AN EXPERIMENTAL DETERMINATION OF THE PERIOD OF ELECTRICAL OSCILLATIONS.¹

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[The following research was carried out in August, 1893, and the manuscript has been out of my possession until within a few weeks. In the meantime extended articles on the same subject have been published by Tellqvist¹ and Seiler.² As however the methods and instruments here employed are quite different from theirs, and as the oscillations studied by me were of much shorter period, and free from the influence of the dielectric in mica condensers, and furthermore as the method also lends itself to a determination of the important quantity " v ," in a way not depending on determinations of the Ohm, I feel warranted in publishing it unchanged.]

The question whose solution is here attempted concerns the experimental verification of the formula giving the period of the oscillations of an alternating current arising when a charged condenser is discharged through a circuit containing self-induction. Any such investigation will necessarily take as a point of departure the classical researches of von Helmholtz³ on Electrical Oscillations, published over twenty years ago, and continued in an exhaustive manner by Schiller.⁴ It consequently remains only to measure absolutely the quantities involved, according to the system of electrical units with which we are now familiar.

Such is the scope of the present investigation. The method, though similar to those of von Helmholtz and Schiller, makes use of different instruments designed and constructed for the purpose, and the experimental arrangement is quite different. In the experi-

¹ This investigation was awarded the Elihu Thompson Prize. See *Science*, Vol. I, p. 190 and p. 294; also *Nature*, Vol. 51, p. 324. (Ed.)

² Tellqvist, *Wied. Ann.* 60, p. 248, 1897. Ueber die Oscillatorische Entladung von Condensatoren.

³ Seiler, *Wied. Ann.* 61, p. 30, 1897. Ueber Oscillationen bei der Ladung von Condensatoren und ihre Anwendung zur Bestimmung des Selbstpotentials beliebiger Leitungssysteme.

⁴ Helmholtz, Ueber Electriche Oscillationen, *Ges. Werke*, I., p. 531.

⁵ Schiller, *Pogg. Ann.* 152, p. 535, 1874.

ments of Schiller a primary current flowing through an induction coil was broken by a falling pendulum, and the potential-difference at the ends of the secondary coil connected with a condenser were measured at subsequent instants by means of a quadrant electrometer. I have also measured the potentials by means of an electrometer, but have avoided the use of an induction coil, in order to simplify the conditions. It is known that the layers of a coil of wire in which a current flows act as a condenser, of which it is impossible to calculate the capacity, and in the case of a long induction coil of fine wire this capacity may be considerable in amount. This accordingly introduces an element of doubt, which it is desirable to eliminate. Now any coil of wire having perceptible self-induction will in some degree possess this undesirable capacity, but it may be made small by properly choosing the coil.

Coefficients of self-induction, as well as capacities, are ordinarily measured in terms of resistances and times, so that Messrs. Ayrton and Perry were led to propose the name *secohm* for what is now generally known as the henry. In order to avoid this dependence upon the value of the ohm, I wound a coil, the coefficient of self-induction of which could be calculated directly from its geometrical measurements. For the condenser, for a similar reason, and in order to be free from effects due to electric absorption, dielectric hysteresis, and conductivity, I made use of an air-condenser whose capacity is known in electrostatic measure from its geometrical measurements. In this manner, besides verifying the formula, I obtain a method of determining " ν ," the ratio between the two electric units.

THE INSTRUMENTS.

The pendulum interrupter¹ of von Helmholtz is well adapted to its purpose, but I determined to attempt to improve it in several respects. In the first place, the falling body, the pendulum, exposes a large and irregular surface to the air, so that the air resistance must be considerable. Secondly, the time-value of the readings of the instrument is invariable, and the absolute value is difficult to determine. The determination of Schiller by Pouillet's galvano-

¹ Vid. Schiller, *loc. cit.*

metric method is inconvenient and doubtful. I accordingly determined to replace the pendulum by a body falling freely, of such a shape that the air resistance would be negligible, or at least constant, so that the time-values could be directly determined from the height of fall, which might be varied. The instrument (Fig. 1),

Fig. 1a.

which will be referred to as *the interrupter*, consists of a solid bar *B* (firmly secured to the wall by two heavy brackets, in which it could be placed vertically by screws), carrying three carriages, two of which slide vertically upon the bar, while the third is screwed to it near its lower end. The upper carriage, *A*, carries an electromagnet, *M*, which supports the projectile, *P*. The proper design of the latter to insure an accurately straight fall was the most difficult point of the whole research, and it was many months before the interrupter was deemed sufficiently perfect for its work. The lower end of the core of the electro-magnet was of the shape shown in Fig. 1a, the conical cavity in the end having a more acute angle than the cone upon the end of the projectile, in order that the projectile upon breaking the current might fall directly without sliding down sidewise upon the cone, and thus being diverted from its proper course. The arrangement first tried of a sharp end to the core and a hollow end for the projectile never succeeded in giving a straight fall.

The projectile (shown half size in Fig. 1 b¹) was accurately turned of steel and its lower portion was hollow and filled with mercury. In its end was inserted a cylindrical piece of agate. The projectile weighed in the neighborhood of five hundred grams. At the lower part of its course the projectile struck off the two contact-levers 1 and 2, the former being held by the

Fig. 1b.

¹ Since replaced by a longer projectile, shown in the photograph.

carriage *C*, secured in place by three set-screws, the latter being movable on a slide *G* by means of a micrometer screw *S*, upon the fixed carriage *D*. The contact levers, Fig. 1c, were made of hardened steel, as light as was consistent with rigidity, and pivoted upon axes turning upon pointed screws *a*, finished by grinding. Contact was made with platinum pointed screws *c*, clamped with

Fig. 1c.

nuts, carried upon standards standing upon a separate piece of ebonite, *e*. In order that the levers might be held against the contacts with the smallest possible, invariable force; instead of springs small permanent magnets, *m*, were used, cut in the form of screws, so that their distances from the levers might be adjusted to give just the proper pressure. The whole contact apparatus stood upon a base of ebonite,

e'. The projectile was caught, after passing through the apparatus, in a wooden cylinder lined with velvet and chamois skin. The contact-levers after being struck off, were caught in spring clips *S*, to prevent them from rebounding and to connect them to earth. The results attained by the performance of this apparatus may be judged by the fact that on several days the position of the zero, determined by the method described later, did not vary by an amount corresponding to an interval of one five-millionth of a second.

The condenser was a plate-condenser, exactly similar to that used and described by Himstedt¹ in his determination of "v," and consisted of two steel plates half a meter in diameter and seventeen millimeters thick, constructed for me by Messrs. Breithaupt und Sohn in Cassel, Germany. They were ground plane, nickel-plated and polished on one side, and separated by sets of three small cylinders cut from plates of plane and parallel glass. These cylinders before use were cleaned in alcohol and distilled water in which they were left for some time, and then carefully dried with filter-paper, as described by Himstedt.²

¹ Himstedt, Eine Bestimmung der Grösse "v," Wied. Am. 29, p. 560.

² The condenser was supported upon ebonite cones resting upon three glass legs, about 60 cm. from the floor, and over two meters from any object.

The capacity was calculated by the formula given by Kirchhoff,¹ and amply confirmed experimentally by Himstedt, by comparison with a guard-ring condenser.

$$K = \frac{r^2}{4d} + \frac{r}{4\pi} \left[1 + \log_e \frac{16\pi r(b+d)}{d^2} + \frac{b}{d} \log_e \frac{b+d}{b} \right]$$

where r is the radius and b the thickness of the plates, d their distance apart.

The electrometer was a modified Mascart-Kelvin quadrant electrometer, reconstructed in the workshop of the university. The suspension was a long quartz fiber of such a diameter as to give a period of twelve seconds for a half swing. The whole instrument was carried in a thin glass jar partly filled with sulphuric acid and serving as Leyden jar. From the acid the needle was charged by means of a platinum wire just dipping into the acid, and furnishing damping enough to bring the needle to rest in four or five minutes, the nearest interval, accordingly, at which observations could succeed each other. For a reason given later, it was essential that the equality of the coefficients of induction between the needle and quadrants should be capable of accurate adjustment, and hence beside the coarse adjustment possible by sliding one quadrant in and out by hand, the opposite quadrant was given a slow motion by means of a micrometer screw, as in the Kelvin electrometer. The cylinder carrying the torsion head was also revolved by a micrometer screw, in order to bring the needle to precisely the right angular position. The torsion head, and hence the needle hanging from it, was adjustable in all three coördinates, and a last adjustment was to be found in the levelling screws supporting the instrument.

The instrument was observed by a telescope and scale at a distance of three and a-half meters, and the sensibility was such that with the needle charged by fifty cells of water-battery, the deflection given by seven Leclanché cells was 105 mm.

The self-induction was furnished by a coil of copper wire insulated with a double winding of silk, wound in a groove two centimeters wide in a circular frame of wood, constructed of a great many

¹ Kirchhoff, Monatsber. and Berl. Acad., 1877, p. 144.

tion, j , rising from the slide. The other slide was generally clamped by a tightening screw. The two slides carried the two plates g and g' , and the three mirrors 1, 2 and 3. A monochromatic or white light was placed at l , and the eye at c . When the adjustments were made with monochromatic light, fringes were seen in g , and as the screw S was turned one fringe passed a marked point for every half wave-length advanced by the screw. When the fringes were brought to that point where they were most distinct (maximum visibility) white light was substituted for monochromatic, and on turning the screw slowly, the few colored fringes, which alone exist in white light, were soon found. The central white fringe makes an admirable zero position, and on taking a reading was always brought to a fixed mark on the mirror. To measure a cylinder the fringes were first brought to the marked position, and a reading taken on the screw head, which read to thousandths of a millimeter. The cylinder to be measured was then inserted between the point of the screw and the projecting piece j , and the screw turned back until the central fringe again fell on the mark. A second reading was then taken, and the difference of the two gave the thickness of the cylinder. Each cylinder was measured four times, being turned end for end each time, and the zero point was found before and after each set.

The results for all three different thicknesses are given.

	$d_1 = 6_1 \text{ mm.} +$		
	a	b	c
Readings	225.0 μ	219.0	225.0
	223.5	218.0	223.0
	223.0	221.0	223.0
	222.0	219.0	223.5
Means	223.4	219.2	223.5
	Mean of three cylinders 222.0		
	Zero reading 80.0		
	$d_1 = 6.1420 \text{ mm.}$		

$$d_2 = 3.8 +$$

	<i>a</i>	<i>b</i>	<i>c</i>
Readings	016.0 μ	013.0	011.0
	016.0	012.0	011.0
	016.0	012.0	011.0
	016.0	012.0	011.0
Means	016.0	012.25	011.0

Mean of three cylinders 0133

Zero reading 1590

$$d_2 = 3.8543 \text{ mm.}$$

$$d_3 = 1.9 +$$

	<i>a</i>	<i>b</i>	<i>c</i>
Reading	108.0	109.0	115.0
	108.0	109.0	114.5
	108.0	110.0	114.0
	108.0	111.0	114.0
Means	108.0	109.75	114.37

Mean of three cylinders 110.71

Zero reading 161.50

$$d_3 = 1.9492 \text{ mm.}$$

The screw was calibrated by measuring the thickness of one of the shortest cylinders in six successive places on one centimeter of the screw, the zero being shifted by moving the slide *B* the required distance each time, and reclamping it. The error of the screw was found too small to take into account. Finally, the calibrated part of the screw was compared with a centimeter of the meter bar.

To the capacities as calculated from these data were added small corrections for the dielectric constant of the glass cylinders, 5.2 mm. in diameter, for which purpose the dielectric constant of the glass was assumed to be 5. The values of the capacities obtained were :

$$K_1 = 275.56 \text{ cm.}$$

$$K_2 = 428.145$$

$$K_3 = 826.50$$

The coefficient of self-induction of the coil was calculated from the formula given by Lord Rayleigh.¹

$$L = 4\pi an^2 \left[\log_e \frac{8a}{r} + \frac{1}{12} - \frac{4}{3} \left(\vartheta - \frac{\pi}{4} \right) \cot 2\vartheta - \frac{1}{3} \pi \operatorname{cosec} 2\vartheta \right]$$

¹ Rayleigh, On the Absolute Value of the *B. A.* Unit, Phil. Trans., 1882, Pt. II.

$$- \frac{1}{6} \cot^2 \vartheta \log_e \cos \vartheta - \frac{1}{6} \tan^2 \vartheta \log_e \sin \vartheta \Big]$$

where n is the number of turns, a is the mean radius of the coil, r the diagonal of the rectangular cross-section, and ϑ the angle made by r with the plane of the coil. A calculation by the formula (58) p. 170, Mascart and Joubert, *Traité d'Electricité et Magnétisme*, confirmed this result.

The diameter of the coil was measured by the cathetometer and the circumference of the groove before and after winding was measured with a steel tape. The mean radius was thus found to be $a = 22.878$ cm.

The width of the channel being 2 cm., and the depth of the winding 1.766 cm.,

$$r = 2.668 \text{ and } \vartheta = 48^\circ 32' 44''$$

There were 572 turns, so that the final result is

$$L = 3.1811 \times 10^8 \text{ cm.}$$

The resistance of the coil was found to be 55.95 ohms.

The time value of one division of the head of the micrometer screw of the interrupter was calculated from the height of fall of the projectile and the value of the acceleration of gravity. It was intended to make a chronographic determination of this element, but the time set for the completion of the investigation did not permit of this. The accuracy of this means of determination is, however, sufficient for the present purpose.

The acceleration of gravity was calculated from the formula

$$g = 980.6056 - 2.5028 \cos 2\lambda - .000003 \, h$$

with $\lambda = 42^\circ 17' 20''$, $h = 16000$ cm. giving

$$g = 980.321 \text{ cm./sec.}^2$$

The height of fall was found by means of a cathetometer by the Société Genèvoise secured to the wall close by the interrupter. The micrometer of the upper telescope was set upon the sharp edge of the piece of agate in the end of the projectile when hanging from the electro-magnet, the micrometer of the lower telescope being at the same time set upon the upper edge of the lever of fixed height, which was struck by the projectile, the distance was then

measured upon a meter bar attached to the brackets supporting the interrupter. This distance was found to be invariable during the course of the experiments and was

$$h = 73.405 \text{ cm.}$$

The time of traversing the vertical distance Δh is $\Delta t = \frac{\Delta h}{\sqrt{2gh}}$ if we may neglect Δh in comparison with $4h$, but as this was not the case in all the experiments, the term involving the second power of Δh in the development of $t = \sqrt{\frac{2h}{g}}$ was made use of, giving

$$\Delta t = \frac{\Delta h}{\sqrt{2gh}} \left(1 - \frac{\Delta h}{4h} \right).$$

The factor of correction thus obtained was $\left[1 - \frac{d}{1321200} \right]$ where d is the reading of the drum a of the micrometer. The micrometer screw, by the well-known firm of Brown and Sharpe, was compared with the standard centimeter and found to be correct with a pitch of one-half millimeter, and the drum was divided into 225 parts. Accordingly τ , the value of one division, is

$$\tau = .00000058577 \text{ sec.,}$$

or 1707.1 divisions equals one one-thousandth of a second.

EXPERIMENTAL ARRANGEMENTS.

Several different arrangements of the electrical connections were successively tried, and they will be described in turn, although they

Fig. 3 A.

were eventually abandoned for the last one devised. In all the figures (Fig. 3) B denotes the battery, R the resistance of the coil of self-induction L , K the condenser, K_0 the electrometer, 1 and 2 respectively the upper and lower contact levers, and E the earth connection, which was very thoroughly made by soldering in several places to a gas pipe.

In the first and simplest arrangement A , one pair of quadrants of the electrometer is connected permanently to the earth. As long

as the contact 1 is closed, the condenser and electrometer, which is connected in parallel with it, are short-circuited, both plates being at the potential of the earth. When 1 is broken, the battery begins to charge the condenser through the resistance R and self-induction L , and the potential is represented at any subsequent time t by

$$v = v_0 \left(1 - e^{-\frac{Rt}{2L}} \cos \sqrt{\frac{1}{KL} - \frac{R^2}{4L^2}} t \right)$$

At any time t the contact 2 is broken, the electrometer is insulated, and the needle swings out under the influence of the charge that it had at potential v at the time t . The time t is vanishingly small in comparison with the time of swing of the electrometer needle, which being, therefore, at rest during the whole time t has a constant capacity K_0 , which is to be added to that of the condenser K . After the electrometer is insulated and begins to swing, to be sure its capacity changes, but its charge remains constant, and experiment shows that the deflections are proportional to the potentials of the quadrant at the instant when contact 2 was broken. The method of verifying this statement by a calibration will be described hereafter.

The objections to this method are : first, that it is not a zero method, and although the curve of potential (a) might be carefully observed, the determination of the time of oscillation, which is the quantity required, would be difficult. Furthermore, in order to observe any considerable number of oscillations, the damping must be small, and the resistance R must be so small that the battery would be likely to polarize. Lastly, for the same reason, the resistance of the contact 1, which cannot be assumed to be absolutely zero, would have a ratio to R that could not be neglected, and hence the electrometer would not stand exactly at zero before the break.

Fig. 4.¹

It now occurred to me that since the deflections of the electrom-

¹ The letters a, b, c, d , in Figs. 4 and 5, refer to the arrangements A, B, C, D .

eter are proportional to the differences of potentials of the two pairs of quadrants, a zero method might be obtained by suddenly connecting the other pair of quadrants, which in $\mathcal{A}$ is permanently connected to earth, to a source of constant potential, and making this equal to the potential $\mathbf{v}$ expected at the end of the time t . To this end it was first necessary to ascertain whether the needle would really stand at rest when the quadrants were both electrified to the same potential other than zero. In order to attain this result it was necessary to adjust the positions of the quadrants and needle very carefully.

Let us consider the conditions of equilibrium of a slightly unsymmetrical electrometer. If the potentials of the two quadrants and the needle be $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ respectively, the charges c_1 , c_2 , c_3 , we have

$$\begin{aligned} c_1 &= q_{11}\mathbf{v}_1 + q_{12}\mathbf{v}_2 + q_{13}\mathbf{v}_3 \\ c_2 &= q_{12}\mathbf{v}_1 + q_{22}\mathbf{v}_2 + q_{23}\mathbf{v}_3 \\ c_3 &= q_{13}\mathbf{v}_1 + q_{23}\mathbf{v}_2 + q_{33}\mathbf{v}_3 \end{aligned}$$

where under the ordinary conditions of use, namely, small deflections ϑ ,

$$\begin{aligned} q_{11} &= c_1 + a_1\vartheta & q_{12} &= c_4 \\ q_{22} &= c_2 - a_2\vartheta & q_{13} &= c_5 - a_1\vartheta \\ q_{33} &= c_3 + (a_1 - a_2)\vartheta & q_{23} &= c_6 + a_2\vartheta \end{aligned}$$

The energy of the system is

$$\begin{aligned} W &= \frac{1}{2} \sum c\mathbf{v} = \frac{1}{2} q_{11}\mathbf{v}_1^2 + \frac{1}{2} q_{22}\mathbf{v}_2^2 + \frac{1}{2} q_{33}\mathbf{v}_3^2 \\ &\quad + q_{12}\mathbf{v}_1\mathbf{v}_2 + q_{23}\mathbf{v}_2\mathbf{v}_3 + q_{13}\mathbf{v}_3\mathbf{v}_1 \\ &= \frac{1}{2}\mathbf{v}_1^2 [c_1 + a_1\vartheta] + \frac{1}{2}\mathbf{v}_2^2 [c_2 - a_2\vartheta] + \frac{1}{2}\mathbf{v}_3^2 [c_3 + (a_1 - a_2)\vartheta] \\ &\quad + c_4\mathbf{v}_1\mathbf{v}_2 + \mathbf{v}_2\mathbf{v}_3 [c_6 + a_2\vartheta] + \mathbf{v}_1\mathbf{v}_3 [c_5 - a_1\vartheta]. \end{aligned}$$

The couple tending to turn the needle is,

$$\theta = \frac{\partial W}{\partial \vartheta} = \frac{1}{2}a_1\mathbf{v}_1^2 - \frac{1}{2}a_2\mathbf{v}_2^2 + \frac{1}{2}\mathbf{v}_3^2(a_1 - a_2) + \mathbf{v}_3 [a_2\mathbf{v}_2 - a_1\mathbf{v}_1],$$

and if $\mathbf{v}_1$ and $\mathbf{v}_2$ are made equal, $\mathbf{v}$,

$$\theta = \frac{1}{2}\mathbf{v}^2 [a_1 - a_2] + \mathbf{v}_3\mathbf{v} [a_2 - a_1] + \frac{1}{2}\mathbf{v}_3^2(a_1 - a_2)$$

Thus we see that unless $\mathbf{v} = \mathbf{v}_3$ the needle will not remain at rest unless $a_1 = a_2$. In order to make the adjustment, we may make $\mathbf{v} = 0$, and take readings with $\mathbf{v}_3$ first zero and then with a large

value. In order to thoroughly test the above general formula two commutators were arranged, so that a given potential $\mathbf{v}$ might be applied either positively or negatively to either quadrant, while the other was connected to earth. Four different deflections were thus obtained whose differences gave the relative values of a_1 and a_2 . The quadrants were then moved until these values were equal, which was finally given a last test by charging them to the same high potential, and then reversing its sign. The difference of the last two readings is equal to $2(a_1 - a_2)\mathbf{v}\mathbf{v}_3$ and could be made imperceptible when $\mathbf{v}$ was ninety volts, while $\mathbf{v}_3$ was the potential due to one hundred water elements.

This point having been established, the arrangement shown in *B* was tried. In order to change the potential of the second quadrants immediately from 0 to $\mathbf{v}$ they were connected to the lever of a relay *r*, whose back contact was connected to earth while the front contact was connected to a variable point *P* be-

Fig. 3 *B*.

tween the resistances R_1 and R_2 whose potential was $\mathbf{v}_0 \frac{R_1}{R_0 + R_2}$ where $\mathbf{v}_0$ is the electromotive force of the battery. The circuit of the relay was closed by the lever 1 being struck into the clip *c*, and so little time was lost that in a test where the quadrants 1 were connected directly to 1' instead of to 2' and charged instantaneously through a small resistance without self-induction to the full value $\mathbf{v}_0$, which was made the potential of *P* also, the deflection on dropping the projectile was not over one scale-division, whereas the potential applied to one quadrant alone would have driven the mirror off the scale.

This arrangement promised to work well, and was a zero method, but one circumstance induced me to modify it; namely, after the contact 2 is broken, the lever 2 remains at the potential $\mathbf{v}_0$ and acts inductively upon the support 2', thus affecting the charge of the quadrant. Hence it was desirable to put the lever 2 to earth imme-

diately after breaking. To this end the arrangement was modified as shown in *C*. The quadrants and condenser were both charged

Fig. 3 C.

at the beginning instead of at the end, and while quadrant 1 and the condenser discharged through R , the potential of quadrants 2 was shifting from V_0 to zero. It must be observed that this shifting did not take place

until after the contact 2 was broken, so that the capacity of the electrometer K_0 was not disturbed. The clip c' put the lever 2 to earth immediately after breaking, so that there was now no disturbing action upon the electrometer. By this arrangement was for the first time obtained a state of things where a zero potential might be observed, which is certainly to be desired on the score of simplicity, while at the same time the course

Fig. 5.

of the phenomenon corresponds exactly to that proposed in stating the question, viz.: the discharge of a condenser by an oscillating current. This arrangement suggested the arrangement that was finally adopted, which enabled me to dispense with the complication of the relay. In the final form, *D*, the two quadrants and condenser are originally at the same high potential, which is how-

ever only the fraction $\frac{R_1}{R_1 + R_2}$ of the whole potential V_0 . By the introduction of a resistance of one thousand ohms at R_2 the influence of the resistance of the contact 1 is eliminated. On breaking the contact 1 the quadrants 2 discharge instantaneously through the resistance R_1 (its time-constant of discharge KR_1 being only one twenty-five-millionth of a second) while the condenser and quadrant 1 discharge through the resistance $R + R_1$. By turning the micrometer screw the points of zero potential are easily obtained,

one-fifth of a division being sufficient to cause a change of sign of the deflection.

The arrangement shown in *E* was that used in finding the zero point, namely that at which both contacts were broken together. The condenser was removed, so that the time-constant was negligibly small, and the screw moved until the deflection on dropping just ceased.

The method was so sensitive that the zero could be determined to one-tenth of a drum-division, a change of one division being sufficient to change the deflection from zero to off the scale.

Fig. 3 *E*. Fig. 3 *D*.

The same arrangement was used in making the calibration above spoken of, the contacts being separated so far that there was no doubt that the full potential v_0 was attained. The potential used was then varied by moving the point *P*, and the deflections were found, as stated, proportional to the potentials. This calibration was, however, no longer needed in the method adopted.

THE EXPERIMENTS.

Before beginning the final experiments the electrometer was taken down and refilled with fresh, pure sulphuric acid, which had been boiled with ammonium sulphate. The electrometer was set up on a shelf between the cathetometer and the interrupter, and the short wire connected with the screw *2'* was shielded by a tin tube connected to earth (not shown in the photograph). The adjustment of the quadrants was next made, for which a potential v of eighty or ninety volts was employed. The leakage of both sets of quadrants was then determined, and found to be too small to influence the results. The zero point of the interrupter was determined before and

after each set of oscillation experiments. The times of oscillation were determined for the electrometer and coil alone, and for the three different condensers used. In observing, a zero point not too near the origin was first observed, and then the one next it, and it

$a_1 + nT_1 = l_1$, to determine T_1 and a_1 .					Giving $T_1 = 67.756$ with a mean error of $\mu_1 = \pm .0473$.
n	l_1'	l_1 Obs.	l_1 Calc.	Obs—Calc.	
10	761.5	761.00	761.508	— .508	
11	829.5	828.92	829.071	— .151	
21	1509.5	1507.60	1506.634	+ .966	
31	2187.5	2183.70	2184.198	— .498	
$a_2 + nT_2 = l_2$.					$T_2 = 77.695$ mean error $\mu_2 = .0587$
2	235.2	235.14	233.595	+ 1.545	
3	311.2	311.10	311.290	— .190	
12	1009.5	1008.66	1010.550	— 1.890	
22	1789.8	1787.20	1787.505	— .305	
32	2570.5	2565.30	2564.460	+ .840	
$a_3 + nT_3 = l_3$.					$T_3 = 100.478$ mean error $\mu_3 = .192$
4	532.1	531.40	532.516	— 1.116	
5	637.2	636.85	633.090	+ 3.760	
15	1642.5	1640.30	1637.874	+ 2.426	
26	2745.0	2739.10	2743.131	— 4.031	
$a_0 + nT_0 = l_0$.					$T_0 = 42.2006$ mean error $\mu_0 = .243$
1	104.0	103.98	102.428	+ 1.552	
2	145.5	145.47	144.629	+ 0.841	
8	394.0	393.85	397.833	— 3.983	
18	822.0	821.43	819.839	+ 1.591	

was in each case noted whether the potential was rising or falling, so that the number of the oscillation might be determined without doubt. Zero points ten, twenty or thirty half-oscillations farther on were then observed, and the results all combined by the method of least squares. The results follow. In the first column is given n , the number of the zero-point, counting from the first which occurs as zero. In the second column is the observed reading l' in drum divisions, from an arbitrary zero. In the third column are the

readings l reduced by the factor given above. In the fourth column are the readings calculated from the value derived from the reduction by least squares, and in the next column are the residual errors.

The differential equation giving the current or potential in terms of the time is
$$L \frac{d^2x}{dt^2} + R \frac{dx}{dt} + \frac{\nu^2}{K} x = 0$$

where K is measured electrostatically, giving as a solution,

$$x = Ae^{-\frac{R}{2L}t} \cos \left(\sqrt{\frac{\nu^2}{KL} - \frac{R^2}{4L^2}} t - \varepsilon \right)$$

and for the time of a half period

$$T = \frac{\pi}{\sqrt{\frac{\nu^2}{KL} - \frac{R^2}{4L^2}}}$$

In the present investigation, the resistance and capacity are so small that the second term of the denominator is quite negligible in comparison with the first, and we may use the simpler expression

$$T = \frac{\pi}{\nu} \sqrt{KL}$$

Putting¹ $a = \frac{\nu^2 T^2}{\pi^2 L}$ we have four equations to determine a and K_0 .

$$\tau T_n = \frac{\pi}{\nu} \sqrt{(K_n + K_0)L} \text{ or } \frac{\nu^2 \tau^2}{\pi^2 L} T_n^2 - K_0 = K_n$$

As there is no reason for assuming that there was any difference in the accuracy of the determination of the three different capacities, but as the times of oscillation were determined with different degrees of precision, weights were assigned to the four equations in accordance with the mean errors given above, and values of K_0 and a determined.

$$a(67.756)^2 - K_0 = 275.56$$

$$a(77.695)^2 - K_0 = 428.145 \quad \text{giving } K_0 = 182.02$$

$$a(100.478)^2 - K_0 = 826.50 \quad \log a = 9.0002922 - 10$$

$$a(42.2006)^2 - K_0 = 0.$$

From a , ν was calculated as

¹ If T is measured in drum-division, and τ is the time value of one division.

$$\frac{\pi}{\tau} \sqrt{aL} = \nu = 3,0259 \cdot 10^{10} \text{ cm./sec.}$$

The fact that the value of “ ν ” obtained should be in error by a large fraction of one per cent. is not surprising, as the investigation was not undertaken with a view to the determination of this quantity, but only of verifying the formula for the period of the oscillation, as proposed by the committee. This verification I hope to have satisfactorily given. The discrepancy in the value obtained is probably due to inaccuracy in the measurement of the quantities involved in the coefficient of self-induction of the coil, which was not of the best possible form for accurate measurement. In order to determine “ ν ” accurately by this method, it would only be necessary to use several different coils of larger cross-section, and in addition to make a chronographic determination of the time-constant of the interrupter. Apparatus for this purpose has been designed and constructed, and, although the time at my disposal did not permit of further experiments in the time allowed, this will probably be carried out in a future investigation.

Clark University, April, 1898.

Fig. 1c.