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36. On Involution Ranges

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36. On Involution Ranges.

(i.) The following is a simple geometrical proof of the theorem that pairs of points in a line, called conjugates of each other, AA', BB', CC', DD' , etc., are in involution* if $OA \cdot OA' = OB \cdot OB' = \text{etc.} = \text{constant}$, O being some fixed point in the line, called the centre of the involution.

Set off OA', OB' on any line making an angle with OA .

Then, since $\frac{OA}{OB'} = \frac{OB}{OA'}$, AB' is parallel to $A'B$.

$$\therefore \frac{AB}{OA} = -\frac{A'B'}{OB'}, \quad \frac{CD}{OC} = -\frac{C'D'}{OD'}, \quad \frac{OA}{AD} = -\frac{OD'}{A'D'}, \quad \text{and} \quad \frac{OC}{CB} = -\frac{OB'}{C'B'}.$$

Compounding, we have $\frac{AB \cdot CD}{AD \cdot CB} = \frac{A'B' \cdot C'D'}{A'D' \cdot C'B'}$
 $\therefore (ABCD) = (A'B'C'D')$,

using the ordinary notation for cross ratios. Thus the cross ratio of any four points is equal to that of their conjugates.

(ii.) The converse is also easily proved.† Let the cross ratio of any four points whatever of the pairs AA', BB' , etc., be equal to that of their conjugates. Take any point V not in AB , and draw the circles $AA'V, BB'V$ to meet in Q ; draw QV to O , and determine C_1 so that

$$OC \cdot OC_1 = OV \cdot OQ = OA \cdot OA' = OB \cdot OB';$$

then by (i.),

$$(AA'BC) = (A'AB'C_1);$$

but by hypothesis

$$(AA'BC) = (A'AB'C'),$$

since the cross ratio of any four is equal to that of their conjugates; hence

$$(A'AB'C_1) = (A'AB'C'),$$

and therefore C_1 coincides with C' ;

$$\therefore OC \cdot OC' = OA \cdot OA' = OD \cdot OD', \text{ etc.} \quad \text{E. BUDDEN.}$$

37. General proof of the pole and polar property of the circle, or any curve of the second order having the cross ratio property of the circle. Such curves are called below cross-ratio curves.

For proof of Pascal's theorem assumed below, see Richardson and Ramsay's *Mod. Pl. Geometry*, viii. 17; Mulcahy's *Mod. Geometry*, § 27; or Taylor's *Euclid* (Pitt Press), vi. Ad. Pr. 21.

*Pairs of points $AA', BB', \dots$ on the same straight line are said to be in involution when the cross ratio of any four points whatever is equal to that of their conjugates.

†This form of proof is adapted from Mulcahy's *Modern Geometry*, § 35.