ORGANIZATION OF CONGRESS IN SECONDARY VOCATIONAL MATHEMATICS.

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During recent years there has been a demand that public education serve all the people in accordance with their several needs. Vocational schools are being established or courses arranged in the already existing high schools to meet this demand. In such schools and courses the English, history, mathematics, and the sciences are taught with the view of furnishing the students the things they will need in their chosen vocations. Vocational mathematics, then, is that mathematics which will be of most help to the student in his every-day life in the store, factory, on the farm, as a carpenter or in whatever line of work he may take up. It is my purpose in this paper to organize the material for courses in secondary vocational mathematics that will meet the demands of boys and girls who for various reasons are not looking forward to a college education. The discussion of this subject naturally falls into three divisions.

I. The general conditions which make special courses in secondary mathematics desirable.

II. The kind and extent of mathematics for mechanic arts students.

III. The kind and extent of mathematics for agricultural students.

I. GENERAL CONDITIONS WHICH MAKE SPECIAL COURSES
DESIRABLE.

Until recent years only two objects were kept in view in determining what should make up the course in secondary mathematics preparation for college, and the general value of the subjects themselves. Later came in the consideration of preparing for business, or commercial life, which led to the introduction of bookkeeping and commercial arithmetic. Today the course of study must take into account the objects for which the boys and girls are being educated, their capabilities, and the opportunities open to young men and women demanding general and special educational qualifications. How to meet these requirements by giving the students just the kind of mathematics they need is a question of vital importance today.

The American people have always stood for equality of opportunity in every phase of life; and, as a result, a single type of

school has developed without reference to local conditions or personal preference. Since mathematics is an exact science it has been one of the last to yield to the popular demand for a change. Time and experience have, however, brought clearly to view the fact that equality of educational opportunity can be provided only as different needs of the community and of particular groups of children are taken into consideration. The great number of boys and girls do not enter professional work but go either from the high school or from the grades to the farm, store, or shop. In view of this fact their mathematics should be of such a nature as to prepare them best for the work they are likely to do.

There is a demand today for finishing schools. A great many colleges and cities have already established schools with courses from two to four years in length in which the students may choose such work as appeals to them and will do them the most good after they get out. These schools are demanded by two different classes of students: (1.) The boys and girls of the towns and cities whose parents can not afford to send them through high school and then to college to get training in some industrial line. To this class also belong those who for various reasons have left school on finishing the eighth grade and have gone into shops, or stores, or have taken up some trade. After a year or two they see the need of more training but can not spend the time necessary to complete a college course but will gladly take two or three years provided they can get a course that furnishes them proficiency in their chosen vocation. (2.) In the rural communities in almost every state in the Union there are a great many boys and girls who finish the country schools and would like to go to high school but the literary and classical education does not appeal to them in such a way as to lead them to believe that they will have greater earning capacity after spending two to four years there. They were raised on the farm and want to stay there; hence the township or county high school should offer courses making for intelligence and practical insight and give opportunity for them to gain such control of materials, tools, and scientific facts that their earning capacity is increased and they are better trained for citizenship. These schools should provide agricultural training for the boys and courses in home economics for the girls.

In such schools a new style of mathematics is demanded. The principles of mathematics are the same today as fifty years ago but that does not necessarily mean that the subject should be

treated the same today as then. Students may be taught to add, subtract, multiply, and divide by using numbers in arithmetic or letters in algebra without reference to what they represent. To treat mathematics in this way would be to regard it as a science and would lead largely to a series of mental gymnastics, and the student would get very little insight into the subject. Mathematics should be treated as a science, to be sure, but as a science applied to practical living. From this point of view mathematics is changing from year to year. The graph in algebra was almost unheard of until recent years. Scientific investigators have demanded a compact, accurate way to submit their data to those interested, hence came the use of the graph. It was found to be such a convenient way of presenting results of investigations that today it is used in almost every report sent out to the public. Let an important change occur in the mode of transacting business and the arithmetics will be changed accordingly. The old texts emphasized troy weight, apothecaries' weight, circulating decimals, alligation—medial and alternate—arithmetical and geometrical progression, all of which are antiquated or of questionable value to the ordinary citizen. We are beginning to find in the new text-books such subjects as saving and loaning money, investments, banking, commission, and problems covering the practical phases of poultry husbandry, dairying, carpentry, farm mechanics, and every other subject that touches any considerable number of people in any community. The arithmetics used in a strictly agricultural district or by agricultural students in any secondary school should contain different material from texts used by students in the heart of a great city, or in a purely mining district, or a community where everybody is interested in a particular industry.

Again, for students in secondary schools not all parts of the algebra, geometry, and trigonometry have applications. Students should not be required to take such parts, but topics should be selected which are fundamental in meeting the requirements of the technical studies or will be of practical value. If the selection is wisely made and the material properly arranged, logical training will not suffer. If we teach so that pupils really understand what they are doing then we are training their logical powers and giving them the help in their ethical and philosophical ways of thinking which are so well developed by a study of Euclid. They will also have a ready, usable knowledge of the fundamental problems and processes of the subjects they have taken.

Mathematics has been too abstract and artificial. Aiming at training the pure reason it has got out of touch with facts and for many pupils has degenerated into mere jugglery of symbols without any thinking. It is no wonder then that mathematics fails to interest all but a very few. The theorem of limits usually stated, "If two variables are constantly equal as they approach a limit their limits are equal," which is of great interest to the grown mathematician is for the very best boys very difficult to grasp and if the complete mastery were necessary before taking up subjects that depend upon it most students would be barred from the work entirely. During nine years of experience in teaching geometry in the Kansas State Agricultural College and other good schools to students, whose previous preparation is not questioned, I have not found a single student who could really grasp the meaning of limits. Occasionally one will get a pretty good notion of the meaning by the time he has completed plane and solid geometry. The teachers do not understand it as they should; however, it is not the fault of the teachers nor the students, the trouble lies in the fact that we are trying to do something that can not be done.

Mathematics which heaps up theorem after theorem without a thought of how its results are to be turned to practical account in the acquisition of knowledge in other fields can not possibly lead to any benefits to the student. The value of mathematical knowledge to students in these finishing schools lies in its applications. There is no reason why mathematicians should not devote themselves exclusively to pure mathematics, if they are so inclined, for without question a great many of the valuable discoveries in other sciences have been made with the help of fundamental formulae worked out by pure mathematicians. The trained mathematician has capital but it is useless so long as it is not turned to practical account in other sciences. We must learn to lay more stress upon the ability to use a theorem intelligently than to give a faultless demonstration. Farmers, mechanics, or engineers, do not care about non-Euclidean geometry nor for computations of plane triangles where the sum of the angles is less than 180° , nor any such problems. The engineer especially asks for the ability to use numbers rapidly and accurately in his particular field.

Common sense demands that the mathematics of real life be coördinated with that taught in the schools. In order to accomplish this the teacher with the pupils should visit the

various industries of the vicinity. They will be brought face to face with real problems, not mere statements of problems. Visit stores, elevators, factories, cement workers where walks and foundations are being made. Visit a house or barn under construction and figure out with them the cost, using local prices, of the building from the excavation to the completion. In a large school the mathematics department should work with the shops, drafting room, and the domestic science and domestic art departments. Teachers of the technical subjects should also be in constant touch with the mathematics department anticipating problems which will arise and reporting immediately any weakness shown by a pupil in problems and principle.

We must come to a saner and more practically useful attitude in all our mathematics or we shall be pushed to the wall. Already there is a demand that mathematics be not required in the high schools and this will inevitably come unless the subject is made to appeal more strongly to the ordinary high-school student. I believe if we are all absolutely frank we will admit that geometry as now taught in most schools is of all the subjects the one which we feel of least real use in advanced mathematics; and as now taught, geometry is felt by students and by many teachers to be of least benefit to the student who does not pursue advanced studies. A great many mathematicians and text-book writers have been putting great stress on rigor in elementary geometry. Surely we should not be held to the old Euclidean style of geometry on that account. We should give up the meaningless pretense of teaching rigorous geometry to boys and girls of fourteen and fifteen years of age. That there is a demand for a change in our secondary mathematics is evidenced by its growing unpopularity and from the fact that it is not now considered so essential in every course offered. There is a demand then as indicated above for a new style of mathematics for students who do not go to college.

II. MATHEMATICS FOR MECHANIC ARTS STUDENTS.

What then should be the nature and extent of the mathematics for one of these finishing schools? The answer to this question will depend upon the work for which the student is preparing. The courses should be made to suit the needs of the two classes of students given above. We will first take up the course for the students of the first division.

The student who is preparing for real leadership in the fac-

tory, machine shop, carpentry, contracting, or similar lines of work, if he is to become a leader in his work, should have a rather extensive course in mathematics covering the subjects of algebra, geometry and trigonometry.

ALGEBRA: The elementary algebra making up the first year's work should have for its immediate aim the mastery of the linear and quadratic equation. The equation can scarcely be over-emphasized in the early study of algebra. Real-life problems can thus be introduced almost at the beginning of the study of the subject and long periods of drudgery on the abstract mathematics avoided. The students ought to be made to feel that they are studying a real, live, practical subject and the abstract parts introduced for the sake of proficiency will simply give variety enough to make the study interesting. Where the interpretations of results of a practical nature are very important care should be taken not to bring into the work long and complicated applications of principles. The student will not need to be able to work out long and complex expressions but needs a thorough working knowledge of the simple principles of algebra. A thorough course up to and including the quadratic equation and covering only simple radicals will be sufficient for any demands made upon the subject both in a practical way and as a foundation for work in geometry and trigonometry. The parts of the subject that should be emphasized are: The four fundamental operations, simple parentheses, special rules in multiplication and division, factoring of a simple type leading directly to the solution of the quadratic, simple fractions, ratio and proportion, simple simultaneous equations of two unknowns, graphs, and simple radicals. The equation, as was indicated above, should be the great point of emphasis throughout the course. The work should be more intensive and not so extensive.

The theoretical considerations of the laws of signs, curious combinations of brackets, difficult factoring, complicated fractions, simultaneous quadratics, cube root, complex radicals, and imaginaries should be omitted. The time usually consumed in these subjects should be used in making the students more proficient in what they do take, and in attention to problems they are meeting in their other subjects and will meet in their everyday life. Reasonable skill in straightforward manipulation is certainly expected and demanded but time can be spent to better advantage in making the student familiar with the few simple

devices at his disposal than by dealing with long tedious examples in simplifications of fractions, or with "elegant devices" for solving problems especially constructed for the purpose.

GEOMETRY: In geometry a good many of the recent textbooks writers have done a great deal toward vitalizing the work by arranging the material so as to set the student to work right at the start with ruler and compass. This plan gives the student something tangible to work with and if properly handled can be made of service to the teacher in presenting the problems of superposition in triangles. This construction work should be accompanied by a great number of numerical exercises in which he will make use of his arithmetic and algebra. Excellent opportunities will also be offered here for the use of graphs in the solution of triangles. After several days of this kind of work they should be introduced to demonstrative geometry. In the demonstrative work more emphasis should be placed upon getting the students to think than upon requiring them to get the exact form of the demonstration given in the book. The construction work and the numerical calculations are not to be dropped as soon as the theorems are taken up but the three will go right along together.

Demonstrative geometry develops mathematical thinking better perhaps than any other phase of the work, yet to emphasize that alone makes of it merely a study of logic, for certainly no very great number of the theorems have direct application to the problems of life. Geometry has its practical side which must be emphasized if it is to hold its place in the curriculum. A good many theorems are intuitively evident and the proof may be omitted without destroying the sequence of thought. The "theory of limits" and "maxima and minima" should be left to the students of higher mathematics. These subjects are of no practical value here and are beyond the comprehension of the ordinary student. In solid geometry only a very few of the theorems should be proven in a formal way. It should be largely the mensuration of the simple solid. Solid geometry also offers an excellent field for construction work which will be of use to the student in other lines of work. The numerical problems are of such a practical nature that the teacher will experience no difficulty in keeping the students interested. By reducing the number of theorems almost all the time can be spent at this kind of work.

TRIGONOMETRY: No course in mathematics for a student

preparing for betterment in his vocation is entirely complete without a working knowledge of trigonometry. It opens up a new field of useful knowledge and furnishes powerful tools for the solution of problems involved. The course should consist of the study of the trigonometric functions with the development of the few fundamental formulae needed. Next should come a large number of problems on the right triangle making use of these formulae and the tables of natural trigonometric functions. Logarithms should then be introduced and a large number of applied problems necessary to insure proficiency in the use of logarithms should be based upon the right triangle and the general triangle. All work with complicated identities, multiple angles, and functions of angles greater than those found in the general triangle should be omitted. With the material of the course made up as above students will be intensely interested from start to finish and will never question the utility of the course.

APPLIED MATHEMATICS: The student has now had one year in algebra, a year in geometry, and a third of a year in trigonometry. At least another third of a year should be spent on the applications of the mathematics he has already learned along with perhaps one or two new features of direct value to him. During the entire course the teacher should constantly hold before the pupil the fact that he is studying mathematics. During the first year the principles of algebra have received the greater part of the emphasis, however, when geometry is taken up the algebra and the arithmetic will not be forgotten but used at every place possible. This course should round up his whole mathematical knowledge and show him if he has not already found it out that what he has in his mathematics is some wonderful tools to be used in the solution of problems soon to confront him. He should be given opportunity to make calculations and work problems that will come up in the particular trade for which he is preparing. The use of the slide rule is one of the new features introduced into this course. The whole term's work should be crowded full of everyday problems bringing into play his formulae from arithmetic, his equation and the graph from algebra, his formulae and logical thinking from geometry, and the use of logarithms and the time-saving devices of trigonometry.

III. MATHEMATICS FOR AGRICULTURAL STUDENTS.

The agricultural high school is a finishing school with courses,

as indicated earlier in the discussion, fitted to the needs of boys and girls who find it almost impossible to obtain a college education. They are fitting themselves in the least possible time for the work on the farm. In view of this fact it is entirely proper that most of their time be spent along the line of practical agriculture. A good part of their time will be spent in the laboratory, in the stock-judging pavilion, and in actual contact with the real farm work. If the student has had a pretty thorough course in arithmetic we should be able with one year's work in mathematics to give him a working knowledge of the principles of mathematics sufficient to enable him to solve the problems of everyday life on the farm. The work for the girls should be essentially the same as that for the boys. The majority of the applied problems will be the same. Only problems pertaining to domestic science and domestic art will not be of value to the boys, and very little if any of the work for boys should be eliminated from the course for the girls. So what I shall say concerning the course will apply to the work for both. Assuming, then, that we have one year for mathematics, what shall be the nature of this mathematics? A careful study of the field will convince one, I think, of the fact that arithmetic, algebra, and geometry all contain principles and formulae which will be of practical value in a course of this kind. We will now see what parts of each shall be utilized.

ARITHMETIC: The field of arithmetic seems to be pretty well defined. Since the class will likely be made up of students of different degrees of proficiency in the four fundamental operations, a few days spent in review using as a basis problems taken from the experiences on the farm, will unify the class and insure better work throughout the term. The order to be followed in taking up the subjects has very little chance for variation. The chief point of difference from the old style arithmetic is in the choice of the problems. Agronomy with its problems in crops, soils, and fertilizers, dairying with its tests and feeding problems, constructions of barns, silos, and so on, animal husbandry with its suggestive problems, horticulture, farm mechanics, botany and zoölogy, all offer problems in common fractions, decimals, denominate numbers, percentage, and quite a number of other subjects. In business transactions of interest to the farmer you will cover insurance, interest, mortgages, notes, discount, commission, banking, and so on. The study of the graph and its use in bulletins and

crop reports should be taken up here. Mensuration is another division of the subject for which problems are easily found. In fact there seems to be little difficulty in finding material in arithmetic sufficient to take the third of the year allotted to it. The main trouble has been to get the problems properly graded. They must be graded with a view of developing the subject of arithmetic logically, and also in such a way as to really make the subject-matter developed a unit and worth while to the student. There is at least one text that seems to have accomplished both ends to a fair degree of success. Since this is true I will not attempt to go further into detail in the subject-matter to be considered.

ALGEBRA: The field in vocational algebra for agricultural students has not been so fully explored. In fact I do not know of any text-book that attempts to give a brief course in algebra designed for such students. With one-third of a year at our disposal what should we attempt to do in the subject? If we follow the usual plan of devoting the time to the study of addition, subtraction, multiplication, division, and factoring we will have used all the time at our disposal and the students will not have anything of practical value. This knowledge will also be of very little use in the work in geometry. A course containing the practical things of the subject can be arranged and should contain the following subjects in the order they are given here.

I. The Equation. If not all the members of the class have had the equation in arithmetic develop the idea by use of the balances and by making use of many concrete examples from arithmetic. Just as soon as they have grasped the meaning of the equation start them in with a list of problems involving simple equations. The problems should be of a practical nature and should constantly draw on their fund of knowledge of arithmetic. Problems like the following kind should make up several days work in the course.

(1.) To grow a ton each of corn and clover requires 813 tons of water. The clover requires twice as much water as the corn. How much water does each require?

(2.) A washing machine, linoleum, and range cost a farmer \$91. The linoleum costs twice as much as the washing machine, and the range twice as much as the linoleum. What is the cost of each?

(3.) A good ration for a cow is equal amounts of corn-and-

cob meal and cotton-seed meal, and five times as many pounds of hay as cotton-seed meal making 28 pounds in all. How many pounds of each were fed?

Twelve or fifteen problems of the kind given above drawn from a wide range of subjects will be sufficient here. Then introduce just as briefly as possible the axioms and by means of illustrative examples lead to the matter of transposition in the equation. About fifteen good examples based upon scientific facts, the perimeters of rectangles, and other subjects of interest should be given here. Problems must be carefully selected so as not to involve the use of negative numbers. In every case the problems should be checked, thus developing the idea of substitution which the student needs in the solution of problems by the use of formulae. If the students are not able to handle the solution of the equations after they have gotten them from the given conditions a number of abstract equations should be given sufficient to develop the desired skill.

II. Positive and Negative Numbers. Positive and negative numbers should be introduced by the use of the thermometer, latitude and longitude, debts, and so on. From this study the students may be led almost unconsciously into the subjects of addition, subtraction, multiplication, and division of a simple kind, using as the basis not abstract quantities alone but problems bringing into play their knowledge of arithmetic. They will readily see that it is perfectly natural to count income, rising temperature, travel east or north, and so on, as positive while debts, falling temperature, sailing south or west are treated as negative quantities. After a clear understanding of the method of notation they should then take up a large number of examples involving simple numbers which will give them the four fundamentals along with the laws of signs. The discussion of the laws of signs from a theoretical point of view has no place here. Problems of the following will serve to bring about the desired result.

(1.) The temperature at 7 a. m. was -10 and at noon $+9$. How many degrees warmer was it at noon than at 7 a. m.?

(2.) A man's income for the month was \$140, rent \$30, food \$40, interest and incidental expenses \$35. Using the positive and negative notation find the result at the end of the month.

A long list of simple, well-graded problems will be sufficient to fix in mind the four fundamental operations and the meaning of positive and negative numbers. I would conclude the

chapter with a list of abstract problems covering the four operations in order to insure that all the students have a ready working knowledge and sufficient drill in handling positive and negative numbers.

III. The Fundamental Operations and Factoring. In the last chapter nothing was said about the exponent, power, or coefficient. These definitions with illustrations should introduce this section. Some formal work in addition, subtraction, multiplication, and division of polynomials should then follow. All the problems in multiplication should be simple in nature perhaps not higher than the third degree in the multiplicand and not more than three, usually two, terms in the multiplier. Out of the multiplication will come in natural order factoring and the solution of the equation by factoring. Here will be given an excellent opportunity to put in a long list of practical problems involving addition, subtraction, multiplication, and division, and the solution of an entirely new kind of equation, the quadratic, by means of factoring. The problems should be carefully graded and the quadratic type should be along the line of the following:

(1.) A rectangle 3 feet longer than it is wide has an area of 28 sq. ft. What are its dimensions?

(2.) A field of grain is 32 rods long and 24 rods wide, how wide a strip must a farmer cut by going round and round the field to cut half the grain?

IV. Fractions and their Applications. Fractions should be taken up with a brief review of fractions in arithmetic. If the student has clearly in mind how to reduce fractions in arithmetic to their lowest terms, mixed numbers to improper fractions, improper fractions to mixed numbers and how to multiply, divide, and reduce fractions to a common denominator he will have only a brief transition to the same things in algebra. This transition can best be brought about by means of abstract problems. After this has been done the rest of the work on fractions should be given to the solution of practical problems from the farm and the sciences. Problems bringing into use all the formulae he has heretofore had and will have soon in the different fields of work should be taken up here. The following are examples of a simple kind which may be used as practical problems in fractional equations.

(1.) According to the Arkansas Experiment Station 1 acre of corn pasture produces one-third as many pounds of pork as

an acre of peanut pasture. If an acre of each together produces 416 pounds, what is the value of each as a pork producer?

(2.) It was estimated that a good road built along a quarter section farm increased its value one tenth of itself. The farm sold for \$16,280. What is the estimated value of good roads per acre?

V. Graphs and their Applications. If the graph has not been taken up in arithmetic it should be taken up here in connection with the crop reports. For example let them represent by a diagram or graph the yield of wheat in Kansas for a certain number of years. This may be done by the blocking method at first then by the continuous line. After some work of this kind a typical graph of the variation of temperature for a certain period of time should be studied carefully. They will then be ready to take up the variable and the ordinary principles of graphing. The linear equation in two variables will then be readily graphed and the solution of simultaneous equations in two unknowns which require only approximate results can be solved. A large part of the work in graphs will be based upon observed data from the laboratory, from bulletins and reports of the work at the experiment station, and from government reports. The chapter will be concluded with a list of problems in simultaneous equations in two unknowns by means of elimination and substitution. There should also be a short list of abstract problems for drill but the majority of the sets of equations will be obtained from the applied problems.

VI. Miscellaneous Problems. For the ordinary class there has been enough material given under the five chapters above. If the class is particularly strong more problems may be needed, and I should give a short chapter to problems which will supplement the different subjects and at the same time furnish nothing but material of the most practical nature. It seems to me if the course is closely followed the students will get the things they will need in the course in geometry which is to follow and at the same time will get only the things that they can make use of after they are through school.

APPLIED GEOMETRY: With only one third of a year to spend on geometry we must look only for the applications of the subject and not use any time for demonstration. The work should start with the definitions and illustrations of a straight line, angle, complement, supplement, and conjugate, then follow with a large number of equation problems. This work in equations

will fix clearly in mind the relation of the different angles and will also give a good drill and review of the equation work in algebra. Take up next the definitions of surface, triangle, rectangle, and all the common polygons. A great amount of valuable work can be based upon the right triangle by making use of the carpenter's square in getting the practical problems. Similar triangles and the different regular polygons offer excellent opportunities for applied problems which may be solved either by formulae, or by use of the graph. I should often require problems solved graphically so as always to keep before the students the fact that it is often the easiest and sometimes the only elementary method of solution for certain problems.

The next division of the work should take up the circle and the many problems related to it. Start with the definition. If the idea of the polygon with a large number of sides has been taken up, the area of the sector and finally the area of the circle can be developed intuitionally and logically. From the area of the circle the area of the segment and the area of the circular ring will easily come. Problems on belts, pulleys, cutting speeds, and gears will furnish a list of the everyday problems of life. Occasionally a problem involving the area of an ellipse will occur in practical work so it is well to give them the formula for the area and the meaning of the terms.

Solid geometry should consist entirely of the mensuration of the prism, cylinder, pyramid, cone, frustums, sphere, and the anchor ring. Definitions of each of these should be given at the proper time and an intuitionial development of the formulae given in each case. The last few days of the course in geometry should be spent in a general round up and review of the formulae learned, emphasizing the most important, so that the students will see that they have had something practical and will also have a clearer notion of how they may use what they have learned.