485. The Graphical Solution of a Cubic Equation with Complex Roots

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485. [X. 4. b. β .] *The Graphical Solution of a Cubic Equation with Complex Roots.*

The following is an application to the solution of cubic equations with imaginary roots, of methods already given of the solution of quadratic equations, in Hamilton and Kettle's *Graphs and Imaginaries*. It is there shown that:

(I) If POQ be any parabola (see figure) and a (or AB) is any straight line meeting POQ in imaginary points, the coordinates of the intersections of parabola and line are $(x \pm i\xi, y \pm i\eta)$, if the "shadow parabola" (see below) of the line meets that line in $(A, B) \equiv (x \pm \xi, y \pm \eta)$;

(II) That a meets any circle KOL in the points whose coordinates are $(x \pm i\xi, y \pm i\eta)$ if the "shadow circle" of a meets a in $(x \pm \xi, y \pm \eta)$.

The "shadow parabola" of POQ corresponding to a is an equal parabola touching POQ at T , where the tangent is $\parallel AB$. The "shadow" is upside down as in figure.

The "shadow circle" of KOL corresponding to a has N as centre and touches KOL , where $CN \perp AB$ and $MN = CL$.

Now the roots of the equation

$$x^3 + px + q = 0 \dots\dots\dots(1)$$

are the abscissae, other than the origin, of the intersections of the parabola

$$y = x^2 \dots\dots\dots(2)$$

and the circle $x^2 + y^2 + (p-1)y + qx = 0. \dots\dots\dots(3)$

(See Hardy, *Pure Mathematics*, Examples on Chap. II.)

Let POQ be the parabola (2); and let KOL be the circle (3).

The real root of (1) is the abscissa of K .

Supposing (1) to have imaginary roots, the join of the imaginary intersections of (2) and (3) is a real line parallel to that tangent (TU) to POQ , which makes the same angle with the axis of POQ as KO does.

Draw $CU \perp TU$, and produce to M , where the diameter conjugate to TU meets it.

Mark off $MN = CL$, and draw the circle of centre N and radius NL .

Draw AMB through M and parallel to the tangent to POQ at T , meeting the circle centre N in A, B .

Then if A, B are the points $(x \pm \xi, y \pm \eta)$, the imaginary intersections of (2) and (3) are $(x \pm i\xi, y \pm i\eta)$. Hence the imaginary roots of (1) are $x \pm i\xi$.

For, the "shadow parabola" corresponding to the point T will clearly pass through the points A, B determined as above.

Then by the principle (I) (see above), the points A, B determine the intersections of the line a with the parabola POQ . Also by (II) the same points A, B determine the intersections of a with the circle KOL .

Hence A, B determine the intersections of the parabola POQ with the circle KOL , which are what we require. E. R. HAMILTON.

486. [I. 5.] *The Definition of a Complex Number.*

In the March number of the *Gazette* Dr. Hardy discusses the introduction of complex numbers by means of an operator O , such that $O\{O(a)\} = -a$. He says that this method has been adopted in a recently published book, and develops in detail the objection that $O\{O(a)\} = -a$ does not define any unique operation.

I know of only one recently published book, Dr. Bowley's *Pure Mathematics*, which adopts this method of development, and as it has been reviewed by Dr. Hardy in the *Cambridge Review*, it is doubtless the one to which he refers. But in that book the operator O is not applied to a complex number, but only to real or purely imaginary numbers, so that Dr. Hardy's analysis does not apply.

Of course the development of imaginary and complex numbers in an elementary text-book is, as far as my experience goes, bound to be open to criticism. So is the development of negative numbers, at the stage at which they are generally introduced, unless an unusual amount of space is devoted to it. In the book in question the whole of the mathematics, beyond matriculation standard, which the author considers necessary for the non-specialist is given in 269 pages, so that space must be an important consideration.

In spite of some objections, I have found the "operator" method a useful one for young students, as it helps them to realise that there is a difficulty. They are apt to regard i as "the square root of -1 " and to settle the whole matter in one line. And it is open to doubt whether the idea that $a + ib$ is merely a symbolic equivalent for a pair of real numbers (a, b) gives the best way of introducing complex numbers to young students.

G. W. PALMER.

487. [L¹. 3. d.] *Parabolic Asymptotes.*

To find the parabolic asymptote of

$$(y - 2x)^2(y^2 - x^2) + y^3 + 2x^2y + y^2 + x^2 + ax + by + c = 0.$$