243. Notes on the Nine-Point Circle

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Source: *The Mathematical Gazette*, Vol. 4, No. 66 (Oct., 1907), pp. 122-128

Published by: [Mathematical Association](#)

Stable URL: <http://www.jstor.org/stable/3604289>

Accessed: 29-02-2016 10:57 UTC

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in the same sense, about the Intersection, into the P. Pl. Join EL and draw QR parallel to LE . (Lower diagram.)

FIG. 6.

In Slide VIII. (not reproduced here) Theorem (4) is illustrated by a set (or *pencil*) of lines in the Ground Plane all passing through the same point L in the Foot line and the set (or *pencil*) corresponding to them on the Picture Plane all parallel to EL .
(To be continued.)

MATHEMATICAL NOTES.

243. [K. 2. c.] *Notes on the Nine-Point Circle.*

1. D, E, F being the mid points of BC, CA, AB , to determine a point P on the nine-point circle such that

$$PE = PD + PF \quad (a > b > c).$$

Bisect the arcs DD', EE', FF' in α, β, γ .

Let X, Y, Z be the points of contact of the in-circle, and let the sides of XYZ cut the corresponding sides of DEF in $U_1 U_2 U_3$, so that

$$De' = DX = \frac{1}{2}(b - c),$$

$$Fe = FZ = \frac{1}{2}(a - b),$$

$$Ee = Ee' = \frac{1}{2}b.$$

and

It is noteworthy that $PP_1P_2P_3$ can thus be found independently of ABC or XYZ .

8. As YP passes through U_2 , bisecting DPF ,
 so XP " " U_1 , " EPF (externally),
 ZP " " U_3 , " DPE (externally).

It is easily seen that for U_1 the trilinear coordinates are given by

$$a\alpha : b\beta : c\gamma = -(b-c) : c-a : a-b,$$

$$\text{for } U_2 \text{ the ratios are } b-c : -(c-a) : a-b,$$

$$\text{" } U_3 \text{ " " } b-c : c-a : -(a-b).$$

Whence it follows that

$$AU_2U_3, BU_3U_1, CU_1U_2 \text{ are straight lines,}$$

and that AU_1, BU_2, CU_3 are parallel.

The isogonals to these parallels meet at the point S on the circum-circle, the coordinates of S being given by $a : \beta : \gamma = \frac{a}{b-c} : \frac{b}{c-a} : \frac{c}{a-b}$.

S is the focus of a parabola touching the sides of ABC , and having its axis parallel to AU_1, BU_2, CU_3 .

If AU_2U_3 cut BC in T_1 , and CU_2U_1 cut AB in T_3 , then

$$BT_1 : CT_1 = a-b : a-c,$$

and

$$AT_3 : BT_3 = a-c : b-c.$$

Hence the equation to T_1T_3 is

$$\frac{a\alpha}{b-c} + \frac{b\beta}{c-a} + \frac{c\gamma}{a-b} = 0,$$

to that T_1T_3 is the tangent at P .

Thus the common tangent $T_1T_2T_3$ can be drawn and its point of contact P determined independently of the nine-point and in-circle.

This tangent divides each of the parallels AU_1, BU_2, CU_3 in the ratio 2 : 1.

9. $XYZP, CAT_3T_1$ being associated in- and circum-quadrilaterals for the in-circle, the pairs of diagonals YP, ZX and AT_1, CT_3 have the common