May 19, 1857.

ROBERT STEPHENSON, M.P., President,
in the Chair.

THE following Candidates were balloted for, and duly elected :—
Lieut. GEORGE HENRY BASEVI, WILLIAM CHRISTOPHER BENNETT,
CHARLES PERCY BYSSHE SHELLEY, and JOSEPH WRIGHT, jun., as
Associates.

No. 967.—“On the Disturbances of Suspension Bridges, and the
modes of counteracting them.” By AUGUSTUS STEPHEN LUKIN
and CHARLES EDWARD CONDER.

SUSPENSION bridges, requiring considerably less material for their
construction than girder bridges, and being practicable for far
greater spans, than have ever been attempted with structures of a
more rigid nature, would probably be frequently employed for
railway traffic, as well as for other purposes, were they not so
liable to disturbances of various kinds.

The chief sources of danger arise from the mobility, or flexi-
bility of the suspension chains; and the movements so produced
may be thus enumerated :—

1. Undulations, or distortions produced in the main chains by
an unequally distributed, or a passing load.
2. Effects of agitations, or distortions of the main chains in
vertical planes, reacting on the roadway.
3. Oscillation, or transverse swing.
4. Distortion of the lines of the main chains, when there
happens to be only a small load on the platform, owing to their
tendency to assume the catenary, or other curves natural to
them.
5. The unequal strains in different bays, incident to bridges of
more than one span.

Various modes may be suggested for meeting these difficulties.
The tension and weight of the main chains may be increased to
such a degree, as to reduce, within narrow limits, their liability to
disturbance from a passing load. But, by so increasing the mass
of the chains, there ensues an augmentation of danger to the rods
and roadway, when the chains are swung and agitated by storms.
The question of cost, also, will preclude the employment of much
more material in the suspension chains, than is required for the
support of the maximum load. It is, therefore, only in great
spans, where the chains necessarily become very massive, that

their simple weight can be expected to impart considerable steadiness to the structure.

The reduction of the deflection of the curve will have an important effect, in diminishing the undulations and distortions induced by the suspended load, as well as in lessening the liability to swing. As the deflection is reduced, however, the tension is increased, rendering necessary a corresponding increase in the strength of the chains, so that economy assigns certain limits to this mode of steadying the superstructure.

If attention is directed to the roadway, it will be seen, that there is advantage in giving it a certain degree of stiffness, combined with elasticity; as a pressure on any point of the platform will then be communicated to several pairs of rods, so as, in some degree, to equalise the effects of irregular, or traversing loads. The question next naturally arises, whether the stiffness of the roadway should not be sufficiently increased to insure complete immobility, by resisting the agitations of the chains. Opinions have been given against the advisability of thus combining an absolutely rigid structure with suspension chains, considered simply as a question of permanent stability. An objection also lies against such a bridge in point of economy, since the material employed in giving sufficient strength to the platform might be expected to go far towards the construction of a girder bridge, entirely independent of chains.

The Niagara Falls Suspension Bridge owes the steadiness which it is said to preserve, under railway traffic at a low rate of speed, in a great degree to its having separate platforms for the railway and the roadway; each being suspended from a pair of main chains. One of these platforms being below the other, they are firmly braced together, so as to form in effect a very deep girder. The stiffness of the structure is further secured by stays, some of which extend from the platform to the tops of the towers, and others from the platform to the cliffs below, the peculiar circumstances of this bridge imposing no restrictions as to headway.¹

It remains to be seen, how far the general arrangement of the chains and rods, considered simply, and apart from the question of the stiffness of the roadway, may be so modified, as to reduce some of the disturbances arising from the causes already adverted to. One modification has been proposed, in which four main chains are employed. Each chain rises from the platform level, at one tower, to the summit of the opposite tower, so as to resemble one half of a main chain, for a bridge of double the span.

¹ *Vide* "Memoir of the Niagara Falls Suspension Bridge." By John Roebling, C.E., of the United States. Weale, 1856.

The pair of chains, on each side of the platform, thus cross half way, and they may be connected together at the points where they cross. The suspension rods are hung alternately from one chain and the other. This plan offers certain advantages in correcting the disturbances produced in the chains by irregular loading; the fixing down of the lower ends of the chains to the towers will also greatly reduce the liability to swing. Such a bridge will, however, require an enormous amount of material: as much, in fact, as an ordinary suspension bridge of double the span and the same maximum load, the height of the towers being equal in the two cases. Thus, the metal required for this bridge will be more than enough to construct two ordinary bridges of the same span, and having towers only half the height. This mode of suspension is shown in Fig. 1.

Fig. 1.
Suspension Bridge with Double Chains.

More important advantages are secured by arrangements, in which the suspension rods are sloping instead of being vertical.

Fig. 2.
Suspension Bridge with Inclined Rods.

Fig. 2 represents the common direct suspension by rods alone, which may be regarded as the basis of all systems having inclined rods. In this method the absence of chains entirely removes the liability to deflections and undulations. But the great length of the more central rods, as well as the danger of some rods working loose and throwing the whole duty on the rest, constitute serious objections against the direct mode of suspension.

It thus seems necessary to revert to the chain, as the only means of insuring the constant, equable adjustment of the varying strains, at the same time that it materially shortens the central rods. The combination of a chain with slanting rods

approaches the character of direct suspension, and although not absolutely free from undulations, it is far less liable to them than the usual arrangement. This appears from the consideration, that while the vertical rod communicates any disturbing impulse in a direction approaching that of the radius of the curve, so as to cause extreme deflection and the greatest amount of agitation, the slanting rod transmits its impulses more nearly in the direction of the curve of the chain, the tension of which it increases, without causing much depression, so as to transmit the shock almost immediately to the pier. Again, if the chain be drawn up, or deflected, by any cause, the radiating action of the inclined rod greatly reduces the disturbance caused in the platform. Fig. 3 exhibits the case of a rod so inclined as to extend over half the subtangent of the curve of the main chain, the curve being a parabola. In this instance, as is shown by the calculations in the

MS. Appendix to this Paper, the vertical displacement of the platform is little more than half that of the chain. Another advantage, attending the use of sloping rods, is the economy which it renders practicable in the chain. It is true, that the weight of the rods will be very much greater, than in the common forms of bridge, but this will be much more than compensated by the reduction in the chains, which may be considerably tapered, as they recede from the towers, and material may thus be saved in a much greater degree than can be effected in bridges with vertical rods.

It must, however, be objected to the method of suspending by sloping rods, that the load on the roadway will subject the platform to horizontal strains, resembling those which it produces in the chains. Whether these strains be regarded as extensile, or compressive, the result must be, that the stability of the bridge requires a platform strong enough to sustain either extensile strains, similar to those borne by the lower section of a girder,

of which the middle depth is equal to the deflection of the chain, or else compressive strains, similar to those borne by the upper section of such a girder. Thus the roadway becomes a heavy item in the cost of the bridge, which shares in a great degree the character of a girder suspended from chains.

In seeking a mode of removing this disadvantage, let the direct mode of suspension be again reverted to. Fig. 4 exhibits a mode of direct suspension, in which the system of rods from either pier, instead of stopping at the half span, extends beyond it, to the opposite end of the platform. Thus, each point is supported by a pair of rods converging from the opposite towers, and the horizontal strains being neutralised, the platform is exempted from all duty, excepting its natural office of carrying the load.

Fig. 4.

Suspension Bridge with Inclined Rods, extending beyond the centre of the span.

But the objections made to the common direct suspension will apply with double force to the arrangement now suggested, since twice as many rods must be employed, and most of them will require to be of enormous length, some rods being even longer than the span of the bridge. These considerations show the employment of chains to be absolutely necessary.

The course of inquiry thus naturally leads to an arrangement, in which a double set of convergent rods is attached to a double set of chains, the rods meeting the chains at very acute angles. Each chain will extend from the summit of a tower, to the platform-level at its opposite extremity, and the curves of the chains will cross at the half-span. It is not, however, essential that any chain should extend the whole distance from tower to tower, provided its lower end can be well secured by other means.

A bridge constructed on this principle may be distinguished as a convergent suspension bridge. If the curve of the chains be such as to insure equilibrium, an increased load, equally distributed over the platform, tends merely to render the whole arrangement tighter and firmer, without causing either deflection, or a strain on the platform. It may, however, conduce to economy, to make use, to a certain degree, of the capability of the platform to sustain tension, if by so doing a saving can be effected in the strength of the chains.

The convergent bridge, without requiring a rigid platform, shares all the advantages of bridges with a single set of inclined rods, besides having some peculiar to itself. If a weight impinge on any point of suspension, the disturbance will be shared by two chains instead of one, and the effect will be proportionately diminished, while the fixing down of the lower extremity of each chain, by checking the lateral movement, will reduce the undulations. The fixing down of the chain has also a great tendency to lessen the danger of any transverse swing.

The determination of the inclinations for the rods and the curves for the chains, calculated to insure equilibrium in a convergent bridge, presents a problem of some complexity and admitting of a great variety of solutions. The simplest solution leads to the forms of bridge exhibited in Figs. 5 and 6. It is, however, possible, that other solutions might be obtained which, although not so simple, would enable bridges to be constructed for great spans, at less cost of material, than the designs now exhibited would require. The nature of the problem is shown in the MS. Appendix, and this question appears well worthy of the attention of engineers.

Fig. 5 exhibits a convergent bridge on the arrangement that will here be called Design No. 1. $OQ, O'Q'$, are the main chains, hanging in arcs of parabolas, of which the vertices are at O, O' , at which points, as well as at Q, Q' , they are firmly fixed: the attachments at Q, Q' being to stay chains, and those at O, O' to bolts built into the towers. The inclination of any suspension rod, as PZ is such, that the distance of Z from the opposite tower OQ' is three-quarters the distance of P from the same tower. Two portions $OV', V'O'$ of the platform, each equal to a fourth of the span, are sustained by rods, one out of every pair of which is attached to a main chain, and the other to the pier. The rods attached to the pier are fixed in the same positions as if they were carried up to a continuation of the main chains, beyond the face lines of the towers. The deflection of each chain is equal to one-eighth of the span.

This mode of suspension requires, for ordinary spans, about the same amount of metal, in the rods and the chains, as a common suspension bridge, of which the deflection is one-thirteenth of the span: but as the span increases, the convergent design, No. 1, becomes more and more costly, in comparison with the ordinary bridge.

To economise material, it is requisite to reduce the tension of the chains, by allowing them more flexure. This may be secured by the Design No. 2, Fig. 6. In this design, the portions $V'O, O'V$ of the platform, each equal to one-fifth of the span, are supported by rods radiating from the tops of the piers, on the direct

Fig. 5.

Convergent Suspension Bridge—Design No. 1.

Fig. 6.

Convergent Suspension Bridge—Design No. 2.

suspension principle. The points O, O' are the vertices of parabolas $OQ, O'Q'$ in which the main chains are hung, the convergent rods having the same inclinations as in Design No. 1. Under-stays $OS, O'S'$ pass from the lower extremities of the chains to points S, S' , where they are attached to bolts built into the piers, at a depth below the platform equal to one-tenth of the span. The object of these stays is to guard against the points O, O' being disturbed by any agitation of the chains, as well as to assist in sustaining the horizontal strain caused by the tension of the chains, when the portions $V'O', O'V$ of the roadway are not heavily loaded. When the loads on these portions of the platform are very heavy, the horizontal strains caused at O , and O' may exceed the opposing tension of the main chains. In such cases the excess of strain acts mainly on the portion OO' of the platform bearer, which thus takes the place of a tie-beam. The height above the platform of the towers of Design No. 2, Fig. 6, is two-fifteenths of the span. The increased curvature allowed to the chains effects great economy in this form of bridge, which requires considerably less metal for its construction, than an ordinary bridge, having a deflection equal to one-thirteenth of the span, and capable of sustaining an equal amount of weight, before breaking. To diminish the danger of transverse swing, it is proposed to arrange the chains and rods in battering planes, converging slightly as they approach the platform level.

The proper adjustment of the strains borne by each rod, in every pair, claims especial consideration. This end can be secured by the use of a self-adjusting lever, as shown by bcd in Fig. 7, the positions of the convergent rods being indicated by the lines ab, de , and the point of suspension in the platform hanging from the fulcrum c .

Fig. 7.

The position of the lever being determined on the principles
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explained in the MS. Appendix, it will remain in stable equilibrium, so long as each rod bears its proper share of strain: but if, through a disturbance of the chains, one rod is tightened, and the other rendered slack, the lever will change its position, so as to throw less weight on the slackened chain, and more weight on the chain which is drawn up, thus tending to check the disturbance, at the same time that the play, thus admitted, greatly diminishes the displacement of the platform. When the disturbing cause ceases to operate, the lever will resume its position of equilibrium. These levers must not be considered as indispensable.

The fourth cause of disturbance, mentioned at the commencement of this Paper, arises from the weight of the chains, and is common to all suspension bridges, although not existing in equal degrees in the various designs, nor in the same design for different spans. While the conditions of suspension usually require a parabola as the curve of the main chains, the chains would, if left to themselves, assume catenaries, or if tapering, curves still further removed from the proper lines. As the span is augmented, the appreciable difference between the normal and the natural curve increases, and the mass of the chains rapidly becomes so considerable, in proportion to the suspended load, that it seems no longer safe to neglect the adjustment of the conflicting strains. By a rigid platform of sufficient strength, the main chains might be kept permanently warped from their natural curves. For large spans, however, this would require much weight, as well as strength, in the platform; and in imposing on it this task, much of the strength must be expended that would otherwise be available for other duties. On the other hand, if the platform is not sufficiently stiff, it will be constantly distorted from its true line, when the maximum load is not on the bridge, unless the chains can be caused spontaneously to assume the curve required by the conditions of suspension.

This end can be attained, by ballasting the lower portions of the main chains. A formula is given in the MS. Appendix, for determining the gross sectional weight at every part of a chain, so as to insure its equilibrium in any assigned curve. By these means the steadiness of the chains will be increased, and the bridge be entirely freed from disturbances of the nature referred to.

It may be objected to this expedient, that adding to the weight of the chains causes more strain, and thus renders greater strength necessary. It is, however, clear, that the measure of strength will always be the maximum load; and the stiffness of the roadway, when weighted with the maximum load, would throw vertical pressures on the chains of the same nature, that it is here proposed to apply constantly by means of ballast.

It may, therefore, be inferred, that while the weighting of the chains would cause much less strength to be requisite in the platform, it would not demand any great increase of metal in the chains.

In order to compare the cost in material of convergent bridges and bridges with vertical rods, Table No. 1 (Appendix p. 470) has been computed from formulæ, the proofs of which are given in the MS. Appendix. It is assumed, in these formulæ, that the chains are so tapered as to be of equal strength throughout. This, though not usual in the common suspension bridge with vertical rods, would nevertheless effect an economy of material, and therefore the results are more fairly comparative, than if the chains were taken as of uniform section. Taking the breaking strain of iron at 25 tons per square inch, this Table exhibits the tonnage of iron in rods and chains, for every hundred feet span, up to one thousand feet; the breaking weight being taken at 12 tons for every foot of span, equally distributed over the surface of the roadway, exclusive of the effect of the weight of the chains. No allowance is made for bolts, overlap of links, or stay chains, the object of the Table being simply to facilitate comparison as to cost. The amount of metal is, in every case, divided into that required to support the load, and the metal that has to be added to the chains, to enable them to sustain their own weight, in addition to the greatest load. The first item increases as the square of the span, and the second item a little more rapidly than the cube of the span. As the span approaches certain limits, depending on the strength and weight of the metal employed, it becomes a question of more importance how to sustain the chains, than how to support the load. Four forms of bridge are selected for comparison in this Table; namely:—the two designs for convergent rods; a bridge with vertical rods, having the main chains in parabolas, with a deflection equal to one-thirteenth of the span; and a bridge with vertical rods and intersecting main chains, having a deflection equal to one-eighth of the span. The weights of wrought iron tubular girders up to 300 feet span, exclusive of rivets and joints, are also given, the depth of girder in the middle being from one-fifteenth to one-thirteenth of the span, and the breaking load the same as in the other bridges, inclusive, however, of the weights of the girders themselves.

In order to exhibit the effects of employing a material stronger in proportion to its weight than iron, Table No. 2 (Appendix p. 471) is computed for bridges of the same dimensions and breaking weights, showing the estimated tonnage of steel for chains and rods, taking the breaking strain of steel at 60 tons per square inch. It will be seen, from an examination of the two Tables, that when the stronger metal is employed, the convergent suspen-

sion compares more favourably, in point of economy, with the vertical suspension, than when wrought iron is the material selected for the chains and rods.

The proofs of the formulæ, from which the Tables referred to are computed, as well as the analysis by which the designs for convergent suspension, now explained, were arrived at, being unsuited for insertion in a Paper of this nature, are given in detail in the MS. Appendix.

The fifth source of disturbance adverted to, is that which exists in suspension bridges of more than one bay. If two adjacent bays are unequally loaded, a strain will result, tending to overthrow the middle tower, unless a continuous main chain is adopted, with sufficient play, where it rests on the tower, to allow the unequal tensions to adjust themselves. But as this adjustment will be accompanied by the sinking of the roadway bearing the heavy load, and the drawing up of the more lightly loaded platform of the other bay, a new source of derangement arises, which, in the case of heavy railway traffic, assumes a formidable character.

An inverted system of chains and rods below the roadway, presenting the appearance of arches springing from the lower parts of the piers, might be adopted, in cases where the extent of headway admitted of such an arrangement, so as effectually to prevent, or to reduce, within safe limits, the drawing up of the platform. In large spans, the additional cost of the inverted chains would be far less than would attend the substituting, for two bays, one of more than twice the span. But it is now proposed, not so much to inquire into the means of combating the danger, as to trace how far the cause of disturbance, existing in the inequality of tension induced in the main chains of adjacent bays, may be modified by the mode of suspension. With this view Table No. 3 (Appendix p. 472) is given, showing the strains in tons, at the highest point of the main chain, for the greatest and least loads in different bridges, and exhibiting the ratio of the greatest to the least tension. The least load is taken at $\frac{3}{4}$ of a ton for every foot of span, which is assumed as the weight of the platform alone; and the greatest load is taken at $2\frac{3}{4}$ tons per foot of span. The bridges selected for examination are, an ordinary suspension bridge with vertical rods, and the two designs for bridges with convergent rods, of which the weights are given in Table No. 1. It will appear, on inspection, that in every case the extreme inequalities of tension, in the convergent bridges, are somewhat less than in the vertical-rod bridge of the same span; but in the convergent design, No. 1, Fig. 4, the superiority, in this respect, becomes remarkable, as the span increases.

Having thus briefly glanced at the chief causes of instability in

suspension bridges, with such suggestions for their remedy as seem most feasible, the Authors hope that, however imperfectly the subject may have been treated, it may give rise to discussion and investigation, calculated to elucidate a subject of acknowledged difficulty, but of great practical importance.

The Paper is accompanied by a series of Diagrams from which Figs. 1 to 7 are compiled; and by a MS. Appendix giving the formulæ and calculations referred to in the Paper.

APPENDIX.

TABLE No. 1.

WEIGHT of Wrought Iron, in chains and rods, required for different Suspension Bridges;—the breaking-weight being taken at 12 tons per lineal foot, equally distributed over the platform; and the breaking-weight of wrought iron at 25 tons per square inch.

	Spans in Feet.									
	100	200	300	400	500	600	700	800	900	1000
Common Suspension Bridge.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
Deflection = $\frac{1}{10}$ span.	12·17	49	110	195	304	438	596	779	986	1217
To support load .	0·13	1	4	8	17	29	47	71	102	142
To support chains										
Total . .	12·30	50	114	203	321	467	643	850	1088	1359
Suspension Bridge, with crossed chains and vertical rods.										
Deflection = $\frac{1}{10}$ span.	29·50	118	266	472	738	1062	1446	1888	2390	2950
To support load .	0·76	9	22	53	106	189	309	475	698	990
To support chains										
Total . .	30·26	127	288	525	844	1251	1755	2363	3088	3940
Convergent Bridge, No. 1.										
To support load .	11·4	46	103	183	286	412	560	732	927	1144
To support chains	0·4	4	12	30	60	106	173	267	393	557
Total . .	11·8	50	115	213	346	518	733	999	1320	1701
Convergent Bridge, No. 2.										
To support load .	7·7	31	69	123	192	276	376	491	622	768
To support chains	0·2	2	6	15	29	51	83	126	183	256
Total . .	7·9	33	75	138	221	327	459	617	805	1024
Wrought-iron girder	40	177	398	..	..	..	..	..	..	..

TABLE No. 2.

WEIGHTS (in tons) of Steel required for chains and rods in different forms of Suspension Bridges, the breaking strain being taken at 60 tons per square inch.

	Spans in Feet.									
	100	200	300	400	500	600	700	800	900	1000
Common Suspension Bridge.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
Deflection = $\frac{1}{16}$ span.										
To support load .	5·17	20·7	47	83	129	186	254	331	419	518
To support chains	0·02	0·2	1	2	3	5	8	12	17	24
Total . .	5·19	20·9	48	85	132	191	262	343	436	542
Suspension Bridge, with crossed chains and vertical rods.										
Deflection = $\frac{1}{8}$ span.										
To support load .	12·55	50·2	113	201	314	452	615	803	1016	1255
To support chains	0·14	1·1	4	9	18	31	50	75	108	150
Total . .	12·69	51·3	117	210	332	483	665	878	1124	1405
Convergent Bridge, No. 1.										
Deflection = $\frac{1}{8}$ span.										
To support load .	4·9	19·5	44	78	122	175	238	311	394	487
To support chains	0·1	0·6	2	5	10	17	28	42	61	84
Total . .	5·0	20·1	46	83	132	192	266	353	455	571
Convergent Bridge, No. 2.										
Deflection = $\frac{2}{16}$ span.										
To support load .	3·27	13·1	29	52	82	118	160	209	264	327
To support chains	0·04	0·3	1	3	5	9	14	21	30	42
Total . .	3·31	13·4	30	55	87	127	174	230	294	369

TABLE No. 3.

COMPARATIVE Tensions at highest point of one main chain, for a maximum load of $2\frac{3}{4}$ tons, and a minimum load of $\frac{1}{4}$ ths of a ton, per lineal foot, in various forms of Suspension Bridges, for a breaking load of 12 tons per lineal foot.

	Spans in Feet.									
	100	200	300	400	500	600	700	800	900	1000
Suspension Bridge, Deflection = $\frac{1}{8}$ span.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
Maximum load .	457	936	1437	1961	2507	3078	3676	4298	4947	5624
Minimum load .	133	285	462	662	882	1129	1400	1698	2023	2373
Ratio . =	3·4:1	3·3:1	3·1:1	3:1	2·8:1	2·7:1	2·6:1	2·5:1	2·4:1	2·4:1
Convergent Bridge, No. 1.										
Maximum load .	338	712	1128	1589	2098	2659	3279	3961	4712	5539
Minimum load .	106	248	432	661	939	1268	1656	2106	2625	3220
Ratio . =	3·2:1	2·9:1	2·6:1	2·4:1	2·2:1	2·1:1	2:1	1·9:1	1·8:1	1·7:1
Convergent Bridge, No. 2.										
Maximum load .	270	559	868	1198	1550	1925	2325	2750	3208	3684
Minimum load .	81	178	298	439	603	786	997	1233	1494	1786
Ratio . =	3·3:1	3·1:1	2·9:1	2·7:1	2·6:1	2·4:1	2·3:1	2·2:1	2·1:1	2·1:1

NOTE.—This Table shows the inequality of tension produced in the main chains, on opposite sides of a tower, standing between a fully loaded and an unloaded bay.