On The Use of Anderson's Bridge for the Measurement of the Variations of the Capacity and Effective Resistance of a Condenser with Frequency

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- I. *On The Use of Anderson's Bridge for the Measurement of the Variations of the Capacity and Effective Resistance of a Condenser with Frequency.* By S. BUTTERWORTH, M.Sc., National Physical Laboratory.

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ABSTRACT.

An analysis of the effect of residuals and earth capacities in Anderson's Inductance-capacity bridge is made, and it is shown that if balances are obtained

- (a) by balancing the bridge with *direct* currents ;
- (b) by making the alternating current adjustments by means of a small series resistance (s') and parallel condenser (C') in the condenser arm ;

then the changes required in s' and C' to hold the balance at different frequencies are equal and opposite to the variations of the effective (series) resistance and capacity of the condenser with frequency.

The assumptions made in obtaining the above conclusions are that the residual inductances and resistances of the "non-inductive" arms of the bridge are invariable with frequency and that the resistance of the inductive arm varies as the square of the frequency. No knowledge of the absolute values of the residuals, &c., is required for the method.

The method is illustrated by results obtained with a condenser of capacity $0.5\mu F$, and details are given showing how the chief experimental trouble, viz., drift in D.C. balance owing to temperature variations, may be overcome.

I.—Introduction.

ALTHOUGH Anderson's bridge has been used frequently in precision measurements of inductance and capacity and the effects of the residual inductances of the arms have been discussed,* a valuable property of this bridge seems to have been overlooked. An examination of the equations of balance of this bridge in which the effects of residuals are included shows that if the balancing adjustments are carried out in a particular manner, the only factors which necessitate a readjustment of balance at various frequencies of current are :

- (a) the variation of the capacity of the condenser with frequency,
- (b) the variation of the effective resistance of the condenser with frequency,
- (c) the variation of the inductance with frequency.

The assumptions upon which the above conclusions are based are that the resistances and residual inductances (or capacities) of the "non-inductive" arms are invariable with frequency and that the resistance of the inductive arm has a term varying as the square of the frequency.

The observed variations of adjustment with frequency may therefore be used to determine the frequency variations of capacity and effective resistance of a condenser, the frequency coefficient of the inductance being supposed given. This frequency coefficient may be determined by a measurement of the self capacity of the inductive coil if the coil is of stranded wire. The absolute magnitudes of the residuals need not be known if the only quantities required are the *variations* of capacity and effective resistance of the condenser.

II.

Anderson's bridge is shown in Fig. 1*a* in which X_1 , X_2 , X_3 , X_4 are resistances having small inductances, X_5 is an inductive coil and X_6 is the condenser under test.

* Rosa and Grover, *Bull. Bureau of Standards*, Vol. I., p. 291 (1905).

If X_3 is zero the bridge becomes Maxwell's inductance-capacity bridge (Fig. 1(b)). The latter bridge may also be derived from Anderson's bridge by converting the "star" combination X_1, X_2, X_3 to its equivalent "mesh,"* $X'_1, X'_2, X'_3 = (X_3X_1 + X_1X_2 + X_2X_3)/X_2, X_1, X_3 \dots \dots \dots (1)$ the arm X'_3 acting as a shunt across the detector.

It will first be shown that small constant residuals in the arms X_1, X_2, X_3 convert to small constant residuals in the equivalent arms X'_1, X'_2, X'_3 and then the complete conditions of balance, including residuals, in Maxwell's bridge will be obtained.

Let $X_n = R_n + j\omega l_n \quad (n=1, 2, 3)$

Then $X'_1 = X_1 + X_3 + X_1X_3/X_2$
 $= R_1 + R_3 + j\omega(l_1 + l_3) + (R_1 + j\omega l_1)(R_3 + j\omega l_3)/(R_2 + j\omega l_2)$
 $= R_1 + R_3 + R_1R_3/R_2 + j\omega\{l_1(1 + R_3/R_2) + l_3(1 + R_1/R_2) - l_2R_1R_3/R_2^2\}$

to the first order of small quantities

FIG. 1.—(a) ANDERSON'S BRIDGE. (b) MAXWELL'S BRIDGE.

Hence if l'_1, l'_2, l'_3 are the residual inductances of the equivalent arms X'_1, X'_2, X'_3

$$\left. \begin{aligned} l'_1 &= l_1(1 + R_3/R_2) + l_3(1 + R_1/R_2) - l_2R_1R_3/R_2^2 \\ l'_2 &= l_2(1 + R_3/R_1) + l_3(1 + R_2/R_1) - l_1R_2R_3/R_1^2 \\ l'_3 &= l_1(1 + R_2/R_3) + l_2(1 + R_1/R_3) - l_3R_1R_2/R_3^2 \end{aligned} \right\} \dots \dots \dots (2)$$

If l_1, l_2, l_3 are invariable with frequency, then to the first order of small quantities l'_1, l'_2, l'_3 are also invariable.

For analytical simplicity it is convenient in certain cases to replace residual inductances by residual capacities. Thus if l is the residual inductance of a resistance R , the equivalent residual capacity acting as a shunt is

$$c = -l/R^2 \dots \dots \dots (3)$$

III.—Conditions of Balance in Maxwell's Bridge.

Maxwell's bridge is redrawn in Fig. 2 with the residuals included.

R_1 is a resistance having residual inductance l ; R_2 is an inductive coil of inductance L ; G_2 is a conductance with residual capacity c ; G_1 is a conductance

* G. A. Campbell, Trans. Am.I.E.E., Vol. XXX., p. 891 (1911).

shunting the condenser C , its residual capacity being included in C . The condenser C has leakage conductance g .

If the bridge is earthed at the point E the earth capacities (assumed to be sufficiently represented by condensers connecting the branch points to earth) are either included in c , C or are thrown across the whole bridge.

The vector equation of balance when using currents of frequency $\omega/2\pi$ is

$$(G_1 + g + j\omega C)/(G_2 + j\omega c) = (R_2 + j\omega L)/(R_1 + j\omega l) \quad (4)$$

from which the two conditions of balance are

$$(G_1 + g)R_1 - \omega^2 l C = G_2 R_2 - \omega^2 L c \quad (5)$$

$$C R_1 + l(G_1 + g) = c R_2 + L G_2 \quad (6)$$

FIG. 2.—MAXWELL'S BRIDGE, INCLUDING RESIDUALS.

Now let the parallel combination of capacity C shunted by conductance g be replaced by its equivalent series combination of capacity K in series with resistance s , so that

$$g = s\omega^2 K^2 / (1 + \omega^2 s^2 K^2), \quad C = K / (1 + \omega^2 s^2 K^2) \quad (7)$$

Since normally $s\omega K$ will seldom exceed 0.001, we may write to an accuracy 10^{-6}

$$g = s\omega^2 C^2, \quad K = C \quad (8)$$

and the two conditions of balance are now

$$G_1 R_1 + \omega^2 (C^2 s R_1 - l C + L c) = G_2 R_2 \quad (9)$$

$$C R_1 = L G_2 - l G_1 + c R_2 \quad (10)$$

products of residuals being ignored.

IV.

Apart from the variations of C and s with frequency all the factors in the above equations are invariable with frequency except R_2 and L , and for a stranded wire coil we may write

$$L = L_0(1 + a\omega^2), \quad R_2 = R_0(1 + \beta\omega^2) \quad (11)$$

Suppose now that the condition

$$G_1 R_1 = G_2 R_0 \quad (12)$$

has been satisfied by balancing with *direct* currents. Then, using (11) and (12) and ignoring products of small quantities, (9) and (10) become

$$C^2sR_1 - iC + L_0c = G_2R_0\beta \quad \dots \quad (13)$$

$$CR_1(1 - a\omega^2) = L_0G_2 - iG_1 + cR_0 \quad \dots \quad (14)$$

If, therefore, the alternating current balance is secured by having a small adjustable air condenser C' in parallel with C and a small adjustable resistance s' in series, equations (13) and (14) show that *for all frequencies, $s+s'$ and $C(1-a\omega^2)+C'$ are constant.*

The variations required in s' and C' to restore balance at the various frequencies will therefore be equal and opposite to the variations of s and $C(1-a\omega^2)$.

V.—Bridge Details for Effective Resistance Measurements.

The bridge arrangements when used for the measurement of the variations in effective resistance of $\frac{1}{2}$ microfarad condensers are shown in Fig. 3. The arms corresponding to X_1, X_2 in Fig. 1(a) were similar non-inductive coils each of 100 ohms. The inductive arm was made up of an inductance coil R_2, L and a non-inductive 10-ohm resistance. The coil L consisted of a single layer 7/36 stranded wire coil of

FIG. 3.—ANDERSON'S BRIDGE FOR EFFECTIVE RESISTANCE OF $\frac{1}{2} \mu F$ CONDENSERS.

inductance 780 microhenries and resistance approximately 2.6 ohms. Its self-capacity, including the connecting leads, was 20 micro-microfarads, so that its increase in effective inductance at a frequency of 2,000 cycles per second was $3/10^6$. The arm adjacent to the inductive arm was made up of a non-inductive 10-ohm coil and a non-inductive copper coil R_1 of resistance slightly greater than that of the coil R_2 . The coil R_1 was shunted by a resistance box ($Sh.$). The inductive coil R_2 and the copper coil R_1 were enclosed in the same wool-lined box so that their temperature variations would be as nearly as possible identical, the function of R_1 being to balance any changes of resistance due to temperature variations of R_2 . (See Section VII.)

C was the condenser under test of nominal value $\frac{1}{2}$ microfarad. It was shunted by an adjustable air condenser C' whose capacity ranged from 60 to 260 micro-microfarads, and the combination was in series with a constant inductance rheostat s' ranging from 0.1 to 1.1 ohm which could be read to 0.001 ohm. The resistance R' corresponding to X_3 in Fig. 1(a) was for purposes of preliminary balancing. It could be adjusted by steps of 0.01 ohm to 100 ohms.

VI.—*Modifications for the Measurement of Variations of Capacity with Frequency.*

The above arrangements required modification when the capacity variations were being studied. Any inductance in the condenser arm will introduce a frequency variation in the effective capacity of this arm, so that the presence of the constant inductance rheostat s' in the condenser arm is a disadvantage. However, the direct current balancing condition is no longer of importance, so that if s' is transferred to the inductive arm R_2 and the fine adjustments are made by s' in R_2 and C' in parallel with C , then, since in (14) we may replace R_0 by R_2 to the order of accuracy required, small variations of R_2^* will only introduce second order errors in the capacity equation as R_2 is associated only with the residual capacity c . The capacity arm C may then be made to include only the condenser and its leads, and only a small correction is required for the inductance of the leads.

These modifications were accordingly used when capacity variations were being measured.

VII.—*Temperature Variations.*

The chief source of trouble in the effective resistance measurements was the difficulty experienced in holding the direct-current balance throughout the alternating-current adjustments, the balance tending to drift because of the large temperature coefficient of the resistance of the inductive coil in R_2 . This difficulty was largely overcome by the use of the balancing coil R_1 of Fig. 3, both R_1 and R_2 being shielded from stray temperature variations by enclosing them together. It was found necessary also to shield the leads to these coils. For similar reasons the current supplied to the bridge, whether direct or alternating, was made to have the same R.M.S. value. Even with these precautions a small drift in balance often occurred so that observations were taken in a definite order, and a correction applied for want of D.C. balance. An examination of equation (9) shows that a departure from condition (12) introduces an error in s proportional to $1/\omega^2$, and is thus more serious at the lower frequencies. For this reason it was found advisable to take the lower frequency observations immediately after securing the D.C. balance. Also by deliberately throwing the D.C. balance out of adjustment and observing the change in s' required to re-balance with alternating currents of known frequency, the correction for a given want of balance could be determined. The method is illustrated by the following example.

VIII.—*Example Showing Correction for Drift in D.C. Balance.*

$Sh \doteq 400$ ohms. $R_1 \doteq 3$ ohms.

An increase of 10 ohms in Sh causes an increase in deflection of the D.C. galvanometer of 35 divisions, and to restore the A.C. balance at a frequency of $500 \sim$ per second requires a diminution in s' of 0.129 ohm.

Hence for a want of balance represented by 1 division of the scale of the D.C. galvanometer the correction on $s' = 0.0037$ ohm.

* It is essential, however, that R_2 shall remain of constant inductance throughout the adjustments, so that s' must be of the type specified.

The correction is applied as in the following table :—

Time. Min.	D.C. observations.		A.C. observations.		Deduced.	
	Source.	Galvo. deflections.	Frequency.	s' ohms.	Galvo. zero.	Want of balance.
0	+	+2.0	...	...	...	...
1	—	+4.0	...	...	+2.8	...
2	+	+1.0	...	...	...	—1.8
4	...	...	500	0.520*	...	...
6	+	—5.0	...	...	...	—7.5
7	—	+10.0	...	...	+2.5	...
8	+	—5.0	...	...	...	...

Want of balance at moment of A.C. observation = —4.6.
Therefore corrected $s' = 0.520 - 0.0037 \times 4.6$
= 0.503 ohm

IX.—Typical Results for the Change in Effective Resistance of a 0.5 μF Condenser.

Preliminary adjustments are made by R' and Sh , and the remaining adjustments at all frequencies by s' and C' only, observations being taken as in the above table. The following results were obtained in a typical case :—

Condenser I (0.5 μF).

Bridge current = 30 milliamperes.

Frequency $\sim$ per second...	500	800	1,000	1,600	2,000
s' ohms	0.503	0.597	0.625	0.675	0.685
s ohms	0.287	0.193	0.165	0.115	0.105
Power-factor ωsC	$10^{-4} \times 4.5$	4.8	5.2	5.8	6.6

The values of s have been obtained from the known value of s for this condenser at a frequency of 1,600 $\sim$ per second,† and the law $s + s' = \text{constant}$ for all frequencies.

X.—Typical Results for the Variation in Capacity of a 0.5 μF Condenser.

The rheostat s' is removed from the condenser arm, and placed in series in the arm R_2 . After preliminary adjustments by R' and Sh the fine adjustments and adjustments for different frequencies are made by s' and C' only. The following results hold for condenser I. :—

Frequency. $\sim$ per sec.	C'^* $\mu\mu F.$	$C - C_{1000}$ Uncorrected $\mu\mu F.$	Correction $\mu\mu F.$	$C - C_{1000}$ corrected $\mu\mu F.$
500	132	+46	+3	+49
675	154	+24	+2	+26
830	164	+14	+1	+15
1,000	178	0	0	0
1,170	185	—7	—1	—8
1,510	193	—15	—3	—18
2,000	193	—15	—10	—25

The correction in the fourth column is for the variation of the inductive coil L

* Mean of source and telephone reversals.

† Butterworth, Proc. Phys. Soc., Vol. XXXIII, p. 346, 1921.

with frequency and for the residual inductance of the leads to the condenser. As regards the latter, if λ is the inductance of the leads, the effective capacity of the whole condenser arm is $C(1 + \omega^2\lambda C)$. In the above observations $\lambda = 0.38$ microhenry, so that using $C = \frac{1}{2} \mu F$,

$$\omega^2\lambda C = 30 \times 10^{-6}$$

at a frequency of 2,000 cycles per second. Now the correction due to L (section 5) is 3×10^{-6} and (equation 14) acts oppositely to the λ correction, so that the whole

FIG. 4.—CHANGE OF CAPACITY OF $\frac{1}{2} \mu F$ CONDENSER WITH FREQUENCY.

correction is 27×10^{-6} at 2,000 $\sim$ per second. For a $\frac{1}{2} \mu F$ condenser this leads to a negative correction of $13.5 \mu F$. The corrections for the lower frequencies follow from the square law. The corrected variations are plotted against frequency in Fig. 4. It is seen that the smoothed curve never differs by more than five parts in a million from the observed values.

DISCUSSION.

Prof. FORTESCUE, referring to the typical results for the change in effective resistance of a condenser given in section IX. of the Paper, remarked that telephone engineers usually assume that the power factor of a condenser is constant, and would be specially interested to see that this assumption is incorrect. He enquired as to the type of condenser used.

Dr. D. OWEN congratulated the author on his valuable contribution, and expressed a belief that his methods would be adopted as the standard practice. It would be of interest to apply them to types of bridge other than that of Anderson. He asked if a sufficient number of condensers had been tested to determine whether the frequency coefficient of mica condensers is a definite quantity, or varies from one condenser to another like the temperature coefficient.

Mr. S. BUTTERWORTH, in reply to the discussion, stated that the results quoted in illustration of the method were obtained with a standard mica condenser. The power factors of four other mica condensers were also obtained, and the results varied between 0.0001 and 0.0007. Only one condenser was tested in regard to variation of capacity with frequency. However, a large number of mica condensers have been measured by H. L. Curtis (Bureau of Standards Circular, S.137), and his results are regarded as standard. The results of Curtis depend on the perfection of an air condenser of 0.1 μF capacity.