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Theorem in space of two dimensions.

If P, Q, R, S be four points and if O_1 and O_2 be conjugate points with respect to each member of the system of four-point conics $PQRS$, then

$$O_1[PQRS] = O_2[O_1ABC],$$

where A, B, C are the vertices of the self-conjugate triangle.

Consider the four conics of the above system, viz.: $S_1 \equiv$ the lines APQ, ARS ; $S_2 \equiv BPR, BSQ$; $S_3 \equiv CPS, CQR$; $S_4 \equiv O_1PQRS$.

Then the polar lines of O_1 with regard to S_1, S_2, S_3, S_4 are respectively O_2A, O_2B, O_2C, O_2O_1(2)

Also the polar lines of P with regard to S_1, S_2, S_3, S_4 are respectively PQ, PR, PS , tangent at P to conic O_1PQRS(3)

Hence, by (1), (2), (3),

$$\begin{aligned} O_2[ABCO_1] &= P[QRSP] \\ &= O_1[QRSP] \quad (\text{by cross-ratio property of conics}). \end{aligned}$$

Theorem in space of three dimensions.

Let the two quadrics S and S' have as their intersection the quadriquadric SS' , and let $ABCD$ be the common self-conjugate tetrahedron.

Let P be any given point in space having as the line of intersection of all its polar planes with respect to members of the system $S + \lambda S'$ the line p . Let Q be any point on the quadriquadric and let q in the same way be the line common to all its polar planes.

Consider the four cones of the system $S + \lambda S'$. Then the polar planes of P and Q with regard to these four cones are pA, pB, pC, pD and qA, qB, qC, qD .

Hence, by (1), $p[ABCD] = q[ABCD]$.

But qA, qB, qC, qD are evidently the four planes drawn through the tangent q to the quadriquadric to touch the curve again.

Hence the cross-ratio of the four tangent planes that can be drawn from any tangent to a quadriquadric to touch the curve again is constant.

Also, since $p[ABCD]$ is constant, we see that all the lines p belong to the Tetrahedral Complex defined by the tetrahedron $ABCD$.

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399. [K. 1. γ .] Given $\triangle BAC$; BC fixed in position; angle A of constant size; BA produced to P so that $AP = AC$.

To find the locus of P .

It is a circular arc such that $\hat{BPC} = \frac{1}{2}A$.

Now consider the limiting positions:

- (1) When A coincides with C , P also coincides with C ; $\therefore C$ is on the locus.
- (2) When A coincides with B , AB becomes a tangent at B to the $\odot BAC$, viz. BT , and T is a pt. on the locus if $BT = BC$.

The meaning of the arc TBC of the locus-circle remains to be interpreted.

In the case of the orthocentre of $\triangle ABC$, the complete locus is obtained by including the locus of the orthocentre of $\triangle BDC$ in the segment supplementary to ABC .

In the case of the incentre, the complete locus-circle is obtained by including the excentre.

In this case, if D is a pt. moving on the arc supplementary to ABC , and if DB is produced to P so that $DP = DC$, then the resulting positions of P complete the locus-circle.

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