The SIRIUS Code:
Statistical Inference of physical paRameters of sIngle and mUltiple populations in Stellar clusters

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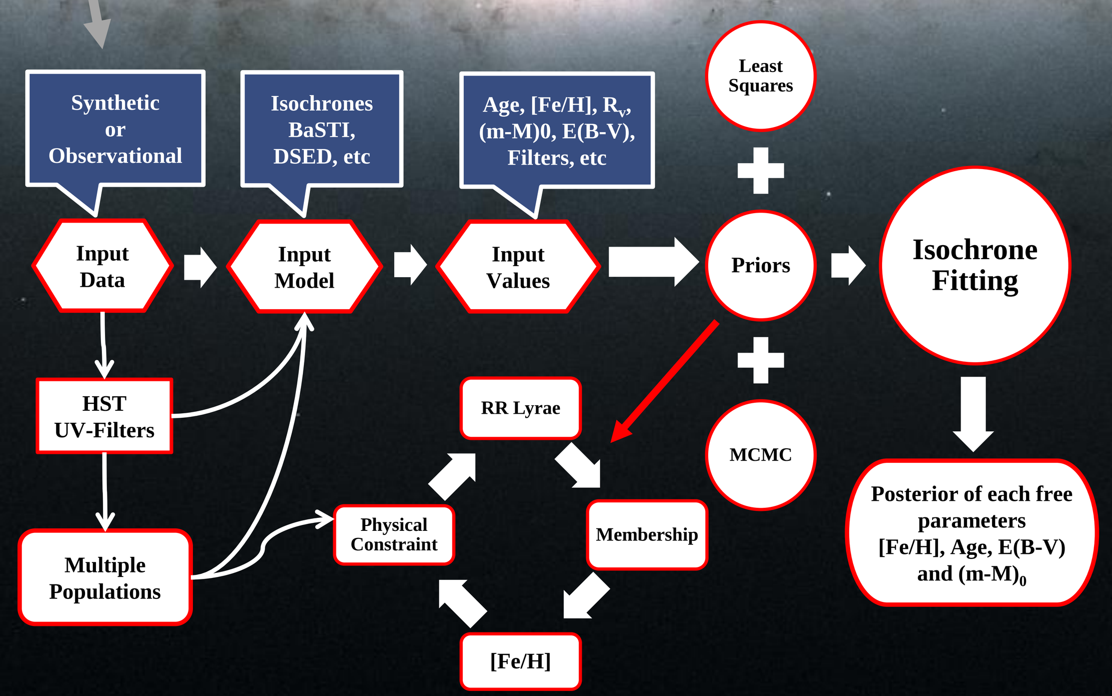
Abstract

Globular clusters (GCs) are fundamental pieces to understand the formation and evolution of the Milky Way. From the color-magnitude diagrams it is possible to get physical parameters such as age, distance, reddening, among others. The wealth of information available in high quality photometric, spectroscopic and/or astrometric data, allows to perfom a comprehensive and self-consistent analysis combining different data and techniques. We are developing a python package based on a Bayesian approach, that we named SIRIUS, to extract as much as possible information of a stellar cluster, in general. In addition, SIRIUS can characterize both simple and multiple stellar populations in a stellar cluster. The validity of Sirius has been tested by analyzing HST, VLT, Gemini, VVV, and Gaia DR2 data of bulge GCs.

1. The SIRIUS code

SIRIUS is a Python code based on a Bayesian approach of Isochrone fitting which can be applied to determine density of probabilities of the cluster physical parameters using the Metropolis-Hastings algorithm of Markov chain Monte Carlo[1]. Also, SIRIUS makes use of Machine Learning Classification methods to isolate the multiple generations, when it is available.

SIRIUS is designed to analyse different photometric systems with diverse stellar evolutionary models (BaSTI, DSED, etc). On the other hand, this code possesses the advantage of analyzing both synthetic and real CMDs (Kerber et al. 2007).



2. Theoretical Background

- Statistical approach from Bayes' theorem:

$$P(\varphi|D) = \frac{P(D|\varphi) \times P(\varphi)}{P(D)}$$

- φ and D represent the parameter space (model) and the data (observational), respectively.
- $P(D|\varphi)$ - Likelihood distribution
- $P(\varphi)$ - Priors (distributions *a priori* about parameter space).
- $P(D)$ - Marginal distribution, it can be used to normalize the likelihood. Once it does not depend on the parameter space, we can assume equal to 1.
- $P(\varphi|D)$ - Posterior distributions (distributions *a posteriori* of the observational data)

The Bayes' theorem can be written in logarithm as:

$$\ln P(\varphi|D) = \ln P(D|\varphi) + \ln P(\varphi)$$

Likelihood function: $\ln P(D|\varphi) = \chi^2_{\mathrm{Color}} + \chi^2_{\mathrm{Magnitude}}$

e.g. : $\chi^2_{Color} = \sum_{i=1}^{N} \frac{-1}{2} \left(\frac{Color_i^{OBS} - Color_i^{ISO}}{\sigma_i^{Color}} \right)^2$

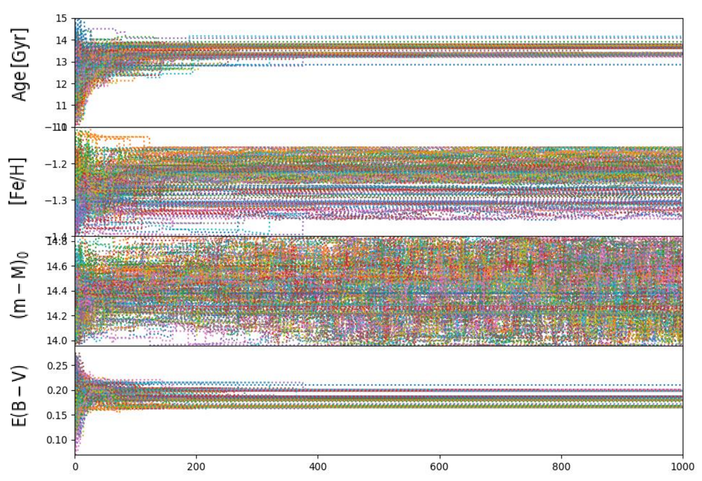


Fig 1: Path of the "walkers" in the parameters space. It is possible to see that in a spacific "step" the walkers do not change their positions.

- Markov Chain Monte Carlo (MCMC) is able to explore the parameter space looking for the best probability distribution of a problem.
- The algorithm uses "walkers" that are solutions of a free parameter, that can move in a grid with a specific value of "steps".
- The combination of walkers and steps can ensure or not the convergence of the solution. The convergence can also be reached combining many chains.

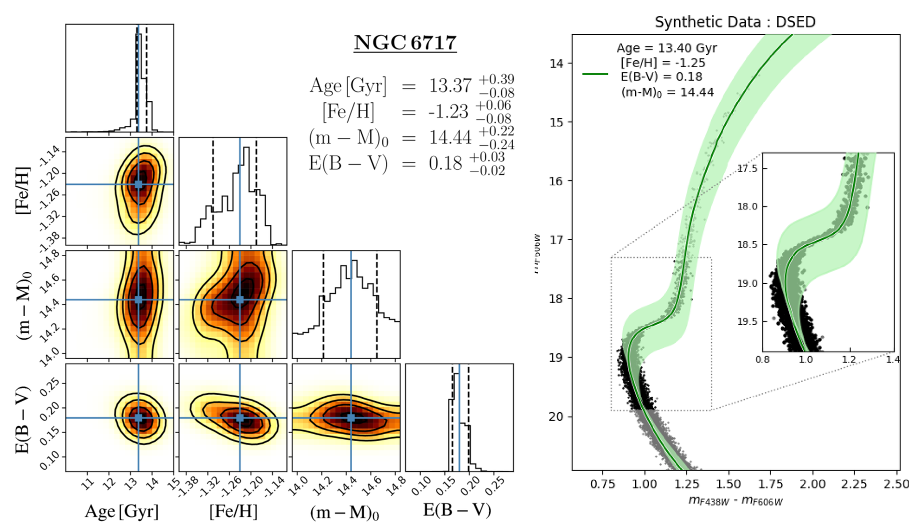


Fig 2: Example solution using SIRIUS. Corner-Plot shows the posteriors distributions for 4 free parameters. Best solution (solid line) and 1 sigma solution region (green region) for Synthetic Data.

- The corner-plot represents the hyperspace of parameters in a 2D projection.

➢ **Input of the synthetic CMD:**

- ➢ **Age = 13.5 Gyr**
- ➢ **[Fe/H] = -1.25 dex**
- ➢ **E(B-V) = 0.18 mag**
- ➢ **(m-M)0 = 14.4 mag**

3. Multiple Populations – HST UV-Filters (See R. A. P. Oliveira's poster)

To separate the multiple populations we use:

- Chromosome Map[3] diagram for MS and RGB.
- Two-Color diagram for SGB[4].
- Gaussian Mixture models to estimate the center of the star distribution.

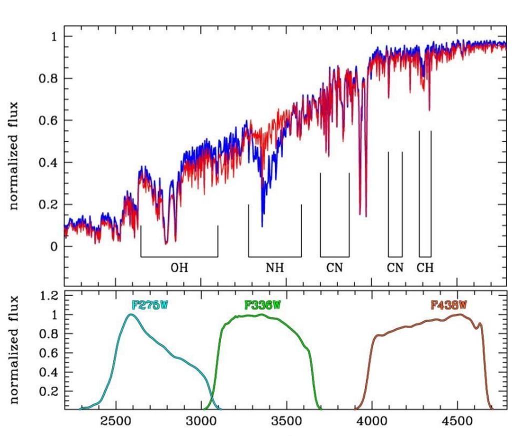
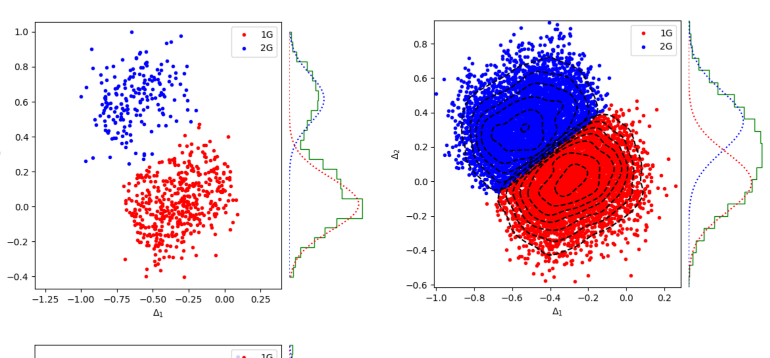


Fig. 3: HST UV filters F275W, F336W and F438W compared with the Cold Stars spectra and their molecular bands of OH, NH, CN and CH (Piotto et al. 2015).

First Generation (1G)
Second Generation (2G)



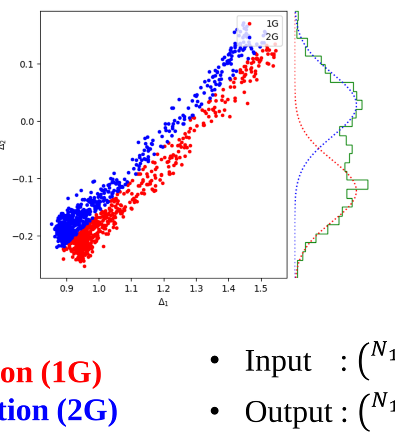


Fig 4: Split generations in RGB (top left panel), MS (top right panel) and SGB (left bottom panel) for a Synthetic cluster with multiple generations.

- Input : $\left(^{N_{1G}}/_{N_{\mathrm{Total}}}\right)_{\mathrm{RGB}} = 0.69$
- Output : $\left(^{N_{1G}}/_{N_{Total}}\right)_{RGB} = 0.70 \pm 0.05$

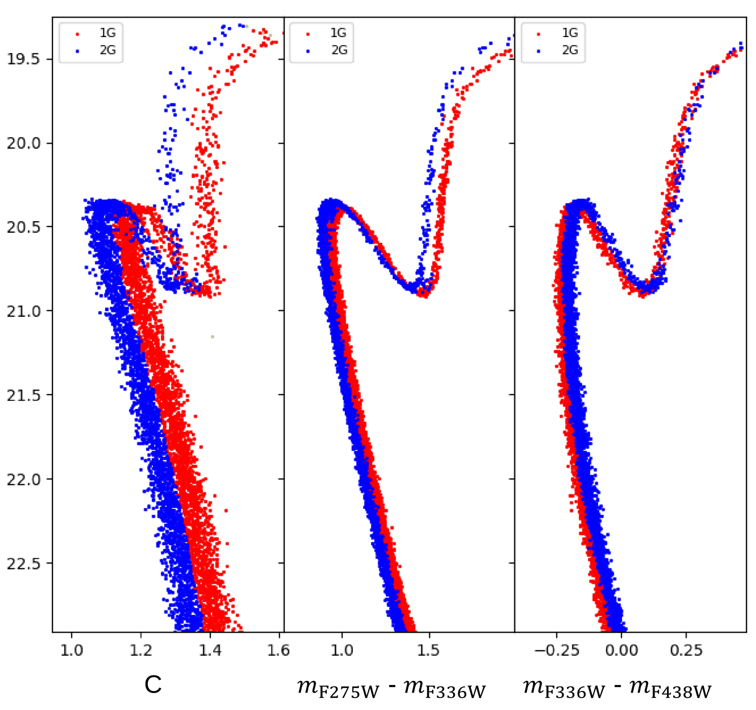


Fig 5: CMD with distinction of the multiple generations for a Synthetic cluster. Left panel represents the CMD with the pseudo-color C that is the combination of the color of the middle panel and the right panel.

4. First Result: HP1

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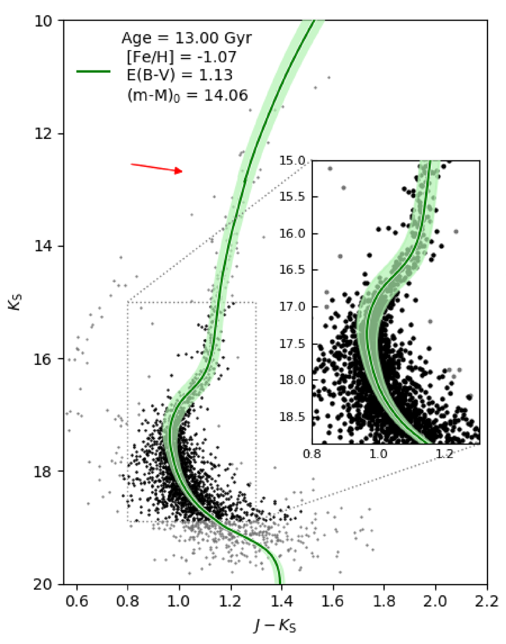
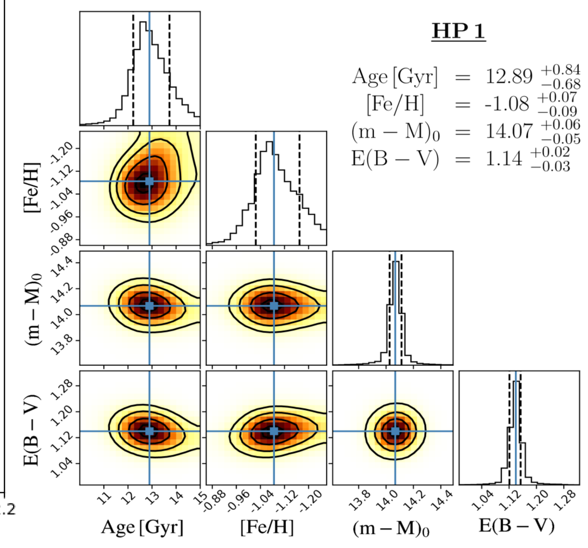
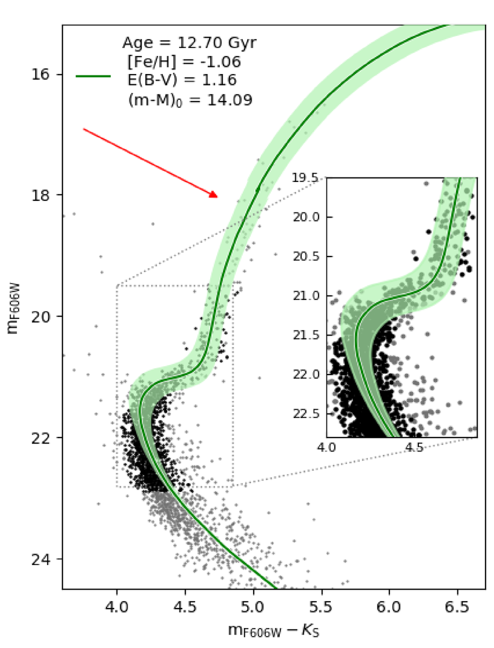
- Multi-epoch analysis with HST and GSAOI data.
- The CMDs K_S vs. J-K_S and m_{F606W} vs. m_{F606W} -K_S were used, F606W from HST and K_S and J from 2MASS.
- Stellar evolutionary models from BaSTI[6] and DSED[7] were applied.



- **GSAOI CMD:**





- **HST+GSAOI CMD:**

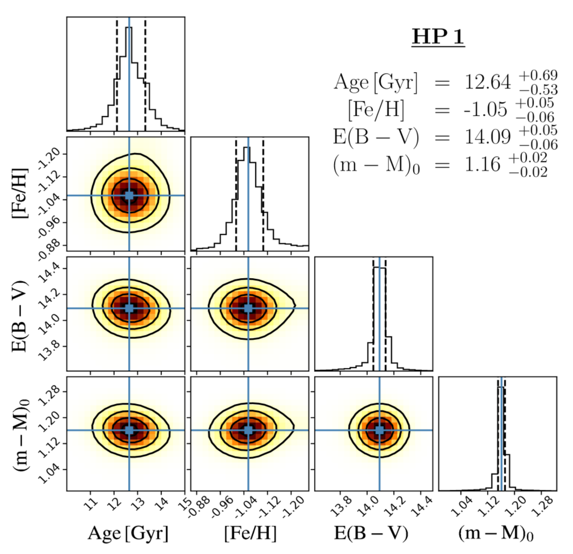


Fig 6: Results for Ks vs. J-Ks (GSAOI) and V vs. V-Ks (HST+GSAOI) for DSED. The lef panels show the region of most probabilities solutions and the best solution (green line), and the right ones represent the posteriors distributions of probabilities of the free parameters.

Tab 1: Average results (DSED & BaSTI* models)

[Fe/H]	Age (Gyr)	$(m-M)_0$	$d_\odot$	E(B-V)
$-1.09^{+0.07}_{-0.09}$	$12.75^{+0.86}_{-0.81}$	$14.10^{+0.06}_{-0.05}$	$6.59^{+0.17}_{-0.15}$	$1.15^{+0.02}_{-0.02}$

BaSTI* is the BaSTI results after the expected correction for the atomic diffusion (Δage ~ 0.90 Gyr).

5. Conclusions

We have developed the SIRIUS code, a python package to estimate physical parameters of stellar clusters.

Even though SIRIUS was designed and successfully tested to analyze globular clusters in the Milky Way, it is so flexible that can be extended to analyse stellar cluster in general. A future implementation for SIRIUS is to include another stellar evolutionary model, e.g. Victoria Regina. Also, we are using SIRIUS to analyaze clusters from the HST UV-Legacy Survey (see Oliveira et al. Poster).

References

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