

ART. XLIII.—*A Curious Inversion in the Wave Mechanism of the Electromagnetic Theory of Light*; by C. BARUS.

1. THE account given by the equations of the electromagnetic theory of light as to the motion, is without ambiguity; but when one endeavors to reconstruct such a wave graphically one is apt to fall into a trap such as will make the wave run backwards. To briefly summarize the formulæ relevant to the present paper, I will refer to the mode of derivation in which the conception of a displacement current conditioned by the time variation of electric force is first clearly put. Then the fundamental equation for electric flux in the direction of an axis ( $Y$ ) in terms of the line integral of magnetic force ( $a, \gamma$ ) around that axis is in the usual notation

$$\frac{K}{V} \frac{\partial Y}{\partial t} = \frac{\partial a}{\partial z} - \frac{\partial \gamma}{\partial x} \quad (1)$$

provided the Hertzian device of measuring electric quantities electrostatically and magnetic quantities electromagnetically is introduced for symmetry. Here  $V$  is the normal velocity of light,  $K$  the specific inductive capacity of the medium. A clockwise curl of magnetic force around the electric axis is therefore in question.

On the other hand, the fundamental surface integral of magnetic flux in the direction of an axis ( $z$ ), is expressed in terms of the line integral of electric forces around that axis by

$$-\frac{\mu}{V} \frac{\partial \gamma}{\partial t} = \frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} \quad (2)$$

where Hertz's method has again been used and where  $\mu$  is the magnetic permeability of the medium. The curl of electric force around the magnetic axis is here counter-clockwise. Further equations need not be adduced since the electric forces  $X$  and  $Y$  in directions  $x$  and  $y$  and magnetic forces  $a, \gamma$  in directions  $x$  and  $z$  at the point  $x, y, z$  and at the time  $t$ , suffice for the present purposes.

From these equations to the differential equations of wave motion, known in form and treated with success even in D'Alembert's time, is not an arduous span. Making it one obtains in the usual way

$$\frac{\partial^2 Y}{\partial t^2} = \frac{V^2}{\mu K} \nabla^2 Y,$$

and

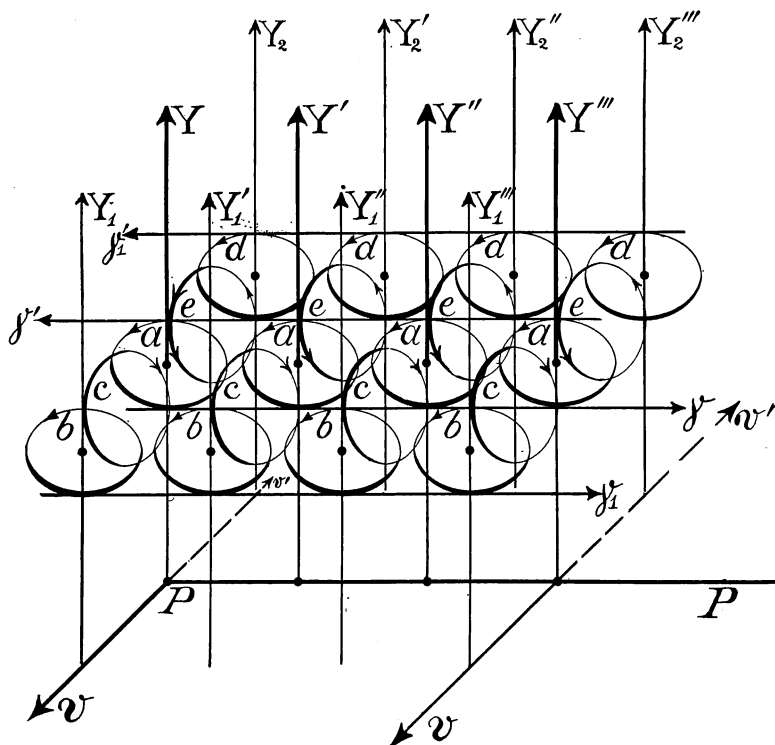
$$\frac{\partial^2 \gamma}{\partial t^2} = \frac{V^2}{\mu K} \nabla^2 \gamma,$$

two equations precisely alike except as to the variables  $Y$  and  $\gamma$ . The clockwise and counter-clockwise contrast of (1) and (2) has vanished.  $V/\sqrt{\mu K}$  is seen to be the common velocity of the two waves, if waves there be. Applied to plane wave fronts parallel to  $yz$  one obtains

$$Y = A \sin 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right)$$

$$\gamma = NA \sin 2\pi \left( \frac{t}{T} - \frac{x}{\lambda} \right)$$

where  $N$  is the index of refraction,  $T$  the period and  $\lambda$  the wave length in the given medium of permeability  $\mu$  and specific inductive capacity  $K$ . The two waves differ merely and naturally in amplitude. They are alike in phase. Nothing is left to recall the opposite signs of (1) and (2). The direction



of common advance is  $+x$ , when the direction of positive electric force is  $+y$  and of magnetic force  $+z$ , in english or clockwise coördinates. The wave follows the motor rule which,

given in Fleming's lucid way, points the first finger of the *left* hand in the direction of magnetic force, the second finger in the direction of electric current, leaving the thumb in the direction of advance of the wave.

2. Suppose now one endeavors to make a straight-forward step-for-step construction of the electric wave, as is done in the annexed diagram, where full arrows denote magnetic circuits and forces, open arrows the corresponding electric quantities.

Let  $Y, Y' \dots$  in the original plane  $PP$  of the wave front, denote the direction of increasing electric force, and therefore of electric polarization and of electric displacement currents. Each current is accompanied by a clockwise whirl,  $a, a, \dots$ , of magnetic force encircling it. These forces neutralize each other in the regions between the electric forces, but they have a common direction  $\gamma$  and  $\gamma'$  on either side of the wave front  $PP$ . Now let the resulting magnetic forces  $\gamma$  and  $\gamma'$  in their turn be treated individually and each provided with a counter clockwise whirl of electric force, polarization and current, as shown by  $c, c, \dots$  around  $\gamma$  and  $e, e, \dots$  around  $\gamma'$  respectively. The curls so obtained both tend to neutralize the original electric forces  $Y, Y', \dots$ , and to replace them by two new planes of electric force or wave fronts  $Y_1, Y_1' \dots$  and  $Y_2, Y_2' \dots$  etc. Clearly electric forces above and below the plane  $\gamma, \gamma'$  would also tend to neutralize each other.

Each of the new electric forces  $Y_1, Y_1'$  and  $Y_2, Y_2', \dots$  is however again to be provided with a clockwise magnetic whirl  $b, b \dots$  and  $d, d \dots$  respectively, which together would tend to neutralize  $\gamma$  and  $\gamma'$  and evoke the new magnetic forces  $\gamma_1$  and  $\gamma_1'$ . Hence by a construction quite recalling Huyghens's principle a new set of electric and magnetic forces  $Y_1$  and  $\gamma_1$ , and  $Y_1'$  and  $\gamma_1'$  respectively, have been obtained from the original  $Y$  and  $\gamma, \gamma'$ . The process may be indefinitely repeated, provoking an advance of the wave front in both directions,  $v$  and  $v'$ , from the original wave front  $PP$  and at right angle to this plane. In other words, the advance in the figure is to and from the reader.

But unfortunately both waves, if the forces  $Y$  and  $\gamma$  are correctly drawn, run in the wrong direction! This may easily be tested by using the left hand as above stated. If the direction of advance  $v$  and the electric forces are correct, the magnetic force is in the wrong direction; or finally for correct advance and magnetic force, the electric forces turn out negative. True, since we have two waves moving from each other, the discrepancy could be reconciled by coupling the forces of either wave front with the direction of the other, though such counsel is hardly compatible with sobriety. The very predic-

tion of two waves might be an uncomfortable consideration. In fact only if we accompany the electric current with a counter-clockwise magnetic whirl, and the magnetic flux with a clockwise electric whirl—only by reversing the relations of electric and magnetic field, which everybody is in the habit of associating with electromagnetic phenomena, can we make the wayward wave run right.

3. To arrive at an insight into the conditions here presented one may note that if the  $Y$  forces were constant, there would be no magnetic field in the dielectric due to the occurrence of the electric excitation. Since the magnetic forces  $\alpha, \beta, \gamma$ , are in this case partial differential coefficients of a potential function, there may be a magnetic field due to extraneous causes. This however has nothing to do with the immediate problem. In the diagram  $Y, Y', \dots$  would alone occur unaccompanied by magnetic whirls. In the second place if  $\delta Y/\delta t$  is constant a magnetic field must be evoked in the dielectric; but this field, apart from the manner of its origin, will during the period of constant  $\delta Y/\delta t$  be invariable in intensity and position. It will be a stationary magnetic field, the lines of which are fixed both as to their geometry and their number. As such it is entirely unable to evoke displacement currents. In the above diagram this would mean that, although the forces  $Y, Y', \dots$  the whirls  $\alpha, \alpha, \dots$  and in consequence the forces  $\gamma, \gamma'$  exist, the electric whirls  $e, e \dots$  and  $c, c \dots$  do not exist.

Finally let  $\delta^2 Y/\delta t^2$  have a significant value, positive or negative. The values  $\delta Y/\delta t$  will then be variable. Displacement currents will therefore be evoked in the dielectric, and the electric whirls  $c, c \dots$  and  $e, e \dots$ , the resultant electric forces  $Y_1, Y'_1, \dots$  and  $Y_2, Y'_2, \dots$ , the magnetic whirls  $b, b \dots$  and  $d, d, \dots$  etc., will in general all have to be entered in the diagram. One may conceive the disturbance to be transferred from  $PP$  in both directions indefinitely. It need not however be a harmonic disturbance.

If however the time variation of  $Y, Y', \dots$  is simple harmonic, then  $\delta^2 Y/\delta t^2$ , etc., are essentially negative and negatively increasing of the  $Y$ 's themselves are essentially positive and increasing; and vice versa. Hence  $\gamma$  is essentially decreasing if  $Y$  is essentially increasing in the lapse of time. The magnetic polarization is therefore falling off. The effect of an increasing  $Y$  is thus virtually to superpose on any existing magnetic field, new forces  $\gamma$  and  $\gamma'$  which have directions respectively *opposite* to those given in the diagram. These new forces  $\gamma$  and  $\gamma'$  then constitute a *counter-clockwise* whirl of magnetic forces  $\alpha, \alpha \dots$ , around the electric forces  $Y, Y', \dots$ , thus wholly reversing the directions of the parts  $\alpha, \alpha \dots$ ,  $\gamma$  and  $\gamma'$  of the diagram.

The same result may be reached somewhat differently: constant  $\delta Y/\delta t$  is equivalent to a fixed magnetic polarization in circuit  $a$ , the value of which may be either large or small but which is clockwise around  $Y$ . Retarded variation of  $\delta Y/\delta t$ , such for instance as occurs in simple harmonic motion, is thus equivalent to a waning of the clockwise magnetic polarization; or from another point of view, it is equivalent to an increase of counter-clockwise magnetic polarization around  $Y$ , hence to a series of counter-clockwise whirls of magnetic force around  $Y$ ,  $Y'$  . . . yielding  $\gamma$  and  $\gamma'$  reversed as resultants.

4. Turning now to the values of  $\gamma$  and  $\gamma'$  which by the present inferences are *opposite* in direction to those given in the diagram, and are increasing in value in this reversed direction at the retarded rate demanded by the simple harmonic equation, it will be expedient to carry forward the argument in the step for step fashion of the preceding paragraph. If  $\gamma$  were constant it would be unaccompanied by an electrical effect, if we except the case that  $X$ ,  $Y$ ,  $Z$ , may in such a case have a potential due to extraneous causes of no relevant interest. If  $\gamma$  is variable but  $\delta\gamma/\delta t$  constant, the result would be an electromotive force curled counter-clockwise around  $\gamma$ . There would be a curled electric polarization of the dielectric but no displacement current around the axis of magnetic force. Finally if  $\delta^2\gamma/\delta t^2$  is significant displacement currents must occur and be maintained.

But for the case of an inherent simple harmonic law originating in  $Y$  and transferred to  $\gamma$ , the time variation of  $\gamma$  is always retarded from an absolute point of view. Hence the counter-clockwise electric polarization around  $\gamma$ , reversed, is falling off; in other words, clockwise electric polarization around  $\gamma$  reversed is increasing. Virtually therefore a clockwise displacement current is running around  $\gamma$  reversed, i. e. a counter-clockwise current must encircle the  $\gamma$  as drawn in the diagram. Hence both the whirls  $c, c$  . . . and  $e, e$  . . . , are correctly drawn in the diagram for positive increasing  $Y$ ,  $Y'$  . . . and the new electric excitations ( $c, e$ ) tend both to weaken  $Y$ , and to evoke the new wave front  $Y_1$  and  $Y_2$  as shown.

These forces  $Y_1$  and  $Y_2$  are then to be treated each in its turn in the same way as  $Y$ , reproducing the same results indefinitely. Thus the following paradoxical rule relative to the construction of the mechanism of a simple plane electric wave would seem to follow: Provide the original electric flux ( $Y$ ) with a counter-clockwise magnetic curl ( $a$ ); provide the resulting magnetic fluxes ( $\gamma, \gamma'$ ) each with a clockwise electric curl ( $c$  and  $e$ ); and so forth indefinitely. For such a wave the electric and magnetic forces and the advance of the wave will be in correct directional relations (§ 1, end). The wave would

advance in one direction only if for  $Y$  we insert the corresponding gradient  $\delta Y/\delta x$  in the direction of advance.

Mathematically stated, if  $Y$  is simple harmonic and positive,  $\delta^2 Y/\delta t^2$  will be negative and  $\delta^4 Y/\delta t^4$  positive again. If therefore clockwise curls are associated with positive fluxes, the magnetic curl around  $Y$  conditioned by  $\delta^2 Y/\delta t^2$  must be counterclockwise, whereas the electric curl around  $\gamma$  or  $\gamma'$  conditioned by  $\delta^4 Y/\delta t^4$  must be clockwise.

5. With the wave disposed of, the question nevertheless remains as to what will happen if the original impulses  $Y$ ,  $Y'$  . . . are not simple harmonic but of a character to make  $\delta^2 Y/\delta t^2$  positive. If then  $\delta^4 Y/\delta t^4$  is also positive,  $\delta^2 \gamma/\delta t^2$  is positive and increasing and the above diagram is a correct representation of the case. Energy would be transferred from the original plane  $PP$  in both directions  $v$ ,  $v'$ , not however as a wave but as simple flux in the direction of the energy paths  $v$ ,  $v'$ , until the original stress  $Y$  breaks down or follows some other law. It seems to me that the case is not without subtlety and that this nonharmonic rush of electric energy in a direction opposite to the usual energy path cannot be excluded. Its duration would be short since  $Y$ ,  $Y'$  . . . are to remain finite.

Suppose now that such a rush and breakdown occurs irregularly: the irregularity would be accentuated since the energy paths in the rush and the breakdown would in a measure be in opposite directions. Energy so transferred could neither be easily reflected nor refracted and it certainly could not be brought to interference or polarization. In proportion as the rush and breakdown became more and more rhythmic or wave-train-like, reflection, etc., would eventually manifest themselves. One is tempted therefore to associate these occurrences with the recent advances in electro-optics, particularly since the deflecting effect of a magnet on the transient electric whirls or their resultants  $Y$ , and therefore on the energy path, is not out of the question. During the rush, in other words, the direction of electric current in the whirls or the fluxes  $Y$  remains unchanged for relatively long periods, whereas in an ordinary wave these directions are regularly and rapidly reversed in the lapse of time.

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