

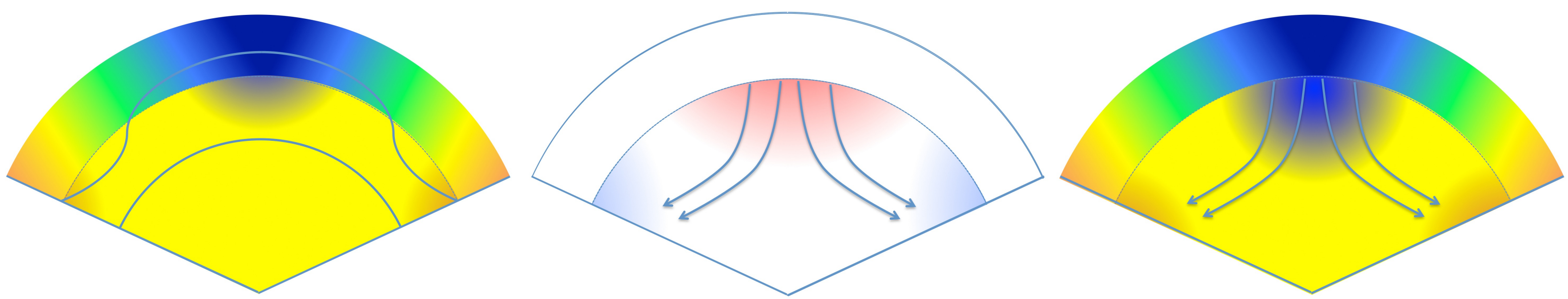
- If "standard" shear instabilities don't work, what else could do the trick?
- MESA has a number of built-in routines to calculate diffusion coefficients from other rotationally-induced or shear-induced processes
  - ES: Eddington-Sweet flows (rotation & shear induced)
  - SSI: Secular Shear Instability (shear-induced)
  - SH: Solberg-Høiland (shear-induced)
  - GSF: Goldreich-Shubert-Fricke (shear-induced)
  - ST: Spruit-Taylor (shear-induced, magnetic).

6. Other shear-induced instabilities (abridged)

Eddington-Sweet flows

- Rotation (or differential rotation) causes isobars and isotherms to deviate from sphericity: $\bar{T}(r) \rightarrow \bar{T}_{new}(r) + \tilde{T}(r,\theta)$ and same for p.
- Assuming thermal equilibrium implies that large-scale meridional flows must be generated to compensate for the offset. These are Eddington-Sweet flows.

$$\rho c_p \mathbf{u}_{ES} \cdot \nabla s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \chi \frac{\partial \tilde{T}}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \chi \frac{\partial \tilde{T}}{\partial \theta}\right)$$

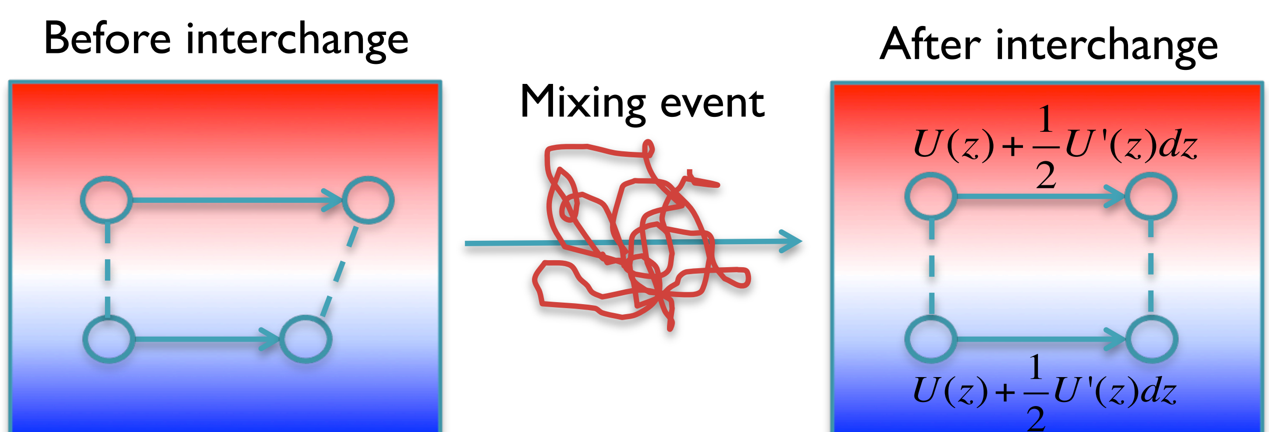


- This leads to large-scale advective transport, which cannot (therefore should not!) be modeled as a diffusion process. See Zahn (1992) for detail. HLW2000/MESA still do it, however.

Other shear-induced instabilities (abridged)

Secular shear instability.

- The Richardson Criterion for DSI is very stringent because it assumes that the exchange of the parcels is fast, so one has to pay a hefty price in potential energy to make it happen.
- But if the exchange is slow, temperature can diffuse out of the parcels, and equalize with the background before the exchange is finished. We still have to worry about chemical stratification, however, but that is not always important.
- This relaxes the Richardson criterion significantly



Other shear-induced instabilities (abridged)

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- But if the exchange is slow, temperature can diffuse out of the parcels, and equalize with the background before the exchange is finished. We still have to worry about chemical stratification, however, but that is not always important.
- This relaxes the Richardson criterion significantly
- As with most of these instabilities, very little is known about induced mixing. Use MESA/HLW2000 at your own risk.
- However the "relaxed" stability criterion used is not un-reasonable, so using the routine can at least give a good idea as to when the instability may occur.

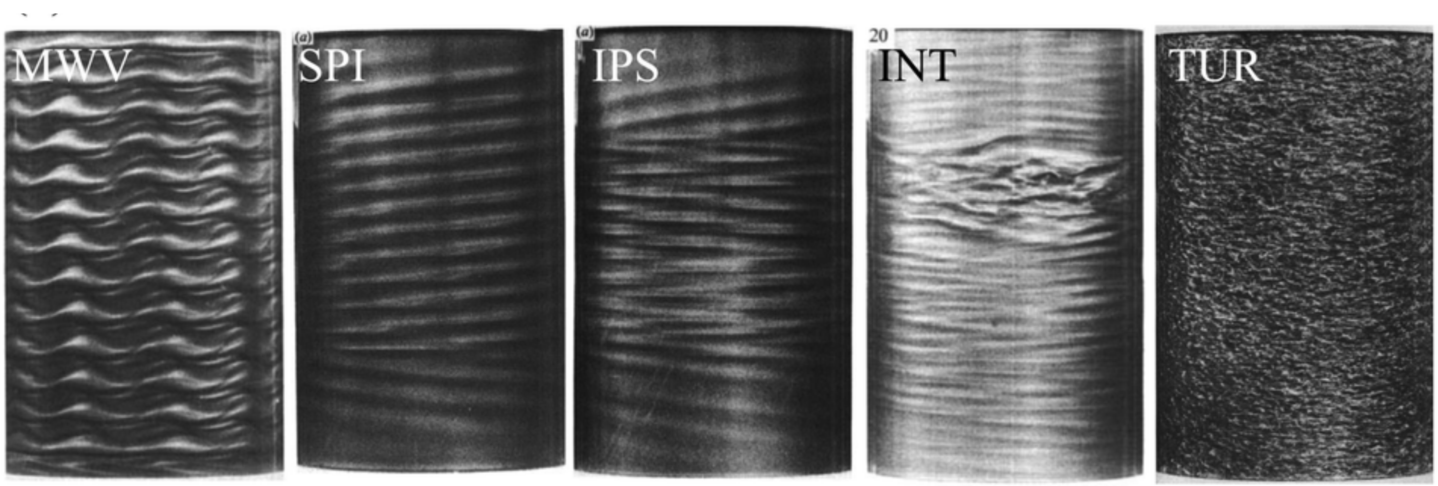
Other shear-induced instabilities (abridged)

Solberg-Høiland instability

- In unstratified fluids, a powerful "centrifugal" instability can occur if the angular momentum ever decreases outward.

$$\frac{d}{dr}\left(r^2\Omega\right) < 0$$

Rayleigh criterion



Fardin et al. 2014

Other shear-induced instabilities (abridged)

Solberg-Høiland instability

- In unstratified fluids, a powerful "centrifugal" instability can occur if the angular momentum ever decreases outward.

$$\frac{d}{dr}\left(r^2\Omega\right) < 0 \qquad \text{Rayleigh criterion}$$

- In stratified fluids, this becomes the Solberg-Høiland criterion, which (as in the case of the Richardson criterion) captures the stabilizing effect of buoyancy.

$$N_T^2 + N_\mu^2 + \frac{1}{r^3}\frac{d}{dr}\left(r^2\Omega\right)^2 < 0$$

- As with most of these instabilities, very little is known about induced mixing. Use MESA/HLW2000 at your own risk.

Other shear-induced instabilities (abridged)

Goldreich Schubert Fricke instability

- The GSF instability is a double-diffusive version of the Solberg-Høiland instability, where the two (or more) competing gradients are an unstable gradient of angular momentum and a stable gradient of entropy or composition. In fact,

Ledoux criterion for thermo-compositional convective instability	$\longleftrightarrow$	Solberg-Høiland criterion for thermo-centrifugal instability
Standard convection vs. fingering convection.	$\longleftrightarrow$	Solberg-Høiland instability vs. GSF instability.

- As with most of these instabilities, very little is known about induced mixing. Use MESA/HLW2000 at your own risk.

Other shear-induced instabilities (abridged)

Spruit-Tayler mechanism

- Magnetic fields can also transport angular momentum by
  - Causing instabilities that drive turbulence
  - Being themselves sources of angular momentum transport (via Maxwell stresses).

$$\frac{D}{Dt}(r^2\Omega) = \frac{\partial}{\partial m}\left((4\pi r^2 \rho)^2 \nu_{AM} r^2 \frac{\partial \Omega}{\partial m} + C r \rho B_r B_\phi \right)$$

Add turbulent transport here

Maxwell stress. Constant C depends on field geometry.

  - Note: the Maxwell stresses don't look anything like the turbulent diffusion term.

Other shear-induced instabilities (abridged)

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Add turbulent transport here

Maxwell stress, depends on field geometry.

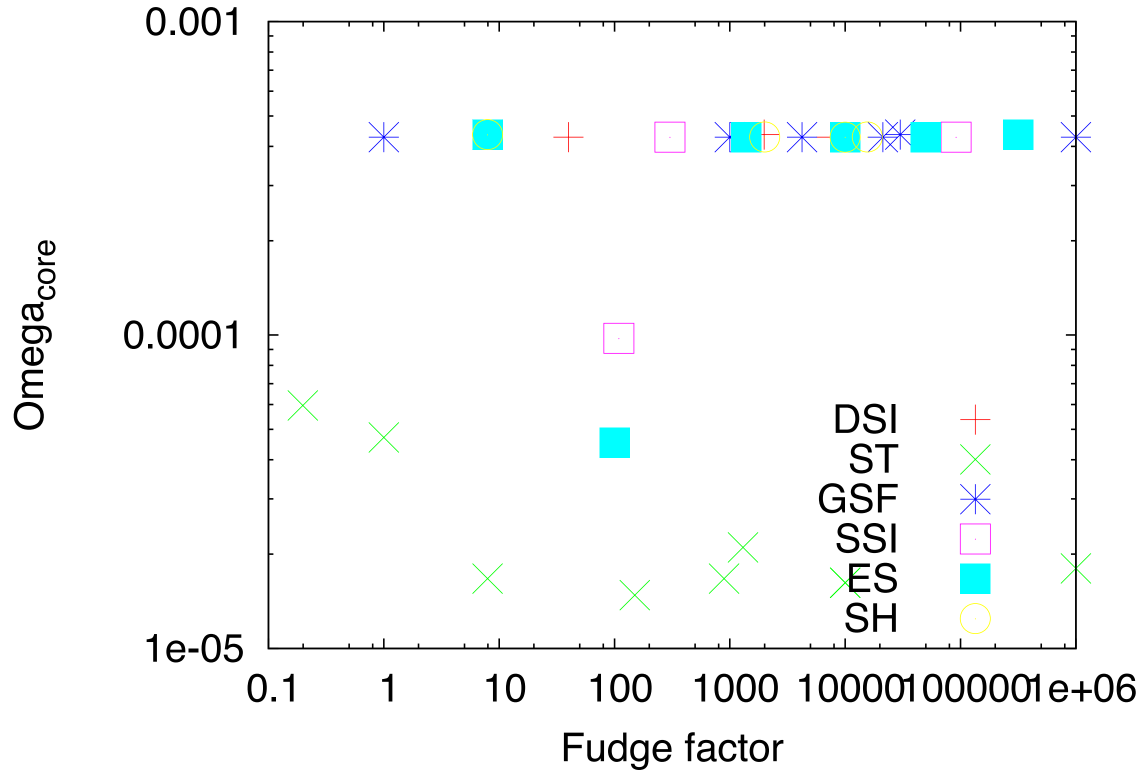
  - Note: the Maxwell stresses don't look anything like the turbulent diffusion term.
  - However, field and shear interact nonlinearly with each-other, so there is presumably a relationship between the two.

Other shear-induced instabilities (abridged)

Spruit-Tayler mechanism

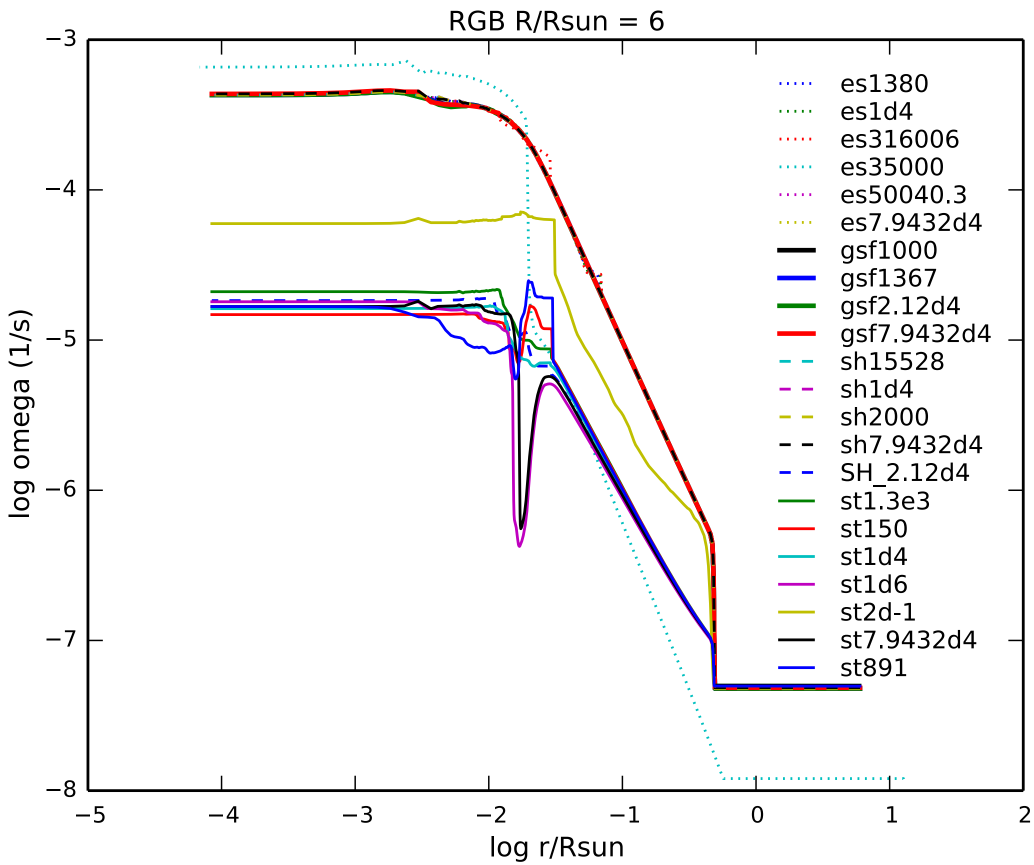
- Magnetic fields can also transport angular momentum by
  - Causing instabilities that drive turbulence
  - Being themselves sources of angular momentum transport (via Maxwell stresses).
- Tayler (1973+) studied instabilities of various magnetic field configurations in stellar radiative zones.
- Spruit (2002) proposed that these could drive a dynamo (self-sustaining process of magnetic field generation) within the radiative zone. This is controversial.
- Spruit, and later Heger, Woosley & Spruit (2005) proposed an associated angular momentum "mixing" coefficient from this mechanism. (Use at your own risk, etc…).

Results.



0.001
0.0001
1e-05
Omega$_{core}$
0.1
1
10
100
1000
10000
100000
1e+06
Fudge factor
DSI
ST
GSF
SSI
ES
SH

Results.



RGB R/Rsun = 6
log omega (1/s)
log r/Rsun
−3
−4
−5
−6
−7
−8
−5
−4
−3
−2
−1
0
1
2
es1380
es1d4
es316006
es35000
es50040.3
es7.9432d4
gsf1000
gsf1367
gsf2.12d4
gsf7.9432d4
sh15528
sh1d4
sh2000
sh7.9432d4
SH_2.12d4
st1.3e3
st150
st1d4
st1d6
st2d-1
st7.9432d4
st891

Make-up-your-own prescription!

In general: Regardless of the driving mechanism:

- We know that shear probably drives the instability.
- We know that stratification probably quenches the instability.
- This balance is probably well-measured by the Richardson number.
- We want to create a diffusion coefficient that has the dimensions of cm^2/s (as all the diffusion coefficients do).

Possible very general prescription based only on dimensional arguments above:

$$\nu_{AM,shear} = C_{AM} \frac{\kappa_T}{Ri^{\alpha}} \quad \text{and} \quad D_{shear} = C_{mix} \frac{\kappa_T}{Ri^{\alpha}}$$

where C is either a constant, or a function of other non-dimensional parameters.

Note that with $\alpha = 1$ we recover prescription from Zahn (1974) for Secular Shear Instability, partially validated numerically by Prat & Lignieres (2014).

4. Activity #3: Modifying the dynamical shear instability routine in MESA.

Part 1

- Test the effectiveness of the different AM transport prescription in MESA.. Do any of them explain observations?
- Collate and study results on Google spreadsheet.

Part 2

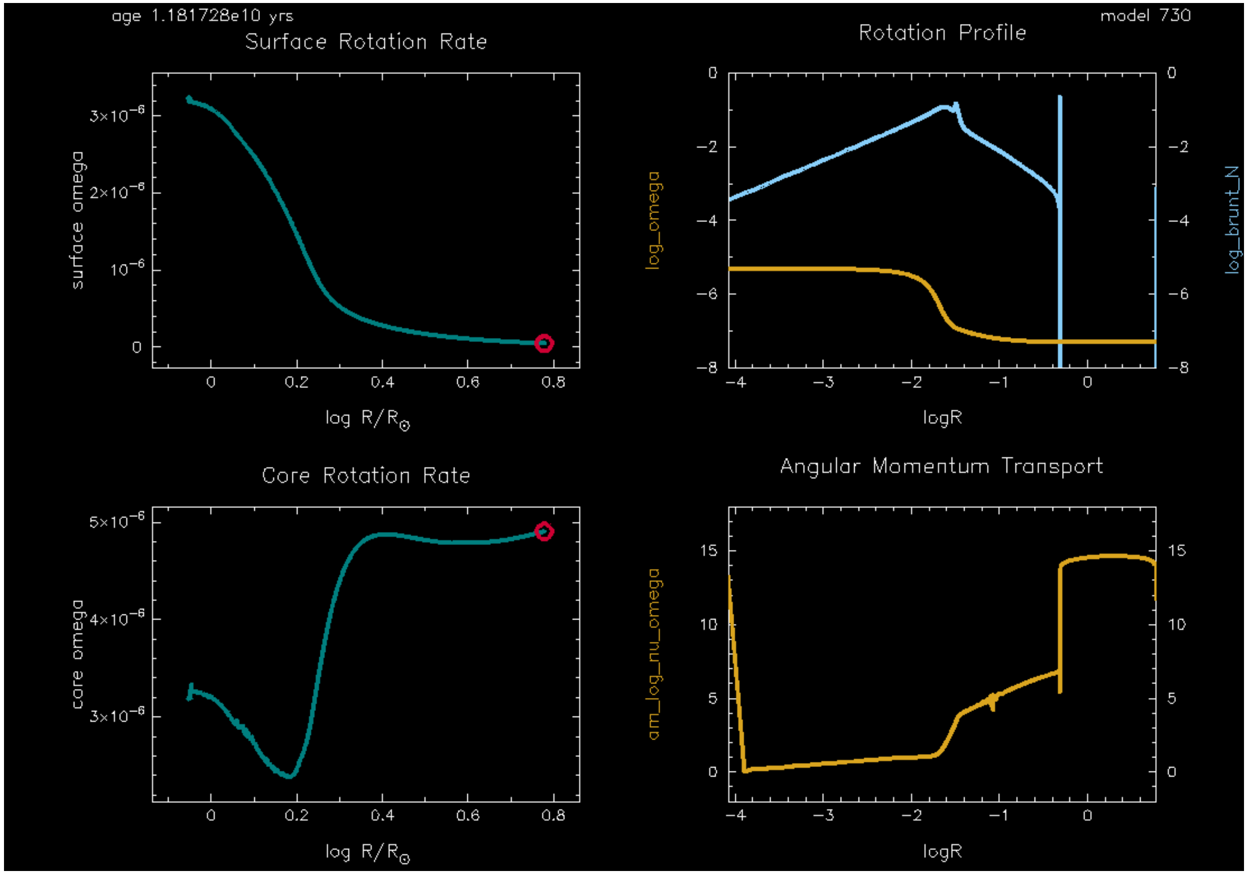
- Using `run_star_extras`, implement your own prescription for `am_new`. Think about the stability criterion (if you want to implement one), and the mixing coefficient. You can use formula on previous slide, or any other of your choice.
- Collate and study results. Do any of them explain observations?

Part 3 (If there is time)

- Turn compositional mixing back on, and add the rotationally induced mixing corresponding to your new model. See what happens to abundances.

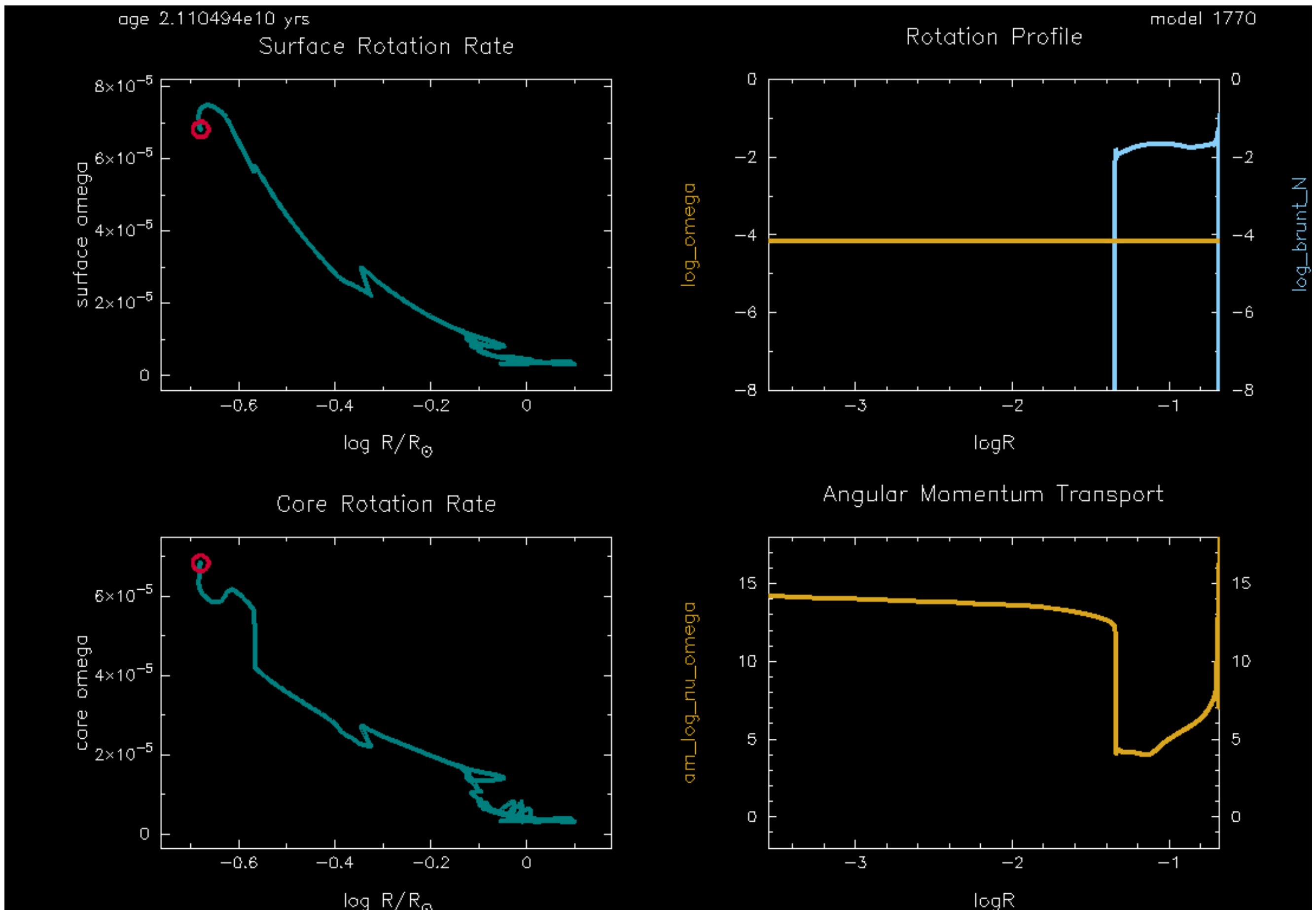
Result.

Example run with only new angular momentum mixing implemented, with alpha = 0.5 and C = 1000.



Result.

Example run with both new angular momentum mixing implemented, and equivalent D_{mix}, with alpha = 0.5 and C = 1000.



Result.

Something terribly wrong is happening: the star becomes core convective (see Brunt-Vaisala frequency for instance, is 0 in most of the star now).

This is because with an equivalent compositional mixing coefficient, hydrogen gets mixed back into the core very efficiently, and H-burning reignites.

This shows that in order to explain RGB stars, we both need

- Extremely efficient AM transport but
- Only weak compositional transport.

Solution to problem (AM mixing only)

Sample code to be added to `run_star_extras.f` can be found in `run_star_extras_solution.f`

Also do not forget to set `use_other_am_mixing = .true.` in your inlist