IV.—WILLIAM OF OCKHAM ON CONTINUITY.

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THE work of William of Ockham on Continuity and Infinite Divisibility does not contain any statements of philosophical importance which are unknown to us now : but it is at least historically interesting, in view of the fact that it was done in the early fourteenth century. And there are interesting parallels to be found between the ideas of Ockham and Mr. Bertrand Russell : sometimes there is even a close likeness between the actual sentences of the ninth question in the first book of Ockham's *Quotlibeta* and those in Mr. Russell's *Our Knowledge of the External World*. I shall attempt to give, without much commentary, a short summary of Ockham's treatment of continuity. The problem for him arises in that of the infinite divisibility of space : but he gives the clearest indications that the conceptions he uses are valid also for time, and he seems to imply that they are valid for all forms of continuity.

Two sentences are specially interesting, that in which Ockham declares it impossible for those who do not admit infinite divisibility to reply to Zeno's argument about Achilles, and that in which he distinguishes "compact series" from "things juxtaposed". The whole of his treatment of continuity is based upon the idea that there is always a third point between any two in a line or that there is not a "certain number" of units in a continuity. In the reply to objections there is a hint at a theory of sense-data in continuous visions which seems important : for Ockham seems to regard the conception of the true continuum (compact series) as a sufficient explanation of change. As for his method, the whole *questio* fills six columns of print in the edition of 1491. We must therefore allow for the abruptness and apparent neglect of detailed proof : for we have not here what Ockham would have said in his lectures, but only a summary of his theory. He begins with mathematical proofs of the infinite number of points in a line, and then proceeds to deal with the Achilles argument of Zeno.

Then follow replies to objections. The whole is very abruptly stated and the proofs of his statements are sometimes rather indicated than expressed. In some places the contractions in the text are so obscure that it is uncertain whether we have the exact wording.

The general result of this *quæstio* will be seen to be in harmony with the realism (in the modern sense of the word) of Ockham's theory of the individual. It has a close connexion with his treatment of "magnitude" in the *De Sacramento Altaris* and his treatment of "the infinite regress" at the beginning of the *Centiloquium Theologicum*. But here I shall attempt only a summary of the *quæstio* in the *Quotilibeta*, "Utrum linea componatur ex punctis".

After a preliminary indication, in the form of a proof, that the line does consist of points, followed by the remark that Aristotle (VI. Physicorum) is opposed to this opinion, Ockham indicates that the real problem cannot be solved if it is phrased thus. The treatment of Aristotle as an authority may be compared with Ockham's sentence in the *Commentary on the Sentences*—"quicquid de hac re dicit Aristoteles non curo," which gives one an unconventional view of the mediæval treatment of "the philosopher". There were at least some in the fourteenth century who would not be put off by having a great name quoted against them. As for the problem of line and points, Ockham sees that the real issue is concerned with the sense in which the line may be said to consist or be made up of anything.

He proceeds, therefore: "Here we must first express the true meaning of the problem, which is this: whether any parts of a line are indivisible: and if such be the meaning, I say that *no part of the line is indivisible, nor is any part of any continuum indivisible*".¹ The point is not, then, a minimum length, or the number of points in a line is infinite. Ockham's words continue thus—"This is proved as follows. If any part were indivisible (or if there were a finite number of points in the line), then the side of a square would be equal to its diagonal; and therefore the diagonal would be commensurable with the side. The consequence is clear; for, from every point in the diagonal a line may be drawn at right angles to the side. This follows from the fact that a line can be drawn from every point in one of the sides at right angles to (every point in) the other side.

¹ Hic primo dandus est intellectus quæstionis, qui est iste: utrum aliquæ partes lineæ sint indivisibiles. Et sic intellecta quæstione, dico quod nulla pars lineæ est indivisibilis, nec etiam pars quæcunque cuiuscunque continui est indivisibilis.

Indeed on the above hypothesis there *is* such (a line). And every such line is drawn through a point in the diagonal. Therefore from every point in the diagonal there is a line at right angles to the side.¹ If, therefore, there are six points in the diagonal there must be six in the side." The conclusion is that there is no indivisible in the line, or that a point is not a minimum length. The proof consists of two parts: (a) If the points in CD and EF, opposite sides of a square, are not infinite, then from every one point in CD let a line be drawn at right angles to every point in EF (the number being equal): then ED contains the *same number* of

points. But if it be said that there may be a point A in the diagonal ED through which *no* line from CD to EF passes, then (b) let a line AB be drawn at right angles to CD from the point A. If B is *not* a point from which a line has been already drawn to EF, then we have not done what we supposed in (a), and if it *is*, then the line AB is the same as one line of the many from CD to EF, since all lines at right angles to CD must be parallel.²

Secondly, says Ockham, if points were minimum lengths then "there would be the same number of points in a greater as in a smaller circle. For from any point in the greater circle a straight line may be drawn to the common centre of both. Indeed there *is* such a line, if the hypothesis (of points in the circle) is correct. And these lines do not meet except in the centre. Therefore, either two lines drawn from two points next one another (immediate) in the greater circle will pass through *two* points in the smaller circle or through *one*. If the former, the proposition is proved; for then there

¹ I give the original in case a better translation may make it seem less abrupt. "A quolibet puncto diametri ad costam contingit protrahere rectam lineam. Quod probatur quia a quolibet puncto costæ unius ad quodlibet punctum costæ alterius contingit protrahere rectam lineam. Immo ita esset de facto posita ypotæsi. Et quælibet talis linea protrahitur per aliquod punctum diametri. Ergo a quolibet puncto diametri ad costam est aliquis linea recta."

² I owe to the assistance of Prof. Percy Nunn my restatement of Ockham, but of course this does not make him responsible for the cumbrous manner I have used.

are as many points in the smaller circle as there are in the greater. If the latter, then these lines never meet again." The last sentence in the Latin is obscure, but that is obviously the meaning. Further proofs are given by increasing the number of concentric circles. The conclusion that the point is not a minimum length, or that the number of points in a line is *infinite*, is stated; and then Ockham remarks: "But it does not follow from the difference of these lines and these circles *in infinity* (of points) that there is no part in the greater which has no distinct part in the smaller corresponding to it".

The problem is then made more general by the introduction of Zeno's argument concerning Achilles, which is abruptly stated as "that the swiftest mover will never catch up the slowest" (*argumentum Zenonis secundum quod mobile velocissimum nunquam attingeret mobile tardissimum*). "This argument implies certain presuppositions," says Ockham. "The first is that always the nearer part of space is reached before the farther. The second is that *indivisible* space is not reached except in an instant."¹ "From both which it also follows that no body, if it is moved through such space, would reach in one instant anything but one *indivisible* (part of) space. I then argue thus" . . . and there follows the argument that Zeno's proof depends on supposing that there is a one-one correspondence between indivisible units of space and indivisible units of time. "And if you say that it holds also against those who suppose the continuum to be infinitely divisible, I reply: there is no part of space which is *first* reached in the moment, so that any part (of this part) is covered in that moment. And on this supposition Zeno's argument does *not* hold against those who deny indivisibles as it does against those who believe in them. . . . But, of course, I grant that in any instant some distance is covered, and then a part of it was covered before, and a part of that before, and so on *infinitely*: and so the swift overtakes the slow over the same distance by infinite moments." Then comes the abrupt statement—"he who denies indivisibles can solve Zeno's puzzle, and he who supposes indivisibles can not". The positive part of Ockham's thesis ends with the remark that the conception of infinity will solve all problems arising from change.

Next follow replies to objections. These objections are not fully stated, but are left to be implied in the phraseology of the reply. Some are peculiarly mediæval but not, I

¹ Or "the nearer part is covered," . . . "indivisible space is covered". See text at the end of this article.

think, entirely without modern parallels. They run thus : first, there are indivisibles ; for what about the soul ? To which Ockham replies : " I am not referring to what is not quantitative ". Secondly, it is said that God could make a sphere commensurable with a plane tangential to it. Ockham replies that he could not. Thirdly, against what objection is not quite clear, he says : " We must distinguish the continuum (compact series) from things juxtaposed ".¹ Fourthly, it is asked whether the whole is greater than its part ; and Ockham replies : " That depends what you mean by greater. For if you mean that the whole contains a certain definite number of parts more than *the* part, it is not true. But if you mean that both part and whole may be infinitely divided and yet may be commensurable, then the whole is (in infinite divisibility) greater than the part. " Fifthly, it is said that according to Ockham if God had been dividing from the beginning of the world he would still be at it. Ockham admits the idea and refuses to view it as absurd (*non esset adhuc completa divisio*). A sixth objection refers to conventional lengths such as a yard, which is obviously frivolous. A seventh refers to the Immaculate Conception of the Virgin Mary, which to modern ears may have little indeed to do with infinite divisibility : but Ockham asserts that she never was in Original Sin in any part of an instant. To an eighth implied objection he replies that God could not make an angel pass from one point to another without passing through infinite points between.

Ninthly, as to sense-data, " as the shape of an object is continuously increased (? by coming nearer and nearer), so the visions of white succeed one another *continuously*, and as we need not suppose indivisible units of change, so we need not suppose indivisible units of vision ". That is to say, as I understand him, there is always a sense-datum perceivable between (in time) the first perceived and any other. This is (1) the reply to Bergson's objection against the " cinematograph " idea of vision, and (2) it gives the distinction between the cinema-pictures and actual human vision. For there are *not a certain number* of visions of white or of shapes perceived in change.

Finally, Ockham replies to the general question whether a line consists of points. He says a fallacy is involved in the implied argument : and to this he answers that (1) between the first point and any other there is always a third, but that (2) not *any* point lies between the first and any other (*i.e.* as

¹ *Hæc est differentia inter continuum et contiguum quod partes continui faciunt unum per se, partes contigui non.*

I understand him,—there is no *second* point in the line, but there is a first).

It is perhaps possible, even from this short *questio*, to see why Ockham was known as the "Venerabilis Inceptor". He seems to have had in his philosophy all the prestige of tradition and all the promise of a new beginning.

EXTRACTS FROM THE TEXT OF OCKHAM.

On Zeno:—

. . . Si stat argumentum Zenonis secundum quod mobile velocissimum nunquam attingeret mobile tardissimum. Ista ratio supponit aliqua fundamenta. Unum est quod semper pars propinquior spatii prius attingitur quam remotior. Aliud est quod indivisibile spatium non attingitur nisi in instanti. Ex quibus etiam sequitur quod nullum corpus si moveretur super tale spatium attingeret in instanti nisi unum individuum in spatio. Tunc arguo sic: In instanti in quo acquirit tardum unum indivisibile in spatio quæro quid acquirit velox: aut solum unum indivisibile, et tunc velox nunquam attingeret tardum; aut acquirit duo indivisibilia, et per consequens per secundam suppositionem prius acquirit unum quam aliud, ergo non acquirit in instanti quia in isto non est prius.

Si vero dicis quod istæ passionēs velocitas et tarditas non conveniunt motui alicuius indivisibilis, contra idem arguo per omnia, (?) si corpus divisibile moveatur super (?) spatio composito ex indivisibilibus.

Et si dicis quod illud argumentum Zenonis videtur c. te esse ponendo continuum componi ex semper divisibilibus, dico quod in quolibet instanti est mobile in alio et alio spatio, sed non est dare aliquam partem spatii quæ sit primo acquisita in instanti ita quod quælibet eius pars acquiratur in isto instanti. Et si hoc poneretur, non concluderet argumentum Zenonis contra negantes indivisibilia sicut contra ponentes. Quod patet quia in illo instanti in quo iam tardum aliquas partes spatii acquisivit secundum (?) se totum, aut velox acquirit æqualem partem, et tunc nunquam attingeret tardum, aut acquirit maiorem, et tunc per suppositionem prius acquirit unam partem illius quam aliam, et per consequens non acquirit in instanti. Et ideo dico quod in instanti non acquiritur aliqua pars nec loci nec alterius formæ per alium modum (?) secundum se totam, ita quod quælibet pars acquiratur in isto instanti. Sed concedo quod in quolibet instanti aliquid acquiritur et tunc pars illius acquisita fuit prius acquisita et pars illius prius et sic in infinitum. Et sic mobile tardum præcedit velox in isto spatio per infinita instantia. Patet manifesta. Unum (?) impossibile est dare aliquid primo acquisitum in instanti per quemcunque modum. Sed quodcunque acquiritur in tempore acquiritur. Et quæcunque pars eius in tempore acquiritur. Et ideo dico quod velox in eodem tempore plures partes eiusdem quantitatis spatii pertransit quam tardum: immo negans indivisibilia potest argumenta Zenonis solvere, et nullo modo ponens ea. Ista solutio est generalis ad omnia argumenta quæ fiunt de formis acquisitis per motum.

Is the whole greater than the part?

Dico quod hæc propositio totum habet partes plures quam pars potest dupliciter intelligi: uno modo quod pars habeat aliquem certum numerum partium et quod partes totius excedunt in aliqua certa multitudine. Et ille intellectus est falsus: quia implicatur quod totum et pars habeant aliquam certum numerum, quod falsus est. Alio modo quod pars totius habeat partes non tot in aliquo certo numero quam plures, quia infinitas,

et quod totum habeat tot partes et adhuc aliquas alias. Et sic concedo quod totum cœlum habeat plures partes quam pars.

Sense-data :—

Ad aliud dico quod visiones albedinis sibi succedunt continue, sicut forma continue augetur. Et ita sicut propter motum augmentationis non oportet ponere talia indivisibilia ita nec propter visiones.

General reply :—

Ad principale dico quod solvendum est per fallacia figuræ dictionis : quia hæc est concedenda—inter primum punctum et quodlibet aliud est aliquid punctus medius ; sed hæc est neganda—aliquis punctus est medius inter primum et quodlibet aliud.