3. DS ANDROMEDAE CURVES ANALYSES: EXTENDED DISCUSSION VERSION

Here we describe the modeling procedure, the weighting of the data, curves and models, and multiple stages of improving and testing the models in detail. We discuss first the initial trials in which the temperature of the hotter star was kept fixed at the expected temperature of stars of its reported spectral type as well as those in which both temperatures were adjusted, and the number of curves which were simultaneously modeled for these and other trials (Section 3.1); second, models with different interstellar extinction and metallicity in various combinations (3.2); third, models that included 3rd light (3.3); fourth, models that treated the RADS and non-RADS data differently, either by running them in separate bands or by weighting one relative to the other (3.4); fifth, models with spots on one or both components (3.5); sixth, models with different passband absolute calibration constants and with different metallicities (3.6); seventh, models run with detailed reflection options of two and three reflections, with and without convective atmosphere values for albedo and gravity darkening parameters, A_2 and g_2, respectively, and with non-synchronous rates, i.e., with $F_{1,2} \neq 1$ (3.7); eighth, semi-detached binary models (3.8); ninth, the effects of adjusting the period variation parameter, P-dot (3.9); and tenth, the effects of including the relatively sparse U data (Section 3.10). Note, here we use the term “model” to mean any particular configuration of parameters and with any suite of data in a WD run or sequence of runs.

3.1 Modeling Procedure and Two-Temperature Adjustment Models

The 2013 version of the Wilson Devinney program (Wilson & Van Hamme 2013 and references therein), hereafter WD, was used for all the modeling described here. An input file for Model 41, one of many converged models for the Differential Corrections routine (hereafter, DC) and the data (Table 15) are available in the Zenodo repository package also. The sample input file itself excludes the 94 sparse and relatively noisy *U* observations that were used mainly as a check in some runs, but these are in the package. Identification of other quantities in the DC input file may be found in Wilson & Van Hamme (2013). In WD, there are general *level weights*, more specific *curve weights*, and, for each datum, an individual weight. All are entered in the DC input file. The level weight switch, labeled "NOISE" in WD, was set at 1, appropriate for photon statistics, for all runs**.** For each iteration of each run, the DC output file provides two tables to measure the goodness of the differential corrections fitting: "Standard Deviations for Computation of Curve Dependent Weights" for each curve, and the "Input-Output in F and D formats." As indicated in the title of the first table, the program allows for the weighting of each curve; these weights are computed internally from the curve sigmas given in this table and that are entered directly into the DC input file for the following run [magnitudes are converted to light units, erg $cm^{-3}s^{-1}$, for photometric data; and in a specified "VUNIT" (here, 100 kms^{-1}) for RV data]; they are to be manually entered into the DC input file for the following run. These weights are critical to the adequate relative weighting of the RV and photometric data. A software switch (*KSD* = ***1***) when set in the input file to automatically update these curve weights after each iteration during a run; multiple iterated corrections of the main set, and subsets of uncorrelated or weakly correlated parameters, were always obtained. The second table includes a list of input and output parameters and their standard errors, and, at the foot of this table, "the mean residual for input values," a mean weighted residual of all the curves, in which the weights are applied so that both light and radial velocity curves contribute appropriately to the resulting, mixed-unit, mean. This mean weighted residual, $<r_{in}>$, is the fitting datum that we have used to assess the overall goodness of fit of each converged solution.

Parameter correlations were dealt with in two ways. First, the Marquardt (1963) damping constant, λ, was set to 10^{-6} to lessen the effects of parameter correlations in the full set on the damped least-squares results. Second, subsets of only weakly-correlated parameters were adjusted in each run. We rarely needed to use the subsets, as the multiple iteration operation produced full convergence within six runs. Each run consisted of 30 or 40 iterations. We considered full convergence to be achieved when all the adjustments became smaller than the probable errors, i.e., less than 2/3 of the standard deviations listed as the uncertainties in the DC output files.

In all cases, we adjusted the semi-major axis, a; the systemic or gamma velocity, V_{sys}; the orbital inclination, i; the temperature of the cooler star, T_2; the modified Kopal potentials, $\Omega_{1,2}$ (or $Omega_{1,2}$); the mass ratio, $q = M_2/M_1$ (where star *1* is the *primary* star, here taken as that eclipsed at primary minimum, thus the ostensibly hotter component when the depths are unequal); the epoch, t_0, specifying the instant of a particular conjunction and primary minimum central eclipse; and the orbital period, P. In many runs, T_1, was adjusted also. In some runs the passband luminosities, $L^{\lambda}{}_1$, were adjusted; when these were not adjusted, the logarithm of the distance, *log d*, was adjusted. In the bulk of the runs, the 3rd light parameter, $\ell_3{}^{\lambda}$, was adjusted [see Wilson & Van Hamme (2013, Chapter 6, p.11) for the distinction between "luminosity" and "light" as used in WD]. The initial values of the limb-darkening coefficients were taken from Van Hamme's (1993) table via a desk-top GUI devised by Dirk Terrell (1995ff) that allows temperature, *log g* and metallicity [Fe/H] to be entered. It produces coefficients for each of three types of limb-darkening. We used the logarithmic form ($LD_{1,2} = -2$) for all trials, internally computed beyond the first iteration, and our "Van Hamme-Terrell tables" for initial values. They generally closely matched the flux-weighted limb-darkening coefficients produced by the LC-routine, which was run after every final DC convergence. For almost all trials, the albedo parameters, ALB1,2, were fixed at 1.000, appropriate for the radiative envelopes of stars earlier than the Sun in spectral type. In a later section (3.8), we discuss trials where these were set to values appropriate for convective envelopes.

The initial series of analyses were begun with the BVR_CI_C and radial velocity (RV) data listed in Schiller (1986), with initial values for the parameters from SM88. We labeled this configuration Model 0. As work progressed, the above authors became aware of the work of ThMA and obtained permission to make use of his radial velocity data described in Section 2.2. Both sets of radial velocities were incorporated in the modeling, and these data were used in the preliminary results reported by us in 2014 (Milone & Schiller 2015), but it soon became clear that ThMA's RVs alone would provide greater precision in the dynamical parameters (a, V_{sys}, i, q, $\Omega_{1,2}$). From Model 21 on, the Mellergaard Amby (2011) RVs alone were used in the modeling. In the earliest runs the temperature of the hotter star was fixed at 6775 K, the temperature adopted by Schiller (1986) and Schiller & Milone (1988) from the early-F spectral classification and color index of the system according to Table 1 of Popper (1980). Other fixed values of T_1 were 6795 K, merely to test the effect of this small change on the results, and 6964 K, found in the newer tables and formulae of Flower (1996), as corrected by Torres (2010)]. Later, when L^{λ}_1 was adjusted in 4-passband runs, T_1 values were fixed at the means of 2- or 3-passband runs in which $log\ d$ and both temperatures were adjusted. The fixed T_1 runs, in which T_2 was always adjusted, we refer to as "1T" models. The modeling thus consisted of two types of runs: those involving all four passbands, in which the passband luminosities, T_2, and the other parameters were adjusted but not the $log\ d$ parameter; and those involving only two or three passbands, in which both T_1 and T_2 and the $log\ d$ parameter, along with other parameters except the passbands luminosities, were adjusted. The reason for the latter scheme was the temperature-distance (T – d) theorem[1] as discussed by Wilson (2007, 2008, Sections 2-4), which, to avoid under- or over-conditioned circumstances, places limits on the number of passbands run simultaneously if *log d* is one of the adjusted parameters. Within this operational dichotomy, several groups of models were explored. Models in which both temperatures and *log d* were adjusted, we label “2T” Models.

3.2 Interstellar Extinction and Metallicity Test Models

A series of runs (Models 1-27) were undertaken to test the effects of different interstellar extinction coefficients, A_V, and metallicity, [M/H], across the ranges of these quantities reported in various studies of the cluster NGC 752. Friel & Janes (1993) from high-resolution spectroscopy of eight F stars near the Main Sequence turn-off found [Fe/H] = -0.09(5); Demarque et al. (2004) derived E_{BV} = 0.035, [Fe/H] = -0.05; Sesito et al. (2004) from high-resolution spectroscopy of G giants found [Fe/H] = +0.01(4); Bartasiute et al. (2007) determined from seven-color Vilnius system photometry of 65 stars that [Fe/H] = -0.14(3) for a group of 65 stars and a value of -0.06 for DS And, itself; Carrera & Pancino (2011) in a chemical abundance analysis of six clusters adopted for NGC 752 E_{BV} = 0.038(2), implying A_V = 0.118(6), and [Fe/H] = +0.08(4) with a systematic error of 0.03; and Twarog et al. (2015) on the basis of intermediate and narrow-band photometry of 68 “highly probable F dwarf members,” derived E_{BV} = 0.034(4) and a final value of [Fe/H] = -0.03(2). Three values of both quantities were tested in early trials: A_V = 0.075, 0.100, 0.125 magn. and [Fe/H] = -0.1, 0, +0.1, in all combinations. The resulting effects on the parameters were found to be slight, if not

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[1] “The T-d theorem can be stated as: EB light curves can yield temperatures of both stars and distance if and only if the light curves are standardized, two or more substantially different photometric bands are fitted, and radial velocities determine the absolute length scale.” (Wilson 2008).

insignificant, and the effects on the fittings, negligible. For example, the weighted means of the distance and the corrected distance modulus for each assumed extinction value obtained in these early trials (and, in following parentheses, their uncertainties in units of the last decimal place), were:

$Av = 0.075$ (Models 22, 23, 24): 437(3) pcs, $(M\text{-}M)_0 = 8.202(12)$;

$Av = 0.100$ (Models 19, 21, 30): 438(2), $(M\text{-}M)_0 = 8.209(11)$; and

$Av = 0.125$ (Models 25, 26, 27): 441(3), $(M\text{-}M)_0 = 8.195(18)$.

The weighted means of the distance and the corrected distance modulus for each metallicity case were:

$[M/H] = -0.1$ (Models 21, 23, 26): 433(1) pcs, $(m\text{-}M)_0 = 8.183(4)$;

$[M/H] = 0.0$ (Models 19, 22, 25): 436(2), $(m\text{-}M)_0 = 8.198(9)$; and

$[M/H] = +0.1$ (Models 14, 24, 27): 441(4), $(m\text{-}M)_0 = 8.224(17)$.

Models with $A_V = 0.125$ and $[M/H] > 0$ yielded larger mean residuals and so were slightly less favored. The trials suggested an A_V in the range 0.075 and 0.100 and a metallicity between -0.1 and 0.0, in accord with the trend of the cluster studies. It should be noted that the models used in the early trials (to Model 27) did not include adjustments for 3rd light. In most of the later runs, A_V was fixed at 0.100, and [M/H] was fixed at 0, the solar value. In one model (12) we attempted to adjust A_V, but this produced anomalously wide ranges of temperatures [6800 - 7800 K, for a mean of 7691(133) K for T_1, and 5800 - 6500 for a mean of 6418(122) for T_2], and A_V [-0.01 to 0.54 with a mean of 0.47(8)] were produced. The parameters for Model 12 are considered outliers and are not included in the group averages discussed in a later section. In further efforts to determine A_V, we carried out two sets of grid determinations, adjusting all parameters [a, V_{sys}, i, $T_{1,2}$, $\Omega_{1,2}$, q, t_0, P, $log\ d$, and ℓ_3^{λ})], for the suite of *VB* and RV data, where each individual RADS datum was given the weight of 0.0933 and 0.0883 for the *V* and *B* RADS data sets, respectively, and each non-RADS datum was given unit weight. The value of A_V was not adjusted within the run but was changed after each converged result for the following run. A_V values were stepped from 0.05 to 0.15 in units of 0.0125. A fitting of the squares of the mean residuals to a parabola yielded a minimum at $A_V = 0.1059$, justifying the use of the values 0.1 for the bulk of the previous trials. These conclusions were further tested when the effects of using different calibration constants, namely those indicated in Table 3 of Wilson & Van Hamme (2013) were explored. Those trials are described further in Section 3.6 below. With the adopted solar abundance and $A_V = 0.100$ interstellar extinction values, iterations between the 4-passband, L^{λ}_1 - adjusted runs of Model 41 and three-passband, $log\ d$ - adjusted runs were performed in the Model 28 series to obtain fully consistent values of T_1 and L^{λ}_1 in the final averages of the runs where $log\ d$ was adjusted. This was not strictly necessary, as not having fully consistent L^{λ}_1 and L^{λ}_2 input values does not affect any other parameters, and absolute passband luminosities in physical and solar units are in fact computed in the LC routine, along with other absolute parameters, but the input values of L^{λ}_1 as well as the corresponding computed values of L^{λ}_2 are listed in the output of the LC routine, hence it is useful to have them appear consistent with other radiative quantities. If they are consistent, then, the relative passband luminosities for the luminosity parameter (with WD-computed L^{λ}_2) will be the same as that computed from the passband luminosities in the LC-routine output file, $L^{\lambda}_1/(L^{\lambda}_1+L^{\lambda}_2) = \mathscr{L}_{\lambda 1} / (\mathscr{L}_{\lambda 1} + \mathscr{L}_{\lambda 2})$, where $\mathscr{L}_{\lambda 1}$ & $\mathscr{L}_{\lambda 2}$ are listed both in units of solar passband luminosities and absolute, cgs units.

3.3 Third-Light Models

As no previous work had produced evidence of 3rd light in the DS And system, we did not immediately explore that possibility in the present study. However, in checking over a problem we had with data input, R. E. Wilson ran one of our input files adjusting 3rd light and found it to be significant. Subsequently, from Model 28 onward, with a few exceptions mentioned later, 3rd light was adjusted, and, in nearly every case where this was done, it was found to be significant at about the 10% level in each passband, including *U*. This result implies that DS And appears slightly dimmer and farther than we at first reported: an average detached-model distance of 436(1) pc (Milone, Schiller & Amby 2015), where the uncertainty is the mse of the mean, signifying mainly the lack of dispersion among models. The estimated systematic error is an order of magnitude greater. It is important to note, however, that most of the adjusted parameters were not significantly affected by the adjustment of 3rd light.

3.4 RADS and non-RADS Photometry Treatment Models

The RADS and non-RADS (hereafter, *R* & *n-R* in this section and in some tables) data were obtained at slightly different epochs over the 1982-83 observing season (see Section 2.1), and these contributed unevenly to the folded light curves. Accordingly, experiments were undertaken to see if separating the R & n-R data or weighting one set relative to the other would improve the fittings. *R* & *n-R* data in each passband were entered as separate bands in Models 28A and 37-40. The ratio of the sigmas in each passband of the *R* & *n-R* light curves in Models 37 (RADS) and 38 (non-RADS) were used as the basis to compute and apply a relative weighting to the RADS data for some runs. Curve weights (see Section 3.1) that are produced in each run for each succeeding run are applied to each band of all runs of all models. In Models

40a-f (the $I_C R_C$, VB, $I_C V$, $I_C B$, $R_C V$, and $R_C B$ runs, respectively), the *R* & *n-R* data were divided into separate bands and the sigma of each band was inserted into the next run's DC input file, with unit individual weights in all bands. The parameter means of these models, which we call Model 40, produced the smallest mean residual of any of the models, with extremes of $2.77 \mathrm{x} 10^{-8}$ for Model 40a and $4.02 \mathrm{x} 10^{-8}$ for Model 40b, and an average $<r_{in}> = 3.53(20)\ \mathrm{x} 10^{-8}$ for Model 40 (see Table 2). In Model 28B, three-passband suites with combined R & n-R data, an individual weight equal to the RADS/non-RADS ratio of the inverse square of the mean of the full curve sigmas was applied to each individual RADS datum; non-RADS retained individual unit weight. The mean residuals for stipulated phases of maximum light only, as provided also in the DC output file's curves weights table, were not used in the production of these weights because of a relative dearth of non-RADS data (typically a few points) in the maxima. All individual weights for non-RADS data were set equal to 1. Models 37 (RADS) and 38 (non-RADS) runs yielded mean sigmas for the *IcRcVB* and $RV_{1,2}$ curves, respectively, of: [(2.522, 2.846, 4.434, 5.315) x 10^{-7} erg $\mathrm{cm}^{-3}\mathrm{s}^{-1}$, and (4.511, 2.103) $\mathrm{x}10^{-2}$ in units of 100 kms^{-1}]; and [(1.233, 1.127, 1.326, 1.580) x 10^{-7} erg $\mathrm{cm}^{-3}\mathrm{s}^{-1}$, and (4.582, 2.128) $\mathrm{x}10^{-2}$ 100-kms^{-1}]. The relative weights for the RADS data follow: 0.2391, 0.1569, 0.0933, and 0.0884, for the I_C, R_C, V, and B data, respectively. The scheme achieved improvements in the mean standard deviations of the fittings, if no dramatic improvement in the appearance of the light curves plots, and the fitting precision was better than that for most but not the best models. For the Model 28B runs, the mean residual, $<r_{in}> = 4.396 \mathrm{x}\ 10^{-8}$.

The corresponding curve sigmas for the Model 40 runs are discussed in Section 5. For completeness and to test the solution robustness in the case of a violation of the T - d theorem, we also ran all four passbands, again divided into *R* & *n-R* bands (for a total of eight bands) and with all data of unit weight: Model 40". The Model 40" fittings were not quite as good as those with fewer bands, possibly due to over-conditioning, but the resulting system parameters are consistent. All adjusted parameters for each of the Models 40a, b, c, d, e, f, 40", and 40 converged runs are presented in Table 2. The m.s.e.s are below each weighted mean and the errors computed from the inverse square root of the sums of the squares of the inverse standard deviations of each parameter follow in brackets. The adopted uncertainty is the larger of the two in each case; systematic error is not included. The values for 3rd light (suitably normalized by the sum of the light of both stars and third light) are listed for both the RADS and then the non-RADS bands in the lower part of the table. The weighted means of the combined *R* & *n-R* sets are given in footnote *a* of Table 2. The data and the light- and radial velocity curves for Models 40a and 40b, are plotted in Figure 1A-1C. The *VB* passbands with the weighted RADS data and the non-RADS data combined, were used in the A_V grid searches (see Section 3.2). The A_V grid fitting curve is shown in Figure 2. The fittings for the *V* and *B* passbands of the optimum grid-fit for $A_V = 0.1059$ and [M/H] = 0, Model 28C1b and the $I_C R_C$ passband counterpart Model 28D1b, described in Section 3.6, were the templates for the additional trials described in Sections 3.7 and 3.8. The light curve fittings show that the R & n-R data are not fully consistent. Modeling them separately, as in our Model 40" runs, or modeling them together but applying weighting factors to the RADS bands, produce fitting curves that overall do not appear to match the data perfectly. The non-RADS data have much less scatter, but much less phase coverage at maximum light and none at secondary minimum. The separated and the weighted RADS data treatments yielded the smallest mean residuals of all models, and the overall effects on the parameters were not great, as we will show.

Table 2. DS And Model 40 Adjusted Parameters [a]

Model (Pbs)/ Param.	40a (IcRc)	40b (VB)	40c (IcV)	40d (IcB)	40e (RcV)	40f (RcB)	40" (IcRcVB) (8 bands)	40 <IcRcVB> (cols 2-7)
a	5.941	5.923	5.945	5.918	5.938	5.910	5.877	5.930
(Rsun)	0.036	0.035	0.034	0.037	0.034	0.037	0.039	0.006[15]
V_{sys}	+8.09	+8.21	+8.16	+8.09	+8.17	+8.08	+8.05	+8.13
(km/s)	0.56	0.63	0.58	0.57	0.59	0.58	0.60	0.02[24]
i	89.47	88.86	89.01	88.71	89.56	89.81	89.44	89.35
(degs)	0.53	0.37	0.46	0.45	0.40	0.26	0.28	0.19[16]
T_1	6884	7052	7014	7027	7162	7091	7059	7056
(K)	81	30	38	21	51	20	16	21[12]
T_2	5752	6019	5983	5894	6087	5968	5987	5971
(K)	66	31	38	36	46	29	21	33[15]
Ω_1	3.566	3.577	3.580	3.572	3.579	3.567	3.551	3.574
	0.017	0.015	0.016	0.017	0.015	0.017	0.016	0.003[7]
Ω_2	4.274	4.321	4.304	4.319	4.314	4.271	4.238	4.303
	0.068	0.051	0.063	0.072	0.054	0.059	0.052	0.009[24]
q	0.659	0.656	0.663	0.653	0.661	0.650	0.642	0.657
	0.009	0.009	0.009	0.010	0.009	0.010	0.010	0.002[4]
t_0	.403315	.401926	.402397	.403477	.402252	.403277	.402898	.402813
HJD	.001093	.001232	.001150	.001119	.001149	.001112	.001130	.000265
fractn								[465]
P(d)	188143	189627	189052	188180	189142	188299	188576	188697
1.0105^+	001137	001301	001206	001171	001211	001171	001196	000247
								[488]
log d	2.666	2.675	2.676	2.668	2.692	2.682	2.675	2.678
(pcs)	0.011	0.006	0.008	0.008	0.008	0.006	0.004	0.004[3]
$(m_0\text{-}M)$	8.329	8.373	8.380	8.338	8.462	8.412	8.376	8.390
(magn)	0.056	0.030	0.040	0.042	0.039	0.028	0.020	0.018[15]

Table 2[a], continued

Model (pbs) Param.	40a (IcRc)	40b (VB)	40c (I_CV)	40d (I_CB)	40e (R_CV)	40f (R_CB)	40" (I_CR_CVB) (8 bands)	40 $<I_CR_CVB>$ (cols 2-7)
$\ell_3(I_C)_R$	0.113	...	0.100	0.086	...	...	0.103	0.100
(ℓ_{1+2+3})	0.021	...	0.019	0.023	...	...	0.008	0.007[12]
$\ell_3(R_C)_R$	0.112	...	...	...	0.099	0.101	0.090	0.103
(ℓ_{1+2+3})	0.019	...	...	...	0.015	0.013	0.007	0.004[9]
$\ell_3(V)_R$	...	0.099	0.112	...	0.106	...	0.105	0.103
(ℓ_{1+2+3})	...	0.010	0.018	...	0.014	...	0.006	0.004[7]
$\ell_3(B)_R$	...	0.097	...	0.097	...	0.109	0.104	0.101
(ℓ_{1+2+3})	...	0.009	...	0.022	...	0.012	0.006	0.004[7]
$\ell_3(Ic)_n$	0.110	...	0.097	0.083	...	...	0.097	0.098
(ℓ_{1+2+3})	0.021	...	0.019	0.023	...	...	0.007	0.008[12]
$\ell_3(Rc)_n$	0.116	...	...	...	0.100	0.104	0.092	0.105
(ℓ_{1+2+3})	0.020	...	...	...	0.015	0.013	0.007	0.004[9]
$\ell_3(V)_n$	...	.0.084	0.098	...	0.092	...	0.092	0.088
(ℓ_{1+2+3})	...	0.009	0.018	...	0.015	...	0.006	0.004[7]
$\ell_3(B)_n$	...	0.083	...	0.086	...	0.097	0.090	0.088
(ℓ_{1+2+3})	...	0.009	...	0.022	...	0.012	0.006	0.005[7]
<res.> x 10^{-8}	2.7720	4.0207	3.3160	3.8900	3.2863	3.8837	3.5380	3.5281

[a] The Model 40 entries in the last column are weighted means of Models 40a through 40f; and the m.s.e.s of the means, below each entry**;** those from the combination of parameter s.d.s follow in brackets. They indicate only the dispersion among the runs and contain no systematic error estimates (see Sections 3.4 and 4). The 3rd light parameter values have been normalized by the total system light at phases 0.25 & 0.75. The first four ℓ_3(pb) rows are RADS bands values, the last four are non-RADS bands values. The weighted means of both bands are: I_C:0.099(1); R_C: 0.104(1); V: 0.095(7); B: 0.094(7).

Nevertheless, the RADS data are relatively sparse in the first maximum of the I_C curve, and although suggestive of an asymmetry in the maxima (the O'Connell effect --- see Davidge & Milone 1984), the enhancement at this maximum (0.25^p) is absent in the R_C light curve. SM88 suggested that this and mid-eclipse discrepancies were due to variability in the system on time scales shorter than the two months required to observe the full light curve. Attempts to model the I_C anomaly have not been satisfactory, given that at least two light curves are required to correctly model the base temperatures of the two stars and *log d*. A new and complete light curve in this passband would be desirable.

3.5 Spot Models

Spots were introduced to model the largest RV1 residuals in Figure 1A as well as the slight light curve enhancements in Figures 1B and 1C and described in Section 3.4. In spot modeling it is typical to place cool spots on the cooler star, but in this system that model did not converge. Thus, to improve the fitting of the RV_1 data in the first quadrature by weighting some disk grid elements to simulate a velocity distortion, a cool spot was placed on star 1 (in Models 42A, 42B, and 43r) and for this spot, start and stop times were set for the RV data interval only.

For the most noticeable enhancement in maximum I (that following primary minimum) seen in the I_C light curve (Figure 1B), a hot spot was placed on star 1 in Models 42-50.

In one series, star spots were added and usually all four parameters of a spot were adjusted. In two-spot models, however, usually only one parameter was adjusted for each spot, cyclically iterated until consistency was obtained. The Julian dates of the onset and end of the spots were varied prior to runs to test the effects of spots on each of the observing segments, but no improvements were found.

In Model 42A, the spot parameters were entered, but no parameters were adjusted; in Model 42B, the stellar and system parameters were varied, but not the spot parameters; in Model 43r, the spot parameters were adjusted, but serially, i.e., not all simultaneously. The latter spot parameters were relatively poorly determined, and the RV fitting not noticeably improved, so the 2-spot models were abandoned.

In Model 51 a hot spot was placed on star 2's opposite hemisphere.

For completeness, in Model 52 a single cool spot was placed on star 1 but on the opposite hemisphere from that assumed for models with a hot spot on star 1. Star 2 could be expected to have a deeper convection zone, if present, than star 1, but a model with a cool spot on star 2 failed to converge.

Third light was removed for Models 47 and 48 to see if spots alone could provide a better fit. None of the spot models, with or without 3rd light, improved the fit significantly.

The complete spreadsheet Table 3 in the Zenodo repository shows spot parameters of all models. The average values of spot model parameters with a hot spot on star 1 were as follows: ϕ (co-latitude, measured from a pole), 74(2) deg; λ (longitude, increasing CCW from the line of centers), 284.4(4) deg; ρ (spot radius), 109.3(4) deg; t_f (temperature factor),1.0055(4). These quantities should be understood merely as means of 15 models in which parameters of this type of spot were adjusted; they correspond **to the** adjusted parameters of no single converged run. Other averaged parameters of this group of models are listed in Column 7 of Table 3; note the large $<r_{in}>$ value. The adopted model does not include spots.

Table 3. DS And Adjusted Parameters Summary[a]

Group/ Parameter	2-T models	ARV models	Av=.1 models	Solar [M/H]	3rd light	spot models	non-sp models	R/nR cases	Wtd means
a	5.862	5.925	5.933	5.933	5.937	5.948	5.934	5.917	5.940
(R_{sun})	0.015	0.006	0.007	0.007	0.005	0.003	0.007	0.011	0.004
V_{sys}	+7.34	+8.13	+8.14	+8.14	+8.15	+8.16	+8.15	+8.11	+8.15
(km/s)	0.22	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01
i	87.43	88.49	88.92	88.87	89.55	89.08	89.23	89.23	89.32
(deg)	0.30	0.26	0.24	0.25	0.07	0.33	0.22	0.08	0.17
T_1	7059	7063	7067	7064	7070	7053	7071	7046	7064
(K)	6	8	9	9	8	11	12	21	3
T_2	5931	5929	5928	5921	5962	5946	5926	5954	5950
(K)	13	17	23	25	08	14	34	18	6
Ω_1	3.542	3.593	3.595	3.595	3.592	3.608	3.587	3.575	3.595
	0.012	0.004	0.004	0.005	0.006	0.005	0.005	0.012	0.005
Ω_2	4.252	4.355	4.334	4.338	4.286	4.352	4.299	4.282	4.305
	0.034	0.017	0.018	0.019	0.008	0.033	0.014	0.015	0.010
q	0.624	0.656	0.658	0.658	0.659	0.656	0.658	0.654	0.657
	0.008	0.002	0.002	0.002	0.002	0.001	0.002	0.003	0.001
log d	2.657	2.664	2.670	2.669	2.690	2.670	2.677	2.675	2.673
(pc)	0.004	0.004	0.004	0.005	0.008	0.009	0.003	0.004	0.003
t_0	.399622	.402897	.402789	.402793	.402663	.402679	.402751	.402719	.402756
2436142+	.001099	.000071	.000071	.000071	.000058	.000098	.000070	.000083	.000056
P (d)	92165	88605	88693	88703	88776	88791	88733	88711	88729
1.010514+	1189	58	57	57	33	60	62	61	40
<r>x10^{-8}	7.907	9.469	8.858	8.789	9.337	10.511	8.623	5.198	8.587

[a] Parameters of all models are available in large spreadsheet tables (see text)**.** Below each mean is the m.s.e. of the mean, the formal, internal error, signifying mainly the dispersion among models of that group with no estimate of the significant systematic error. Column 10 shows grand means of the weighted means of columns 3 through 9, and the same qualification for its m.s.e.s applies. The square root of the sum of the inverse squares of the m.s.e.s of the means is usually smaller; for parameters *a* to *log d* these are, respectively: *a*, 0.002; V_{sys}, 0.004; *i*, 0.05; T_1, 4; T_2, 6; Ω_1, 0.002; Ω_2, 0.005; *q*, 0.001, *log d*, 0.002, t_0, .000028; *P*, 19 x10^{-8}, respectively.

3.6 Passband Absolute Calibration Constants and Metallicity Tests

The DDE algorithm deals with absolute units and requires a calibration constant for each passband. The calibration constants, [called *CALIB* in Wilson & Van Hamme (2013)] are given in cgs flux units (erg $s^{-1}cm^{-3}$) with the number of significant figures supplied in the original publications. The constants for *V* and *B* that we assumed in all previous trials were provided by Wilson (2015) privately, as the best available, namely, 0.36949 for *V*, and 0.62350 for *B*. For the Cousins I_C and R_C passbands the constants from Bessell (1979), 0.122 for I_C and 0.225 for R_C, were adopted. They are needed when log d is an adjusted parameter.

The effects of changing both metallicity and the passband calibration constants were explored in Models 28C and 28D**.** The A_V = 0.1059 grid value result, described in Sections 3.2 and 3.4, was adopted for these trials. The calibration test series made use of only two passbands, ensuring strict adherence to the T - d theorem (Wilson 2008; see Section 3.1). The designations, calibration constants and the sources from which they were derived were taken from Table 3 of Wilson & Van Hamme (2013). For each of the listed models, runs were made for each of the three metallicities -0.1, 0, 1, and +0.1, signified by suffixes a, b, and c, respectively, in the tables**.** For *V* and *B*, respectively:

28C1: 0.36949, 0.62350 (Wilson (2015), as given above;

28C2: 0.378, 0.688 (Johnson 1965,1966);

28C3: 0.361, 0.660 (Bessell 1979);

28C4: 0.3631, 0.632 (Bessell, et al. 1998); and

28C5: 0.36895, 0.6266 (Wilson et al. 2010).

For I_C and R_C, they are, respectively:

28D1: 0.122, 0.225 (Bessell 1979), as given above; and

28D2: 0.1126, 0.2147 (Bessell et al. 1998).

The mean residuals for the models designated above are shown in Table 4. The last two columns of Table 4 contain averages for the calibration set, i.e., averaged across the metallicities, and the m.s.e. of the mean, respectively. The mean residuals of the three metallicity model runs for each otherwise identical model with the same calibration constant are not significantly different. There is also no significant difference in the mean input residuals between the runs with the Wilson (2015) and the Wilson et al. (2010) calibration constants, but the former's mean residuals are consistently smaller among different metallicity trials. The mean residuals for the other pairs of calibration constants are significantly larger. We can also confirm that the metallicity has a relatively small effect on the mean residuals and that the Wilson (2015) calibration constants with solar metallicity produces the smallest mean residuals. In Table 5 column headers labeled "28 Cn" or "28Dn" refer to the model series with *VB* and $I_C R_C$ passbands, respectively, and "n" refers to the nth set of calibration constants listed in Column 1 of Table 4. The suffixes "a," "b," and "c" again designate metallicities -0.1, 0, and +0.1, respectively. The parameter means from each calibration-constant run, averaged across the three metallicity runs, are given in columns 2-8 of Table 5; the parameter means of each metallicity run in the Model 28C series, averaged across all five calibration-constant runs, are given in columns 9-11. Below each model's weighted mean is the weighted m.s.e. of the errors of each run. The consistency among many of the parameters **is** striking; however, it must be remembered that the runs are not fully independent and the m.s.e.s of the means are also formal, internal errors, only.

Table 4
Models 28Cn and 28Dn Metallicity and Calibration Constant Trial Results[a]

Model	[M/H]	$<r_{in}>$ x 10^{-8}	Mean cal n$<r_{in}>$x 10^{-8}	e_{Mean} x 10^{-8}
28C1a	-0.1	4.71649	4.70260	0.01145
28C1b	0.0	4.71141		
28C1c	+0.1	4.67989		
28C2a	-0.1	5.18844	5.20268	0.00784
28C2b	0.0	5.20415		
28C2c	+0.1	5.21546		
28C3a	-0.1	4.98328	4.98632	0.00283
28C3b	0.0	4.99198		
28C3c	+0.1	4.98370		
28C4a	-0.1	4.76459	4.76320	0.00779
28C4b	0.0	4.77593		
28C4c	+0.1	4.74907		
28C5a	-0.1	4.73450	4.72445	0.01073
28C5b	0.0	4.73586		
28C5c	+0.1	4.70300		
28D1a	-0.1	2.98538	2.99006	0.00349
28D1b	0.0	2.98791		
28D1c	+0.1	2.99689		
28D2a	-0.1	2.87757	2.87095	0.00387
28D2b	0.0	2.87112		
28D2c	+0.1	2.86416		
< 28Cb> runs	0.0	...	4.88387	0.09422
< 28Db > runs	0.0	...	2.92951	0.08258

[a] Columns 4 and 5 show the mean residuals and their uncertainties of the three metallicity models (suffixes a, b, and c, resp.) for each calibration constant. The mean residuals for the Model 28Ca,c and 28Da,c runs are: 4.87746(9138), 4.86622(10268) x 10^{-8}; and 2.93146(7623), 2.93052(9385) x 10^{-8}, respectively.

(This is true, as well, of all the columns of Table 3, which lists the model means of each group and the group means.). No weighting was used for the averaged results for each metallicity given in the last three columns. The parameters most impacted by variations in model, metallicity, and calibration constants are the stellar effective temperatures, T_1 and T_2 and to a lesser extent, ℓ_3 and *log d*, that is, the radiative properties parameters, as expected (the distance scales with the square root of the calibration constant). The strongest cause of parameter variation in $T_{1,2}$ is the calibration constant. The Johnson, Bessell, and Bessell et al., calibration constant runs yielded temperatures that were incompatible with the early F observed spectral type of the system, and expected F1 or F2 for the hotter component and likely early G for the second component, so their use here can be rejected as unphysical, further supporting the choice of the Wilson (2015) *V* and *B* calibration constants. For the I_C and R_C passbands, the constants of Bessell et al. (1998) produced consistently smaller mean residuals compared with the Bessel (1979) calibration constants, but yielded temperatures that were again too high. The resulting high temperatures skew the means across the models as can be seen in the last three columns of Table 5. The m.s.e. of $T_{1,2}$ are ~100 and 80 K, and those of *log d* ~0.012, providing upper limits for systematic error due to calibration constant selection.

3.7 Detailed Reflection Model Tests

Following up a comment by one of us (SF) that the secondary star may exhibit a large reflection effect due to the proximity in this system of a larger and hotter companion, we ran the template model 28C1b with enhanced reflection (Wilson 2010), *mref* = 3, keeping the number of reflections at (*nref* =) 2. This model (28C1bR) showed significant changes to some of the

Table 5[a]
DS And [M/H] and Cal. Test Models 28Cn,Dn Adjusted Parameter Means[a]

Model/	28C1	28C2	28C3	28C4	28C5	28D1	28D2	[M/H]	[M/H]	[M/H]
Parameters								-0.1	+0.0	+0.1
								<Cn>[b]	<Cn>[b]	<Cn>[b]
a	5.956	5.956	5.958	5.956	5.957	5.939	5.944	5.956	5.957	5.957
(Rsun)	0.002	0.001	0.001	0.001	0.000	0.002	0.002	0.002	0.001	0.002
V_{sys}	+8.35	+8.35	+8.37	+8.36	+8.34	+8.07	+8.10	+8.35	+8.35	+8.37
(km/s)	0.01	0.00	0.01	0.02	0.06	0.01	0.01	0.00	0.01	0.01
i	89.46	89.78	89.42	89.12	89.15	88.93	89.09	89.88	89.11	89.27
(degs)	0.10	0.19	0.25	0.33	0.10	0.06	0.20	0.10	0.10	0.24
T_1	7052	7510	7539	7327	7088	6817	7275	7318	7287	7305
(K)	13	13	14	74	13	6	14	114	100	105
T_2	5876	6210	6230	6079	5903	5731	6081	6061	6039	6080
(K)	24	10	10	52	24	8	3	88	77	68
$Omega_1$	3.570	3.569	3.575	3.567	3.570	3.569	3.568	3.572	3.571	3.567
	0.002	0.001	0.001	0.003	0.001	0.001	0.002	0.001	0.002	0.002
$Omega_2$	4.289	4.271	4.288	4.286	4.295	4.285	4.281	4.279	4.286	4.293
	0.005	0.003	0.005	0.005	0.007	0.002	0.004	0.002	0.004	0.008
q	0.664	0.665	0.666	0.665	0.662	0.658	0.663	0.665	0.664	0.663
(M_2/M_1)	0.001	0.000	0.001	0.000	0.002	0.001	0.002	0.000	0.001	0.002
t_0	.4014	.4013	.4013	.4013	.4015	.4033	.4032	.4014	.4014	.4013
HJDfr.	.0001	.0000	.0000	.0001	.0001	.0000	.0000	.0000	.0000	.0001
P (d)	19025	19031	19035	19039	19018	18813	18822	19079	19025	19037
1.0105+	00005	00002	00003	00013	00006	00002	00002	00002	00004	00011
log d	2.681	2.733	2.746	2.706	2.685	2.658	2.715	2.706	2.710	2.715
(pcs)	0.003	0.003	0.003	0.001	0.002	0.002	0.000	0.013	0.012	0.013

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[a] Mean standard errors are given for each model below the parameters. The inverse square root of the sums of the inverse squares of the individual run errors is usually similar but sometimes may exceed the m.s.e. of the mean. The robustness of most parameters is striking but $T_{1,2}$ are particularly sensitive to choice of calibration constant, as the m.s.e.s of their means indicate.

[b] The mean parameters for the same [M/H] are for 28C models only.

parameters, namely T_2 = 5707(30), t_0 = .3944(16), P = 1.01051977(17), and the (un-normalized) 3rd light parameters $\ell_3(V)$ = 2.49(35) x 10^{-6}, and $\ell_3(B)$ = 3.05(41) x 10^{-6}, when compared with the corresponding parameters for Model 28C1b, namely, 5864(31), .4015(13), 1.01051902(13), 3.02(41) x 10^{-6} and 3.71(48) x 10^{-6}, respectively. The overall fit was no better, however. The mean residual for this detailed reflection model was 4.758 x 10^{-8} whereas $<r_{in}>$ = 4.711 x 10^{-8} for Model 28C1b. As the temperature of star 2 had now slipped below that of the Sun (in WD2013, T_{sun} = 5779 K is assumed), Model 28C1bR was rerun with convective envelope coefficients A_2 = 0.500 and g_2 = 0.32, and designated Model 28C1bRC. This did not improve the fitting ($<r_{in}>$ = 4.964 x 10^{-8}). Because the duration of a run of detailed reflection models scales heavily with the number of grid elements on the components, we changed the number of high and low precision grid elements on the two stars from (60, 60; 30, 30) to (30, 30; 15, 15), for Model 28C1bRCsg. This made insignificant changes to the parameters and to their uncertainties, resulting in $<r_{in}>$ = 4.981 x 10^{-8}. To test this effect of the insertion of non-synchronous rotation factors (Model28C1bRCsgF12). we set $F_1 = P_{orb}/P_{rot,1}$ = 1.047 and $F_2 = P_{orb}/P_{rot,2}$ = 1.006, corresponding to v_1 = 103 and v_2 = 63 kms^{-1}, slightly off synchronism. No improvement in fitting ($<r_{in}>$ = 4.980 x 10^{-8}) or significant additional changes in the parameters resulted. Then, running this last model with an additional reflection, *nref* = 3, Model 28C1bRCsgF12R3 again produced no additional, significant changes and only slight improvement in $<r_{in}>$ = 4.943 x 10^{-8}. This last model was rerun with the original number of grid elements, with no significant differences in parameters or their errors and $<r_{in}>$ = 4.928 x 10^{-8}. Because the results are so similar for detailed reflection model tests, only the means of the adjusted parameters of the last two trials are included in Table 6 (column 2).

3.8 Semi-detached Test models

Star 1 has been consistently the larger component in all the detached model results, i.e., its modified Kopal potential is closer to that of the inner Lagrangian surface (in this case, Ω_{inner} = 3.1793) than is that of star 2. Initially, therefore, we tested semi-detached (SD) models in mode 4, in which the potential of Star 1 is not adjusted but internally set at that of its inner Lagrangian surface, (and the rest of the full suite of parameters adjusted). The Model 28C1b two-passband (*V* and *B*) model was the template. This model, 28C1bF12sD, with $\Omega_1 = \Omega_{\mathrm{inner}}$, did not converge.

After all the trials described in previous sections were completed, the apparently anomalous luminosity of the secondary component, to be discussed in Section 6, compelled us to reconsider the semi-detached models to see if a radically different model would not only provide better fittings to the data but would also help with the secondary luminosity problem descry bed in Section 6.. We again used Model 28C1b with detailed reflection, and convective envelope parameters for A_2 and g_2, as the template. This time we ran tests in mode 5, in which Ω_2 is not adjusted (but calculated internally in DC) while Ω_1 is adjusted. Initially, no solution was forthcoming.

Following ten 40-iteration runs in which there was no main-set convergence, so that the best fitting iteration or the best fitting subset of only weakly correlated parameters was used to correct the input parameters for the next run, only two parameters failed to converge: i and ℓ_3. In cases where only the i parameter fails to converge, a grid method is sometimes successful, and this was planned. However, given the possibility that 3rd light might not be necessary for this model, $\ell_3^{V,B}$ were fixed at zero and i was adjusted along with all the other parameters. This proved successful: convergence was achieved in four additional runs. This we designate as Model 53_SD5a. The parameters for this model are listed in column 3 under the header “53a” in Table 6. Following this result, we then used the Model 28D1b template (with I_C and R_C passbands), again not adjusting 3rd light, fixed at 0. Convergence was achieved after 15 runs. We designate this as Model 53_SD5b, and in the header of Table 6 this is designated "53b". The two sets of converged SD models differ significantly from each other and from the detached solutions.

To check the robustness of the SD solutions, third light was again adjusted in the Model 53_SD5b template file and after four additional runs, convergence was achieved for the I_CR_C passband suite, Model 53_SD5b1 (identified as header "53b1" in Table 6). For the *VB* passband suite, convergence was also found, Model 53_SD5a1 (identified as header "53a1" in Table 6), but in this case, the ℓ_3^V value was not significant and the ℓ_3^B value only marginally so.

The temperatures of both components in all the SD5 models studies thus far were larger than for the detached models; but the previous SD5 converged models retained convective envelope parameters A_2 and g_2 Setting both parameters to 1.00, appropriate for hotter stars with radiative envelopes (leaving aside the issue of the appropriateness of such an envelope for a lobe-filling star) we obtained convergence for the I_CR_C passband suite in the next run; this is Model 53_SD5b2 ("53b2" in the Table 6 header). No convergence was found for the *VB* passband suite for the radiative atmosphere case. We had previously determined that a slightly better mean residual was obtained for the *VB* suite of 28C1b models by restricting the range of phases around quadrature from 0.10^P to 0.06^P for the calculation of the standard errors at maximum light. The I_CR_C suite models had shown no such improvement, so the range was left at 0.1^P for the 2bD1b template models. To ascertain that this difference did not impact the parameters significantly, Model 53_SD5a1 was rerun with the range reset to 0.10^P; this is Model 53_SD5a2. (The Table 6 headers are designated "53a1" and "53a2", respectively). The results are not significantly different. For the *VB* suite Model 28C1b run, $<r_{in}> = 4.711 \times 10^{-8}$ whereas for Models 53_SD5a,

Table 6
DS And Adjusted Parameters of Detailed Reflection and Semi-Detached Models[a]

Model[a]/ Par.	28...R3	53a	53a1	53a2	53b	53b1	53b2
a (R_{sun})	5.947	6.305	6.300	6.302	6.424	6.016	5.968
	0.030	0.042	0.042	0.042	0.048	0.029	0.050
V_{sys}	+8.33	+8.18	+8.17	+8.17	+7.62	+8.08	+8.16
(km/s)	0.46	0.42	0.42	0.42	0.72	0.52	0.52
i	86.84	72.60	72.74	72.74	73.49	85.09	89.70
(degs)	0.18	0.10	0.24	0.24	0.13	0.47	1.10
T_1	7043	7347	7116	7116	7923	7386	7337
(K)	23	11	43	43	27	79	80
T_2	5712	5908	5722	5722	6657	6232	6388
(K)	21	23	38	38	22	64	67
$Omega_1$	3.559	3.927	3.940	3.939	4.611	5.266	5.086
	0.011	0.030	0.033	0.034	0.049	0.082	0.039
$Omega_2$[b]	4.437	3.205	3.206	3.205	3.135	3.135	3.209
	0.034	…	…	…	…	…	…
q	0.676	0.677	0.677	0.677	0.722	0.680	0.681
(M_2/M_1)	0.006	0.008	0.008	0.008	0.010	0.006	0.009
t_0	.3939	.3957	.3957	.3956	.4043	.4024	.3975
HJDfr.	.0012	.0013	.0013	.0013	.0014	.0013	.0014
P (d) 1.0105+	19829	19627	19627	19631	18717	18916	19425
	00128	00130	00131	00130	00148	00151	00151
log d	2.667	2.723	2.692	2.682	2.782	2.750	2.764
(pc)	0.005	0.003	0.008	0.008	0.005	0.010	0.008

[a] Model28…R3 results are the means of small-and large grid size Model 28C1bRCR3 runs; errors are the inverse square root of the sums of the inverse squares of the run errors. Columns 3-8 are individual run results from Mode 5 semi-detached models in the "53a" (*VB*) and "53b" ($I_C R_C$) series. See Section 3.8 for more detail.

[b] $Omega_2$ values are fixed in mode 5; the listed values are from the DC and LC output files.

[c] HJDfr = fraction of the heliocentric Julian date.

a1, a2, $<r_{in}>$ = 7.634, 6.933, and 6.930 x10^{-8}, respectively. For the $I_C R_C$ suite Model 28D1b run, $<r_{in}>$ = 2.988x10^{-8} whereas for Models 53_SD5b, b1, b2, $<r_{in}>$ = 4.305, 3.147, and 3.255 x10^{-8}, respectively. Although the uncertainties of some parameters, such as i, t_0, and P may be smaller in some semi-detached models, those of others, such as i, $Omega_1$ or q may be larger in others, as Table 6 shows. In all cases, even with detailed reflection and consideration of convective vs. radiant envelope parameters, the mean residuals for the converged semi-detached models exceeded those of corresponding detached models. These results confirm the impression given by the rounded, transit-like primary minimum and the occultation-like secondary minimum seen clearly in the SM88 plots. We therefore favor the adopted detached model for the present data suites.

3.9 Period Variation Trials

Given the presence of 3rd light, template Models 28C1b and 28D1b were rerun with the period variation or P-dot (for dP/dt) parameter adjusted along with all the other parameters. Both sets of trials showed significant dP/dt terms, but the latter only marginally, and they disagree. Only the period and epoch, among the other parameters, were significantly changed. For Model 28C1bP, involving the *VB* passbands, the mean residual of fit $<r_{in}>$ = 4.462 x 10^{-8} and the relevant parameters are: t_0 = 2436142.2819(183); P_0 = 1.01053838(295); and dP/dt = -1.393 (211) x10^{-9} d/d. When run with detailed reflection, *nref* = 3, Model 28C1bPR yielded: $<r_{in}>$ = 4.481 x 10^{-8}; t_0 = 2436142.2798 (185); P_0 = 1.01053864 (299); and dP/dt = -1.405 (214) x10^{-9}. For Model 28D1bPR, involving the $I_C R_C$ passbands, the corresponding quantities were: $<r_{in}>$ = 2.879 x 10^{-8}; t_0 = 2436142.4493(204); P_0 = 1.01051138(329); and dP/dt = +5.330 (2356) x10^{-9}. Generally, an added adjusted parameter produces an improvement in fitting error, even if uncertainties in individual parameters increase. This is the case here, as a comparison with results mentioned in the previous section indicates. The results for the *VB* and $I_C R_C$ suites are significantly different, casting uncertainty on the reality of the dP/dt term and making any determination of parameters of a third body, if any, a matter for future investigations.

What has been found to date is not supportive: an examination of all the times of minimum over the interval 1932 to 2015 has been carried out by Nelson (private correspondence 2018; data available in the link, http://www.aavso.org/bob-nelsons-o-c-files). With weighting of 0.1 applied to visual and photographic data and 1 for photoelectric and CCD times of minima, he found a marginally significant but very small, quadratic term, corresponding to dP/dt = +2.715(1151) $\times 10^{-11}$ d/d.

3.10 Effects of U data Inclusion

The 94 *U* observations obtained by SJS at the McDonald Observatory and not used in earlier runs were added to the input data suite of a Model 41 template file. The U data were confined to the primary minimum and shoulders of adjacent maxima. At the time these runs were made, the “C1” Linux cluster at the University of Calgary was to be decommissioned within a few weeks, so identical runs were made also on the “Storm” cluster, transferred to the University of Calgary from the Compute Canada consortium. The final converged results from the two clusters were not significantly different, as expected. The modeling tests were conducted in two stages. First, all five passbands as well as the two radial velocity bands were run with L_1 but not *log d* adjusted, along with the rest of the parameters. The final converged values of the parameters from two sets of nearly identical results obtained on the C1 and Storm servers, Models 55C1 and 55S, respectively, were then used as starting parameters for subsequent runs of suites of two-band photometric and the RV curves, namely Models 55a ($I_C R_C$), 55b ($I_C V$), 55c ($I_C B$), 55d ($I_C U$), 55e ($R_C V$), 55f ($R_C B$), 55g ($R_C U$), 55h (*VB*), 55i (*VU*), and 55j (*BU*). The *U* bands were run with absolute calibration constant 0.4221 (Wilson, 2015). When these were completed, weighted means of the adjusted and absolute parameters were obtained. As the difference between the adjusted parameters' values obtained from each run was less than 1% in all cases, the determined uncertainties in the DC fittings were used to obtain the final weighted means for the parameters, giving the set of means we call Model 55. Models 55e and 55j, however, yielded radiative parameters (T and ℓ_3) that exceeded Chauvenet's criterion. Deleting the parameters of these two sets yielded the set of means we call Model 55'. The mean residuals, $<r_{in}>$, were 4.484 (146) x 10^{-8} and 4.509(168) x 10^{-8} for Models 55 and 55' respectively. These may be compared with the results of Model 40, 3.528 x 10^{-8}. The largest differences among the parameters between Models 55 and 55' are in T_1 = 7052(50) and 7025(38) K; T_2 = 5876(47) and 5838(37) K, for Models 55 and 55' respectively, where the errors are the m.s.e.s of the weighted means. These and other radiative quantities, such as relative, i.e., normalized, 3rd light, are seen to agree within errors. The values for the latter are: $\ell_3(R_C)$ = 0.116(5) and 0.112(5); $\ell_3(V)$ = 0.112(3) and 0.113(5); $\ell_3(B)$ = 0.098(3) and 0.101(1); $\ell_3(U)$ = 0.122(9) and 0.133(12), for Models 55 and 55' respectively. For completeness, the Model 55 result for $\ell_3(I_C)$ = 0.112(5). The *log d* values were: 2.687(10) and 2.675(7) for Models 55 and 55', respectively. Comparing the results with those of Model 40 (Table 2), the largest difference is for T_2 (5971 K), a difference of about 2σ; similar results can be seen for 3rd light in the R_C,*V*, and *B* passbands, and the *a*, *Vsys*, and *q* parameters, but there is no difference as large as 3σ. Among the curve–dependent parameters, we determined the relative luminosity in the U band, $L_1/L_{1+2}(U)$ = 0.912(3), and normalized 3rd light, $(\ell_3/\ell_{1+2+3})^U$ = 0.122(9), where the errors are m.s.e.s of the means.

We conclude that the exclusion of the U data for the bulk of the trials has had minimal impact on the adjusted parameters, and the increased uncertainty in many parameters when they are included, justifies their exclusion from the bulk of the modeling trials. Additionally, the exercise provided *U* curve parameters, shown in the complete Table 3 in this supplementary package.

This concludes our extended discussion of the DS Andromedae differential corrections runs and models. The summary and significance of the adjusted parameter run results are given in Section 4, and the summary and significance of the absolute parameter LC run results follow in Section 5, of the published paper.

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