ART. X.—*Recent Gravity Work in the United States*;* by
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THIS paper gives a summary of the gravity work done by the Coast and Geodetic Survey during the past few years. This work comes under two general heads. First, the instruments and the field work and methods; and, second, the reduction and discussion of the observations.

In 1891 the Survey began the use of two sets of half-second pendulums.† They proved to be economical in their operation in the field and gave accurate results in the relative determination of the intensity of gravitation. The pendulum is essentially a flat stem, supporting a bob, with devices at the other end of the stem (called the pendulum head) to support the whole upon an agate knife edge fastened to the pendulum case. In the head is fastened rigidly an agate plane which rests upon the knife edge. As originally designed, the knife edge was in the head and the plane attached to the case. By having the plane in the head a more nearly invariable length of pendulum was secured, for any wear on the planes would have negligible effect on the period of oscillation.

Before beginning a swing of a pendulum the air in the case is practically exhausted (to about 60 millimeters of mercury). With a low air pressure a long period of swinging may be had with a small arc of oscillation. By means of a break-circuit chronometer, a flash apparatus and two mirrors, one on the case and the other fastened to the pendulum head, the observations are made by noting the time of a coincidence between the beat of the pendulum and that of the chronometer. If two such coincidences are observed, one can then obtain the period of the pendulum with reference to the chronometer time. The pendulum is so made that its period is slightly greater or less than a half second and a coincidence between the beat of the pendulum and that of the chronometer occurs every five or six minutes. The rate of the chronometer is obtained by observations with an astronomic transit on stars as they cross the meridian. Another piece of apparatus now in use is the interferometer. This is used instead of the static method formerly employed to get the flexure or movement of the pendulum case due to the horizontal force applied by the swinging pendulum. The case moves with the pendulum and tends to lengthen the period of oscillation.

The use of the interferometer to determine flexure was proposed by Mr. John F. Hayford while he was Inspector of

* Read before the Philosophical Society of Washington, May 6, 1911.

† See Appendix 15, C. and G. S. Report for 1891, and Appendix 1, Report for 1894.

Geodetic Work. The necessary investigations and experimental work were done by former Assistant William H. Burger, in the fall of 1907, assisted by Mr. Hayford and Mr. E. G. Fischer, the Chief of the Instrument Division. The instrument was made in 1908 at our office. The interferometer has been used successfully during the past five seasons in the field. A monograph on this subject, written by Mr. Burger, appears as Appendix 6 of the report of the Superintendent for the fiscal year 1909-10, which is now available for distribution.

Previous to the year 1909 the relative intensity of gravitation had been determined at 47 stations in the United States: 37 were established by former Assistant G. R. Putnam, 3 by Assistant Edwin Smith, 4 by former Superintendent Dr. T. C. Mendenhall, and 1 by the late Assistant E. D. Preston. One of the 47 is the station at the Coast and Geodetic Survey Office, and another at the Smithsonian Institution.*

Each of the stations is referred to the pendulum pier at the Coast and Geodetic Survey Office as the base. The value for this station is 980.111 dynes and was determined relatively in 1900, using Potsdam, Germany, as the base.† The value at Potsdam was determined by the absolute method, using the reversible pendulum. Additional gravity work was taken up in 1909. It was started under the direction of Mr. Hayford, as Inspector of Geodetic Work, with Mr. Burger in immediate charge of the party making the observations. When Mr. Hayford left the Survey in the fall of 1909, the work was continued under the speaker. Mr. Burger has established 35 stations, Assistant H. D. King 21 stations, and the field work is now being done by Aid T. L. Warner, who will make observations at from 15 to 20 stations.

The field work during the present campaign will end at the close of this calendar year, by which time the value of the intensity of gravitation will have been determined at about 75 stations during the three years, 1909-11. These, with the 47 old stations, make a total of about 120. In making the selections, uniformity of distribution has been the principal aim, but consideration has also been given geology, elevation, erosion, and deposition of material. With the exception of the introduction of the use of the interferometer in measuring the flexure, the methods of conducting the recent work are practically those used by former Assistant G. R. Putnam.‡

Three pendulums, constituting a set, were used at each station.

* The results are published in Appendix 15, Report for 1891; Appendix 1, Report for 1894; Appendix 6, Report for 1897; Appendix 4, Report for 1899.

† See p. 355, Appendix 5, Report for 1901.

‡ See Appendix 1, Report for 1894.

The General Instructions for Gravity Determinations state, in part:

(1) The determination of the intensity of gravitation shall commence with standard determination of time, and shall end with a second determination of time, made not less than 42 hours later than the first. Between these time determinations the pendulums shall be kept swinging as nearly continuously as is feasible.

(2) A standard determination of the intensity of gravitation shall consist of not less than six swings of which the probable error of the mean result is not less than $\pm .004$ dyne.

(3) Each pendulum shall be swung in the direct position only, and two successive swings are ordinarily to be made with each pendulum. The standard conditions for each swing shall be a pressure (reduced to zero degrees centigrade) of 60 millimeters, and total arc at beginning 5 millimeters. These standard conditions are to be approached as closely as is feasible without the expenditure of extra time for that purpose.

(4) The latitude of each gravity station must be determined within 1'.

(5) The elevation of each gravity station must be determined within 50 feet, unless to secure this degree of accuracy would require the expenditure of much time and money.

Such computations were made in the field as were necessary to ascertain whether the required degree of accuracy was obtained.

If it is assumed that there is no error in the value 980.111 dynes for the intensity of gravitation at the base station, then the average actual error in the values for the other stations is about .003 dyne, with a maximum error, in rare cases, of about .010 dyne. The probable error of the absolute determination of gravity at Potsdam is $\pm .003$ dyne. The probable error of the determination of the value for the Coast and Geodetic Survey station, relative to Potsdam, is about $\pm .002$. Hence, the probable error of the base station is about $\pm .004$ and the average probable error of the other stations in the United States is $\pm .005$ dyne. It is interesting to compare this accuracy with that of the stations determined at sea by Dr. Hecker. It is probable that the uncertainty in the value of the intensity of gravitation at an ocean station is, in some cases, as much as .10 dyne.

Hayford applied Pratt's hypothesis of isostasy in his investigations of the figure of the earth. It is a well-known fact that the plumb line is attracted by the known excesses of mass above the theoretical sea surface, and is repelled by the defect of mass in the oceans. Hayford showed by solution B, in his "Supplemental Investigation in 1909 of the Figure of the

Earth and Isostasy" that the apparent deflections are increased by considering only the attraction, positive and negative, of the topography alone. In his solution H the effect of the negative attraction of the defect of mass under the continents was applied in addition to the effect of the topography, as was also the positive attraction of the excess of mass under the oceans, and the deflections were greatly reduced.

The theory of isostasy postulates that each topographic feature is compensated for by a variation from the normal density below sea level. The crust of the earth under a land area has a deficiency of mass and that under a water area has an excess of mass. It is assumed that the mass is the same in each unit column extending from the actual surface of the earth, land or water, to a certain depth below sea level. This statement is based upon the theory that there is complete local compensation uniformly distributed with respect to depth. Of course, the actual condition deviates somewhat from this ideal. The compensation for a feature, a separate mountain mass for example, is not necessarily directly under it, nor is the compensation distributed absolutely uniformly and absolutely limited by a certain depth. The second investigation of the figure of the earth by Hayford gave 120.9 kilometers as the most probable limiting depth. He predicted that "future investigations will show that the maximum horizontal extent which a topographic feature may have and still escape compensation is between one square mile and one square degree. The introduction of the theory of isostasy to the investigations doubled the weight of the resulting value for the flattening of the earth, which was found to be $1/297.0 \pm 5$.

Having met with such success in the investigations of the figure of the earth, it was decided to apply the theory of isostasy in the reductions of the gravity stations in the United States. This was done for 56 stations by Hayford in 1909. A preliminary report upon the results of his investigation was read at the meeting of the International Geodetic Association at London and Cambridge in 1909.*

The intensity of gravitation increases with the distance from the equator, it decreases with the increase in elevation above sea level and is affected by the topography of the world and its compensation.

The gravity for any latitude is given by Helmert's formula of 1901:

* Published in *Comptes Rendus des Séances de la seizième Conférence générale de L'Association Géodésique Internationale réunie à Londres et à Cambridge du 21 au 29 Septembre 1909*. Rédigés par le Secrétaire perpétuel H. G. Van de Sande Bakhuyzen. 1^{er} Volume : Procès-verbaux et rapports des Délégués sur les travaux géodésiques accomplis dans leurs pays, pp. 354-390.

$$\gamma_0 = 978.046(1 + .005302 \sin^2 \phi - .000007 \sin^2 2\phi).$$

The value sought is γ_0 . The constant 798.046 is the theoretical value of the intensity of gravitation at sea level at the equator, unaffected by topography. The coefficient .000007 was adopted by Helmert from the theoretical investigations of Weichert and Darwin, while the coefficient .005302 resulted from a discussion by Helmert of about 400 selected gravity stations. These stations were selected from a total of about 1600 stations and were those which showed the smallest anomalies. In latitude 39° , approximately that of Washington, there is a change of .0013 dyne for 1 minute change in latitude.

The formula $C_u = -.0003086 H$, in which H is the elevation above sea level in meters, is used to get the correction for elevation of the stations. It follows that a change of .001 dyne is caused by changing the elevation about 3 meters.

The corrections for latitude and elevation, only, are used in the Free Air reduction, which ignores the attraction of the topography altogether. In this reduction it is implied that there is complete isostatic compensation at the depth zero.

The Bouguer reduction applies corrections for latitude, elevation, and for the topography without compensation. The new, or Hayford, method applies these three corrections and also a correction for the isostatic compensation.

It is not necessary to discuss in detail the formulæ used to obtain the attraction of the topography and compensation. They are based upon the fundamental principle that the attraction of any elementary mass dm for a mass of one gram at the station of observation is in dynes $\frac{kdm}{D^2}$, where k is the gravitation constant, in the C. G. S. system, namely, $6673 (10^{-11})$.* All other quantities must, therefore, be expressed in that system. D is the distance from the station to the elementary mass. The vertical component at the station of this attraction is

$$kdm \frac{\sin \frac{\theta}{2}}{4r^2 \sin^2 \frac{\theta}{2}},$$

in which r is the radius of the earth, considered as a sphere (6370 kilometers), and θ is the angle at the center of this earth subtended between the station and the elementary mass (the station and the elementary mass being considered at sea level).

* See page 153 of "The Century's Progress in Applied Mathematics" by R. S. Woodward, in Bulletin of the American Mathematical Society, vol. vi, No. 4, pp. 133-163.

The attraction at the station of an elementary mass which is higher than the station is given by the expression

$$kdm \frac{\sin \frac{\theta}{2} - \sin^{-1} \frac{h \cos \frac{\theta}{2}}{\sqrt{D^2 + h^2 + 2Dh \sin \frac{\theta}{2}}}}{D^2 + h^2 + 2Dh \sin \frac{\theta}{2}}$$

In this h is the difference in elevation between the elementary mass and the station.

The attraction at the station of an elementary mass which is lower than the station is given by the expression

$$kdm \frac{\sin \frac{\theta}{2} + \sin^{-1} \frac{h \cos \frac{\theta}{2}}{\sqrt{D^2 + h^2 - 2Dh \sin \frac{\theta}{2}}}}{D^2 + h^2 - 2Dh \sin \frac{\theta}{2}}$$

For masses near the station the following formula was used, which gives the attraction of a mass having the form of a right cylinder, upon a point outside the cylinder and lying in its axis produced :

$$\text{Attraction in dynes} = k2\pi\delta \{ \sqrt{c^2 + h^2} - \sqrt{c^2 + (h+t)^2} + t \}.$$

δ is the density of the material, c is the radius of the cylinder, t is an element of the cylinder and h is the distance from the attracted point, the station, to the nearest end of the cylinder.

For a mass which has the form of a cylindrical shell, which is the difference in the effect of two solid cylinders of the same length, having different radii c_1 and c_2 , this expression was used :

Attraction in dynes =

$$k2\pi\delta \{ \sqrt{c_2^2 + h^2} - \sqrt{c_1^2 + h^2} - \sqrt{c_2^2 + (h+t)^2} + \sqrt{c_1^2 + (h+t)^2} \}.$$

In order to apply the formulæ to the computation of the effect of topography and the compensation, the whole surface of the earth was divided into zones, each having the station as the center. Each zone was divided into equal compartments by radial lines from the station. The division adopted is shown in the following table :

Designation of zone	Inner radius of zone. Meters	Outer radius of zone. Meters	Number of compartments
A	0	2	1
B	2	68	4
C	68	230	4
D	230	590	6
E	590	1280	8
F	1280	2290	10
G	2290	3520	12
H	3520	5240	16
I	5240	8440	20
J	8440	12400	16
K	12400	18800	20
L	18800	28800	24
M	28800	58800	14
N	58800	99000	16
O	99000	166700	28

	Degrees, minutes and seconds	Degrees, minutes and seconds	
18	1° 29' 58"	1° 41' 13"	1
17	1 41 13	1 54 52	1
16	1 54 52	2 11 53	1
15	2 11 53	2 33 46	1
14	2 33 46	3 03 05	1
13	3 03 05	4 19 13	16
12	4 19 13	5 46 34	10
11	5 46 34	7 51 30	8
10	7 51 30	10 44 00	6
9	10 44 00	14 09 00	4
8	14 09 00	20 41 00	4
7	20 41 00	26 41 00	2
6	26 41 00	35 58 00	18
5	35 58 00	51 04 00	16
4	51 04 00	72 13 00	12
3	72 13 00	105 48 00	10
2	105 48 00	150 56 00	6
1	150 56 00	180 00 00	1

It was found most convenient, in making the computations, to have the radii of some zones in meters and others in degrees and minutes of a great circle. All of the zones combined together cover the earth completely. For each zone a special reduction table was prepared, giving the relation between the mean elevation of each compartment of that zone, and the effect of the topography and compensation in that compartment upon the intensity of gravitation at the station. The mechanical part of the computation was performed by the use

of transparent celluloid templates on which the boundary lines of the various zones were drawn. These templates, made to proper scale, were superimposed on the contour maps.

The following are condensed tables for zones E and K, which are hollow cylinders. The corrections are given in the fourth decimal place in dynes.

Zone E. Inner radius, 590 meters. Outer radius, 1280 meters. Eight compartments.

		Correction for elevation of station.—									
Mean elevation.	Correc- tion	Station above compartment.					Station below compartment.				
		200 feet	600 feet	1000 feet	1800 feet	3000 feet	200 feet	600 feet	1000 feet	1800 feet	3000 feet
—500	—17	—1									
—400	—13	—1	—3								
—300	—9	—1	—3	—4							
—200	—4	—1	—2	—4	—4						
—100	—1	—1	—2	—2	—3						
—50	—1	0	—1	—1	—2	—1					
Fathoms											
0	0	0	0	0	0	0					
400	+1	+1	+2	+3	+4	+3	—1				
800	+3	+1	+4	+6	+6	+5	—1	—5			
1200	+7	+2	+4	+7	+7	+4	—2	—7	—12		
2000	+16	+2	+5	+7	+6	+1	—2	—9	—16	—30	
3000	+28	+2	+4	+5	+2	—4	—2	—9	—17	—35	—56
4000	+37	+1	+3	+3	—1	—9	—1	—7	—14	—32	—60
5000	+45	+1	+2	+1	—4	—13	—1	—6	—13	—29	—57
6000	+51	+1	+1	0	—6	—16	—1	—5	—11	—26	—51
8000	+60	0	—1	—2	—9	—20	0	—4	—9	—22	—43
10000	+66	0	—1	—3	—10	—22	0	—4	—8	—19	—38
15000	+73	0	—1	—4	—12	—24	0	—3	—6	—16	—32

Zone K. Inner radius, 12400 meters. Outer radius, 18800 meters. 20 compartments.

		Correction for elevation of station.—							
Mean elevation.	Correc- tion	Station above compartment.				Station below compartment.			
		600 feet	1800 feet	4200 feet	7200 feet	600 feet	1800 feet	4200 feet	7200 feet
—1200	+1	—1	—1						
—800	+1	0	—1	—1					
—400	+1	0	—1	—1	—1				
Fathoms									
0	0	0	0	0	0				
1000	—1	0	0	+1	+1	0			
2000	—1	0	+1	+1	+2	0	—1		
6000	—2	+1	+2	+3	+6	0	—2	—4	
10000	—1	+1	+2	+6	+9	0	—2	—6	—11
13000	+1	+1	+3	+7	+11	—1	—3	—8	—13
15000	+3	+1	+3	+7	+12	—1	—4	—9	—14

The first column of the table for zone E gives the mean elevation of the compartment in fathoms or feet, the second the correction for topography and its compensation, assuming the station to be at the same elevation as the compartment, the third to seventh columns give a correction to the values in column 2 due to the elevation of the station above the compartment, and the last five columns the correction to the quantities in the second column should the station be below the mean elevation of the compartment. The tables for the other lettered zones are practically the same in form as those for zones E and K, while the tables for the numbered zones are materially different owing to the absence of any correction for elevation of the station above or below the compartments, and due also to the fact that they are computed by a different formula.

The following table gives the values for the separate zones for six typical stations :

Correction for Topography and Isostatic Compensation—Separate Zones.

Zone	Kansas City. No. 39	Pike's Peak. No. 43	Colorado Springs. No. 42	Grand Junction. No. 46	Beau- fort. No. 18	San Francisco. No. 54
A	+ 2	+ 2	+ 2	+ 2	+ 1	+ 2
B	+64	+ 72	+ 68	+ 68	0	+48
C	+97	+157	+160	+160	0	+35
D	+72	+310	+306	+282	0	+15
E	+30	+540	+408	+336	0	+ 1
F	+16	+602	+331	+239	0	0
G	0	+512	+192	+120	0	0
H	0	+443	+127	+ 66	0	0
I	0	+412	+ 78	+ 33	0	0
J	-16	+201	- 11	- 7	0	- 2
K	-20	+ 79	- 71	- 65	0	- 1
L	-24	- 32	-147	-132	0	- 7
M	-53	-290	-369	-391	0	-14
N	-47	-334	-342	-374	+ 4	+15
O	-55	-329	-339	-349	+45	+99
18	-10	- 68	- 68	- 74	+14	+24
17	-10	- 68	- 69	- 73	+16	+21
16	-10	- 68	- 69	- 72	+20	+20
15	-11	- 64	- 68	- 66	+27	+19
14	-11	- 59	- 62	- 65	+34	+17
13	-20	- 83	- 81	-101	+52	+25
12	-13	- 48	- 48	- 55	+36	+23
11	-14	- 30	- 30	- 34	+29	+21
10	-14	- 17	- 17	- 15	+19	+14
9	- 8	0	0	+ 3	+14	+10
8	- 1	+ 9	+ 9	+ 11	+14	+15

Correction for Topography and Isostatic Compensation—Separate Zones.
(Continued.)

Zone	Kansas City. No. 39	Pike's Peak. No. 43	Colorado Springs. No. 42	Grand Junction. No. 46	Beau- fort. No. 18	San Francisco. No. 54
7	+ 8	+ 7	+ 7	+ 7	+ 5	+10
6	+ 9	+ 9	+ 9	+ 9	+ 6	+ 9
5	+11	+ 9	+ 9	+ 9	+ 8	+ 9
4	+ 7	+ 8	+ 8	+ 8	+ 7	+ 8
3	+ 5	+ 5	+ 5	+ 5	+ 6	+ 5
2	+ 3	+ 3	+ 3	+ 3	+ 3	+ 4
1	+ 1	+ 1	+ 1	+ 1	+ 1	+ 1
Total,	-12	+1891	- 68	-511	+361	+446

The six stations in the above table were selected as representing typical classes. Kansas City is an inland station, far from the coast of the continent and not in a mountainous region. Pike's Peak is a station on a high mountain summit in the interior of the country. Colorado Springs is a station near the foot of the same mountain and at the edge of a great plain. Grand Junction is a station in the interior of the continent in a deep valley among high mountains. Beaufort and San Francisco are coast stations near deep water.

It is a most interesting fact and one brought out for the first time by Hayford, that there is a change of sign in the combined effect of topography and compensation near zone J, outer radius about 12 kilometers. This change is due to the fact that the effect of the compensation for more distant zones is greater than that of the topography. It will be seen that the effect of the distant topography and its compensation, beyond zone J for example, is in some cases entirely too large to be neglected. At Pike's Peak, it is -136 dyne and at San Francisco it is $+034$ dyne.

The results of the computations for certain stations are shown in the following two tables. In the first ϕ is the latitude, λ is the longitude, and γ_0 is the theoretical value for the latitude computed by Helmert's formula. H is the elevation of the station above sea level in meters. The other headings are self-explanatory. It will be noticed that the effect of topography and compensation at some stations is very small, zero in two cases, and large at others. For Pike's Peak it is nearly 200 dyne.

Number of station	ϕ	λ	H meters	γ_0	Correction for elevation	Correction for topography and compensation
25	40 21.0	74 39.5	64	980.212	— .020	+ .013
26	40 44	74 02	11	980.248	— .003	+ .008
27	40 48.5	73 57.7	38	980.254	— .012	+ .011
28	42 16.5	71 48.5	170	980.386	— .052	+ .018
29	42 21.6	71 03.8	22	980.393	— .007	+ .013
30	42 22.8	71 07.8	14	980.395	— .004	+ .010
31	45 11.2	67 16.9	38	980.649	— .012	+ .010
32	42 27.1	76 29.0	247	980.402	— .076	+ .005
33	41 30.4	81 36.6	210	980.317	— .065	.000
34	39 08.3	84 25.3	245	980.105	— .076	+ .002
35	39 28.7	87 23.8	151	980.135	— .047	+ .001
36	41 47.4	87 36.1	182	980.342	— .056	+ .007
37	43 04.6	89 24.0	270	980.458	— .083	+ .003
38	38 38.0	90 12.2	154	980.061	— .048	+ .001
39	39 05.8	94 35.4	278	980.101	— .086	— .001
40	38 43.7	98 13.5	469	980.069	— .145	— .004
41	38 54.7	101 35.4	1005	980.085	— .310	.000
42	38 50.7	104 49.0	1841	980.080	— .568	— .007
43	38 50.3	105 02.0	4293	980.079	— 1.325	+ .189
44	39 40.6	104 56.9	1638	980.153	— .505	— .015
45	38 32.6	106 56.0	2340	980.053	— .722	— .001
46	39 04.2	108 33.9	1398	980.099	— .431	— .051
47	38 59.4	110 09.9	1243	980.092	— .384	— .643
48	39 50.8	111 00.8	2191	980.168	— .676	+ .024
49	40 46.1	111 53.8	1322	980.250	— .408	— .041
50	44 43.3	110 29.7	2386	980.607	— .736	+ .038
51	44 44.2	110 42.0	2276	980.608	— .702	+ .031
52	44 33.4	110 48.1	2200	980.592	— .679	+ .028
53	47 39.6	122 18.3	58	980.872	— .018	— .020
54	37 47.5	122 25.7	114	979.986	— .035	+ .045
55	37 20.4	121 38.6	1282	979.947	— .396	+ .120
56	47 36.5	122 19.8	74	980.867	— .023	— .018

The second table gives the number and name of the station, the computed gravity at the station, the observed gravity and the anomaly.

Number and name of station		Computed gravity at station g_c	Observed gravity at station g	$g-g_c$
25	Princeton, New Jersey	980.205	980.177	— .028
26	Hoboken, New Jersey	980.253	980.268	+ .015
27	New York, New York	980.253	980.266	+ .013
28	Worcester, Massachusetts	980.352	980.323	— .029
29	Boston, Massachusetts	980.399	980.395	— .004

Number and name of station	Computed gravity at station	Observed gravity at station	$g-g_c$
	g_c	g	
30 Cambridge, Massachusetts.....	980·401	980·397	—·004
31 Calais, Maine.....	980·647	980·630	—·017
32 Ithaca, New York.....	980·331	980·299	—·032
33 Cleveland, Ohio.....	980·252	980·240	—·012
34 Cincinnati, Ohio.....	980·031	980·003	—·028
35 Terre Haute, Indiana.....	980·089	980·071	—·018
36 Chicago, Illinois.....	980·293	980·277	—·016
37 Madison, Wisconsin.....	980·378	980·364	—·014
38 St. Louis, Missouri.....	980·014	980·000	—·014
39 Kansas City, Missouri.....	980·014	979·989	—·02
40 Ellsworth, Kansas.....	979·920	979·925	+·005
41 Wallace, Kansas.....	979·775	979·754	—·021
42 Colorado Springs, Colorado....	979·505	979·489	—·016
43 Pike's Peak, Colorado.....	978·943	978·953	+·010
44 Denver, Colorado.....	979·633	979·608	—·025
45 Gunnison, Colorado.....	979·330	979·341	+·011
46 Grand Junction, Colorado....	979·617	979·632	+·015
47 Green River, Utah.....	979·665	979·635	—·030
48 Pleasant Valley Junction, Utah	979·516	979·511	—·005
49 Salt Lake City, Utah.....	979·801	979·802	+·001
50 Grand Canyon, Wyoming....	979·909	979·898	—·011
51 Norris Geyser Basin, Wyoming	979·937	979·949	+·012
52 Lower Geyser Basin, Wyoming	979·941	979·931	—·010
53 Seattle, Washington (Univ't'y)	980·834	980·732	—·102
54 San Francisco, California....	979·996	979·964	—·032
55 Mount Hamilton, California...	979·671	979·659	—·012
56 Seattle, Wash'gton, High School	980·826	980·724	—·102

As the mean with regard to sign of 54 of the 56 stations (the two Seattle stations were rejected on account of their very large anomalies) was —·008 dyne, a correction of this amount was applied to the constant (978·046) of Helmert's formula. For all of the 56 stations considered as a single group the means for the three reductions are as follows:

	Anomaly		
	(New Method)	(Bouguer)	(In free air)
Mean with regard to sign...	—·004	—·056	·000
Mean without regard to sign	·018	·066	·030
With the two Seattle stations omitted the means are:			
Mean with regard to sign...	·000	—·053	+·005
Mean without regard to sign	·015	·064	·027

It is seen that the anomalies by the new method are decidedly smaller than by the two older methods of reduction.

An analysis of the anomalies showed that there is only slight, if any, connection between the size and sign of the

anomaly and the topography. The nearly complete disappearance of the association between anomalies and topography is a proof that the condition called isostasy exists, and that the assumption that the compensation is complete and uniformly distributed with respect to depth from the surface to a depth of 113.7 kilometers is a close approximation to the truth. This value for the limiting depth of compensation was used instead of 122 kilometers (the latest value) as the computations were begun before the latter value was available.

The facts and conclusions stated above are for the 56 gravity stations in the United States used in the reduction made by Mr. Hayford in 1909. He and the speaker have nearly completed an enlargement of the first investigation with the addition of 33 stations, making 89 in all. It is expected that the results of this investigation will be published by the Coast and Geodetic Survey during this year. This investigation will verify the conclusions arrived at from the first one. Certain new features have been added. One is a study to show whether the theory of local distribution of the compensation is nearer the truth than what may be considered regional distribution. Another is an analysis of the anomalies to show whether there is any connection between the size and sign of the residual and the geologic formation. Though the number of stations is too small to justify the statement that there is a positive connection, yet the evidence is strong that there is a relation. This is especially the case with regard to the oldest and most recent formations.

It has been found that the results from the gravity investigations are in almost perfect harmony with those of Hayford's investigations of the figure of the earth and isostasy. Those who have read his second publication may remember the illustration on which are shown by ovals certain areas of excessive and defective mass, as indicated by the anomalies of the deflections. Gravity stations have been established in many of these areas and in each case the gravity anomaly by the Hayford method is of the same sign as that indicated by the deflections.

To sum up—the work done by the Coast and Geodetic Survey proves conclusively that a close approximation to the condition of complete isostasy exists. The introduction of the theory of isostasy into the reduction of gravity observations has made the anomalies very small. Further investigations of deflections of the vertical and of gravity observations may show how the compensation is distributed and how nearly it is complete. With increased knowledge of the distribution of compensation the figure and size of the earth can be more accurately determined.

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