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To cite this article: B.N. Chuckerbutti M.Sc. (1922) LXV. On the deformation of the “rings and brushes” as observed through a spath hemitrope , Philosophical Magazine Series 6, 43:255, 560-567, DOI: [10.1080/14786442208633911](https://doi.org/10.1080/14786442208633911)

To link to this article: <http://dx.doi.org/10.1080/14786442208633911>

Published online: 08 Apr 2009.

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LXV. *On the Deformation of the "Rings and Brushes" as observed through a Spath Hemitrope.* By B. N. CHUCKERBUTTI, M.Sc., Assistant Professor of Physics, Calcutta University*.

[Plate IX.]

1. *Introduction.*

CRYSTALS are frequently found which are obviously of a composite character, that is, are composed of more than a single individual crystal of the same substance and in which the parts belonging to different individuals are united in a definite and regular manner, the peculiar mode of union being characteristic of the substance. The true nature of the composite structure is often betrayed by the presence of *re-entrant angles*, forming notches (as in Diamond crystals), arrow-head shapes (as in Gypsum), knee-shapes (as in Cassiterite), cruciform (as in Staurolite), or heart-shapes (as in Fluorspar); but not infrequently the two or more individuals are so intimately blended, that the appearance at first sight is that of a single individual crystal.

The law defining the manner in which the composition occurs is usually stated as follows:—

The two individuals are first supposed to be arranged parallel to each other, with one face of each in mutual contact, and then in order to produce the twin, one individual is supposed to be rotated to 180° , upon a plane, which is called the *twin-plane*, and about the normal to the plane as axis, which is called the *twin-axis*. But it is necessary to point out here, that in the actual process of the formation of the twins, no such two different crystals are brought into contact and then one turned through 180° , as stated above. The orientation takes place amongst the crystalline molecules of their own accord during the process of formation. Hence, the *twin-plane* is not necessarily always the plane of contact. In general, the plane of twinning may be any actual or possible face, except obviously a plane of symmetry of the crystal.

The twinning may be either of Primary or of Secondary origin. The former refers to the case when the formation takes place of itself without the intervention of some external agency during the process; but when the twinning takes place subsequently to the original formation of

* Communicated by the Author.

the crystal or crystalline mass, by means of external pressure, it is called secondary. Closely related to the cleavage-direction in their connexion with the cohesion of the molecules of a crystal, are the *Gliding planes* (Gr. Gleitflächen) or directions parallel to which a slipping of the molecules may take place under the application of an external force. This molecular slipping may be attended by a rotation through 180° of the molecules and the resulting twin formed.

According to the mineralogists, Iceland-spar belongs to a class of crystals which always produce twinning lamellæ, that is, one twinning plane in the case of this crystal is always followed by another at a very short distance so that a thin layer of similarly oriented molecules is formed. These are found abundantly in nature and, moreover, the pressure upon the cleavage-fragment of Iceland-spar results in the formation of a number of thin laminæ in twinning position to the parent mass. Secondary twinning lamellæ are often observed in natural cleavage masses of calcite.

The explanation of the mechanism of formation of twin crystals is rather difficult. Lord Kelvin*, however, has touched upon the problem. He has given a very lucid exposition of the whole process of crystal-building from a solution. In the particular class of crystals producing twins, he considers the constituent molecules symmetrical on the two sides of a plane passing through itself and also on the two sides of a plane perpendicular to this plane, that is to say, his crystalline molecules are egg-shaped. A real crystal which is growing by addition to a face would give layer after layer regularly. But, if *by some change in internal circumstances*, the molecules that would go to the formation of a layer are all oriented to 180° with respect to the molecules that formed the previous layer, a twinning plane is formed, and if the remaining layers form in the same way as the last mentioned one, then we shall get a crystal having two different parts separated by a twin-plane between. If, again, the process of orientation continues only to some layers and then, due to the *re-establishment of the initial conditions*, the layers form as at the start, we shall get two portions to the two sides with similarly oriented molecules, enclosing a thin layer in which the molecules are turned through 180° . The case will resemble that of Iceland-spar. But the main difficulty in the explanation is to understand *what is that change in the circumstances* that causes the crystalline molecules to turn through 180° in the process of

* Baltimore Lectures, p. 629, Art. 37-39.

crystal formation, and how is it in the case of Iceland-spar that *the initial conditions are re-established* after a thin layer is formed under the changed conditions. The difficulty is still greater when we come to consider the case of repeated twinning as in the case of potassium chlorate crystals, where it is found that a very large number of twinning layers may be formed with a most surprisingly regular periodicity and constancy of thickness. Thus there is a good deal in regard to the mechanism of formation of these twinning layers that is as yet but imperfectly understood.

2. *Optical behaviour of a Spath Hemitrope.*

On looking at a source of light through a twin crystal of Iceland-spar, generally it is found that three images of the source are formed, the central one of which is always stationary. If the source of light is an incandescent electric lamp, then the images are beautifully coloured, the nature of the colour changing with the orientation of the plane of the crystal to the incident beam. It is also observed that the two outside images are polarized in perpendicular planes. On rotating the crystal, the two outside images are found to rotate about the central one, and in the course of a revolution there are positions in which both of them disappear, the remaining one becoming the most brilliant for the time.

The author has also found that when a beam of light is allowed to fall upon a crystal cut perpendicular to the axis and also polished in such a way that the lines, in which the twinning lamina cuts the planes of the main crystal, are on the surface, some interesting diffraction effects are observed. Fringes are noticed in the region of the transmitted light which show a remarkable asymmetry. The edges of the lamina also appear to be luminous. The phenomena appear to belong to the class of laminar diffraction effects, but there are certain features regarding them of which the explanation is not clear. The author is at present engaged in a fuller study of these effects, and the investigation mentioned above will probably prove of considerable interest in relation to the determination of the optical nature of the twinning layer.

On examining the crystal with convergent plane polarized light and between crossed nicols, the ordinary "rings and brushes" are found to be distorted, the amount and the nature of the distortion changing with the orientation of the crystal. However, in some cases, the distortion more or less resembles that produced by the superposition of a

quarter-wave mica-plate upon a crystal of Iceland-spar cut perpendicular to the axis, so that the plane containing the optic axes of the mica-plate makes an angle of 45° with the vibration planes of the crossed nicols.

The explanation of the phenomena of refraction has been given in text-books on Optics*. There, the formation of the three images and the polarization are all explained. The disappearance of the outside images in certain positions, as stated before, is connected with the orientation of the optic axis of the intervening layer to that of the main crystal.

The problem of the form of the rings and brushes through a twin crystal has been treated mathematically, but the case has always been considered where a twinning plane separates two crystals. The problem of the two similar crystal wedges separated by a thin twinned lamina inserted between them in an inclined position appears not to have been solved, and an investigation in this direction forms the subject-matter of the present paper.

3. *Explanation of the Photographs.*

The crystal that was employed in the following experiments was cut perpendicular to the axis, and the portions free from the twinning layer showed the ordinary "rings and brushes" peculiar to crystals cut in such a manner. The angle which the twinning plane makes with the faces of the crystal is determined by observing the marginal outline of the plane as seen on looking through the edge of the crystal.

The Hemitrope, as examined with tourmaline tongs, shows very beautiful distortions and changes in the usual circular ring-system. It is difficult to photograph the successive stages completely, and it is found that the changes repeat themselves four times during a turn of the crystal through 2π , when the tourmalines are crossed,—the position for which most of the observations were made. The rings lose their circular nature altogether, and they will be found to be elliptic as if pressed up and down or sideways. Moreover, the change due to the introduction of the thin twin plane is most prominent near the central part of the system. These changes at the centre mixing up with the crosses make the central portion look rather cumbrous.

Figs. 1 to 4 (Pl. IX.) represent the changes for a rotation of the crystal through $\pi/2$, when the analyser has its

* Mascart, *Traité d'Optique*, vol. ii. p. 192.

plane of polarization vertical and the polarizer horizontal. In fig. 1 the crystal is placed so that the twinning layer extends vertically up and down, inclined in such a manner that its sections with the faces of the crystal are vertical straight lines. Light is always allowed to fall *normally* upon the first face of the crystal, as otherwise the three images of which mention has been made before will make the system more complicated. Figs. 2, 3, 4 are obtained by rotating the crystal to the left in its own plane till the rotation amounts to $\pi/2$, when the system corresponds with fig. 1 but rotated through 90° . As we proceed further beyond $\pi/2$, the figures 2, 3, 4 come in the reverse order, fig. 4 being followed by figs. 3 and 2 and at π we get back to fig. 1. On rotating further, figs. 2, 3, 4 come in turn, similarly as in the first quadrant, and the figure at $3\pi/2$ corresponds with the one at $\pi/2$. The order of succession of the system along the last quadrant corresponds with that of the second quadrant. Figs. 5 to 8 were obtained by rotating the tourmalines in some position other than the crossed and at the same time rotating the crystal also. In general, the systems are rather cumbrous in these cases and the four cases are selected out, as showing the changes rather systematically. Figs. 7 and 8 have the peculiarity that they are complementary in nature to figs. 4 and 1 respectively, although they were obtained when the tourmaline planes were not parallel but inclined at an angle of about 45° .

4. *Physical Theory.*

In general, if δ be the path retardation of two rays due to their passage through a uniaxal crystal the equation to the generating curve to the isochromatic surface is

$$\{(\mu_0^2 - \mu_e^2)y^2 + \delta^2\}^2 = 4\mu_0^2 \delta^2 (x^2 + y^2),$$

where the optic axis lies along the axis of X.

In case of a regular crystal of thickness T, the section of the isochromatic surface by a plane $x=T$ gives the equation for the rings, and the points giving the same path-retardation are equidistant from the centre of the system. Now the interposition of the thin layer which again has its optic axis inclined to the optic axis of the main crystal, causes different path-retardation upon the interfering rays, so that the points of equal path-retardation are not equidistant from the centre, and as a result we should expect distorted curves.

Let

$$l\xi + m\eta + n_1\zeta = 1,$$

$$l\xi + m\eta + n_2\zeta = 1,$$

be the equations of the two waves in unit time after passing through a point o of the surface of the crystal.

[n_1 and n_2 are the reciprocals of the intercepts made on the normal (axis ζ) to the plate through the point o by the refracted waves in unit time after passing through o .]

If θ be the azimuth of the plane of incidence with respect to that of $\xi\zeta$ and v be the velocity of light in air, then

$$l = \frac{\sin i \cdot \cos \theta}{v}, \quad m = \frac{\sin i \cdot \sin \theta}{v},$$

where i = angle of incidence.

Suppose, that the plane $\xi\zeta$ contains the greatest axis oz of the ellipsoid of polarization, the plate being on the positive side of ζ , then if ox , oy , oz be the axes of the ellipsoid and $zo\xi = \chi$, we get by transformation from the axes of optical symmetry to the new axes,

$$x = \xi \cos \phi \cos \chi - \eta \sin \phi + \zeta \cos \phi \sin \chi,$$

$$y = \xi \sin \phi \cos \chi - \eta \cos \phi + \zeta \sin \phi \sin \chi,$$

$$z = -\xi \sin \chi + \zeta \cos \chi.$$

The equation to the wave surface, referred to the axes of optical symmetry is

$$\frac{a^2 x^2}{\sigma^2 - a^2} + \frac{b^2 y^2}{\sigma^2 - b^2} + \frac{c^2 z^2}{\sigma^2 - c^2}.$$

From the condition that the plane $l\xi + m\eta + n\zeta = 1$ should touch the wave surface in the new system of coordinates, we find

$$\begin{aligned} \frac{\delta}{T} = & - \frac{\sqrt{v^2 - a^2 \sin^2 i}}{a} + \frac{(a^2 - c^2) \sin \chi \cos \chi \cos \theta \sin i}{a^2 \cos^2 \chi + c^2 \sin^2 \chi} \\ & + \frac{\sqrt{(a^2 \cos^2 \chi + c^2 \sin^2 \chi)(v^2 - c^2 \sin^2 i) - c^2(a^2 - c^2) \sin^2 \chi \cdot \cos^2 \theta \cdot \sin^2 i}}{a^2 \cos^2 \chi + c^2 \sin^2 \chi}, \end{aligned}$$

where $\delta = vT(n_2 - n_1)$.

Hence, for a crystal cut perpendicular to the axis ($\chi = 0$),

$$\therefore \frac{\delta}{T} = \{ \sqrt{v^2 - c^2 \sin^2 i} - \sqrt{v^2 - a^2 \sin^2 i} \} / a.$$

Let T, t be the thicknesses of the main crystal, and of the thin twin layer respectively, and δ_1 and δ_2 be the corresponding path retardations. In the present case, if we assume that the twin layer is also cut perpendicular to the optic axis, then from the angle (30°) at which its plane cuts the faces of the crystal, we must have $\chi = 60^\circ$ for the thin layer. So that the expression for the retardation in terms of the refractive indices takes the form,

$$\frac{\delta_2}{t} = \sqrt{\mu_e^2 - \sin^2 i} + \frac{\mu_0^2 - \mu_e^2}{\mu_0^2 - 3\mu_e^2} \cdot \sqrt{3} \cdot \sin i \cdot \cos \theta \\ - \frac{\sqrt{\mu_0^2 - 3\mu_e^2 - 4\mu_0 \sin^2 i \cos^2 \theta}}{2\mu_0(\mu_0^2 + 3\mu_e^2)}.$$

For the rays which fall normally upon the face of the crystal (*i. e.*, the rays which make an angle of 30° with the normal to the twin layer),

$$\delta_2 = t \{ 1.3186 + 0.05 \cos \theta + 0.0028 \cos^2 \theta \}$$

and $\delta_1 = T\mu_0 \{ \mu_0 - \mu_e \}.$

Moreover,

$$\delta_1 = T\mu_0 \left\{ \frac{(\mu_0^2 - i_1^2)^{\frac{1}{2}}}{\mu_0} - \frac{(\mu_e^2 - i_1^2)^{\frac{1}{2}}}{\mu_e} \right\},$$

where i_1 = angle of incidence (very small).

The thickness of the twin layer as seen under a high power microscope and estimated with the help of a micrometer eyepiece is about 2λ . So that

$$\delta_2 = 2\lambda(1.3186 + 0.05 \cos \theta + 0.0028 \cos^2 \theta).$$

Now as the azimuth angle increases from 0° to 90° the value of δ_2 diminishes. On the other hand, δ_1 goes on increasing as the angle of incidence is increased, and is constant for the same angle of incidence.

If r be the radius-vector of the interference minima or maxima, then we may put

$$r^2 = k(\delta_1 + \delta_2),$$

where k is a constant. Hence we can expect the elliptic form of the curves, the radius-vector for an azimuth of 90° being the least. Moreover, at a distance from the centre where i is great, δ_1 will be very great in comparison to δ_2 , which latter may then be safely neglected, the radius-vector in those cases being given by $r^2 = k\delta_1$.

Another important feature of this thin twin plane is the asymmetric laminary diffraction* pattern to be observed near the two edges of the twin plane, with a beam of light incident normally upon the surface of the crystal. This problem, however, is under investigation.

The experimental work described in this paper was carried out in the Palit Laboratory of Physics and the writer wishes to express his thanks to Prof. C. V. Raman for kindly providing all facilities for work.

University College of Science, Calcutta.

24th August, 1921.

LXVI. *Coupled Vibrations by means of a Double Pendulum.*

By A. L. NARAYAN, M.A.†

[Plates X. & XI.]

FOR the production of high-frequency oscillations required in radio-telegraphy, one of the most important elements in the oscillation circuit is the oscillation-transformer, which is simply a magnetically-coupled oscillating circuit possessing a primary and a secondary each containing inductance and capacity in series. In order to elucidate and interpret the theory of these electric circuits and illustrate how, when oscillations are excited in one circuit, these two circuits act and react on each other so that the result is that oscillations of two different periods are set up in both circuits, one greater than and the other less than the natural period of either circuit taken separately, mechanical systems were described by Prof. Thomas Lyle, *Phil. Mag.* vol. xxv. pp. 567-572 (April 1913): Prof. Barton and Miss Browning, *Phil. Mag.* vol. xxxiv. p. 246 (1917), vol. xxxv. p. 62 (1918), and vol. xxxvi. p. 36 of the same year; and Mr. Jackson, *Phil. Mag.* March and September 1920.

In this paper the author describes an entirely different kind of mechanical system, which gives results very similar to those obtained by Prof. Barton and Miss Browning, and Mr. Jackson, in the papers referred to above. At the same time, the whole arrangement is more neat and much simpler than the very elaborate one adopted by Mr. Jackson, and the method of photographing the vibrations in the present case is essentially different from that of the previous authors.

The coupled system treated in these pages consists, as shown in the two diagrams (Pl. X.), of two rigid pendulums A and B,

* P. N. Ghosh, *Proc. Roy. Soc. A*, vol. xcvi. (1919).

† Communicated by the Author.