ART. XV.—*The Counter-twisted Curl Aneroid*,* by CARL BARUS, Hazard Professor of Physics at Brown University.

INTRODUCTORY.

1.—It seems plausible to argue that much goes on in the atmosphere immediately related to pressure which the ordinary pneumatic barometer merely integrates, and of which it can give no detailed account. I refer both to the changes referable to the gusty character of the wind † and to pressure variations of a more subtle nature,‡ the origin of which may be considered in relation to the earth's magnetic and electrical field.

2.—The problems, therefore, are beyond the immediate scope of instruments of large mass like the ordinary mercury or water barometers. The conditions to be fulfilled are (1) great sensitiveness and (2) instantaneous indications; (3) registry subject to corrections of a purely scientific kind. It is when these three conditions are simultaneously demanded that the problem becomes formidably difficult. No matter what form of mechanism is selected, one is brought face to face at once with viscosity and with the thermal variations of both viscosity and of elasticity.

I desire in this paper to find out how far one can go with suitable modifications of the Bourdon tube. In certain measurements§ of high pressure made with such a tube coiled helically I received much encouragement, inasmuch as the instrument could be read off closely enough, without the aid of subsidiary mechanism. The difficulties which I then encountered were purely technical. In flattening and coiling the necessarily heavy tube, I had to remove the temper at a sacrifice of strength and resilience, and the gauge actually burst at 1000 atmospheres. I abandoned it simply because of the difficulty in fashioning this cumbersome apparatus in the laboratory.

In relation to low pressures, however, all of these difficulties fall away at once, and it becomes merely a question of patience to find the limit of constancy and precision to which a gauge of this kind can be pushed.

The conditions of sensitiveness were discussed not long ago

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† Cf. S. P. Langley: "The Internal Work of the Wind," Smithsonian contributions, No. 884, 1893, Washington, D. C.

‡ Considerations of this kind originated, I believe, with the late Prof. Balfour Stewart. Recently the subject has been attacked more seriously, notably by Prof. F. H. Bigelow.

§ Barus: Bulletin U. S. Geological Survey, No. 96, p. 29, 1892. Cf. Proc. American Acad., xxv, p. 106, 1890.

by Prof. Worthington,* and in a more elaborate article by Prof. Greenhill.† There is not much succor to be gained from theory. The most lucid expression of the case is due to Lord Rayleigh.‡ His explanation meets the conditions of the present paper very fully, since the gauges are to be constructed with sharp edges and a spindle-shaped section, in order that the changes of shape occurring may be pure bending. § 8. Stretching would necessarily introduce resistances large as compared with the feeble forces to be measured. Since in the flexure of an inextensible surface, the product of the principle radii of curvature at a point of the surface remains constant (Gauss), any uniform sectional flattening of the walls of the tube due to increased external pressure must be compensated by an increased curvature of the axis; and vice versa.

If the edges are sharp, there seems to be no advantage in increasing the height of the section, for the elastic resistances increase at the same time as the external pressure. There is, on the other hand, a disadvantage in wide tubes, for the length of the helix is thereby necessarily increased and the curl made more cumbersome, while the shape of the section cannot long remain regularly and uniformly arched.

SIMPLE CURLS.

3. *Apparatus.*—There is, therefore, a demand for extremely thin metallic tubes, flattened nearly to the point of actual contiguity of the walls and left with almost sharp edges. Tubes made of metal as thin-walled as 0.01^{cm} are manufactured abroad;§ but American makers usually fall short of this remarkable accomplishment. I found, however, that it was quite possible to derive full advantage from the American tubes by *dissolving* off the outside layers in a suitable acid bath.

The tube at my disposal was made of copper, being like the material I formerly used in my calorimetric work|| This metal is elastically unpromising, apart from the technical advantages of smooth solution in diluted nitric acid, of easy flattening and bending, and of a small modulus of elasticity. Results, however, which can be obtained with copper tubes, can be much improved by the use of other more suitable metals, and for this reason the data of sensitiveness and constancy obtained would be an index of the possibilities of the gauge erring markedly on the side of safety.

Two methods suggest themselves for flattening: the tube

* Worthington: *Nature*, xli, p. 296, 1890.

† Greenhill: *Nature*, xli, p. 517, 1890.

‡ Rayleigh: *Proc. Roy. Soc.*, Dec., 1888.

§ I owe most of my information in relation to thin tubes to Prof. Langley.

|| Barus: *Proc. Am. Acad.*, xxvi, pp. 316–317, 1891. The liquid was examined in a very thin tubular helix of copper, to insure rapid cooling in the calorimeter.

may either be compressed between steel rollers, then neatly coiled on a steel mandrel at the lathe with the turns one or two millimeters apart, so that the finished tube has the general form of a curl, and finally dissolved to the required thinness of wall in the nitric acid bath; or the tube may first be dissolved to the required lightness, then flattened and coiled as stated. The second of these methods is the best, though it requires a long tubular bath with a cistern at the top to avoid spilling the acid during the more or less violent effervescence in solution. The ends of the tube are stopped with corks, covered over with a cement of resin and beeswax, and all imperfect parts of the tube are similarly protected. The progress of solution is tested by taking weights from time to time, and when the tube is so light as to begin to float, it is suitably held down. A spring balance arrangement attached to the tube and out of the reach of the acid fumes, obviates the need of repeatedly removing the tube from the bath. Holes are sometimes eaten through the walls at defective places, probably from galvanic action. Should this occur, the tube must be cleaned at once, and cut apart at the defective place.

The first of the methods sketched above is more convenient, as the curl requires a much smaller bath. Unfortunately solution is most active at the edges; the tube becomes very fragile and is in danger of fissuring on continued use.

Tubes rolled down and coiled under a tense pull are apt to be quite closed. The way in which this can be remedied by *counter-twisting* will be described below (§ 8, et seq.), where closed tubes may even be an advantage. But in this place it is best to stretch two fine wires of brass or copper (say 0.02^{cm} in diameter) from end to end of the tube, so that complete closure is avoided.

An interesting structural result was observed in this work. Open helices, with the turns about 0.3^{cm} apart, after solution became closed helices with the turns all but touching at the edges, showing that the layers of the metal were subjected to unequal strains with a maximum of deformation at the outside.*

As a rule tubes must first be cleansed internally (to remove pulverulent or fatty matter) with ammonia, hydrochloric acid and water, and then thoroughly dried by a current of warm air. The finished curl is closed at one end, with the other

Simple curled Bourdon tube.

* Compare this with the gradual loss of explosive tendency of a Prince Rupert drop, when the external layers are successively etched off (Barus and Strouhal).

soldered to a suitable terminal tube to connect it with the air pump. The accompanying figure 1 shows the curl ready for experiment; *a* is the terminal tube leading to the air pump, *bb* the curl aneroid, *c* a mirror or other form of index, to register the rotation of the lower end of the curl relatively to the upper, when the air within is being exhausted. For convenience in drawing only a few turns of tube are shown in the figure.

4. *Results.*—In the first experiments the object sought was some *guidance* as to the effect of varying the diameter of the curls. Copper tube, with walls 0.025^{cm} thick, was flattened till the spindle-shaped section was about 0.5^{cm} high and 0.08^{cm} wide, then wound so as to make an open helical spring. The diameter of the curl was successively reduced by winding on smaller mandrels, and the corresponding sensitiveness was found by comparing the curl with a mercury gauge when both were joined to the receiver of a Sprengel pump. The index at the bottom end of the centered curl moved over a graduated circle about a foot in diameter and divided in quarter degrees.

Table 1 contains an example of the results. The curl lengths are approximate, due to unavoidable irregularities of winding. In addition to the direct readings I will give the pressure difference in centimeters of mercury per degree of deflection (i. e. per degree of deviation between the terminal tangents of the helix), and the same datum taken per turn of the curl and per centimeter of the length measured along the turns of the helix. In general pressure denotes the difference of pressure between the outside and the inside of the tube and is always given in centimeters of mercury.

It appears therefore, that within the range of accuracy of these experiments, the untwisting of the curl in degrees of arc is very nearly proportional to the pressure difference in cm. of mercury, remembering that the pressure on the outside of the tube acts on the cistern of the barometer. In the second place, the sensitiveness of the coil for a given length of tube is somewhat greater for small radii than for large radii; thus for the diameter 3.1^{cm} , a degree of arc corresponds to 4.07^{cm} of mercury, whereas for the diameter 2.0^{cm} , a degree corresponds to 3.5^{cm} of mercury. The changes, however, are small and irregular and possibly largely influenced by temperature, which was not taken. At all events, the data for cm. of mercury per degree of deflection, per turns of curl, increases so rapidly, that in view of the objectionably great length of curls of small diameter, they offer no advantage. As a rule the diameter 2.8^{cm} will be adhered to in the present paper.

this Journal, xxxii, p. 182, 1886); and with another result (this Journal, xxxiv, p. 183, 1887), in which the limit of torsional resilience of soft iron was reached whenever the obliquity of the external fiber of the wire exceeded about 0.003 radians.

TABLE 1.—*Behavior of Curls of different diameters, for the same length of tube coiled to a helix.*

Description of curl.*	Mercury Mano- meter. cm	Inflection of curl. ° arc	Cm. of Hg per degree of arc.	Cm. of Hg per degree per turn.	Cm. of Hg per degree per cm. of diameter.
Diam. of curl 3.1 ^{cm}					
Turns of curl 21	76.7	18.7	4.07	85	1.3
Length of curl 18 ^{cm}	67.9	16.6	4.07		
	61.5	15.0	4.08		
	54.3	13.2			
	41.5	10.1			
	32.1	7.8			
Diam., 2.8 ^{cm}	76.3	21.1	3.55	87	1.5
Turns, 24	70.5	19.5	3.55		
Length of curl 19.5 ^{cm}	65.3	18.0	3.57		
	59.8	16.4	3.63		
	52.5	14.4			
	47.4	13.0			
	41.4	11.3			
	29.3	8.0			
Diam., 2.0 ^{cm}	60.3	17.4	3.45	121	1.7
Turns, 35	52.0	14.9	3.49		
Length of curl 22 ^{cm}	31.0	8.9			
	0.0	0.0			

Practically, in estimating the effect produced on thinning the walls by etching, I may therefore either express the number of cm. of mercury which correspond to a degree of arc per turn of the tube, in which case the same diameter should occur throughout or be specified; or I may make similar reference to the diameter; or finally (and probably best), I may state the centimeters of mercury per degree of deflection, of the coil per unit (cm.) of length of the tube used in winding the curl. Lumping the results of Table 1, where the length of tube used was about 210^{cm} throughout, it appears that in these initial experiments a pressure of about 780^{cm} of mercury would produce 1° of deflection between the ends (tangents) of a tube 1^{cm} long, and having the size and thickness specified.

The endeavor was now made to dissolve down this curl, but the action was carried too far and it was eventually lost in the process. After about $\frac{1}{3}$ of the weight had been etched off however, 61^{cm} of mercury corresponded to a degree of twist per turn for the diameter of curl 2.9^{cm}, or 550^{cm} of mercury per linear cm. per degree, showing a decided gain over the general data† of Table 1.

5. Another curl, No. V, was now wound, etched off and tested with the following results, after having been exhausted for some time. The original weight was 39^g and after etching

* The length of the curl and the length of the tube used in winding the curl must be carefully distinguished.

† A slight digression was made in coiling the next tube like a watchspring; but the experiment showed no special advantages.

20g, so that the walls must have been reduced in thickness from 0.025^{cm} to about one-half this value,

Diam. of curl, 2.8 ^{cm}	Pressure, 71.9 ^{cm} , Hg	Deflection, 111°
Turns of curl, 22	59.5 ^{cm}	97°
Length of curl 14.7 ^{cm}	0.0	20°
	(later) 0.0	0°
	(next day) 0.0	-2°
Pressure per degree of arc per turn, 14.6 ^{cm} , Hg.		
Pressure per degree per linear cm., 130 ^{cm} , Hg.		

There has therefore been a considerable gain in sensitiveness,* inasmuch as the deflection of one degree between the ends of a linear centimeter of the coiled tube is now equivalent to 130^{cm} of mercury. A new feature has been introduced in the apparent occurrence of marked viscosity, as shown by the gradual displacement of the "zero" reading for pressure difference. The cause of this will be further studied, but it is probably due to the tendency of external pressure to quite close the tube,† so that it takes the air some time to reënter.

This curl was now cut in two parts, V_a and V_b , of 9.5 and 11 turns respectively, in order to test whether the air had quite reached the extreme parts of the tube. An examination showed the following results:

		Pressure:	Deflection:
V_a	Diameter of curl, 2.88 ^{cm}	72.0 ^{cm} , Hg	49.6°
	Turns of curl, 9.5	64.3	43.7°
	Length of curl,	55.8	38.3°
		33.3	23.4°
		0.0	1.6°
	(later)	0.0	0.0°
	Pressure per degree per turn, 14.0 ^{cm} .		
V_b	Pressure per degree per linear cm., 122 ^{cm} .		
	Diameter of curl, 2.8 ^{cm}	62.7 ^{cm} , Hg	56.3°
	Turns, 11	53.9	49.6°
	Length,	46.2	43.8°
		35.6	35.3°
		0.0	7.9°
	(later)	0.0	0.0°
	Pressure per degree per turn, 14.3 ^{cm} .		
	Pressure per degree per linear cm., 127 ^{cm} .		

These results agree substantially with the data for the uncut curl, the discrepancies being due to the difficulties of estimating the number and diameter of the turns and to the occurrence of viscosity. It is seen that viscosity has in no way disappeared, though it is much more marked in V_b than in V_a .

* Data of the table taken in accord with other similar experiments on the same curl.

† There may also be a displacement of the zero due to friction of the walls of the tube on each other. Such displacement, however, would probably be permanent.

From this it follows that the viscosity in question must be in large measure *apparent* but not real. While V_b was preserved for other purposes, V_a was now further etched off. In so doing some of the turns had to be sacrificed, being eaten through. The results in these cases were more irregular, and apparent viscosity more marked. I will not therefore give them in full. After the first solution, 9 turns of V_a showed 13^{cm} pressure per degree per turn, or 107^{cm} Hg per degree per linear cm. After the second solution V_a showed 8.7^{cm} pressure per degree per turn, or 77^{cm} pressure per degree per linear cm. It was impossible to carry the solution further, because the edges were too thin to withstand flexure without fissuring; but the walls were 0.01^{cm} in thickness, and might easily have been reduced. cf. § 3.

Summary.—Remembering, therefore, that if viscosity were excluded, the curl would have been more sensitive, I may state that it takes from 50^{cm} to 75^{cm} of flat copper tubing, the walls being 0.01^{cm} thick, to make a curl aneroid such that a degree of arc shall correspond to 1^{cm} of the barometer. If, therefore, the reading be made with mirror and telescope, with the scale at a distance of 286^{cm} from the mirror, 10 linear cm. of the scale correspond to 1^{cm} of the barometer. If the tube be wound on a mandrel a little over 2.8^{cm} in diameter, 7 or 8 turns would suffice, and the length of the curl need therefore not exceed 8^{cm} . Hence, 10 such curls joined in series end to end and suitably supported, would show 100^{cm} at the telescopic scale per cm. of the barometer, and the curl would be less than a meter long. § 12.

COUNTER-TWISTED SYSTEMS.

7.—In the results thus far, the object has been merely to exhibit the possibilities of the curl aneroid. If the apparatus is to attain precision, the apparent viscosity must be brought quite under control, and the effect of the temperature of the medium evaluated and reduced to the smallest margin. I have in a measure fulfilled both these conditions by using counter-twisted systems in the way presently to be specified. As a first step in this direction, I will cite some data obtained with curl VIII, in which two fine brass wires (diameter 0.02^{cm}) were drawn through the tube before flattening and coiling. The walls were 0.013^{cm} thick, and all but 5.2 turns were lost during solution.

Diam. of curl,	2.8^{cm}	Pressure, 73.7^{cm} , Hg.	Deflection, 38.3°
Turns,	5.2	63.2	33.4°
Length of curl, 3.3^{cm}		48.5	26.0°
		0.0	0.6°
		0.0	0.0°
Pressure per degree, per turn, 10.4^{cm} .			
Pressure per degree, per linear cm., 91^{cm} .			

The small residue of viscosity left here is very probably not even yet a true phenomenon, i. e. it is due to strictures which prevent the free passage of air, and to friction between the contiguous walls of the tube and wire; but the improvement over the preceding cases is obvious.

8.—To enter into this question somewhat more fully and from a different point of view, a wider tube was selected (the walls of which were but 0.12^{cm} thick), flattened and coiled *without* closing the section. Indeed a blunt edge was left and the section was about 0.06^{cm} broad and 1.6^{cm} high. The data for this coil (IV) were as follows:

Diam. of curl, 3.2 ^{cm}	Pressure, 75.9 ^{cm} , Hg.	Deflection, 11.1°
Turns of curl, 4.5	69.5	10.3°
Length of curl, 14.0 ^{cm}	57.8	8.8°
	41.3	6.6°
	36.8	5.9°
	0.0 zero.	0.0°
	Pressure per degree, per turn, 32.5 ^{cm} .	
	Pressure per degree, per linear cm., 330 ^{cm} .	

In this curl there is no evidence of viscosity, but the relation of pressure-difference and deflection is not quite linear, as was the case with sharp-edged coils. The data are mean ratios. The sensitiveness (330^{cm}, Hg, per degree per linear cm.) is of low order, in spite of the thin walls (0.012^{cm}) and broad tube. § 2.

This curl was now replaced on the metallic mandrel, and the edges hammered quite sharp from end to end. On removing the curl from the mandrel I found that *no air* could be sucked through it. The walls, therefore, overlapped each other, *imperviously* to air. When, however, the curl was somewhat uncoiled in the hands, the air came through quite freely. This suggested a novel method of making the curl aneroid, requiring no inclosed wires, and partaking of other advantages, since the uncoiling can be done with a suitable spring. In a counter-twisted system of this kind:

(1.) The sharp-edged coils can be opened by an amount compatible with the free access of air. Therefore this system is adapted for extreme sensitiveness.

(2.) The system is differential; or, in other words, the *differences* of viscosity and of elasticity of curl and spring, and the *differences* of the thermal variations of these quantities come into play. Thus if a spring and curl could be made having the same effective viscosity* and the same thermal coefficients of viscosity and elasticity, respectively, *the system would be perfectly elastic and independent of temperature*; or,

* Depending therefore both on the material and on the lengths of the two helices.

(3.) If the adjusted curl aneroid, exhausted and therefore twisted by external pressure, be untwisted to the same degree by the spring, the curl is now nearly without strain, excepting such of higher orders; and the viscosity and thermal relations of viscosity and rigidity of the counter-twisting spring alone come into play. Hence a metal of low rigidity may be chosen for making the curl, while the counter-twisting spring is selected for high viscosity. One would use, for instance, hard steel annealed at 400° or even a sufficiently thick quartz fiber.

It is clear that the counter-spring must be weak, so that it may make a number of rotations for each rotation of the curl; for the resistance of the counter-spring is incremented at the expense of the deflection of the curl. Hence a long fiber, or spiral spring, or a watch spring is adapted. Finally, to prevent knotting of the spring, a weight is (*temporarily*) added to the system, preferably suspended in a basin of water to deaden vibrations. The curl aneroid has now taken the form of figure 2, where *cc* is the curl, communicating at one end, *k*, with the air pump. The other end is closed and carries the horizontal, radial stylus *f*, to which the stiff wire *dd* of the spring *bb*, the mirror *e* and the damping plate *g* are attached symmetrically to the curl. The plate *g* is of lead, thus serving as a weight (as well as a damper) to pull out the spring *bb*. A dish *h* (shown with *g* in cross section) containing a liquid, surrounds *g*. The spring *b* is attached above to a rigid clamp at *a*, by means of a stiff wire *l*, the clamp (not shown) being so constructed that the spring can be both raised and twisted.

The weight *g* and the damping arrangement are only permissible in calibration work like the present. If the registry of the aneroid is to be instantaneous, the mass must be kept down to the lowest limit possible, and no ballast is to be attached. Hence a delicate watch-spring, or a double helical spring is preferably employed, and a suitable method of damping vibrations must be investigated. An elastic fiber fastened at both ends, with the end of the helix attached near the middle, is also contemplated.

Counter-twisted system of curled Bourdon tube, spring and appurtenances.

9.—To obtain preliminary evidence, I took curl IV, just discussed (§ 8), though it had become somewhat leaky from hard usage. The counter-twisting was done by a long steel wire with the following results:

Turns uncoiled from 4.5 to 3.7.	{	Pressure per degree, per turn, 13.4 ^{cm} , Hg.
		Pressure per degree, per linear cm., 167 ^{cm} .
Turns uncoiled from 4.5 to 3.0.	{	Pressure per degree, per turn, 5.1 ^{cm} , Hg.
		Pressure per degree, per linear cm., 79 ^{cm} .

In all these cases the deflections were instantaneous, and there was no apparent viscosity. The curl itself has been improved from one of low sensitiveness (330^{cm}) to nearly the same range of high sensitiveness (77^{cm}), actually obtained in § 6 for extremely thin copper tubes.

This result deserves special study; but it already appears that so long as the coil is sharp-edged in section—so long as the strain is a case of nearly pure bending—the coil increases in sensitiveness as its spindle-shaped section is more highly arched. The smaller the medial radius of curvature of a right section of the tube, the greater proportionately is its variation for the same pressure-difference; and the greater proportionately must the corresponding variations of the longitudinal radius of curvature (coil radius) also be, since the product of two radii is to remain constant. Hence *a rotundly arched, spindle-shaped section, maintained in an excessively thin-walled tube, is compatible with the greatest amount of rotation at the registering index.*

10.—A copper tube having walls 0.01^{cm} thick was flattened and rolled down as usual. The curl (No. VII) contained 9 turns and it was 2.8^{cm} in diameter and about 12.5^{cm} long, each turn in section being 0.8^{cm} high, with blunt edges about 0.05^{cm} wide. So constructed, the sensitiveness (conformably with the data in § 8) was low, for the pressure in cm. Hg per degree per turn was 50, and the pressure per degree per linear cm. of the curl 440^{cm}.—in spite of the thin copper walls stated. This tube was now hammered flat and sharp-edged on the steel mandrel. A part, VII_b, free from flaws was then cut from this tube and used in the following tests, Table 2. For want of better material a helical spring of brass spring wire was used in the counter-twisting and the system was weighted to prevent knotting (cf. § 8).

These results are given graphically in the chart figure 3, where the ordinates are pressures in cm. of mercury, corresponding to a deflection of 1° for a length of 1^{cm} measured on the turns of the curl. The abscissas show the amount of counter-twisting at the lower end of the system in degrees. The sensitiveness therefore increases in marked degree with

the amount of counter-twisting, and apparently reaches a limit at about 150° . From here on, however, the evidences of permanent set were met with. They are in the direction of the uncoiling, showing the spring to have acted on the curl. At the end of the experiment, when the spring was released, about 90° of permanent set had been imparted to the curl. I believe therefore, that the curl could have been made even more sen-

Chart showing the inverse sensitiveness (ordinate) of a counter-twisted system varying with the amount of counter-twist (abscissa).

sitive than shown in figure 3, if its material had been resilient and of high viscosity. At all events, the datum reached (50^{cm} Hg corresponding to a degree of deflection between the ends of a linear cm. of the turns of the curl) is markedly in excess of the results given at the end of §7, while the apparent viscosity of the metal has now no serious significance. Indeed since the viscous set above was actually negative (uncoiled curl), whereas in the simple curl (§5 et seq.) it is positive, a virtual elimination of viscosity is clearly possible.

TABLE 2.—*Showing the increased sensitiveness resulting from increased counter-twisting.*

Turns of VII ₆ No.	Counter-twist in degrees of arc.	Pressure per degree, per turn. cm., Hg.	Pressure per degree, per linear cm. cm., Hg.	Remarks.
2.5	0	13.6	120	No viscosity.
2.3	60	9.9	95	" "
2.1	100	6.6	69	" "
1.9	140	4.3	50	" "
1.7	180	4.3	56	{ Curl shows negative viscosity. Curl shows 90° set after releasing spring.
2.3 (released)	0	10.0	98	

11.—I want finally to put the remarks made in the above paragraphs to an actual test in continuous series of measurements with mirror and scale. A figure of the apparatus is given in § 8. A curl, No. II, was wound for the purpose, of copper tube. A very small leak was purposely left, so that the exhausted curl might gradually (several days) fill with air, while comparison with the attached mercury manometer were in progress. Preliminary tests made on the graduated circle showed the following results:

Description.	Pressure.	Deflection.	Pressure	Pressure	Pressure
	cm., Hg.	°arc.	per degree.	per degree, per turn.	per degree, per linear cm.
Diameter curl, 2.8 ^{cm}					
Turns, 15.5	76.80	84.75	0.926	14.1	124
Length, 12 ^{cm}	57.92	64.80	0.894		
	40.70	45.75			
	0.0	0.0			

The relation is not linear, though the discrepancy may be thermal. The curl is only *moderately* sensitive (for in § 10 this sensitiveness has been doubled), and not quite free from viscosity. It is therefore interesting to see in how far precision can be obtained with this short and not very favorable coil, when suitably counter-twisted.

The spring used in counter-twisting was of brass spring wire, 0.05^{cm} diameter. It consisted of 31 turns about 1.5^{cm} in diameter and when stretched was 1.7^{cm} long. The counter-poise weighed something over 25^g.

The system was adjusted by first exhausting the curl aneroid to a nearly perfect vacuum, after which the mirror was brought back to its original position (that is before exhausting) by the spring. There was thus but little strain left in the curl. The method adopted was therefore the third in § 8.

A thermometer hung beside the mercury manometer and the curl, giving the temperature of both, though not closely enough (as observation showed). A fine bulb thermometer placed within the coils is essential. The scale was at a distance 3.5^{met} from the mirror.

Of the two continuous series of observations made I shall, for brevity, only give an example of part of the second. The first is defective in temperature. Table 3 contains the results, showing in the first three columns the height (*h*) of the mercury in the manometer (reduced to 0° C.), the temperature (*t*) of the air near the curl, and the reading (*s*) in the telescope of the deflections of the curl. Observations were made in groups of four or more, during different parts of the day for different days. Temperature was very variable, thus adding to the severity of the test.

The column headed *B* is the virtual barometer, to be obtained in a way presently to be given.

TABLE 3.—*Showing the behavior of the curl aneroid as compared with the mercury manometer.*

<i>b</i>				<i>t</i>				<i>B</i>			
Manometer.	Curl.	Tem-	Manometer	Manometer.	Curl.	Tem-	Manometer	Manometer.	Curl.	Tem-	Manometer
cm., Hg.	cm.	perature.	at 25°.	cm., Hg.	cm.	perature.	at 25°.	cm., Hg.	cm.	perature.	at 25°.
		° C.	cm., Hg.			° C.	cm., Hg.			° C.	cm., Hg.
75.49	54.75	25.6	75.53	75.69	58.80	29.0	75.99	75.49	54.75	25.6	75.53
74.89	49.40	25.8	74.95	75.16	54.30	28.8	75.44	74.89	49.40	25.8	74.95
74.58	46.70	26.0	74.65	74.36	46.30	28.4	74.61	74.58	46.70	26.0	74.65
73.95	40.30	26.3	74.05	72.97	32.00	28.0	73.19	73.95	40.30	26.3	74.05
75.58	56.50	27.3	75.75	76.06*	57.40	23.1	75.92	75.58	56.50	27.3	75.75
75.38	54.60	27.3	75.55	75.29	50.70	23.1	75.15	75.38	54.60	27.3	75.55
74.95	50.60	27.5	75.14	74.72	45.00	23.2	74.59	74.95	50.60	27.5	75.14
74.66	47.90	28.0	74.88	74.02	38.40	23.6	73.92	74.66	47.90	28.0	74.88
75.59	57.60	29.2	75.90	76.01	58.00	23.5	75.90	75.59	57.60	29.2	75.90
75.15	54.00	29.6	75.50	75.69	54.90	23.5	75.58	75.15	54.00	29.6	75.50
74.22	44.70	30.0	74.59	75.18	50.00	23.5	75.07	74.22	44.70	30.0	74.59
73.89	41.60	29.8	74.24	74.76	45.80	24.2	74.70	73.89	41.60	29.8	74.24
75.57	57.90	30.2	75.96	76.46†	60.70	21.8	76.22	75.57	57.90	30.2	75.96
75.35	56.50	30.2	75.74	75.70	54.55	22.3	75.50	75.35	56.50	30.2	75.74
74.95	52.30	30.0	75.32	75.22	50.50	22.8	75.05	74.95	52.30	30.0	75.32
71.92	21.10	29.6	72.25	73.94	37.85	23.4	73.82	71.92	21.10	29.6	72.25
75.54	57.45	29.4	75.87	76.36	63.00	26.0	76.44	75.54	57.45	29.4	75.87
74.95	52.40	29.2	75.26	75.86	57.30	26.1	75.92	74.95	52.40	29.2	75.26
74.45	46.60	29.2	74.76	75.42	54.50	26.3	75.52	74.45	46.60	29.2	74.76
74.02	42.70	29.2	74.33	74.78	47.60	26.2	74.87	74.02	42.70	29.2	74.33
75.64	58.50	29.4	75.97	76.34‡	57.60	20.8	76.02	75.64	58.50	29.4	75.97
75.16	54.30	29.4	75.49	75.93	54.65	21.0	75.63	75.16	54.30	29.4	75.49
74.33	45.80	29.1	74.63	75.49	51.10	21.2	75.20	74.33	45.80	29.1	74.63
73.38	36.10	29.0	73.67	74.86	45.20	21.6	74.61	73.38	36.10	29.0	73.67

* One day after.

† Three days after.

‡ Four days after.

If the deflections (*s*) are plotted as a function of the pressure (*b*), a series of detached lines are obtained, which usually lie at some distance apart and are not quite straight even for a single group. Inspection shows these discrepancies to be principally due to temperature. It is not at once obvious how the temperature coefficient is to be computed, unless it be assumed, conformably with § 4, that the corrected loci are really straight.

The following is a method of calculating the temperature corrections without entering into unduly complex computations: Let *s* be the deflection (scale reading) at the pressure *b* and the temperature *t*; let *s'*, *b'*, *t'* have a similar relation, and let *s*₀, *c* and *α* be constants. For any two groups, put

$$\begin{aligned}(s + s_0)(1 + \alpha t) &= bc, \\ (s' + s_0)(1 + \alpha t') &= b'c.\end{aligned}\quad (1)$$

If by aid of the chart of detached lines just referred to, the values of *b* be taken for *s* = *s'*, then

$$\alpha = \frac{b' - b}{bt' - b't} \quad (2)$$

Proceeding thus, I found $\alpha = 0.001$; and with this value I reduced the observations by placing (1) under the form

$$s = \frac{b}{1 + \alpha t} c - s_0 = Bc - s_0 \quad (3)$$

Hence s is a linear function of B ; and this postulate can be tested graphically without computing c and s_0 . B is given in the last column of Table 3.

The conditions under which these data were obtained were very unfavorable. As a result of the delicate suspension, the jar of wagons and cars passing the laboratory often made it impossible to read the scale millimeters in the telescope. Moreover I was unprepared for so large a temperature coefficient, α , as has just been adduced; and I did not therefore take such special precautions against currents of air which should have been taken, seeing that the thin metallic helix adjusted itself at once to temperature, whereas the thermometer follows sluggishly after. Nor was the latter placed near enough to the coil. Hence since a single degree of temperature corresponds to nearly 0.1^{cm} of the barometer, discrepancies* of half a millimeter of pressure must have been incident to the work, and the table and chart bear this assertion out. Points which lie out of position are isolated in each group and the discrepancy is the result of a current of air.

In spite of the unfavorable choice of metals (copper and brass) the behavior of the copper curl is thus seen to be satisfactory so far as the present purposes go: for a development of principles of construction has only been aimed at. The telescope reading for this small curl is 9.8^{cm} per cm. of the mercurial barometer.

12. *Conclusion.*—Having reached this stage of progress, it seemed expedient to break off the work, for it would not be worth while to proceed with fine measurement without making the system of more highly viscous material at the outset. In brief: tubing preferably of resilient brass 3–4 millimeters in diameter or less, with walls 0.01^{cm} thick or less (by solution) is desirable. The tube should be heated in steam to remove excesses of drawn strain and thereafter flattened and coiled until the walls all but touch. The counter-twisting spring is to be either helical or watchspring-shaped, and of steel annealed at 400°C. , or a suitable long quartz fiber. A long curl would not be self-sustaining; but it could be made so without interfering with its free action, by a series of equidis-

* In the face of the large thermal effect discussed in the text it would be useless to endeavor to evaluate the effect of viscosity. Nor would a metal of low viscosity, like a spring of drawn brass, be chosen in tests of a final character.

tant radial spokes attached at the inner face of the curl with their central ends fastened to a fiber of silk.

If, therefore, the conditions investigated at the end of §§ 6, 10, be called to mind, a non-differential curl aneroid (§ 8, case 3) less than a meter long, provided with a mirror for registry, will give account of variations of atmospheric pressure of a thousandth of a millimeter of the barometer, provided the mounting is sufficiently free from tremor, and temperature be kept constant to a few thousandths of a degree during the interval of observation.

These conditions will be much less severe if the parts of the counter-twisted system are especially chosen (as stated in § 8, case 2) and twisted with reference to viscosity, rigidity and temperature. Indeed the chief result of the present paper is the exhibition of the properties of the counter-twisted system. A continuous mechanism has been brought forward which not only minimizes the hurtful effects of viscosity and of the thermal changes both of viscosity and of rigidity, but which accomplishes these desirable results in such a way as to remarkably increase the sensitiveness of the instrument.

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